Labor markets with search frictions

- Economic fortunes of most individuals largely determined by labor market experiences - finding and losing jobs, dynamic paths of wages, employers
- Important aspects of these issues are not easily addressed using the competitive paradigm of supply and demand in a frictionless market - where a worker can costlessly and immediately choose to work for as many hours as he wants at the market wage.

- What determines the length of unemployment spells?
- Why do unemployed workers sometimes choose to remain unemployed by turning down job offers?
- How can we simultaneously have unemployed workers and job vacancies?
- How can apparently homogeneous workers in similar jobs earn different wages?
- What are the trade-offs faced by firms in paying different wages?
- How do wages and turnover interact?

Search theory has provided a rigorous yet tractable framework to answer these questions.

Central to this approach: there are important frictions in the labor market.

- It takes time for a worker to find a (good) job and for a firm to fill a vacancy.
- There are rents to jobs: if one took an employer and a worker and forcibly separated them one or both of them would be strictly worse off.

Sources of frictions

- Information imperfections
- Heterogeneities
- No centralized market where all sellers and buyers of labor meet and trade at a single wage, as in competitive equilibrium.
- By relaxing assumptions of frictionless trade, search models allow us to think about unemployment and wages in a different light.

1 Basic job search

Rogerson Shimer and Wright, JEL 2005

Single worker searching for a job in discrete time, taking market conditions as given.

- Maximize $E\sum_{t=0}^{\infty}\beta^t x_t$.
  x_t : income at t; $x = w$ if employed at wage w and $x = b$ if unemployed.
  $\beta \in (0,1)$: discount rate;

- With probability λ : an unemployed individual receives one job offer each period from a known distribution $F(w)$.
  With probability $1-\lambda$: no wage offer.

- Jobs last forever

Bellman equations

$$W(w) = w + \beta W(w) \tag{1}$$

$$U = b + \beta\lambda \int_0^\infty \max\{U, W(w)\}\, dF(w) + \beta(1-\lambda)U \tag{2}$$

- $W(w)$ is the payoff from working at wage w;
- U is the payoff from searching for work: earning b and waiting for the next offer.

Generalize discrete-time model: allow the length of a period to be Δ.

The probability to receive a job offer in each period is now $\lambda\Delta$;
and let $\beta = 1/(1 + r\Delta)$.

Expressions (1) and (2) can be rewritten as

$$
\begin{aligned}
W(w) &= \Delta w + \frac{1}{1+\Delta r} W(w) && (3)\\
U &= \Delta b + \frac{\Delta\lambda}{1+\Delta r}\int_0^\infty \max\{U, W(w)\}\, dF(w) + \frac{1-\Delta\lambda}{1+\Delta r} U. && (4)
\end{aligned}
$$

Rearranging:

$$
\begin{aligned}
rW(w) &= (1+\Delta r)\, w && (5)\\
rU &= (1+\Delta r)\, b + \lambda \int_0^\infty \max\{0, W(w) - U\}\, dF(w). && (6)
\end{aligned}
$$

Continuous time: take limit $\Delta \to 0$

$$rW(w) = w \tag{7}$$

$$rU = b + \lambda \int_0^\infty \max\{0, W(w) - U\}\, dF(w). \tag{8}$$

- U : lifetime value of being unemployed;
- rU : flow value of being unemployed (sum of instantaneous payoff b, plus the expected value of any changes in the value of the worker's state, i.e. probability that he gets an offer λ, times the expected increase in value associated with the offer, which may also be rejected).
- $W(w)$ is strictly increasing: unique reservation wage $w_R : W(w_R) = U$
- The worker should reject $w < w_R$ and accept $w \geq w_R$.

Substituting $rU = w_R$ in (8) gives:

$$w_R = b + \lambda \int_{w_R}^{\infty} [W(w) - U] dF(w). \tag{9}$$

Substituting $W(w) - U = (w - w_R)/r$ gives the reservation wage equation

$$w_R = b + \frac{\lambda}{r} \int_{w_R}^{\infty} (w - w_R) dF(w). \tag{10}$$

Alternative representation:

$$w_R = b + \frac{\lambda}{r} \int_{w_R}^{\infty} [1 - F(w)]\, dw, \tag{11}$$

obtained from (9) after integrating by parts (noting $W'(w) = 1/r$).

- w_R increases with b, λ and mean wage offer, and decreases with r.

Some implications of simple model:

- It takes time to find an acceptable job. Rate at which unemployed workers find jobs: $h = \lambda[1 - F(w_R)]$ ("hazard rate"). Unemployment duration is given by $1/h$.

- Where one ends up is at least partially a matter of luck. Otherwise similar agents may end up with different wages.

- Even this basic model makes predictions about variables that would be extremely difficult to generate using any theory that did not incorporate frictions.

2 Worker turnover

Relax the assumption that jobs last forever

- in the US, from 1994 to 2003, 6.6% of employment relationships ended in a given month: 40% of workers switched employers, the rest either became unemployed or left the labor force (Fallick and Fleischman, Federal Reserve Board of Governors Working Paper 2004-34)
- in the UK, from 1997 to 1999, 5.8% of employment relationships ended in a given quarter: 55% switched employers, the rest became nonemployed (Manning, Monopsony in Motion 2003, Table 4.7)
- We now generalize the simple framework to capture these transitions.

2.1 Transitions to unemployment

Jobs end for exogenous reason according to a Poisson process with parameter δ:

$$rW(w) = w + \delta[U - W(w)]. \tag{12}$$

The reservation wage still satisfies $W(w_R) = U$. Using same algebra:

$$w_R = b + \frac{\lambda}{r+\delta} \int_{w_R}^{\infty} [1 - F(w)]\, dw. \tag{13}$$

- δ affects w_R by raising the effective discount rate to $r + \delta$.
- w_R falls with δ, as returns to job search are more heavily discounted.
- A worker now goes through repeated spells of employment and unemployment: when unemployed, he gets a job at rate $h = \lambda[1 - F(w_R)]$, and when employed he loses the job at rate δ.

2.2 Job-to-job transitions

To explain how workers change employers without an intervening spell of unemployment: on-the-job search.

New offers arrive at rate λ_0 while unemployed and λ_1 while employed. Each offer is an i.i.d. draw from $F(w)$.

Bellman equations

$$rU = b + \lambda_0 \int_{w_R}^{\infty} [W(w) - U] dF(w) \tag{14}$$

$$rW(w) = w + \lambda_1 \int_0^{\infty} \max\left\{0, W(w') - W(w)\right\} dF(w') + \delta[U - W(w)] \tag{15}$$

- Employed workers switch jobs whenever $W(w') \geq W(w)$, i.e. $w' \geq w$.
- $W(w)$ is increasing in w, thus there exists a unique w_R: $W(w_R) = U$.

Evaluating (15) at $w = w_R$ and combining it with (14):

$$w_R = b + (\lambda_0 - \lambda_1) \int_{w_R}^{\infty} [W(w) - U] dF(w). \tag{16}$$

Note $w_R > b$ if and only if $\lambda_0 > \lambda_1$. If a worker get offers more frequently when employed than when unemployed, he is willing to accept wages below b (option value of being employed).

Use integration by parts (and note that $W'(w) = \{r + \delta + \lambda_1[1 - F(w)]\}^{-1}$ by differentiating (15)):

$$w_R = b + (\lambda_0 - \lambda_1) \int_{w_R}^{\infty} \frac{1 - F(w)}{r + \delta + \lambda_1[1 - F(w)]} dw. \tag{17}$$

Special cases:

- $\lambda_0 > \lambda_1 = 0$ yields (13).
- $\lambda_0 = \lambda_1 = \lambda$ yields $w_R = b$

- rich model on labor market experiences
- predictions on relationship between wages, tenure and separation rates
- aggregate unemployment:

$$\dot{u} = \delta(1-u) - \lambda_0 \left[1 - F(w_R)\right] u$$

in steady state:

$$u = \frac{\delta}{\delta + \lambda_0 \left[1 - F(w_R)\right]} \tag{18}$$

- wage distributions

$$N(w) = \frac{\delta \left[F(w) - F(w_R)\right]}{\left[1 - F(w_R)\right] \{\delta + \lambda_1 [1 - F(w)]\}} \tag{19}$$

(using $u\lambda_0 \left[F(w) - F(w_R)\right] = (1-u)N(w) \{\delta + \lambda_1 [1 - F(w)]\}$ in steady state, and (18))

- So far: single worker problem, taking all conditions as given. Partial equilibrium model in which the arrival rate and the distribution of wage offers are fixed.
- For some issues one may want to know how these are determined as an equilibrium outcome, and in particular how they are affected by labor market conditions or policy.
- Need therefore to model two main aspects:
  1. how workers and firms meet [λ]
  2. how wages determined [$F(w)$]

1 Random matching and wage bargaining

(Pissarides 2000, chapter 1)

- Modelling device that captures the idea of frictions: Matching Function
- The MF gives the number of jobs formed at any moment in time as a function of the number of workers looking for jobs, the number of firms looking for workers, and possibly other variables.
- Very useful device: it captures the implications of costly trade without the need to make information imperfections or heterogeneities explicit
- Black-box function.
- Its usefulness crucially depends on microfoundations and empirical viability

1.1 Steady state unemployment

Job creation: Matching function

$$M = m(\text{unemployment}, \text{vacancies})$$

with M increasing and concave in both arguments and displaying CRS.
Therefore (dividing all terms by labor force L):

$$m = m(u, v) \tag{1}$$

Other properties: $m(0, v) = m(u, 0) = 0$.

Note: only the unemployed search.

Assume all jobs and workers are identical

- Vacancies are filled at Poisson rate

$$\frac{m(u,v)}{v} = m\left(\frac{u}{v}, 1\right) \equiv q(\theta)$$

where $\theta = v/u \equiv$labor market tightness; $q'(\theta) < 0$; $q(0) \to \infty$; $q(\infty) \to 0$. and $\eta(\theta) \equiv |q'(\theta)\theta/q(\theta)| \in (0,1)$. Mean vacancy duration: $1/q(\theta)$.

- Unemployed workers find job at Poisson rate

$$\frac{m(u,v)}{u} = m\left(1, \frac{v}{u}\right) \equiv \theta q(\theta)$$

with $\partial\left[\theta q(\theta)\right]/\partial\theta > 0$, elasticity $1-\eta(\theta)$. Mean unemployment duration: $1/\theta q(\theta)$.

- Matching probabilities depend on θ, which in turn depends on search decisions
  - Endogenous arrival rate of job offers $\lambda = \theta q(\theta)$
  - Trade externalities
  - Equilibrium inefficiency?

Job destruction: exogenous job destruction rate δ. Separations: $\delta(1-u)$

Steady state:

$$\begin{aligned} \dot{u} &= \delta(1-u) - \theta q(\theta) u = 0 \\ u &= \frac{\delta}{\delta + \theta q(\theta)} \end{aligned} \tag{2}$$

- Beveridge curve: downward sloping relationship between u and v

1.2 Firms

Small firms with identical technology:
each firm has one job, occupied and producing or vacant and searching.

- Bellman equation for a firm with a filled job

$$rJ(\pi) = \pi + \delta\,[V - J(\pi)] \tag{3}$$

where $\pi = p - w$, p denotes output (labor productivity), and V is the value of a job vacancy.

- Bellman equation for firm with vacant job

$$rV = -pc + q(\theta)[J(\pi) - V] \tag{4}$$

where pc is the cost of keeping an open vacancy for a job producing p.

- Free entry condition: $V = 0$. Thus

$$J(\pi) = \frac{pc}{q(\theta)} \tag{5}$$

Due to frictions there are rents to jobs.

- Substituting (5) into (3) gives the job creation condition

$$w = p - \frac{(r+\delta)pc}{q(\theta)} \tag{6}$$

It is a marginal condition for labor demand.

- Wages are equal to labor productivity minus the expected capitalized value of hiring costs.
Due to search frictions, $w < p$.

1.3 Workers

Unemployed and searching or employed and producing.

Bellman equations:

$$rU = b + \theta q(\theta)\left[W(w) - U\right] \tag{7}$$

$$rW(w) = w + \delta\left[U - W(w)\right] \tag{8}$$

Solving for rU and $rW(w)$:

$$rU = \frac{(r+\delta)b + \theta q(\theta) w}{r + \delta + \theta q(\theta)}$$

$$rW(w) = \frac{\delta b + \left[r + \theta q(\theta)\right] w}{r + \delta + \theta q(\theta)}$$

Note that if $w > b$ and $r > 0$, employment is preferred to unemployment.

1.4 Wage determination

Job matches produce economic rents, which are shared between firms and workers.

The rent sharing rule is determined by the generalized Nash bargaining solution:

$$w = \arg\max\,[W(w) - U]^{\beta}\,[J(\pi) - V]^{1-\beta} \tag{9}$$

where β is the relative measure of worker bargaining strength.

The solution to (9) satisfies

$$W(w) - U = \beta\,[W(w) + J(\pi) - U - V]\,. \tag{10}$$

Substituting $W(w)$ using (8), $J(\pi)$ using (3) and $V = 0$:

$$w = rU + \beta(p - rU). \tag{11}$$

Then substitute (5) into (10), solve for $W(w) - U$ and substitute into (7):

$$rU = b + \frac{\beta}{1-\beta} pc\theta \tag{12}$$

Finally use (12) to eliminate rU from (11) and obtain the wage equation

$$w = (1-\beta)b + \beta p(1 + c\theta). \tag{13}$$

Wages increase with β, b, p and θ.

Note that $pc\theta = pcv/u$ is the total hiring cost per unemployed worker. Thus workers are rewarded for saving of hiring costs that the a firm enjoys when a job is formed.

1.5 Equilibrium

Three equations in three unknowns u, θ, w

$$u = \frac{\delta}{\delta + \theta q(\theta)} \tag{BC}$$

$$w = p - \frac{(r+\delta)pc}{q(\theta)} \tag{JC}$$

$$w = (1-\beta)b + \beta p(1 + c\theta) \tag{WS}$$

JC and WS jointly determine w and θ. Substituting the resulting value of θ into BC gives equilibrium unemployment.

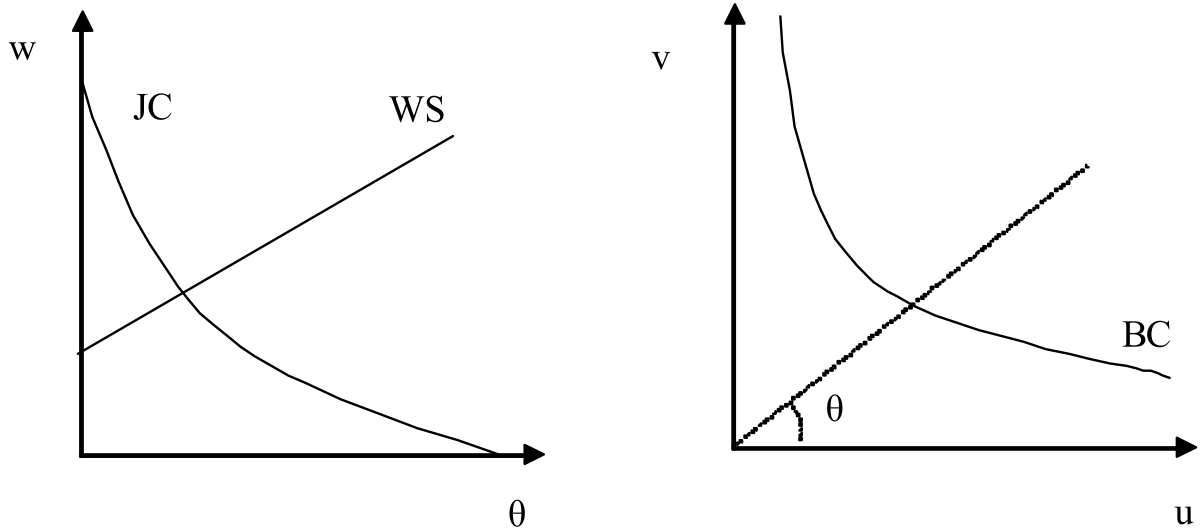

Figure 1: Equilibrium wages, tightness and unemployment

1.6 Exercises

(1) Increase in p (2005 exam, question 2)

- What is the likely impact of an increase in labour productivity on wages, labor market tightness and unemployment?
- Is the impact of higher labour productivity on equilibrium unemployment a desirable property of long-run equilibrium? Why?
- What is preventing the full adjustment of wages to labour productivity? Can you think of an extension to the simple model in which wages fully absorb productivity changes, and equilibrium unemployment is unaffected by productivity growth?

(2) Increase in b

What is the likely impact of an increase in unemployment compensation on wages, labor market tightness and unemployment?

(3) Fall in matching rate [$q(\theta)$ falls at given θ]

What is the likely impact of a deterioration in the matching rate on wages, labor market tightness and unemployment?

What kind of shock may generate such outcome?

2 Match-specific productivity

(Pissarides 2000, chapter 6)

In previous model: it takes time to match, but every job worker contact leads to a match and the wage is the same in every match. The wage distribution is degenerate at w. [why? all jobs are identical and wages are solution to the same sharing rule in all jobs]

- A simple way of generating wage dispersion consists in introducing a distribution of productivity values
- with this extension: not every match yields the same wage and not every contact results in a match.

- Upon meeting: worker and firm sample a job productivity value p from some known distribution $G(p)$ and then decide whether to form and match or keep searching using a reservation rule (p_R).

- If they form a match: surplus is shared according to the Nash bargaining solution, and wages depend on job productivity $w(p)$

- Model more complicated, as both the value of unemployment and the value of a vacancy depend on the *expected* value of job matches.

- p_R increases with θ, as workers are more choosy when it is easier to receive a job offer. But when p_R increases, vacancies are harder to match and job creation falls. Thus there exists a unique equilibrium in p_R and θ.

- There exists a non-degenerate wage distribution $N(w)$, truncated at $w(p_R)$.
- The hazard rate is now $h = \theta q(\theta)[1 - G(p_R)]$.
- Suppose b increases: p_R increases, $\theta q(\theta)$ falls, and h falls. From worker perspective, this closely resembles the simple single-agent problem, but both the arrival rate and the wage distribution are now endogenous.

3 Endogenous separations

(Mortensen and Pissarides RES 1994; Pissarides 2000, chapter 2)

In the simple model the separation rate δ is exogenous. All shocks affect equilibrium via job creation (unless shock is precisely on δ). But evidence shows that both job creation and job destruction respond to shocks.

- Davis Haltiwanger and Schuh (1996): in the US cyclical patterns in JD are stronger than in JC
  Shimer (2005): JC accounts for 3/4 of unemployment fluctuations
  Elsby, Michaels and Solon (2007): JC and JD play roughly equal roles

- Extension: the separation rate can be endogenized by allowing productivity and wage changes on-the-job

- The resulting framework captures endogenously the flows into and out of unemployment
- Let p be the current output in a match: at Poisson rate δ we get a new draw p' from $G(p')$.
- When output falls below some reservation value p_R the job is destroyed. Note: $J(p_R - w(p_R)) = 0$.
- Separations: $\delta G(p_R)(1-u)$
- p_R increases with θ, as higher θ implies higher wages at given p, and thus lower $J(p-w(p))$. Jobs are destroyed at higher output threshold. But when p_R increases, jobs last shorter and job creation falls. Thus there exists a unique equilibrium in p_R and θ.
- There exists a non-degenerate wage distribution $N(w)$, truncated at $w(p_R)$.
- Rich literature that uses this model to study the behavior of job and worker flows across countries as well as over the business cycle.

4 Efficiency

(Hosios RES 1994; Pissarides 2000, chapter 8)

- There are trade externalities in search equilibrium. Transition rates $q(\theta)$ and $\theta q(\theta)$ depend on what other agents decide to do.
- An agent's decision to engage in search generates: positive externalities on the opposite side of the marker; negative externalities on the same side of the market.
- Equilibrium θ depends on wages. Is the Nash bargaining solution able to internalize search externalities and ensure efficiency.
- No reason in principle why wages should internalize externalities.
  - Externalities are generated during search - while wages are only negotiated after a match is made. Agents who are negotiating are not likely to take into consideration the search costs of those still unmatched.

Social output per job: p

Each unemployed enjoys b

Total search costs: $pc\theta u$

Social planner problem in infinitely-lived economy:

$$\begin{aligned} \max_{\theta} \Omega &= \int_0^{\infty} e^{-rt} \left[p(1-u) + bu - pc\theta u \right] dt \\ s.to &: \dot{u} = \delta(1-u) - \theta q(\theta) u \end{aligned} \tag{14}$$

Note that the planner takes search frictions as given (constraint) and is not interested in distributive matters (wages) but only in allocative efficiency (max net social output).

Set the Hamiltonian:

$$H = e^{-rt}\left[p(1-u) + bu - pc\theta u\right] + \mu\left[\delta(1-u) - \theta q(\theta)u\right] \tag{15}$$

First-order conditions:

$$\begin{aligned} H_\theta &= 0 \leftrightarrow -e^{-rt}pcu - \mu u q(\theta)\left[1-\eta(\theta)\right] = 0 & (16)\\ H_u &= -\dot{\mu} \leftrightarrow -e^{-rt}(p - b + pc\theta) - \mu\left[\delta + \theta q(\theta)\right] = -\dot{\mu} & (17)\end{aligned}$$

where $\eta(\theta) = -q'(\theta)\theta/q(\theta)$.

From (16): $\mu = -\frac{e^{-rt}pc}{q(\theta)[1-\eta(\theta)]}$. In steady state: $-\dot{\mu} = r\frac{e^{-rt}pc}{q(\theta)[1-\eta(\theta)]}$.

Substitute these in (17) and obtain

$$[1-\eta(\theta)]\,(p-b)-\frac{\delta+r+\eta(\theta)\theta q(\theta)}{q(\theta)}pc=0 \tag{18}$$

Compare with equilibrium θ in decentralized model (combining (6) and (13))

$$(1-\beta)(p-b)-\frac{\delta+r+\beta\theta q(\theta)}{q(\theta)}pc=0 \tag{19}$$

(18) and (19) coincide when $\eta(\theta)=\beta$ (Hosios condition). This condition is feasible, as both $\eta(\theta), \beta \in (0,1)$, but nothing is ensuring it in decentralized equilibrium (in particular, β is a constant parameter while $\eta(\theta)$ is endogenous).

- Equilibrium unemployment is inefficient in general:
  inefficiently high or inefficiently low?
- If $\beta > \eta(\theta)$, unemployment is “too” high;
  if $\beta < \eta(\theta)$, unemployment is “too” low (think of wage response to θ).

- Note that $\eta(\theta)$ = elasticity of vacancy duration wrt the vacancy rate and $1 - \eta(\theta)$ = elasticity of unemployment duration wrt the unemployment rate (size of negative externalities).

- If $\eta(\theta)$ is high: firms are causing more negative externalities to other firms than workers are doing to other workers. Efficiency requires that firms should be "taxed", and a larger share of surplus (β) goes to workers.

- Also: $-1 + \eta(\theta)$ = elasticity of unemployment duration wrt vacancies and $-\eta(\theta)$ = elasticity of vacancy duration wrt unemployment (size of positive externalities).

- If $\eta(\theta)$ is high: the unemployed are causing more positive externalities to firms than firms are doing to unemployed workers. Efficiency requires that firms should be "taxed", and a larger share of surplus (β) goes to workers.