Optimal Unemployment Insurance over the Business Cycle

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Unemployment insurance debate

1. UI provides a safety net

2. UI reduces job search and raises unemployment

3. UI raises wages and raises unemployment

4. job search is irrelevant if firms do not hire much

- public-finance approach: $1 + 2$

- our approach: $1 + 2 + 3 + 4$
Public-finance approach [Baily, 1978]

- workers are initially unemployed
- workers search for a job with some effort
- workers find a job at rate $f$ per unit of effort
- workers are risk averse but no self-insurance
- job-search effort is unobservable
- **limitation:** $f$ is a fixed parameter
Our approach

- matching model of unemployment with firms
- job-finding rate $f$ depends on tightness $\theta$
- $\theta = \text{recruiting effort} / \text{job-search effort}$
- $\theta$ depends on UI + business cycle
- **contribution**: optimal UI formula in sufficient statistics when $f$ responds to UI + business cycle
Outline

1. General matching model
2. Optimal UI formula
3. Specific matching models
4. Quantitative exploration
A static model

- measure 1 of identical workers, initially unemployed
- measure 1 of identical firms
- workers and firms meet on frictional labor market
- **tightness** $\theta = \text{recruiting effort/job-search effort}$
Summary of matching frictions

- unobservable job-search effort: $e$
- job-finding rate per unit of effort: $f(\theta)$
- **job-finding probability**: $e \cdot f(\theta)$ with $f' > 0$
- employees = $[1 + \tau(\theta)] \cdot$ producers
- recruiters = $\tau(\theta) \cdot$ producers with $\tau' > 0$
- workers like $\theta$, firms dislike $\theta$
Workers

- given $\theta$ and UI, choose $e$ to maximize

$$
\begin{align*}
&v(c^u) + e \cdot f(\theta) \cdot [v(c^e) - v(c^u)] - k(e) \\
&\text{consumption utility} \quad \text{utility gain from search} \quad \text{search cost}
\end{align*}
$$

- effort supply $e^s(\theta, UI)$ determines optimal effort:

$$
k'(e^s) = f(\theta) \cdot [v(c^e) - v(c^u)]
$$

- labor supply $l^s(\theta, UI)$ determines employment rate:

$$
l^s(\theta, UI) = e^s(\theta, UI) \cdot f(\theta)
$$
Firms

- number of employees \( l > \) number of producers \( n \)
- given \( \theta \) and wage \( w \), choose \( n \) to maximize

\[
y(n) - w \cdot [1 + \tau(\theta)] \cdot n
\]

production wage of producers + recruiters

- labor demand \( l^d(\theta, w) \) gives optimal employment:

\[
y' \left( \frac{l^d}{1 + \tau(\theta)} \right) = \left[ 1 + \tau(\theta) \right] \cdot w
\]

MPL matching wedge real wage
Government

- UI provides $c^u$ to unemployed workers
- UI provides $c^e > c^u$ to employed workers
- generosity of UI is replacement rate:

$$R \equiv 1 - \frac{c^e - c^u}{w} = \text{labor tax rate} + \text{benefit rate}$$
Equilibrium

- take UI policy as given
- equilibrium is \((\theta, w)\) such that supply = demand:

\[ l^s(\theta, UI) = l^d(\theta, w) \]

- 2 variables, 1 equation: wage \(w\) is indeterminate
- take general wage schedule: \(w = w(\theta, UI)\)
- **equilibrium tightness is** \(\theta(UI)\)
Equilibrium in \((l, \theta)\) plane

Labor supply

Labor demand

Equilibrium

Unemployment

Labor market tightness

\(\theta\)

Employment
Outline

1. General matching model
2. **Optimal UI formula**
3. Specific matching models
4. Quantitative exploration
Government’s problem

- Choose UI to maximize welfare

\[ l \cdot v(c^e) + (1 - l) \cdot v(c^u) - k(e) \]

- Subject to budget constraint

\[ l \cdot c^e + (1 - l) \cdot c^u = y \left( \frac{l}{1 + \tau(\theta)} \right) \]

- Subject to \( e = e^s(\theta, UI), l = l^s(\theta, UI), \theta = \theta(UI) \)
Social welfare maximization

- Lagrangian: \( \mathcal{L} = \text{welfare} + \phi \cdot \text{budget} \)
- first-order condition \( \frac{d\mathcal{L}}{dUI} = 0 \) implies
  \[
  \left. \frac{\partial \mathcal{L}}{\partial UI} \right|_\theta + \left. \frac{\partial \mathcal{L}}{\partial \theta} \right|_{UI} \cdot \frac{d\theta}{dUI} = 0
  \]
- \( \frac{\partial \mathcal{L}}{\partial UI} \bigg|_\theta = 0 \) is Baily formula
- \( \frac{\partial \mathcal{L}}{\partial \theta} \bigg|_{UI} = 0 \) is generalized Hosios condition
- \( \frac{d\theta}{dUI} \) can be expressed in sufficient statistics
Baily formula

■ optimal UI at constant $\theta$ satisfies

$$\frac{R}{1 - R} = \frac{l}{\varepsilon^m} \cdot \left[ \frac{v'(c^u)}{v'(c^e)} - 1 \right]$$

UI generosity                  moral hazard cost                  insurance value

■ $R$: replacement rate of UI

■ microelasticity $\varepsilon^m$: response of unemployment to UI at constant $\theta$ (only search effort responds)
Microelasticity in \((l, \theta)\) plane

![Graph showing labor supply, demand, and equilibrium with high UI.](image-url)
Microelasticity in \((l, \theta)\) plane

- Labor supply with high UI
- Labor supply with low UI
- Microelasticity \(\epsilon^m\)
- Labor demand with high UI
Generalized Hosios condition

- optimal $\theta$ at constant UI satisfies

$$\frac{\Delta v}{\phi \cdot w} + R \cdot \left(1 + \varepsilon^d\right) - \frac{\eta}{1 - \eta} \cdot \tau(\theta) = 0$$

- $\Delta v$: utility gain from employment
- $\eta$: curvature of matching function
- $\varepsilon^d$: discouraged-worker elasticity
- $\tau(\theta)$: business-cycle statistic
Hosios term over the business cycle

Lagrangian $L(UI, \theta)$

Hosios term = 0

Labor market tightness $\theta$

Welfare
Hosios term over the business cycle

Lagrangian $L(UI, \theta)$

Welfare

Labor market tightness $\theta$

Hosios term $> 0$

recession
Hosios term over the business cycle

Welfare

Labor market tightness $\theta$

Lagrangian $L(UI, \theta)$

Hosios term < 0

expansion
Microelasticity and macroelasticity

Equilibrium with high UI

Microelasticity $\varepsilon^m$

Labor supply with low UI

Labor market tightness

Employment
Microelasticity and macroelasticity

- Equilibrium with low UI
- Labor demand with low UI
- Labor supply with low UI

Microelasticity $\varepsilon_m$

Macroelasticity $\varepsilon^M$

Equilibrium

$d\theta < 0$
Microelasticity and macroelasticity

Labor demand with low UI

Employment

Labor market tightness

Equilibrium with low UI

Microelasticity $\varepsilon^m$

Equilibrium with high UI

Macroelasticity $\varepsilon^M$

Labor supply with low UI

Labor demand with low UI

$d\theta = 0$
Microelasticity and macroelasticity

Employment

Labor market tightness

\[ d\theta > 0 \]

Equilibrium with low UI

Microelasticity \( \varepsilon^m \)

Labor supply with low UI

Labor demand with low UI

Equilibrium with high UI

Macroelasticity \( \varepsilon^M \)
Externalities

Equilibrium with low UI

Labor supply with low UI

Labor demand with low UI

Wage externality

Equilibrium with high UI

Labor market tightness

Employment
Elasticity wedge measures $d\theta/dUI$

- macroelasticity $\varepsilon^M$: response of employment to UI in general equilibrium (search effort + $\theta$ respond)

- $1 - (\varepsilon^M / \varepsilon^m) > 0$: lower UI $\Rightarrow$ lower $\theta$

- $1 - (\varepsilon^M / \varepsilon^m) = 0$: UI does not influence $\theta$

- $1 - (\varepsilon^M / \varepsilon^m) < 0$: lower UI $\Rightarrow$ higher $\theta$
Optimal UI formula in general equilibrium

\[ \frac{R}{1-R} = \text{Baily term} + P \cdot \left[ 1 - \frac{\varepsilon^M}{\varepsilon^m} \right] \cdot \text{Hosios term} \]

- \( \frac{\partial \mathcal{L}}{\partial UI} \bigg|_{\theta} = 0 \)
- \( \frac{d \theta}{d UI} \cdot \left[ \frac{\partial \mathcal{L}}{\partial \theta} \bigg|_{UI} \right] \): externality-correction term
- more UI than Baily if \( \frac{d \theta}{d UI} \cdot \left[ \frac{\partial \mathcal{L}}{\partial \theta} \bigg|_{UI} \right] > 0 \)
- more UI than Baily if UI brings \( \theta \) to optimum
Optimal UI formula in general equilibrium

\[
\frac{R}{1-R} = \text{Baily term} + \left( \sum_{i} P_i \cdot \left[ 1 - \frac{\varepsilon^M}{\varepsilon^m} \right] \cdot \text{Hosios term} \right)
\]

- \( R \): replacement rate of UI

- if \( \left[ 1 - \left( \frac{\varepsilon^M}{\varepsilon^m} \right) \right] \cdot \text{Hosios term} > 0 \): UI above Baily

- if \( \left[ 1 - \left( \frac{\varepsilon^M}{\varepsilon^m} \right) \right] \cdot \text{Hosios term} = 0 \): UI at Baily

- if \( \left[ 1 - \left( \frac{\varepsilon^M}{\varepsilon^m} \right) \right] \cdot \text{Hosios term} < 0 \): UI below Baily
Optimal replacement rate vs. Baily rate

\[ 1 - \left( \frac{\varepsilon^M}{\varepsilon^m} \right) \]

<table>
<thead>
<tr>
<th></th>
<th>$-$</th>
<th>0</th>
<th>$+$</th>
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</thead>
<tbody>
<tr>
<td>recession</td>
<td>lower</td>
<td>same</td>
<td>higher</td>
</tr>
<tr>
<td>at Hosios</td>
<td>same</td>
<td>same</td>
<td>same</td>
</tr>
<tr>
<td>expansion</td>
<td>higher</td>
<td>same</td>
<td>lower</td>
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Outline

1. General matching model
2. Optimal UI formula
3. **Specific matching models**
4. Quantitative exploration
### Three matching models

<table>
<thead>
<tr>
<th></th>
<th>Pissarides</th>
<th>Hall</th>
<th>Michaillat</th>
</tr>
</thead>
<tbody>
<tr>
<td>production</td>
<td>linear</td>
<td>linear</td>
<td>concave</td>
</tr>
<tr>
<td>$y(n) = n$</td>
<td>$y(n) = n$</td>
<td>$y(n) = n^\alpha$, $\alpha &lt; 1$</td>
<td></td>
</tr>
<tr>
<td>wage</td>
<td>Nash bargaining</td>
<td>rigid</td>
<td>rigid</td>
</tr>
<tr>
<td>$w = w(\theta, UI)$</td>
<td>$w &gt; 0$</td>
<td>$w &gt; 0$</td>
<td></td>
</tr>
</tbody>
</table>
Pissarides’ model: \(1 - (\varepsilon^M / \varepsilon^m) < 0\)
Pissarides’ model: \[ 1 - \left( \frac{\varepsilon^M}{\varepsilon^m} \right) < 0 \]
Pissarides’ model: $1 - \left( \frac{\varepsilon^M}{\varepsilon^m} \right) < 0$
Hall’s model: \( 1 - \left( \frac{\varepsilon^M}{\varepsilon^m} \right) = 0 \)
Hall’s model: \[ 1 - (\varepsilon^M / \varepsilon^m) = 0 \]
Michaillat’s model: \(1 - \left(\frac{\varepsilon^M}{\varepsilon^m}\right) > 0\)
Michaillat’s model: $1 - \left( \frac{\varepsilon^M}{\varepsilon^m} \right) > 0$
Michaillat’s model: \[ 1 - \left( \frac{\epsilon^M}{\epsilon^m} \right) > 0 \]
Optimal UI in various matching models

<table>
<thead>
<tr>
<th></th>
<th>Pissarides</th>
<th>Hall</th>
<th>Michaillat</th>
</tr>
</thead>
<tbody>
<tr>
<td>wage ext.</td>
<td>yes</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>labor-demand ext.</td>
<td>no</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>$1 - (\varepsilon^M/\varepsilon^m)$</td>
<td>—</td>
<td>0</td>
<td>+</td>
</tr>
<tr>
<td>optimal UI</td>
<td>procyclical</td>
<td>acyclical</td>
<td>countercyclical</td>
</tr>
</tbody>
</table>
Outline

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Empirical strategy

- microelasticity: increase in probability of unemployment when *individual UI* increases
- macroelasticity: increase in aggregate unemployment when *aggregate UI* increases
Elasticity wedge estimates

- Crepon, Duflo, Gurgand, Rathelot, and Zamora [QJE, 2013] for France
  - treatment: job-search assistance
  - labor-demand externality only
  - $1 - (\varepsilon^M / \varepsilon^m) = 0.37 > 0$

- Lalive, Landais, and Zweimüller for Austria
  - treatment: increase UI duration from 52 to 209 weeks
  - labor-demand and wage externality
  - $1 - (\varepsilon^M / \varepsilon^m) = 0.35 > 0$
Optimal UI over the business cycle

Unemployment rate
Replacement rate

\[ 1 - \left( \frac{\varepsilon^M}{\varepsilon^m} \right) = 0.4 \]

\[ 1 - \left( \frac{\varepsilon^M}{\varepsilon^m} \right) = 0 \]
Optimal UI over the business cycle

\[ 1 - \left( \frac{\varepsilon^M}{\varepsilon^m} \right) = 0.4 \]

\[ 1 - \left( \frac{\varepsilon^M}{\varepsilon^m} \right) = 0 \]

\[ 1 - \left( \frac{\varepsilon^M}{\varepsilon^m} \right) = -0.5 \]
Optimal UI over the business cycle

1 - (ε^M/ε^m) = 0.4
1 - (ε^M/ε^m) = 0
1 - (ε^M/ε^m) = -0.5
0.2 < 1 - (ε^M/ε^m) < 0.9
Future research

1. empirical estimates of elasticity wedge $1 - (\varepsilon^M / \varepsilon^m)$

2. optimal macro policies over the business cycle
   - fiscal policy, insurance programs, monetary policy
   - formula for policy $\tau$ takes form

\[
0 = \text{PF term} + \frac{d\theta}{d\tau} \cdot \text{Hosios term}
\]

- PF term $= \frac{\partial SW}{\partial \tau}|_\theta$ and Hosios term $= \frac{\partial SW}{\partial \theta}|_\tau$
- see Michaillat and Saez [2013]
Matching frictions

- measure 1 of workers, initially unemployed
- job-search effort (unobservable): \( e \)
- number of vacancies: \( o \)
- constant-returns matching function: \( m(\cdot, \cdot) \)
- number of matches: \( l = m(e, o) \leq 1 \)
- labor market tightness: \( \theta \equiv o/e \)
- vacancy-filling proba.: \( q(\theta) = l/o = m(1/\theta, 1) \)
- job-finding rate: \( f(\theta) = l/e = m(1, \theta) \)
- job-finding proba.: \( e \cdot f(\theta) \)
Matching cost

posting each vacancy requires \( r \) workers:

\[
\begin{align*}
\text{employees} & = n + r \cdot \frac{l}{q(\theta)} \\
\Rightarrow l \cdot \left[1 - \frac{r}{q(\theta)}\right] & = n \\
\Rightarrow l & = \left[1 + \frac{r}{q(\theta) - r}\right] \cdot n \\
\Rightarrow \text{employees} & = \left[1 + \tau(\theta)\right] \cdot \text{producers}
\end{align*}
\]
Formula in dynamic model

\[ \frac{w}{\Delta c} - 1 \approx \frac{1}{\varepsilon^m} \left( \frac{c^e}{c^u} - 1 \right) + \frac{1}{1 + \varepsilon^d} \left( 1 - \frac{\varepsilon^M}{\varepsilon^m} \right) \times \left[ \frac{\ln(c^e/c^u)}{1 - c^u/c^e} + \left( 1 + \varepsilon^d \right) \left( \frac{w}{\Delta c} - 1 \right) - \frac{\eta}{1 - \eta \Delta c} \frac{w}{u} \tau(\theta) \right] \]

- solve for replacement rate \(1 - (\Delta c / w)\)
- exogenous sufficient statistics: \(\varepsilon^d, \varepsilon^M, \varepsilon^m, \eta, \tau(\theta)/u\)
- \(1 - (\varepsilon^M / \varepsilon^m)\) measures labor-demand & wage externality
- \(\tau(\theta)/u\) measures business cycle
Flows in finite-duration model

- Employed: \( n \)
- Eligible Unemployed: \( x^u \)

Job finding: \( e^u . f(\theta) . x^u \)
Flows in finite-duration model

- Employed: \(n\)
- Eligible Unemployed: \(x^u\)
- Ineligible Unemployed: \(x^a\)
- Job finding: \(e^u.f(\theta).x^u\)
- Ineligibility: \(\lambda x^u\)
Flows in finite-duration model

- Job finding: $e^a f(\theta) x^a$
- Employed: $n$
- Eligible Unemployed: $x^u$
- Ineligible Unemployed: $x^a$
- Ineligibility: $\lambda x^u$
Countercyclical arrival rate of ineligibility

![Graph showing the relationship between unemployment rate and arrival rate of ineligibility]