# A Macroeconomic Theory of Optimal Unemployment Insurance

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**Abstract**

We develop a theory of optimal unemployment insurance (UI) that accounts for workers' job-search behavior and firms' hiring behavior. The optimal replacement rate of UI is the conventional Baily [1978]-Chetty [2006*a*] rate, which solves the trade-off between insurance and job-search incentives, plus a correction term, which is positive when UI brings the labor market tightness closer to efficiency. For instance, when tightness is inefficiently low, optimal UI is more generous than the Baily-Chetty rate if UI raises tightness and less generous if UI lowers tightness. We propose empirical criteria to determine whether tightness is inefficiently high or low and whether UI raises or lowers tightness. The theory has implications for the cyclicality of optimal UI.

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# I. Introduction

Unemployment insurance (UI) is a key component of social insurance in modern welfare states. The microeconomic theory of optimal UI, developed by Baily [1978] and Chetty [2006*a*], is well understood: it is an insurance-incentive trade-off in the presence of moral hazard. UI helps workers smooth consumption when they are unemployed, but it also increases unemployment by discouraging job search. The Baily-Chetty formula resolves this trade-off.

The microeconomic theory of optimal UI, however, accounts only for workers' job-search behavior and not for firms' hiring behavior. This becomes problematic in slumps because when unemployment is high, the effect of UI on welfare can only be properly measured if firms' hiring behavior is accounted for. For instance, some argue that UI has a particularly negative effect of welfare in slumps because UI exerts upward pressure on wages, thereby discouraging job creation. Others argue that in slumps, jobs are simply not available to jobseekers, irrespective of how much they search, so UI does not raise unemployment much.

In this paper we develop a macroeconomic theory of optimal UI, accounting not only for workers' job-search behavior but also for firms' hiring behavior.[1] The theory encompasses the microeconomic insurance-incentive trade-off as well as the macroeconomic influence of UI on firms' hiring decisions. We derive a formula that gives the optimal replacement rate of UI as a function of statistics that can be estimated empirically. Our formula provides guidance to link the generosity of UI to the unemployment rate.[2]

We begin by embedding the Baily-Chetty model of UI into a matching model with general production function, wage mechanism, and matching function.[3] The matching framework is well suited for our purpose because it describes both workers' job-search process and firms'

[1] A few papers study the optimal response of UI to shocks by simulating calibrated macroeconomic models, but by nature the results are somewhat specific to the calibration and structural assumptions about the wage mechanism or production function. See for instance Jung and Kuester [2015].

[2] In the US, the generosity of UI depends on the unemployment rate. UI benefits have a duration of 26 weeks in normal times. The Extended Benefits program automatically extends duration by 13 weeks in states where unemployment is above 6.5% and by 20 weeks in states where unemployment is above 8%. Duration is often further extended in severe recessions. For example, the Emergency Unemployment Compensation program enacted in 2008 extended durations by an additional 53 weeks in states where unemployment was above 8.5%.

[3] Matching models have been used to study UI in other contexts. See for instance Cahuc and Lehmann [2000], Fredriksson and Holmlund [2001], Lehmann and Van Der Linden [2007], and Mukoyama [2013].

hiring process. The labor market tightness, defined as the ratio of vacancies to aggregate job-search effort, is central to our theory. In equilibrium, tightness equalizes labor demand and supply. UI generally affects tightness because labor supply and demand respond to UI.

In matching models the equilibrium is not necessarily efficient. The labor market is efficient if tightness maximizes welfare for a given UI; otherwise, the labor market is inefficient. Tightness is inefficiently high if reducing tightness increases welfare; in that case, unemployment is inefficiently low and firms devote too much labor to recruiting workers. Tightness is inefficiently low if raising tightness increases welfare; in that case, unemployment is inefficiently high and too few jobseekers find a job.

When the labor market is efficient, the optimal replacement rate of UI is given by the Baily-Chetty formula. The reason is that when the labor market is efficient, the marginal effect of UI on tightness has no first-order effect on welfare. Hence, optimal UI is governed by the same principles as in the Baily-Chetty model, in which tightness is fixed. Our theory uses the Baily-Chetty formula as a baseline. This is convenient because the Baily-Chetty formula has been studied extensively, so we have a good idea of the replacement rate implied by the formula.[4]

When the labor market is inefficient, the replacement rate given by the Baily-Chetty formula is no longer optimal. This is our main result, and it has a simple intuition. Consider a labor market in which tightness is inefficiently low. If UI raises tightness, UI is desirable beyond the insurance-incentive trade-off, and the optimal replacement rate of UI is higher than the Baily-Chetty rate. Conversely, if UI lowers tightness, UI is not as desirable as what the insurance-incentive trade-off implies, and the optimal replacement rate of UI is lower than the Baily-Chetty rate. The same logic applies when tightness is inefficiently high.

Formally, we develop a formula that expresses the optimal replacement rate of UI as the sum of the Baily-Chetty rate plus a correction term. The correction term is equal to the effect of UI on tightness times the effect of tightness on welfare. The term is positive if UI brings tightness closer to its efficient level, and negative otherwise. Hence, the optimal replacement rate of UI is above the Baily-Chetty rate if and only if increasing UI brings the labor market closer to efficiency.

[4] See for instance Gruber [1997] and Chetty [2006*a*].

In matching models, increasing UI can raise or lower tightness. UI can lower tightness through a *job-creation effect*: when UI rises, firms hire less because wages increase through bargaining, which reduces tightness. UI can also raise tightness through a *rat-race effect*: if the number of jobs available is somewhat limited, then by discouraging job search, UI mechanically increases the job-finding rate per unit of search effort and thus tightness. The overall impact of UI on tightness depends on which effect dominates. For example, in the model of Pissarides [2000] with wage bargaining and linear production function, only the job-creation effect operates and UI lowers tightness. But in the job-rationing model of Michaillat [2012] with rigid wage and concave production function, only the rat-race effect operates and UI raises tightness.

To facilitate the application of the theory, we express the optimal UI formula in terms of estimable statistics, as in Chetty [2006*a*]. We develop an empirical criterion to evaluate whether tightness is inefficiently low or inefficiently high. The criterion is that tightness is inefficiently low if and only if the value of having a job relative to being unemployed is high enough compared to the share of labor devoted to recruiting. We develop another empirical criterion to evaluate whether UI raises or lowers tightness. The criterion is that UI raises tightness if and only if the microelasticity of unemployment with respect to UI is larger than its macroelasticity. The microelasticity measures the partial-equilibrium response of unemployment to UI, keeping tightness constant, whereas the macroelasticity measures its general-equilibrium response. The microelasticity accounts only for the response of job search to UI while the macroelasticity also accounts for the response of tightness. This criterion is simple to understand. Imagine that UI raises tightness. Then UI increases the job-finding rate, which dampens the negative effect of UI on job search. In that case, the macroelasticity is smaller than the microelasticity.

Our theory can be combined with empirical evidence to determine the cyclicality of optimal UI. Recent empirical work finds that the macroelasticity of unemployment with respect to UI is smaller than the microelasticity, suggesting that UI raises tightness. Using data from the Current Employment Statistics program and the National Employment Survey of the Bureau of Labor Statistics (BLS), we construct a monthly time series for the share of labor devoted to recruiting. This share exhibits wide procyclical fluctuations in the US, suggesting that tightness is inefficiently low in slumps and inefficiently high in booms. This empirical evidence indicates that the correction term in our optimal UI formula is countercyclical. Hence, the optimal replacement rate of UI is below the Baily-Chetty rate in booms and above the Baily-Chetty rate in slumps.

Finally, to quantify the countercyclical fluctuations of the optimal replacement rate of UI, we simulate the job-rationing model of Michaillat [2012]. We opt for this model because unlike other matching models, but consistent with available evidence, it predicts that the macroelasticity of unemployment with respect to UI is smaller than the microelasticity. The simulations indicate that the optimal replacement rate of UI is sharply countercyclical, increasing from below 55% to above 75% when the unemployment rate rises from 5% to 13%.

## II. The Model

This section develops a model of UI. This model embeds the model of UI by Baily [1978] and Chetty [2006*a*] into a matching model of the labor market.[5]

**The Labor Market.** There is a measure 1 of identical workers and a measure 1 of identical firms. Initially, all workers are unemployed and search for a job with effort $e$. Each firm posts $o$ vacancies to recruit workers. The number $l$ of workers who find a job is given by a matching function taking as arguments aggregate search effort and vacancies: $l = m(e,o)$. The function $m$ has constant returns to scale, is differentiable and increasing in both arguments, and satisfies the restriction that $m(e,o) \leq 1$.

The labor market tightness is defined as the ratio of vacancies to aggregate search effort: $\theta = o/e$. Since the matching function has constant returns to scale, the labor market tightness determines the probabilities to find a job and fill a vacancy. A jobseeker finds a job at a rate $f(\theta) = m(e,o)/e = m(1,\theta)$ per unit of search effort; thus, a jobseeker searching with effort $e$ finds a job with probability $e \cdot f(\theta)$. A vacancy is filled with probability $q(\theta) = m(e,o)/o = m(1/\theta,1) = f(\theta)/\theta$. The function $f$ is increasing in $\theta$ and the function $q$ is decreasing in $\theta$. In other words, it is easier to find a job but harder to fill a vacancy when the labor market tightness

[5]See Pissarides [2000] for a classical matching model of the labor market. Our matching model follows the formalism developed by Michaillat and Saez [2015].

is higher. We denote by $1-\eta$ and $-\eta$ the elasticities of $f$ and $q$: $1-\eta = \theta \cdot f'(\theta)/f(\theta) > 0$ and $\eta = -\theta \cdot q'(\theta)/q(\theta) > 0$.

**Firms.** The representative firm hires $l$ workers, paid a real wage $w$, to produce a consumption good. We assume that the firm has two types of employees: some are engaged in production while others are engaged in recruiting. A number $n < l$ of workers are producing a quantity $y(n)$, where the production function $y$ is differentiable, increasing, and concave. A number $l-n$ of workers are recruiting employees by posting vacancies. Posting a vacancy requires $r \in (0,1)$ recruiter, so the number of recruiters required to post $o$ vacancies is $l-n = r \cdot o$. Since hiring $l$ employees requires posting $l/q(\theta)$ vacancies, the number $n$ of producers in a firm with $l$ employees is limited to $n = l - r \cdot l/q(\theta)$. Hence, the numbers of producers and employees are related by

$$n = \frac{l}{1+\tau(\theta)}, \tag{1}$$

where $\tau(\theta) \equiv r/(q(\theta)-r)$ is the recruiter-producer ratio. The function $\tau$ is positive and increasing when $q(\theta) > r$, which holds in equilibrium. The elasticity of $\tau$ is $\theta \cdot \tau'(\theta)/\tau(\theta) = \eta \cdot (1+\tau(\theta))$.

The firm sells its output on a product market with perfect competition. Given $\theta$ and $w$, the firm chooses $l$ to maximize profits $\pi = y(l/(1+\tau(\theta))) - w \cdot l$. The optimal number of employees satisfies

$$y'\left(\frac{l}{1+\tau(\theta)}\right) = (1+\tau(\theta)) \cdot w. \tag{2}$$

At the optimum, the marginal revenue and marginal cost of hiring a producer are equal. The marginal revenue is the marginal product of labor, $y'(n)$. The marginal cost is the real wage, $w$, plus the marginal recruiting cost, $\tau(\theta) \cdot w$.

We implicitly define the labor demand $l^d(\theta, w)$ by

$$y'\left(\frac{l^d(\theta,w)}{1+\tau(\theta)}\right) = (1+\tau(\theta)) \cdot w. \tag{3}$$

The labor demand gives the number of workers hired by firms when firms maximize profits.

**The Unemployment Insurance.** The receipt of UI cannot be contingent on search effort because search effort is not observable. Hence, UI provides all employed workers with consumption $c^e$ and all unemployed workers with consumption $c^u < c^e$, irrespective of their search effort. We use three alternative measures of the generosity of UI: UI is more generous if the consumption gain from work $\Delta c \equiv c^e - c^u$ decreases, if the utility gain from work $\Delta v \equiv v(c^e) - v(c^u)$ decreases, or if the replacement rate

$$R \equiv 1 - \frac{\Delta c}{w}$$

increases.[6] The government must satisfy the budget constraint

$$y(n) = (1-l) \cdot c^u + l \cdot c^e. \tag{4}$$

If firms' profits $\pi$ are equally distributed, the UI system can be implemented with a UI benefit $b$ funded by a tax on wages $t$ so that $(1-l) \cdot b = l \cdot t$ and $c^u = \pi + b$ and $c^e = \pi + w - t$. If profits are unequally distributed, a 100% tax on profits rebated lump sum implements the same allocation.

**Workers.** Workers cannot insure themselves against unemployment in any way, so they consume $c^e$ if employed and $c^u$ if unemployed. The utility from consumption is $v(c)$. The function $v$ is differentiable, increasing, and concave. The disutility from job-search effort, $e$, is $k(e)$. The function $k$ is differentiable, increasing, and convex. Given $\theta$, $c^e$, and $c^u$, a representative worker chooses $e$ to maximize expected utility

$$l \cdot v(c^e) + (1-l) \cdot v(c^u) - k(e) \tag{5}$$

[6] When a jobseeker finds work, she keeps a fraction $\Delta c/w = 1 - R$ of the wage and gives up a fraction $R$ as UI benefits are lost. This is why we can interpret $R$ as the replacement rate of the UI system. Our definition of the replacement rate relates to the conventional definition as follows. Consider a UI system that provides a benefit $b$ funded by a tax $t$ so that $\Delta c = w - t - b$. Our replacement rate is defined as $R = (t+b)/w$. The conventional replacement rate is $b/w$; it ignores the tax $t$ and is not the same as $R$. However, since unemployment is small relative to employment, $t \ll b$ and $R \approx b/w$.

subject to the matching constraint

$$l = e \cdot f(\theta), \tag{6}$$

where $l$ is the probability to find a job and $1 - l$ is the probability to remain unemployed. The optimal search effort satisfies

$$k'(e) = f(\theta) \cdot \Delta v. \tag{7}$$

At the optimum, the marginal utility cost and marginal utility gain of search are equal. The marginal utility cost is $k'(e)$. The marginal utility gain is the rate at which a unit of effort leads to a job, $f(\theta)$, times the utility gain from having a job, $\Delta v$. We implicitly define the effort supply $e^s(f(\theta), \Delta v)$ as the solution of (7). The function $e^s$ increases with $f(\theta)$ and $\Delta v$. Hence, search effort is higher when the labor market is tighter and when UI is less generous.

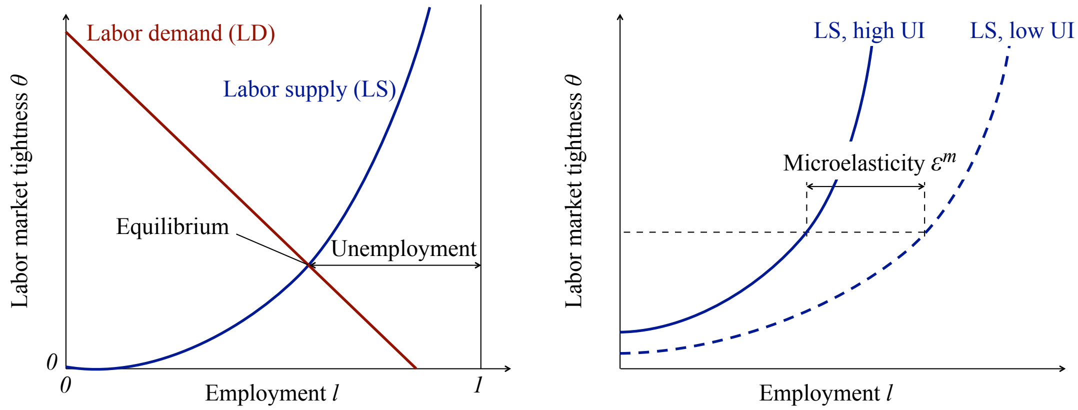
We define the labor supply by

$$l^s(\theta, \Delta v) = e^s(f(\theta), \Delta v) \cdot f(\theta). \tag{8}$$

The labor supply gives the number of workers who find a job when workers search optimally. The labor supply increases with $\theta$ and with $\Delta v$. Labor supply is higher when UI is less generous because search efforts are higher. The labor supply is higher when the labor market is tighter because the job-finding rate per unit of effort is higher and search efforts are higher.

**Equilibrium.** For a given $\Delta v$ and $w$, an equilibrium is a collection of variables $\{e, l, n, \theta, c^e, c^u\}$ such that workers maximize utility given tightness and UI, firms maximize profits given tightness and wage, and the government satisfies a budget constraint. These variables satisfy (1), (2), (4), (6), (7), and $\Delta v = v(c^e) - v(c^u)$. The equilibrium consists of 6 variables determined by 6 equations, so it is well defined.

So far we have taken $\Delta v$ and $w$ as given. After Section III, $\Delta v$ will be determined by the government to maximize welfare, and $w$ will be determined by a wage mechanism. Once $\Delta v$ and




Panel A. Labor market equilibrium

Panel B. Microelasticity of unemployment

Figure 1: Equilibrium in a $(l,\theta)$ plane

$w$ are determined, the key to solving the equilibrium is determining the labor market tightness, $\theta$. Note that in equilibrium, $\theta$ equalizes labor supply and labor demand:

$$l^s(\theta,\Delta v) = l^d(\theta,w). \tag{9}$$

Hence, we determine $\theta$ by solving (9). Once $\theta$ is determined, $l$ is determined from $l = l^s(\theta,\Delta v)$, $e$ from $e = e^s(f(\theta),\Delta v)$, $n$ from $n = l/(1+\tau(\theta))$, and $c^e$ and $c^u$ from the budget constraint (4) and $\Delta v = v(c^e) - v(c^u)$.

The equilibrium is represented in a $(l,\theta)$ plane in Panel A of Figure 1. The labor supply curve is upward sloping, and it shifts inward when UI increases. The labor demand curve may be horizontal or downward sloping, and it responds to UI when the wage responds to UI. The intersection of the labor supply and labor demand curves gives the equilibrium level of labor market tightness, employment, and unemployment.

## III. The Social Welfare Function

We express social welfare as a function of two arguments: the generosity of UI and the labor market tightness. We compute the derivatives of the social welfare function with respect to UI and tightness. These derivatives are the key building blocks of the optimal UI formula derived in Section IV. Following Chetty [2006*a*], we express the derivatives in terms of statistics that can be estimated empirically. Section VII will discuss estimates of these statistics.

We begin by defining the social welfare function. Consider an equilibrium parameterized by a utility gain from work $\Delta v$ and a wage $w$. For a given $\Delta v$, there is a direct relationship between $w$ and the labor market tightness $\theta$. This relationship is given by (9). Hence, it is equivalent to parameterize the equilibrium by $\Delta v$ and $\theta$. Social welfare is thus a function of $\Delta v$ and $\theta$:

$$SW(\theta,\Delta v) = e^s(\theta,\Delta v)\cdot f(\theta)\cdot \Delta v + v(c^u(\theta,\Delta v)) - k(e^s(\theta,\Delta v)), \tag{10}$$

where $c^u(\theta,\Delta v)$ is the equilibrium level of consumption for unemployed workers. The consumption $c^u(\theta,\Delta v)$ is implicitly defined by

$$y\left(\frac{l^s(\theta,\Delta v)}{1+\tau(\theta)}\right) = l^s(\theta,\Delta v)\cdot v^{-1}\left(v(c^u(\theta,\Delta v))+\Delta v\right) + (1-l^s(\theta,\Delta v))\cdot c^u(\theta,\Delta v). \tag{11}$$

This equation ensures that the government's budget constraint is satisfied when all the variables take their equilibrium values. Since $v(c^e) = v(c^u)+\Delta v$, the term $v^{-1}\left(v(c^u(\theta,\Delta v))+\Delta v\right)$ gives the consumption of employed workers.

Next, we define two elasticities that measure how search effort responds to UI and labor market conditions. We use these elasticities to analyze the social welfare function.

**DEFINITION 1.** *The* microelasticity of unemployment with respect to UI *is*

$$\varepsilon^m = \frac{\Delta v}{1-l}\cdot \left.\frac{\partial l^s}{\partial \Delta v}\right|_{\theta}.$$

The microelasticity measures the percentage increase in unemployment when the utility gain from work decreases by 1%, taking into account jobseekers' reduction in search effort but ignoring the equilibrium adjustment of labor market tightness. The microelasticity can be estimated by measuring the reduction in the job-finding probability of an unemployed worker whose unemployment benefits are increased, keeping the benefits of all other workers constant. In Panel B of Figure 1, a change in UI leads to a change in search effort, which shifts the labor supply curve. The microelasticity measures this shift.

The empirical literature does not typically estimate $\varepsilon^m$. Instead, this literature estimates the microelasticity $\varepsilon^m_R$ of unemployment with respect to the replacement rate, $R$. This is not an issue, however, because the two elasticities are closely related. Usually $\varepsilon^m_R$ is estimated by changing benefits $c^u$ while keeping $c^e$ constant. As $\Delta c = (1-R)\cdot w$, $\Delta v = v(c^e) - v(c^e - (1-R)\cdot w))$ and $\partial l^s/\partial R\big|_{\theta,c_e} = -w\cdot v'(c^u)\cdot \left(\partial l^s/\partial \Delta v\big|_{\theta}\right)$. The empirical elasticity $\varepsilon^m_R$ is thus related to $\varepsilon^m$ by

$$\varepsilon^m_R \equiv \frac{R}{1-l}\cdot \left.\frac{\partial(1-l^s)}{\partial R}\right|_{\theta,c_e} = \frac{R\cdot w\cdot v'(c^u)}{\Delta v}\cdot \varepsilon^m. \tag{12}$$

**DEFINITION 2.** *The* discouraged-worker elasticity *is*

$$\varepsilon^f = \frac{f(\theta)}{e}\cdot \left.\frac{\partial e^s}{\partial f}\right|_{\Delta v}.$$

The discouraged-worker elasticity measures the percentage increase in search effort when the job-finding rate per unit of effort increases by 1%, keeping UI constant. In our model, workers search less when the job-finding rate decreases and $\varepsilon^f > 0$; hence, $\varepsilon^f$ captures jobseekers' discouragement when labor market conditions deteriorate. The discouraged-worker elasticity determines the elasticity of labor supply with respect to labor market tightness:

**LEMMA 1.** *The elasticity of labor supply with respect to tightness is related to the discouraged-worker elasticity by*

$$\frac{\theta}{l}\cdot \left.\frac{\partial l^s}{\partial \theta}\right|_{\Delta v} = (1-\eta)\cdot(1+\varepsilon^f)$$

*Proof.* Obvious because $l^s(\theta,\Delta v) = e^s(f(\theta),\Delta v)\cdot f(\theta)$, $\varepsilon^f$ is the elasticity of $e^s$ with respect to $f$, and $1-\eta$ is the elasticity of $f$ with respect to $\theta$. □

Equipped with these elasticities, we can differentiate the social welfare function:

**Lemma 2.** *The social welfare function admits the following derivatives:*

$$\left.\frac{\partial SW}{\partial \theta}\right|_{\Delta v} = \frac{l}{\theta}\cdot(1-\eta)\cdot\phi\cdot w\cdot\left[\frac{\Delta v}{\phi\cdot w}+R\cdot\left(1+\varepsilon^{f}\right)-\frac{\eta}{1-\eta}\cdot\tau(\theta)\right] \tag{13}$$

$$\left.\frac{\partial SW}{\partial \Delta v}\right|_{\theta} = (1-l)\cdot\frac{\phi\cdot w}{\Delta v}\cdot\varepsilon^{m}\cdot\left[R-\frac{l}{\varepsilon^{m}}\cdot\frac{\Delta v}{w}\cdot\left(\frac{1}{v'(c^{e})}-\frac{1}{v'(c^{u})}\right)\right], \tag{14}$$

*where $\phi$ is a harmonic mean of workers' marginal utilities:*

$$\frac{1}{\phi} = \frac{l}{v'(c^{e})}+\frac{1-l}{v'(c^{u})}. \tag{15}$$

*Proof.* We first derive (13). Since workers choose effort to maximize expected utility, a standard application of the envelope theorem says that changes in the effort $e^{s}(\theta,\Delta v)$ resulting from changes in $\theta$ have no impact on social welfare. The effect of $\theta$ on welfare therefore is

$$\frac{\partial SW}{\partial \theta} = \frac{l}{\theta}\cdot(1-\eta)\cdot\Delta v+v'(c^{u})\cdot\frac{\partial c^{u}}{\partial \theta}. \tag{16}$$

The first term is the welfare gain arising from the increase in employment following an increase in $\theta$. Higher employment is beneficial for welfare because it implies that more workers enjoy the high level of consumption $c^{e}$ instead of the low level of consumption $c^{u}$. The first term is obtained by noting that the elasticity of $f(\theta)$ is $1-\eta$ so $e\cdot f'(\theta) = (l/\theta)\cdot(1-\eta)$. This term accounts only for the change in employment resulting from a change in job-finding rate, and not for that resulting from a change in effort. The second term is the welfare change arising from the consumption changes required so that the budget constraint (11) continues to hold after a change in $\theta$.

To compute the consumption change $\partial c^{u}/\partial\theta$, we implicitly differentiate $c^{u}(\theta,\Delta v)$ with respect to $\theta$ in (11). A few preliminary results are helpful. First, (2) implies that $y'(n)/(1+\tau(\theta)) = w$. Second, Lemma 1 implies that $\partial l^{s}/\partial\theta = (l/\theta)\cdot(1-\eta)\cdot(1+\varepsilon^{f})$. Third, the definition of $\Delta v$ implies that $v^{-1}\left(v(c^{u}(\theta,\Delta v))+\Delta v\right)-c^{u} = \Delta c$. Fourth, the elasticity of $1+\tau(\theta)$ is $\eta\cdot\tau(\theta)$ so the derivative of $1/(1+\tau(\theta))$ with respect to $\theta$ is $-\eta\cdot\tau(\theta)/\left[\theta\cdot(1+\tau(\theta))\right]$. Fifth, the derivative of $v^{-1}\left(v(c^{u})+\Delta v\right)$ with respect to $c^{u}$ is $v'(c^{u})/v'(c^{e})$. The implicit differentiation therefore yields

$$\frac{l}{\theta}\cdot(1-\eta)\cdot(1+\varepsilon^f)\cdot(w-\Delta c)-\frac{l}{\theta}\cdot\eta\cdot\tau(\theta)\cdot w=\left(\frac{l}{v'(c^e)}+\frac{1-l}{v'(c^u)}\cdot\right)\cdot v'(c^u)\cdot\frac{\partial c^u}{\partial\theta}.$$

The first term on the left-hand side is the budgetary gain from the new jobs created. Each new job increases government revenue by $w-\Delta c$. The new jobs result both from a higher job-finding rate and from a higher search effort. The term $(1+\varepsilon^f)$ captures the combination of the two effects. The second term on the left-hand side is the loss in resources due to a higher $\theta$, which forces firms to devote more labor to recruiting and less to producing. The entire left-hand side is the change in resources available to the government to fund the UI system after a change in $\theta$. This change in resources dictate the consumption change $\partial c^u/\partial\theta$. Plugging the expression for $\partial c^u/\partial\theta$ into (16), and introducing the variable $\phi$ defined by (15), yields (13).

Next, we derive (14). Following the same logic as above, the effect of $\Delta v$ on welfare is

$$\frac{\partial SW}{\partial\Delta v}=l+v'(c^u)\cdot\frac{\partial c^u}{\partial\Delta v}. \tag{17}$$

The first term is the welfare gain enjoyed by employed workers after a reduction in UI contributions. The second term is the welfare change arising from the consumption changes required so that the budget constraint (11) continues to hold after a change in $\Delta v$.

To compute the consumption change $\partial c^u/\partial\Delta v$, we implicitly differentiate $c^u(\theta,\Delta v)$ with respect to $\Delta v$ in (11). We need two preliminary results in addition to those above. First, the definition of the microelasticity implies that $\partial l^s/\partial\Delta v=[(1-l)/\Delta v]\cdot\varepsilon^m$. Second, the derivative of $v^{-1}(v(c^u)+\Delta v)$ with respect to $\Delta v$ is $1/v'(c^e)$. The implicit differentiation therefore yields

$$\frac{1-l}{\Delta v}\cdot\varepsilon^m\cdot(w-\Delta c)-\frac{l}{v'(c^e)}=\left(\frac{l}{v'(c^e)}+\frac{1-l}{v'(c^u)}\cdot\right)\cdot v'(c^u)\cdot\frac{\partial c^u}{\partial\Delta v}.$$

The first term on the left-hand side is the budgetary gain from the new jobs created by reducing the generosity of UI. This is a behavioral effect, coming from the response of job search to UI. The second term on the left-hand side is the budgetary loss coming from the reduction in the UI contributions paid by employed workers. This is a mechanical effect. Plugging the expression for $\partial c^u / \partial \theta$ into (17), and introducing the variable $\phi$ defined by (15),

$$\frac{\partial SW}{\partial \Delta v} = (1-l) \cdot \phi \cdot \left[ \frac{w}{\Delta v} \cdot \varepsilon^m \cdot R + \frac{l}{1-l} \cdot \left( \frac{1}{\phi} - \frac{1}{v'(c^e)} \right) \right].$$

The definition of $\phi$ implies that $1/\phi - 1/v'(c^e) = -(1-l) \cdot (1/v'(c^e) - 1/v'(c^u))$. Combining this expression with the last equation yields (14). □

## IV. The Optimal Unemployment Insurance Formula

In this section we derive the optimal UI formula, expressed in terms of estimable statistics.

It is well understood that in matching models with unemployment insurance, the equilibrium is generically inefficient. This means that a marginal increase in labor market tightness, keeping the utility gain from work constant, generically has an effect on welfare. [7] If increasing tightness while keeping the utility gain from work constant has no first-order effect on welfare, tightness is efficient. But if the increase enhances welfare, tightness is inefficiently low, and if the increase reduces welfare, tightness is inefficiently high. The equilibrium wage is inefficiently high when the equilibrium tightness is inefficiently low, and it is inefficiently low when the equilibrium tightness is inefficiently high. We will show that the optimal level of UI depends on the effect of tightness on welfare. The following proposition is therefore important because it provides a simple condition to assess the effect of tightness on welfare.

**DEFINITION 3.** *The* efficiency term *is*

$$\frac{\Delta v}{\phi \cdot w} + R \cdot \left(1 + \varepsilon^f\right) - \frac{\eta}{1-\eta} \cdot \tau(\theta),$$

*where $\phi$ is given by* (15).

[7] Our matching model assumes random search—workers apply to firms randomly, irrespective of the wage that they offer. With random search there is no reason for the equilibrium to be efficient, as explained by Pissarides [2000]. Other matching models assume directed search—workers apply to submarkets based on the wage-tightness compromise that they offer. With directed search, the competitive search mechanism of Moen [1997] usually ensures efficiency. But with a UI system, even the competitive search mechanism cannot ensure efficiency because agents fail to internalize the effects of their actions on the government budget constraint [Lehmann and Van Der Linden, 2007]. Hence, inefficiency is generic in matching models with unemployment insurance, whether random or directed search is assumed.

**PROPOSITION 1.** *Consider a marginal increase in labor market tightness, keeping the utility gain from work constant. The efficiency term is positive iff the increase raises welfare, zero iff the increase has no first-order effect on welfare, and negative iff the increase lowers welfare.*

*Proof.* The result directly follows from (13). □

This proposition is closely related to efficiency condition of Hosios [1990]. One difference is that the Hosios condition is a condition on the wage mechanism—it relates workers' bargaining power to the elasticity of the matching function—whereas our efficiency condition is a condition on estimable statistics. Another difference is that the Hosios condition applies to models with risk-neutral workers whereas our condition applies to models with risk-averse workers.

So far, we have parameterized each equilibrium by a pair $(\theta, \Delta v)$. We have used this preliminary analysis to describe the social welfare function and introduce the efficiency term. However, to study optimal UI, we need to account for the equilibrium response of $\theta$ to $\Delta v$. This requires us to specify a wage mechanism. We assume that the wage is determined by a general wage mechanism

$$w = w(\theta, \Delta v).$$

Since all the endogenous variables in the model are a function of $(\theta, \Delta v)$, this mechanism allows the wage to be any function of $\theta$, $\Delta v$, and any other variable. As a consequence, this is the most general wage mechanism possible. With this wage mechanism, the equilibrium condition (9) implicitly defines $\theta$ as a function of $\Delta v$.

The response of the wage to UI has important implications for optimal UI because it determines how labor demand responds to UI. However, we do not need an explicit expression for the wage mechanism. The only information needed is the response of employment to UI, measured by the following elasticity:

**DEFINITION 4.** *The* macroelasticity of unemployment with respect to UI *is*

$$\varepsilon^M = \frac{\Delta v}{1-l} \cdot \frac{dl}{d\Delta v}.$$

The macroelasticity measures the percentage increase in unemployment when the utility gain from work decreases by 1%, taking into account jobseekers' reduction in search effort and the equilibrium adjustment of labor market tightness. The macroelasticity can be estimated by measuring the increase in aggregate unemployment following a general increase in unemployment benefits. Of course, the macroelasticity is endogenous: it may respond to the labor market tightness or the generosity of UI.

The optimal level of UI will depend on the effect of UI on tightness. The following proposition is therefore important because it shows that the effect of UI on tightness is measured by the wedge between microelasticity and macroelasticity of unemployment with respect to UI.

**DEFINITION 5.** *The* elasticity wedge *is* $1-\varepsilon^M/\varepsilon^m$.

**PROPOSITION 2.** *The elasticity wedge measures the response of labor market tightness to UI:*

$$\frac{\Delta v}{\theta}\cdot\frac{d\theta}{d\Delta v}=-\frac{1-l}{l}\cdot\frac{1}{1-\eta}\cdot\frac{\varepsilon^m}{1+\varepsilon^f}\cdot\left(1-\frac{\varepsilon^M}{\varepsilon^m}\right). \tag{18}$$

*The elasticity wedge is positive iff tightness increases with the generosity of UI, negative iff tightness decreases with the generosity of UI, and zero iff tightness does not depend on UI.*

*Proof.* Since $l=l^s(\theta,\Delta v)$, we have:

$$\frac{\Delta v}{1-l}\cdot\frac{dl}{d\Delta v}=\left(\frac{\theta}{1-l}\cdot\frac{\partial l^s}{\partial\theta}\right)\cdot\left(\frac{\Delta v}{\theta}\cdot\frac{d\theta}{d\Delta v}\right)+\left(\frac{\Delta v}{1-l}\cdot\frac{\partial l^s}{\partial\Delta v}\right).$$

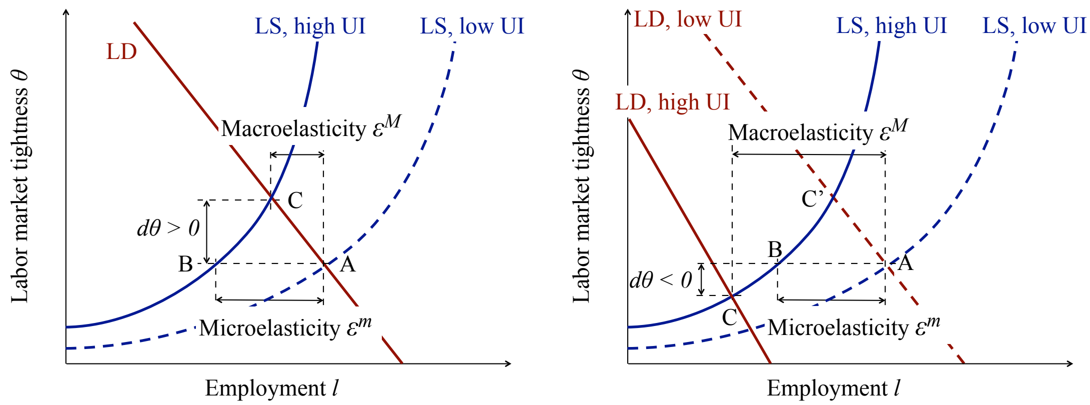
Using $(\theta/l)(\partial l^s/\partial\theta)=(1-\eta)(1+\varepsilon^f)$ from Lemma 1, we obtain

$$\varepsilon^M=\frac{l}{1-l}\cdot(1-\eta)\cdot\left(1+\varepsilon^f\right)\cdot\frac{\Delta v}{\theta}\cdot\frac{d\theta}{d\Delta v}+\varepsilon^m. \tag{19}$$

Dividing this equation by $\varepsilon^m$ and rearranging the terms yields the desired result. □

The proposition shows that a wedge appears between microelasticity and macroelasticity when UI affects tightness, and that this wedge has the same sign as the effect of UI on tightness.

Figure 2 illustrates this result. The horizontal distance A–B measures the microelasticity and the horizontal distance A–C measures the macroelasticity. In Panel A, the labor demand




Panel A. Positive elasticity wedge ($\varepsilon^M < \varepsilon^m$) Panel B. Negative elasticity wedge ($\varepsilon^M > \varepsilon^m$)

Figure 2: The effect of UI on tightness determines the sign of the elasticity wedge, $1 - \varepsilon^M/\varepsilon^m$

*Notes:* This figure illustrates Proposition 2. Panel A considers a downward-sloping labor demand curve that does not respond to UI. Panel B considers a downward-sloping labor demand curve that shifts inward when UI increases.

curve is downward sloping, and it does not shift with a change in UI. After a reduction in UI, the labor supply curve shifts outward (A–B) and tightness increases along the new labor supply curve (B–C). Since tightness rises after the increase in UI, the macroelasticity is smaller than the microelasticity. In Panel B, the labor demand also shifts inward with an increase in UI. Tightness falls along the new supply curve after the labor demand shift (C'–C). In equilibrium, tightness can rise or fall depending on the size of the labor demand shift. In Panel B tightness falls so the macroelasticity is larger than the microelasticity. In Section VI, we will consider specific matching models to describe the mechanisms through which UI affects tightness.

Having analyzed the effects of tightness on welfare and those of UI on tightness, we are equipped to derive the optimal UI formula. The problem of the government is to choose the utility gain from work, $\Delta v$, to maximize social welfare, given by (10), subject to the equilibrium response of tightness, given by (9). The following proposition characterizes optimal UI:

**PROPOSITION 3.** *The optimal replacement rate of UI satisfies the formula*

$$R = \frac{l}{\varepsilon^m} \frac{\Delta v}{w} \left[ \frac{1}{v'(c^e)} - \frac{1}{v'(c^u)} \right] + \frac{1}{1+\varepsilon^f} \left[ 1 - \frac{\varepsilon^M}{\varepsilon^m} \right] \left[ \frac{\Delta v}{w\phi} + \left(1+\varepsilon^f\right) R - \frac{\eta}{1-\eta} \tau(\theta) \right], \quad (20)$$

*where* $\phi$ *satisfies* (15)*. The first term in the right-hand side is the* Baily-Chetty rate*, and the second term is the* correction term.

*Proof.* We define welfare $SW(\theta, \Delta v)$ by (10). The derivative of the social welfare with respect to $\Delta v$ is $dSW/d\Delta v = \partial SW/\partial \Delta v + (\partial SW/\partial \theta) \cdot (d\theta/d\Delta v)$. Therefore, the first-order condition $dSW/d\Delta v = 0$ in the current problem is a linear combination of the first-order conditions $\partial SW/\partial \theta = 0$ and $\partial SW/\partial \Delta v = 0$. Hence, the optimal UI formula is a linear combination of the efficiency condition and the Baily-Chetty formula. Moreover, the efficiency condition is multiplied by the wedge $1 - \varepsilon^M/\varepsilon^m$ because the factor $d\theta/d\Delta v$ is proportional to that wedge.

Equation (18) shows that the labor market tightness variation is given by

$$\frac{d\theta}{d\Delta v} = \frac{1-l}{l} \cdot \frac{1}{1-\eta} \cdot \frac{1}{1+\varepsilon^f} \cdot \frac{\theta}{\Delta v} \cdot \left(\varepsilon^M - \varepsilon^m\right).$$

We combine this equation with the derivatives in Lemma 2 to write the first-order condition $dSW/d\Delta v = 0$. Dividing the resulting equation by $(1-l) \cdot \phi \cdot w \cdot \varepsilon^m \cdot /\Delta v$ yields (20). □

**COROLLARY 1.** *If the labor market equilibrium is efficient, the optimal replacement rate of UI satisfies the Baily-Chetty formula:*

$$R = \frac{l}{\varepsilon^m} \cdot \frac{\Delta v}{w} \cdot \left(\frac{1}{v'(c^e)} - \frac{1}{v'(c^u)}\right). \tag{21}$$

*Proof.* Equation (21) obtains from Propositions 1 and 3. It may not be immediately apparent that (21) is equivalent to the traditional Baily-Chetty formula. The equivalence becomes clear using (12), which allows us to rewrite formula (21) as

$$\varepsilon^m_R = l \cdot \left(\frac{v(c^u)}{v'(c^e)} - 1\right).$$

This is the standard expression for the Baily-Chetty formula. □

Formula (20) shows that the optimal replacement rate of UI is the sum of the Baily-Chetty rate and a correction term. The Baily-Chetty rate solves the trade-off between the need for insurance, measured by $1/v'(c^e) - 1/v'(c^u)$, and the need for incentives to search, measured by $\varepsilon^m$, exactly as in the analysis of Baily [1978] and Chetty [2006*a*]. The correction term is the product of the effect of UI on tightness, measured by the elasticity wedge, and the effect of

Table 1: Optimal replacement rate of UI compared to Baily-Chetty replacement rate

| | | | |
|---|---|---|---|
| | *Panel A. Comparison based on theoretical derivatives* | | |
| | $d\theta/d\Delta v > 0$ | $d\theta/d\Delta v = 0$ | $d\theta/d\Delta v < 0$ |
| $\partial SW/\partial\theta\|_{\Delta v} > 0$ | lower | same | higher |
| $\partial SW/\partial\theta\|_{\Delta v} = 0$ | same | same | same |
| $\partial SW/\partial\theta\|_{\Delta v} < 0$ | higher | same | lower |
| | *Panel B. Comparison based on estimable statistics* | | |
| | Elasticity wedge $< 0$ | Elasticity wedge $= 0$ | Elasticity wedge $> 0$ |
| Efficiency term $> 0$ | lower | same | higher |
| Efficiency term $= 0$ | same | same | same |
| Efficiency term $< 0$ | higher | same | lower |

*Notes:* The replacement rate of UI is $R = 1 - (c^e - c^u)/w$. The Baily-Chetty replacement rate is given by (21). The elasticity wedge is $1 - \varepsilon^M/\varepsilon^m$. The efficiency term is $(\Delta v + k(e))/(\phi \cdot w) + (1+\varepsilon^f) \cdot R - [\eta/(1-\eta)] \cdot \tau(\theta)$. The optimal replacement rate of UI is higher than the Baily-Chetty rate if the correction term in (20) is positive, same as the Baily-Chetty rate if the correction term is zero, and lower than the Baily-Chetty rate if the correction term is negative.

tightness on welfare, measured by the efficiency term. The correction term is positive if and only if increasing UI brings the labor market equilibrium toward efficiency.[8]

There are two situations when the correction term is zero and the optimal replacement rate is given by the Baily-Chetty formula. The first situation is when the labor market tightness is efficient such that the efficiency term is zero. This is the situation described by the corollary. In that case, the marginal effect of UI on tightness has no first-order effect on welfare; hence, optimal UI is governed by the same principles as in the Baily-Chetty framework in which tightness is fixed. The second situation is when UI has no effect on tightness such that the elasticity wedge is zero. In that case, our model is isomorphic to the Baily-Chetty framework, so optimal UI is guided by the same principles.

In all other situations, the correction term is nonzero and the optimal replacement rate departs from the Baily-Chetty rate. The main implication of our formula is that increasing UI above the Baily-Chetty rate is desirable if and only if increasing UI brings the labor market

[8]The response of tightness to UI can be interpreted as a pecuniary externality. The reason is that tightness can be interpreted as a price influenced by the search behavior of workers and influencing welfare when the labor market is inefficient. Under this interpretation, the additive structure of the formula—a standard term plus a correction term—is similar to the structure of many optimal taxation formulas obtained in the presence of externalities.

tightness closer to efficiency. UI brings tightness closer to efficiency either if tightness is inefficiently low and UI raises tightness or if tightness is inefficiently high and UI lowers tightness. In terms of the estimable statistics, UI brings tightness closer to efficiency if the efficiency term and elasticity wedge are both positive or if they are both negative. Table 1 summarizes all the possible situations.

As is standard in optimal tax formulas, the right-hand-side of (20) is endogenous to UI. Even though the formula only characterizes optimal UI implicitly, it is useful. First, it transparently shows the economic forces at play. Second, it gives conditions for the optimal replacement rate of UI to be above or below the Baily-Chetty rate. These conditions apply to a broad range of matching models. Third, the right-hand-side term is expressed with statistics that are estimable. Hence, the formula could be combined with empirical estimates to assess a UI system. This assessment is valid even if the right-hand-side of the formula is endogenous to UI.

An important implication of our formula is that even in the presence of private provision of UI, the public provision of UI is justified. Indeed, small private insurers do not internalize tightness externalities and offer insurance against unemployment at the Baily-Chetty rate. It is therefore optimal for the government to correct privately provided UI by a quantity equal to the correction term. The correction term is positive or negative depending on the level of labor market tightness and the sign of the elasticity wedge.

Our formula reveals some counterintuitive properties of optimal UI. First, even if UI has no adverse on unemployment and $\varepsilon^M = 0$, full insurance is not desirable. Consider a model in which the number of jobs is fixed. Increasing UI redistributes from employed workers to unemployed workers without destroying jobs, but, unlike what intuition suggests, the optimal replacement is strictly below 1. This can be seen by plugging $\varepsilon^M = 0$ and $\varepsilon^m > 0$ in (20). The reason is that increasing UI increases tightness and forces firms to allocate more workers to recruiting instead of producing, thus reducing output available to consumption. In fact, if the efficiency condition holds, UI is given by the standard Baily-Chetty formula and the magnitude of $\varepsilon^M$ is irrelevant.

Second, even in the absence of any concern for insurance, some UI should be offered if UI brings the labor market closer to efficiency. Consider a model with risk-neutral workers and exogenous search effort ($\varepsilon^m = \varepsilon^f = 0$). Formula (20) reduces to $\tau(\theta) = (1-\eta)/\eta$, which is the condition on tightness to maximize output and hence restore efficiency.[9]

Third, even though wages may respond to UI, the response of wages does not appear directly in the formula. It does not matter whether wages change or not because wages do not enter in the government's budget constraint or workers' search decisions. The wage does appear in firms' decisions, but this effect is measured by the macroelasticity. In other words, the elasticity wedge is the sufficient statistics that captures the effects of UI on wages.

## V. Robustness of the Formula

In this section, we show that the optimal UI formula derived in the previous section is quite robust. It continues to hold in a dynamic model that accounts for long-term employment relationships. It also holds when workers can partially insure themselves against unemployment through home production. Finally, it holds when unemployed workers suffer a nonpecuniary cost from being unemployed.

### A. *The Formula in a Dynamic Model*

Section IV considers a static model. However, most employment relationships are long term rather than short term in the US [Hall, 1982]. Here we embed the static model of Section II into a dynamic environment to represent long-term employment relationships. The main difference introduced by the long-term relationships is that at all times, some workers are employed, and only the fraction of workers who are unemployed need to search for a job. In contrast, in the static model, all workers are initially unemployed and need to search for a job. Given this difference, the derivation of the optimal UI formula is slightly modified in the dynamic model.

We work in continuous time. At time $t$, the number of employed workers is $l(t)$ and the number of unemployed workers is $u(t) = 1 - l(t)$. Labor market tightness is $\theta_t = o_t/(e_t \cdot u_t)$. Jobs are destroyed at rate $s > 0$. Unemployed workers find a job at rate $e(t) \cdot f(\theta(t))$. Thus, the

[9] To see this, multiply formula (20) by $\varepsilon^m \cdot (1-R)$. With $\varepsilon^m = \varepsilon^f = 0$ and $\Delta v = \Delta c$ due to risk neutrality, we have $\phi = 1$ and $\Delta v/w = 1-R$. Therefore, $-\varepsilon^M \cdot [(1-R) + R - \eta \cdot \tau(\theta)/(1-\eta)] = 0$. If UI influences tightness, $\varepsilon^M > 0$ and we obtain $\tau(\theta) = (1-\eta)/\eta$.

law of motion of employment is

$$\dot{l}(t) = e(t) \cdot f(\theta(t)) \cdot (1 - l(t)) - s \cdot l(t). \tag{22}$$

In steady state $\dot{l}(t) = 0$. Hence, employment, effort, and tightness are related by

$$l = \frac{e \cdot f(\theta)}{s + e \cdot f(\theta)}. \tag{23}$$

Let $L(x) = x/(s + x)$. The elasticity of $L$ with respect to $x$ is $1 - L$. It is because $l = L(e \cdot f(\theta))$ in the dynamic model instead of $l = e \cdot f(\theta)$ in the static model that the factor $1 - l$ appears in many formulas of the dynamic model.

Firms employ $n(t)$ producers and $l(t) - n(t)$ recruiters. Each recruiter handles $1/r$ vacancies so the law of motion of the number of employees is

$$\dot{l}(t) = -s \cdot l(t) + \frac{l(t) - n(t)}{r} \cdot q(\theta(t)). \tag{24}$$

In steady state $\dot{l}(t) = 0$ and the number of employees is proportional to the number of producers: $l = (1 + \tau(\theta)) \cdot n$, where $\tau(\theta) = s \cdot r / [q(\theta) - s \cdot r]$.[10]

We focus on the steady state of the model with no time discounting. Firms, workers, and government maximize the flow value of profits, utility, and social welfare subject to the steady-state constraints. Given $w$ and $\theta$, the representative firm chooses $n$ to maximize $y(n) - w \cdot (1 + \tau(\theta)) \cdot n$. The optimal employment level satisfies (2). Given $\theta$, $c^e$, and $c^u$, the representative unemployed worker chooses $e$ to maximize

$$l \cdot v(c^e) + (1 - l) \cdot v(c^u) - (1 - l) \cdot k(e) \tag{25}$$

with $l$ given by (23). Routine calculations show that the optimal search effort satisfies

$$k'(e) = \frac{l}{e} \cdot (\Delta v + k(e)). \tag{26}$$

[10]The wedge $\tau(\theta)$ is a different function of the parameters in the static and dynamic environments, but $\tau(\theta)$ should be considered as a sufficient statistic defined as $1 + \tau(\theta) \equiv l/n$. That is, $\tau(\theta)$ is the recruiter-producer ratio.

Finally, the government chooses $c^e$ and $c^u$ to maximize (25) subject to (1), (2), (23), (26), and (4).

Without discounting, the static results are barely modified in the dynamic model. Following the same steps as in the static model, we can show that the formula of Lemma 1 becomes

$$\frac{\theta}{l} \cdot \frac{\partial l^s}{\partial \theta}\bigg|_{\Delta v} = (1-l) \cdot (1+\varepsilon^f) \cdot (1-\eta)$$

The only difference with the original formula is the extra factor $1-l$. In the dynamic environment, equation (19), which links micro- and macroelasticity becomes

$$\varepsilon^M = \varepsilon^m + l \cdot (1-\eta) \cdot \left(1+\varepsilon^f\right) \cdot \frac{\Delta v}{\theta} \cdot \frac{d\theta}{d\Delta v}.$$

As expected, the only difference with the original formula is that a factor $l$ replaces the factor $l/(1-l)$. Accordingly, in the dynamic model, the wedge becomes

$$1 - \frac{\varepsilon^M}{\varepsilon^m} = -l \cdot (1-\eta) \cdot \frac{1+\varepsilon^f}{\varepsilon^m} \cdot \frac{\Delta v}{\theta} \cdot \frac{d\theta}{d\Delta v}.$$

Here again, the only difference with the original wedge is that a factor $l$ replaces the factor $l/(1-l)$. Accordingly, in the dynamic model the optimal UI formula becomes

$$R = \frac{l\Delta v}{\varepsilon^m w}\left[\frac{1}{v'(c^e)} - \frac{1}{v'(c^u)}\right] + \frac{1}{1+\varepsilon^f}\left[1 - \frac{\varepsilon^M}{\varepsilon^m}\right]\left[\frac{\Delta v + k(e)}{w\phi} + \left(1+\varepsilon^f\right) R - \frac{\eta}{1-\eta}\frac{\tau(\theta)}{1-l}\right] \quad (27)$$

with $\phi$ given by (15).

Although it seems that the optimal UI formula is modified in the dynamic model, the formulas in the static and dynamic models are identical once expressed with the correct statistics. The derivation of formula (20) shows that the $\Delta v$ in the efficiency term stands for the utility gap between employment and unemployment. This gap is $\Delta v$ in the static model, where employed and unemployed workers search for jobs, but it is $\Delta v + k(e)$ in the dynamic model, where only unemployed workers search for jobs. The derivation also shows that the $\tau(\theta)$ in the efficiency term is divided by the elasticity of the function $L$, defined such that $l^s(\theta, \Delta v) = L(e^s(\theta, \Delta v) \cdot f(\theta))$. This elasticity is 1 in the static model but $1-l$ in the dynamic model. These differences between the static and dynamic models explain why the $\Delta v$ and $\tau(\theta)$ in the efficiency term of (20) are replaced by $\Delta v + k(e)$ and $\tau(\theta)/(1-l)$ in the efficiency term of (27).

### B. *The Formula With Partial Self-Insurance Against Unemployment*

Section IV assumes that workers cannot insure themselves against unemployment. In reality, workers insure themselves partially against unemployment [Gruber, 1997]. Here we assume that workers insure themselves partially against unemployment with home production.[11]

The UI system provides employed workers with consumption $c^e$ and unemployed workers with consumption $c^u < c^e$. In addition to consuming $c^u$, unemployed workers consume an amount $h$ produced at home at a utility cost $x(h)$. The function $x$ is increasing and convex, and $x(0) = 0$. Workers choose $h$ to maximize the utility when unemployed, $v(c^u + h) - x(h)$. This choice minimizes the utility gain from work, $v(c^e) - v(c^u + h) + x(h)$. Let $\Delta v^h \equiv \min_h \{v(c^e) - v(c^u + h) + x(h)\}$ be the optimal utility gain from work. Let $c^h \equiv c^u + h^s(c^u)$ be the optimal consumption of unemployed workers, where $h^s(c^u) \equiv \text{argmax}_h \{v(c^u + h) - x(h)\}$ is the optimal home production. We also redefine the elasticities $\varepsilon^m$, $\varepsilon^M$, and $\varepsilon^f$ using $\Delta v^h$ instead of $\Delta v$.

When workers have partial access to self-insurance, the optimal UI formula becomes

$$R = \frac{l}{\varepsilon^m} \frac{\Delta v^h}{w} \left[ \frac{1}{v'(c^e)} - \frac{1}{v'(c^h)} \right] + \frac{1}{1+\varepsilon^f} \left[ 1 - \frac{\varepsilon^M}{\varepsilon^m} \right] \left[ \frac{\Delta v^h}{w\phi} + \left( 1 + \varepsilon^f \right) R - \frac{\eta}{1-\eta} \tau(\theta) \right],$$

where $\phi$ satisfies $1/\phi = l/v'(c^e) + (1-l)/v'\left(c^h\right)$. Hence, formula (20) carries over; the only minor modifications required by self-insurance are to change the utility gain from work from $\Delta v$ to $\Delta v^h$ and the marginal utility of unemployed workers from $v'(c^u)$ to $v'(c^h)$. We obtain the formula by repeating all the derivations of formula (20), but replacing $\Delta v$ by $\Delta v^h$.[12]

[11] Home production is a simple representation of all the means of self-insurance available to workers. In practice, workers insure themselves not only with home production but also with savings or spousal income. Analyzing savings or spousal income would be more complex.

[12] Repeating the derivations is straightforward, except for three steps. First, the social welfare function admits a slightly different expression:

$$SW(\theta, \Delta v^h) = e^s(\theta, \Delta v^h) \cdot f(\theta) \cdot \Delta v^h + v(c^u(\theta, \Delta v^h) + h^s(c^u(\theta, \Delta v^h))) - x(h^s(c^u(\theta, \Delta v^h))) - k(e^s(\theta, \Delta v^h)).$$

However, since unemployed workers choose home production to maximize their utility, a standard application of

The availability of self-insurance affects both the Baily-Chetty rate and the efficiency term in the formula. In the Baily-Chetty rate, the need for insurance becomes $1/v'(c^e) - 1/v'(c^h) < 1/v'(c^e) - 1/v'(c^u)$. The need for publicly provided unemployment insurance is lower because unemployed workers are able to insure themselves partially against unemployment. Hence, the Baily-Chetty rate is lower. In the efficiency term, the utility gap between employment and unemployment becomes $\Delta v^h = \min_h \{v(c^e) - v(c^u + h) + x(h)\} < v(c^e) - v(c^u)$. Since the utility gap between employment and unemployment is lower, the efficiency term is lower, which implies that the efficient labor market tightness is lower. Using the logic in Table 1, it follows that the optimal replacement rate is higher if the elasticity wedge is negative but lower if the elasticity wedge is positive. Overall, when self-insurance is available, the optimal replacement rate is unambiguously lower if the elasticity wedge is positive, but the optimal replacement rate may be higher or lower if the elasticity wedge is negative.

### C. *The Formula With a Nonpecuniary Cost of Unemployment*

Section IV assumes that the well-beings of unemployed and employed workers differ only because unemployed workers have a lower income and thus consume less. In reality, controlling for income and many other personal characteristics, unemployed workers report much lower well-being than employed workers [Blanchflower and Oswald, 2004; Di Tella, MacCulloch and Oswald, 2003]; the source of this lower well-being seems to be higher anxiety, lower confidence, lower self-esteem, and depression [Theodossiou, 1998].[13] Here we assume that unemployed workers have utility $v(c^u) - z$, where the parameter $z$ captures the nonpecuniary cost of unemployment. Given the large body of work in medicine, psychology, and economics that documents the large nonpecuniary cost suffered by unemployed workers, it is likely that $z > 0$.

the envelope theorem says that changes in home production $h^s(c^u(\theta, \Delta v^h))$ resulting from changes in $\theta$ and $\Delta v^h$ have no impact on social welfare. Therefore, (16) and (17) remain valid once $\Delta v$ and $c^u$ are replaced by $\Delta v^h$ and $c^h$. Second, since $\Delta v^h = v(c^e) - v(c^u + h^s(c^u)) + x(h^s(c^u))$, the consumption of employed workers is given by

$$c^e(\theta, \Delta v^h) = v^{-1}\left(v(c^u(\theta, \Delta v^h) + h^s(c^u(\theta, \Delta v^h))) - x(h^s(c^u(\theta, \Delta v^h))) + \Delta v^h\right).$$

However, because unemployed workers choose home production to maximize $v(c^u + h) - x(h)$, changes in $h^s(c^u(\theta, \Delta v^h))$ resulting from changes in $\theta$ and $\Delta v^h$ have no impact on $c^e(\theta, \Delta v^h)$. Therefore, the expressions for the consumption changes $\partial c^u / \partial \theta$ and $\partial c^u / \partial \Delta v^h$ remain valid once $\Delta v$ and $c^u$ are replaced by $\Delta v^h$ and $c^h$.

[13] See Frey and Stutzer [2002] for a survey.

In theory, however, it is possible that $z < 0$. In that case, unemployed workers would enjoy nonpecuniary benefits—a benefit could be additional time for leisure.

When unemployed workers suffer a nonpecuniary cost, the optimal UI formula becomes

$$R = \frac{l}{\varepsilon^m} \frac{\Delta v}{w} \left[ \frac{1}{v'(c^e)} - \frac{1}{v'(c^u)} \right] + \frac{1}{1+\varepsilon^f} \left[ 1 - \frac{\varepsilon^M}{\varepsilon^m} \right] \left[ \frac{\Delta v + z}{w \phi} + \left(1+\varepsilon^f\right) R - \frac{\eta}{1-\eta} \tau(\theta) \right].$$

Hence, formula (20) carries over; the only modification required by the nonpecuniary cost of unemployment is to change the utility gain from work from $\Delta v$ to $\Delta v + z$ in the efficiency term.

The nonpecuniary cost from unemployment affects only the efficiency term in the formula, where the utility gap between employment and unemployment becomes $\Delta v + z$. A first implication is that, since the Baily-Chetty rate is independent of the nonpecuniary cost suffered by unemployed workers, the Baily-Chetty rate does not depend on the overall level of well-being of unemployed workers. A second implication is that when unemployed workers suffer a nonpecuniary cost ($z > 0$), the efficiency term and thus the efficient labor market tightness are higher. Using the logic in Table 1, it follows that when unemployed workers suffer a nonpecuniary cost, the optimal replacement rate is higher if the elasticity wedge is positive, but it is lower if the elasticity wedge is negative.

## VI. The Economics of the Elasticity Wedge

Section IV showed that the optimal level of UI depends critically on the effect of UI on tightness, measured by the elasticity wedge, but it remained vague on the economic mechanisms through which UI affects tightness. In this section, we consider four matching models that differ by their wage mechanism and production function to describe possible mechanisms. Table 2 presents the models and summarizes their properties. In these models UI has two possible effects on tightness: a job-creation effect that lowers tightness, and a rat-race effect that raises tightness.

### *A. The Elasticity Wedge in a Standard Model*

The standard model shares the main features of the model in Pissarides [2000] and Shimer [2005]. The production function is linear: $y(n) = n$. When they are matched, worker and firm

Table 2: Effect of UI on labor market tightness in matching models

| | Model | | | |
|---|---|---|---|---|
| | Standard | Rigid-wage | Job-rationing | Aggregate-demand |
| Production function | linear | linear | concave | linear |
| Wage mechanism | bargaining | rigid | rigid | rigid |
| Type of UI effect | job-creation | no effect | rat-race | rat-race |
| Effect of UI on tightness | − | 0 | + | + |
| Elasticity wedge | − | 0 | + | + |

*Notes:* This table summarizes Propositions 4, 5, 6, and 9. The job-creation and rat-race effects are depicted in Figure 3. The elasticity wedge is $1-\varepsilon^M/\varepsilon^m$.

bargain over the wage. The outcome of this bargaining is that the match surplus is shared, with the worker keeping a fraction $\beta \in (0,1)$ of the surplus.[14] The parameter $\beta$ is the worker's bargaining power.[15]

We need to derive the labor demand to analyze the model. We begin by determining the bargained wage. The worker's surplus from a match with a firm is $\mathscr{W} = \Delta v$. The firm's surplus from a match with a worker is $\mathscr{F} = 1-w$ because once a worker is recruited, she produces 1 unit of good and receives a real wage $w$. Since worker and firm split the total surplus from the match, $\mathscr{W}/\beta = \mathscr{F}/(1-\beta)$. Hence, the bargained wage satisfies

$$w = 1 - \frac{1-\beta}{\beta} \cdot \Delta v.$$

Increasing UI raises the bargained wage. The reason is that the outside option of jobseekers increases after an increase in UI, so they are able to obtain a higher wage.

We combine the wage equation with (3) and $y'(n) = 1$ to obtain the labor demand:

$$\frac{\tau(\theta)}{1+\tau(\theta)} = \frac{1-\beta}{\beta} \cdot \Delta v. \tag{28}$$

This equation defines a perfectly elastic labor demand curve in a $(l,\theta)$ plane, as depicted in

[14] In a seminal paper, Diamond [1982] also assumed a surplus-sharing solution to the bargaining problem. If workers and firms are risk neutral, the surplus-sharing solution coincides with the generalized Nash solution. Under risk aversion, these two solutions generally differ. We use the surplus-sharing solution for its simplicity.

[15] To obtain a positive wage, we impose that $\beta/(1-\beta) > \Delta v$.

Panel A of Figure 3. The labor demand shifts downward when UI increases. The reason is that when UI increases, wages rise so it becomes less profitable for firms to hire workers.

Having obtained the labor demand, we can describe the effect of UI on tightness:

**PROPOSITION 4.** *Increasing UI lowers tightness: $d\theta/d\Delta v > 0$. The elasticity wedge is*

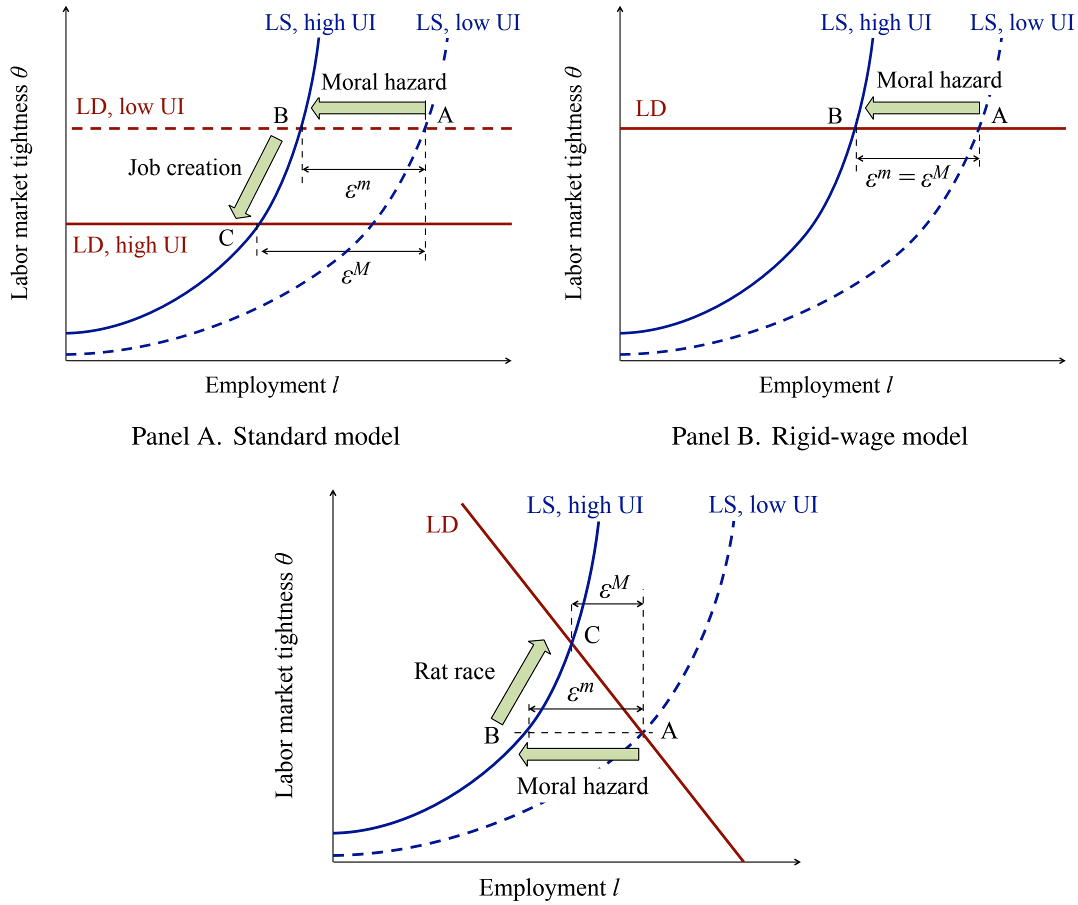
$$1 - \frac{\varepsilon^M}{\varepsilon^m} = -\frac{l}{1-l} \cdot \frac{1-\eta}{\eta} \cdot \frac{1+\varepsilon^f}{\varepsilon^m} < 0.$$

*Proof.* We differentiate (28) with respect to $\Delta v$. Since the elasticities of $\tau(\theta)$ and $1+\tau(\theta)$ with respect to $\theta$ are $\eta \cdot (1+\tau(\theta))$ and $\eta \cdot \tau(\theta)$, we obtain $(\Delta v/\theta) \cdot (d\theta/d\Delta v) = 1/\eta$. It follows that $d\theta/d\Delta v > 0$. Using (18) then immediately yields the expression for the wedge. □

Panel A of Figure 3 illustrates the proposition. After an increase in UI, jobseekers search less, shifting the labor supply curve inward by a distance A–B. In addition, bargained wages increase, shifting the labor demand curve downward, and further reducing employment by the horizontal distance B–C. The total reduction in employment is given by the horizontal distance A–C. Since A–C is larger than A–B, the macroelasticity is larger than the microelasticity.

The standard model nicely captures two effects of UI: the moral-hazard effect and the job creation effect. The moral-hazard effect is the reduction in employment caused by the reduction in search effort, which is not observable and thus a source of moral hazard. The distance A–B measures this effect. The job-creation effect is the reduction in employment caused by the reduction in hiring following the increase in wages. The distance B–C measures this effect. The job-creation effect is the reason why tightness falls when UI increases, and why the macroelasticity is larger than the microelasticity.

The proposition implies that optimal UI is higher than the Baily-Chetty level when tightness is inefficiently high and lower when tightness is inefficiently low. For example, when tightness is inefficiently low, reducing UI lowers wages and increases tightness, which improves welfare beyond the insurance-incentive trade-off.




Panel A. Standard model

Panel B. Rigid-wage model

Panel C. Job-rationing and aggregate-demand models

Figure 3: Effect of UI on labor market tightness and employment in matching models

## *B. The Elasticity Wedge in a Rigid-Wage Model*

The rigid-wage model shares the main features of the model developed by Hall [2005]. The production function is linear: $y(n) = n$. The wage is fixed: $w = \omega$, where $\omega \in (0,1)$. We combine the wage schedule with equation (3) to obtain the labor demand:

$$1 = \omega \cdot (1 + \tau(\theta)). \tag{29}$$

This equation defines a perfectly elastic labor demand in a $(l, \theta)$ plane, as depicted in Panel B of Figure 3. The labor demand is unaffected by UI because the wage does not respond to UI.

Having obtained the labor demand, we can describe the effect of UI on tightness:

**PROPOSITION 5.** *Increasing UI has no effect on tightness. The elasticity wedge is 0.*

*Proof.* Equilibrium tightness is determined by (29). This equation is independent of $\Delta v$. □

Panel B of Figure 3 illustrates the result. Since the labor demand curve is horizontal and independent of UI, UI has no effect on tightness. The only effect at play is the moral-hazard effect, as in the Baily-Chetty framework. The rigidity of wages with respect to UI eliminates the job-creation effect that was present in the standard model. The proposition implies that optimal UI is always given by the Baily-Chetty formula even if the efficiency condition does not hold. Tightness may be inefficient but this inefficiency does not affect optimal UI because UI has no effect on tightness.

## C. *The Elasticity Wedge in a Job-Rationing Model*

The job-rationing model shares the main features of the model developed by Michaillat [2012]. The production function is concave: $y(n) = a \cdot n^{\alpha}$, where $a$ is technology and $\alpha \in (0,1)$ parameterizes diminishing marginal returns to labor. The wage is independent of UI and partially rigid with respect to technology: $w = \omega \cdot a^{\gamma}$, where $\gamma \in [0,1)$ parameterizes the rigidity with respect to technology. If $\gamma = 0$, the wage is fixed: it does not respond to technology. If $\gamma = 1$, the wage is fully flexible: it is proportional to technology.

We combine the wage schedule with equation (3) to obtain the labor demand:

$$l^d(\theta, a) = \left(\frac{\omega}{\alpha} \cdot a^{\gamma-1}\right)^{-\frac{1}{1-\alpha}} \cdot (1+\tau(\theta))^{-\frac{\alpha}{1-\alpha}}. \tag{30}$$

The labor demand is unaffected by UI because the wage does not respond to UI. The labor demand is decreasing with $\theta$. When the labor market is tighter, hiring workers is less profitable as it requires a higher share of recruiters, $\tau(\theta)$. Hence, firms choose a lower level of employment. The labor demand is also increasing in $a$. When technology is lower, the wage-technology ratio, $w/a = \omega \cdot a^{\gamma-1}$, is higher as wages are somewhat rigid, and hiring workers is less profitable. Hence, firms choose a lower level of employment. In the $(l, \theta)$ plane of Panel C in Figure 3, the labor demand curve is downward sloping, and it shifts inward when technology falls.

Having characterized the labor demand, we describe the effect of UI on tightness:

**PROPOSITION 6.** *Increasing UI raises tightness: $d\theta/d\Delta v < 0$. The elasticity wedge is*

$$1 - \frac{\varepsilon^M}{\varepsilon^m} = \left(1 + \frac{\eta}{1-\eta} \cdot \frac{\alpha}{1-\alpha} \cdot \frac{1}{1+\varepsilon^f} \cdot \tau(\theta)\right)^{-1} > 0. \tag{31}$$

*Proof.* The elasticity of $1 + \tau(\theta)$ with respect to $\theta$ is $\eta \cdot \tau(\theta)$. From (30), we infer that the elasticity of $l^d(\theta, a)$ with respect to $\theta$ is $-\eta \cdot \tau(\theta) \cdot \alpha/(1-\alpha)$. By definition, $\varepsilon^M$ is $l/(1-l)$ times the elasticity of $l$ with respect to $\Delta v$. Since $l = l^d(\theta, a)$ in equilibrium, we infer that

$$\varepsilon^M = -\frac{l}{1-l} \cdot \eta \cdot \frac{\alpha}{1-\alpha} \cdot \tau(\theta) \cdot \frac{\Delta v}{\theta} \cdot \frac{d\theta}{d\Delta v}$$

We plug the expression for $(\Delta v/\theta) \cdot (d\theta/d\Delta v)$ given by (18) into this equation and obtain

$$\varepsilon^M = \frac{\eta}{1-\eta} \cdot \frac{\alpha}{1-\alpha} \cdot \frac{1}{1+\varepsilon^f} \cdot \tau(\theta) \cdot \left(\varepsilon^m - \varepsilon^M\right).$$

Dividing this equation by $\varepsilon^m$ and re-arranging yields (31). □

Panel C of Figure 3 illustrates the proposition. After an increase in UI, jobseekers search less, shifting the labor supply curve inward by a distance A–B. Since the labor demand curve is downward sloping and does not respond to UI, the initial reduction in employment is attenuated by a horizontal distance B–C. The total reduction in employment is given by the horizontal distance A–C. Since A–C is smaller than A–B, the macroelasticity is smaller than the microelasticity.

In addition to the moral-hazard effect, the job-rationing model features the rat-race effect, which is not present in the standard model. The rat-race effect is the increase in employment caused by the increase in tightness following the increase in UI. Intuitively, the number of jobs available is somewhat limited because of diminishing marginal returns to labor. Hence, when workers search less, they reduce their own probability of finding a job but mechanically increases others' probability of finding one of the few jobs available. By discouraging job search, UI alleviates the rat race for jobs and increases the job-finding rate per unit of effort and labor market tightness.[16] The distance B–C measures employment gained through the rat-race

[16]The formal argument is as follows. Consider an increase in UI. Imagine that tightness, $\theta$, remained constant.

effect. The rat-race effect is the reason why tightness rises when UI increases, and why the macroelasticity is smaller than the microelasticity.

Proposition 6 implies that optimal UI is lower than the Baily-Chetty level when tightness is inefficiently high and higher than the Baily-Chetty level when tightness is inefficiently low. For instance, when tightness is inefficiently low, increasing UI raises tightness by alleviating the rat-race for jobs, which improves welfare beyond the insurance-incentive trade-off. The proposition also shows that the sign of the elasticity wedge does not depend at all on the rigidity of wages with respect to technology. Even if the wage were completely flexible with respect to technology ($\gamma = 1$), the job-rationing model would feature a positive elasticity wedge and thus a rat-race effect. In a way this is obvious because the response of tightness to UI is independent of the response of tightness to technology. What is critical to obtain a positive elasticity wedge is the rigidity of the wage with respect to UI.

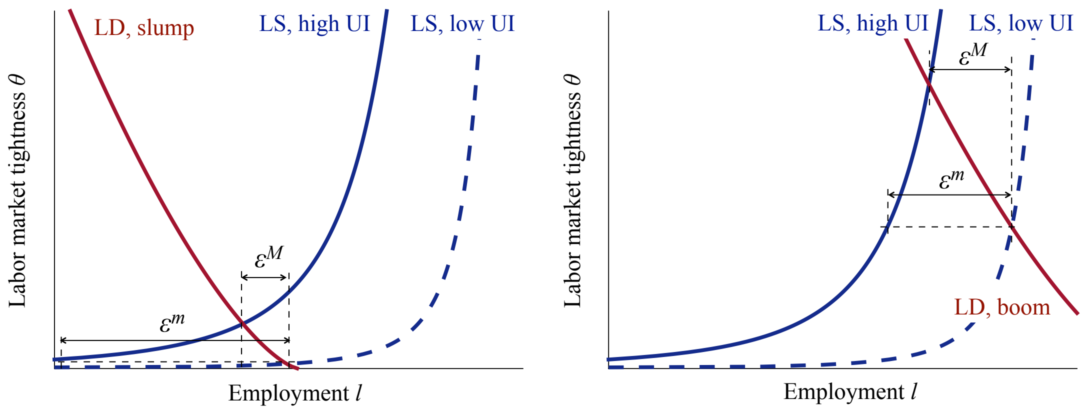
The properties of the labor demand imply that equilibria with low technology are slumps, and equilibria with high technology are booms.

**PROPOSITION 7.** *For a given utility gain from work, an equilibrium with lower technology has lower tightness and lower employment:* $\partial\theta/\partial a\big|_{\Delta v} > 0$ *and* $\partial l/\partial a\big|_{\Delta v} > 0$.

*Proof.* The equilibrium condition is $l^d(\theta, a) = l^s(\theta, \Delta v)$, where $l^d$ is given by (30) and $l^s$ by (8). Implicit differentiation of this condition yields $\partial\theta/\partial a = \left(\partial l^d/\partial a\right) \cdot \left(\partial l^s/\partial\theta - \partial l^d/\partial\theta\right)^{-1}$. We have seen that $\partial l^d/\partial a > 0$, $\partial l^s/\partial\theta > 0$, and $\partial l^d/\partial\theta < 0$. Thus $\partial\theta/\partial a > 0$. The other result follows since $l = l^s(\theta, \Delta v)$ and $\partial l^s/\partial\theta > 0$. □

The proposition says that when technology is low, tightness and employment are low, as in a slump. Conversely, tightness and employment are high when technology is high, as in a boom. The mechanism is simple. When technology is low, the wage-technology ratio is high by wage rigidity, which depresses labor demand and therefore tightness and employment. In

Then the marginal recruiting cost, $\tau(\theta)$, would remain constant. As the wage, $w$, remains constant, the marginal cost of labor, $w \cdot (1 + \tau(\theta))$, would remain constant. Simultaneously, firms would employ fewer workers because workers search less. Hence, the marginal product of labor would be higher because of the diminishing marginal returns to labor. Firms would face the same marginal cost but a higher marginal product of labor, which would not be optimal. Thus, firms post more vacancies and the new equilibrium has higher labor market tightness.




Panel A. High elasticity wedge in a slump

Panel B. Low elasticity wedge in a boom

Figure 4: Countercyclicality of the elasticity wedge, $1-\varepsilon^M/\varepsilon^m$, in the job-rationing model

*Notes:* This figure illustrates Proposition 8. A slump corresponds to an equilibrium with low technology. A boom corresponds to an equilibrium with high technology.

Figure 4, Panel A plots the labor demand curve for a low technology, which represents a slump, and Panel B plots it for a high technology, which represents a boom.

Furthermore, the properties of the labor demand imply that jobs are rationed in slumps. When technology is low enough ($a < (\alpha/\omega)^{\frac{1}{\gamma-1}}$), then $l^d(\theta = 0, a) < 1$ and jobs are rationed: firms would not hire all the workers even if workers searched infinitely hard and tightness was zero. In Panel C of Figure 3, the labor demand curve crosses the x-axis at $l < 1$.

To conclude, we discuss how that the rat-race effect varies over the business cycle. The following proposition establishes that the elasticity wedge is higher in slumps than in booms:

**ASSUMPTION 1.** *The matching function and the marginal disutility of search effort are isoelastic:* $m(e,v) = \omega_h \cdot e^{\eta} \cdot v^{1-\eta}$ *and* $k'(e) = \omega_k \cdot e^{\kappa}$ *for* $\eta \in (0,1)$, $\kappa > 0$, $\omega_h > 0$, *and* $\omega_k > 0$.

**PROPOSITION 8.** *Under Assumption 1, an equilibrium with lower technology has a higher elasticity wedge:* $\partial \left[1-\varepsilon^M/\varepsilon^m\right]/\partial a\big|_{\Delta v} < 0$.

*Proof.* This result follows from combining Proposition 7 with (31), the fact that $\tau(\theta)$ is increasing with $\theta$, and the fact that $\varepsilon^f = 1/\kappa$ and $\eta$ are constant under Assumption 1. □

The proposition shows that the elasticity wedge is higher in slumps than in booms. This implies the rat-race effect is stronger in slumps than in booms. This result appears in Figure 4 by comparing the boom in Panel A to the slump in Panel B. The wedge between $\varepsilon^M$ and $\varepsilon^m$ is driven by the slope of the labor supply relative to that of the labor demand. In a boom, the labor supply is steep at the equilibrium point because the matching process is congested by the large number of vacancies. Hence, $\varepsilon^M$ is close to $\varepsilon^m$. Conversely, in a slump, the labor supply is flat at the equilibrium point because the matching process is congested by search efforts. Hence, $\varepsilon^M$ is much lower than $\varepsilon^m$. Formally, let $\varepsilon^{ls} \equiv (\theta/l) \cdot (\partial l^s/\partial\theta)$ and $\varepsilon^{ld} \equiv -(\theta/l) \cdot \left(\partial l^d/\partial\theta\right)$ be the elasticities of labor supply and labor demand with respect to tightness ($\varepsilon^{ld}$ is normalized to be positive). We could rewrite the elasticity wedge as $1 - \varepsilon^M/\varepsilon^m = 1/\left[1 + (\varepsilon^{ld}/\varepsilon^{ls})\right]$. The elasticity wedge is countercyclical because $\varepsilon^{ld}/\varepsilon^{ls}$ is procyclical.[17]

### *D. The Elasticity Wedge in an Aggregate-Demand Model*

The aggregate-demand model does not appear elsewhere in the literature, but it is useful to establish the robustness of the rat-race effect. This model shows that diminishing marginal returns to labor are not required to obtain a rat-race effect. Here the rat-race effect is present even though the marginal returns to labor are constant. We will see that this effect is present because the general-equilibrium labor demand is downward sloping in a $(l, \theta)$ plane, such that the number of jobs is limited for a given tightness. This model also shows that technology shocks combined with real wage rigidity is not the only mechanism that can generate slumps and booms. In this model, slumps and booms are generated by money-supply shocks and nominal wage rigidity.

We make the following assumptions on the production function and wage schedule. The production function is linear: $y(n) = n$. The nominal wage is independent of UI and partially rigid with respect to the price level $P$: $W = \mu \cdot P^\zeta$, where $\zeta \in [0, 1)$ parameterizes the rigidity of the nominal wage with respect to the price level. The real wage is $w = W/P = \mu \cdot P^{\zeta-1}$.

Because of nominal wage rigidity, it is necessary to define the price-setting mechanism. As in Mankiw and Weinzierl [2011], we assume that workers are required to hold money to

[17]The cyclicality of the elasticity wedge is closely connected to the cyclicality of the public-employment multiplier in Michaillat [2014]. Both rely on the cyclicality of the ratio $\varepsilon^{ld}/\varepsilon^{ls}$. The difference is that the elasticity wedge describes the response to a shift in labor supply whereas the multiplier describes the response to a shift in labor demand.

purchase consumption goods and that the money market is described by a quantity equation: $M = P \cdot y$. The parameter $M > 0$ is the money supply. The quantity equation says that nominal consumer spending is equal to the money supply.[18] Since $y = n$, the quantity equation implies that $P = M/n$. A high number of producers implies high output and, for a given money supply, a low price. In general equilibrium, the real wage therefore satisfies

$$w = \mu \cdot \left(\frac{n}{M}\right)^{1-\zeta}.$$

When the money supply falls or the number of producers rises, the price falls and the real wage rises.

The product market is perfectly competitive; hence, firms take the price as given and (3) remains valid. We combine the wage equation with (3) and $l = (1 + \tau(\theta)) \cdot n$ to obtain the general-equilibrium labor demand:

$$l^d(\theta, M) = M \cdot \mu^{-\frac{1}{1-\zeta}} \cdot (1 + \tau(\theta))^{-\frac{\zeta}{1-\zeta}}.$$

This is a general-equilibrium demand because it takes into account the quantity equation describing the money market. Labor demand decreases with $\theta$, as in the job-rationing model. But the mechanism is different. Higher employment implies more production, lower prices in the goods market, and higher real wages by nominal wage rigidity. Firms are willing to hire more workers only if tightness is lower, which reduces recruiting costs and compensates for the higher real wage. Moreover, the labor demand increases with $M$: After a negative money-supply shock, prices fall. Nominal wage rigidity combined with a lower price level leads to a higher real wage and a higher marginal cost of labor, which leads to lower hiring and higher unemployment. The labor demand slopes downward in a $(l, \theta)$ plane, as depicted in Panel C of Figure 3. The labor demand shifts inward when the money supply decreases, but the labor demand does not shift after a change in UI. Jobs are also rationed in slumps. If money supply is low enough ($M < \mu^{1/(1-\zeta)}$), then $l^d(\theta = 0, M) < 1$ and jobs are rationed: firms would not hire all the workers even if workers searched infinitely hard.

[18] We implicitly assume that the velocity of money is constant and normalized to 1.

It is clear by now that the aggregate-demand model has exactly the same properties as the job-rationing model, except that money-supply shocks and not technology shocks generate fluctuations in tightness and employment. To conclude, we list all the properties of the aggregate-demand model. The interpretations and proofs are the same as in the job-rationing model.

**PROPOSITION 9.** *Increasing UI raises tightness: $d\theta/d\Delta v < 0$. The elasticity wedge is*

$$1-\frac{\varepsilon^M}{\varepsilon^m}=\left(1+\frac{\eta}{1-\eta}\cdot\frac{\zeta}{1-\zeta}\cdot\frac{1}{1+\varepsilon^f}\cdot\tau(\theta)\right)^{-1}>0.$$

**PROPOSITION 10.** *For a given utility gain from work, an equilibrium with lower money supply has lower tightness and lower employment: $\partial\theta/\partial M\big|_{\Delta v}>0$ and $\partial l/\partial M\big|_{\Delta v}>0$.*

**PROPOSITION 11.** *Under Assumption 1, an equilibrium with lower money supply has higher elasticity wedge: $\partial\left[1-\varepsilon^M/\varepsilon^m\right]/\partial M\big|_{\Delta v}<0$.*

## VII. Optimal Unemployment Insurance over the Business Cycle

A common view is that unemployment is efficient on average, but because of macroeconomic shocks, unemployment is inefficiently high in slumps and inefficiently low in booms. This view implies that the efficiency term in our optimal UI formula systematically changes sign over the business cycle. If the elasticity wedge is nonzero, this view therefore implies that optimal UI fluctuates over the business cycle.

In this section we explore this view. We provide empirical evidence that the elasticity wedge is positive, and that the efficiency term is positive in slumps but negative in booms. Our theory therefore suggests that the optimal replacement rate of UI is higher in slumps than in booms. We also find that the fluctuations of the optimal replacement rate over the business cycle are sizable.

### *A. Estimates of the Elasticity Wedge*

To assess empirically whether increasing UI raises or lowers the labor market tightness, we need an estimate of the elasticity wedge. The ideal experiment to estimate the elasticity wedge is a design with double randomization : (i) some randomly selected areas are treated and some are not, and (ii) within treated areas, all but a randomly selected and small subset of jobseekers are treated. The treatment is to offer higher or longer UI benefits. The elasticity wedge can be estimated by comparing the unemployment durations of non-treated jobseekers in non-treated areas to that of non-treated jobseekers in treated areas. We discuss two recent studies that estimate this wedge.

Lalive, Landais and Zweimüller [2013] use a natural experiment that offers the desired design: the Regional Extended Benefit Program implemented in Austria in 1988–1993. The treatment was an increase in benefit duration from 52 to 209 weeks for eligible unemployed workers in a subset of regions. They estimate a positive elasticity wedge, $1-\varepsilon^M/\varepsilon^m = 0.3$.

Marinescu [2014] follows another route to assess the sign and magnitude of the elasticity wedge: she directly estimates the effect of a change in UI on labor market tightness. The changes in UI that she considers are the long UI extensions implemented in the US from 2009 to 2013. Using detailed information on vacancies and job applications from CareerBuilder.com, the largest American online job board, she computes the effect of a change in UI on aggregate search effort, measured by the number of job applications sent, and on vacancies. At the state level, she finds that an increase in UI has a negative effect on job applications but no effect on vacancies. Since labor market tightness is the ratio of vacancies to aggregate search effort, these results imply that an increase in UI raises tightness and thus that the elasticity wedge is positive. Given the measured response of tightness to changes in UI, Marinescu estimates an elasticity wedge of $1-\varepsilon^M/\varepsilon^m = 0.3$, exactly as Lalive, Landais and Zweimüller.

The two previous studies find a positive elasticity wedge when the rat-race and job-creation effects are accounted for. Many papers study these effects in isolation. Several papers find that an increase in the search effort of some jobseekers, induced for example by placement programs, has a negative effect on the job-finding probability of other jobseekers [for example, Crepon et al., 2013]. These findings are consistent with rat-race effects. Several studies use microdata to investigate whether more generous UI benefits affect the re-employment wage. Most studies find no effect on wages or even slightly negative effects [for example, Card, Chetty and Weber, 2007]. Since the job-creation effect operates when an increase in UI raises wages, the absence of effect on re-employment wages suggests that the job-creation effect is likely to be small.[19] To conclude, the absence of evidence of job-creation effect together with the evidence of rat-race effect is further evidence of a positive elasticity wedge.

In sum, the available evidence suggests that the elasticity wedge is positive, with a best estimate of 0.3. We use this estimate to quantity the cyclical fluctuations of optimal UI.

### B. An Estimate of the Efficiency Term

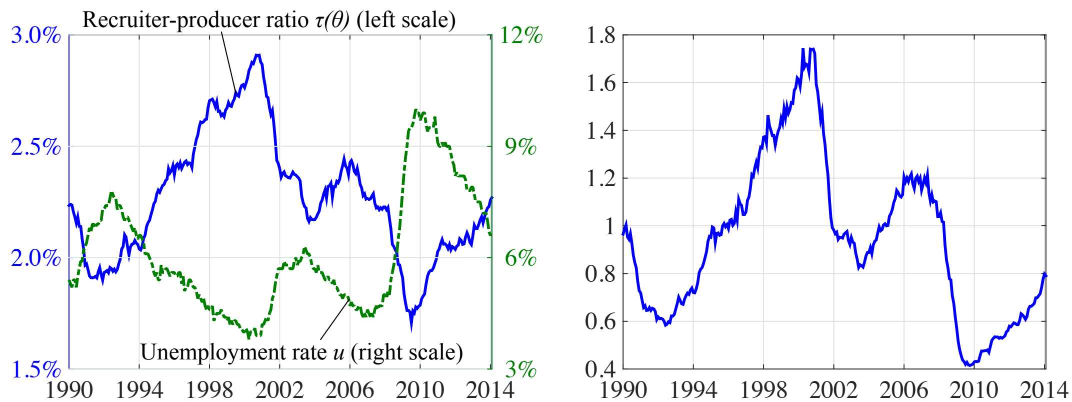
To determine empirically whether the labor market tightness is inefficiently high or low, we measure the efficiency term in US data. In the dynamic environment with home production and a nonpecuniary cost of unemployment, the efficiency term is

$$\frac{v\left(c_t^e\right) - v\left(c_t^h\right) + z_t + k(e_t) + x(h_t)}{w_t \cdot \phi_t} + \left(1 + \varepsilon^f\right) \cdot R_t - \frac{\eta}{1-\eta} \cdot \frac{\tau_t}{u_t}.$$

We start by measuring $[\eta/(1-\eta)] \cdot \tau_t/u_t$. We set $\eta = 0.7$, in line with empirical evidence [Petrongolo and Pissarides, 2001]. We measure the unemployment rate $u_t$ with the seasonally adjusted monthly unemployment rate constructed by the Bureau of Labor Statistics (BLS) from the Current Population Survey (CPS). The challenging part is to measure the recruiter-producer ratio $\tau_t$. We measure $\tau_t$ in three steps.

First, we measure the number $rec_t$ of employees in the recruiting industry. We set $rec_t$ to the seasonally adjusted monthly number of workers in the recruiting industry computed by the BLS from Current Employment Statistics (CES) data. The recruiting industry is the industry with North American Industry Classification System (NAICS) code 56131. Its official name is "employment placement agencies and executive search services". It comprises firms primarily engaged in listing employment vacancies and referring or placing applicants for employment, and firms providing executive search, recruitment, and placement services. The series is avail-

[19] More generous benefits induce longer unemployment durations that may have a negative effect on wages if the duration of unemployment spells affects the productivity of unemployed workers or is interpreted by employers as a negative signal of productivity. It is difficult to disentangle this negative effect from the positive effect of UI on wages through bargaining, which is the relevant effect for our analysis. Schmieder, von Wachter and Bender [2013] attempt such a decomposition in German data by controlling for the duration of the unemployment spell. They find a negative effect of UI on wages through longer unemployment durations but zero effect through wage bargaining.




Panel A. Fluctuations of recruiter-producer ratio

Panel B. Fluctuations of $\frac{\eta}{1-\eta} \cdot \frac{\tau(\theta_t)}{u_t}$

Figure 5: A method to measure the fluctuations of the efficiency term

*Notes:* The time period is January 1990–February 2014. In Panel A the unemployment rate, $u_t$, is the seasonally adjusted monthly unemployment rate constructed by the BLS from the CPS. The recruiter-producer ratio is constructed as $\tau_t = \sigma \cdot rec_t/(l_t - \sigma \cdot rec_t)$, where $rec_t$ is the seasonally adjusted monthly number of workers in the recruiting industry (NAICS 56131) computed by the BLS from CES data, $l_t$ is the seasonally adjusted monthly number of workers in all private industries computed by the BLS from CES data, and $\sigma = 8.4$ is a scaling factor ensuring that the level of the recruiter-producer ratio matches the evidence in Villena Roldan [2010]. In Panel B the solid blue line is $[\eta/(1-\eta)] \cdot \tau_t/u_t$ with $\eta = 0.7$.

able from January 1990 to February 2014. This industry comprises 279,800 workers on average.

Second, we scale up $rec_t$ by a factor 8.4 to measure the total amount of labor devoted to recruiting in the economy. Of course, the employees in the recruiting industry constitute only a small fraction of the workers devoted to recruiting. Scaling up is necessary to account for the numerous workers in firms outside of the recruiting industry who spend a lot of time and effort to recruit workers for their own firm. The scaling factor ensures that the average share of labor devoted to recruiting in 1997 is 2.5%. We obtain this amount from the National Employer Survey (NES) conducted in 1997 by the Census Bureau. The survey gathered employer data on employment practices, especially recruiting. In the 1997 survey, 4500 establishments answered detailed questions about the methods used to recruit applicants. Villena Roldan [2010] analyzes this survey and finds that firms spend 2.5% of their total labor cost in recruiting activities. In other words, 2.5% of the workforce is devoted to recruiting.[20]

[20] In monetary terms, firms spent on average \$4200 per recruited worker in 1997.

Third, we construct $\tau_t$ as

$$\tau_t = \frac{8.4 \cdot rec_t}{l_t - 8.4 \cdot rec_t}, \tag{32}$$

where $l_t$ is the seasonally adjusted monthly number of workers in all private industries computed by the BLS from CES data. Panel A of Figure 5 displays the recruiter-producer ratio $\tau_t$ and the unemployment rate $u_t$. The series $\tau_t$ is very procyclical. Panel B of Figure 5 displays the term $[\eta/(1-\eta)] \cdot \tau_t/u_t$. This term is extremely procyclical: it is four times higher at the peak of the 2001 expansion (when $\tau_t = 1.74$) than at the bottom of the 2009 recession (when $\tau_t = 0.41$).[21]

Next we measure $\left(1+\varepsilon^f\right) \cdot R_t$. UI benefits replace between 50% and 70% of the pre-tax earnings of a worker. Following Chetty [2008] we set the benefit rate to 50%. Since earnings are subject to a 7.65% payroll tax, we set the average replacement rate to $R = 0.5 + 0.0765 = 58\%$. We do not have direct evidence on the discouraged-worker elasticity, $\varepsilon^f$, but $\varepsilon^f$ is closely related to the microelasticity for which we have a lot of evidence. We show in Online Appendix A that a reasonable estimate is $\varepsilon^f = 0.021$. Finally, $(1+\varepsilon^f) \cdot R_t$ is somewhat countercyclical in the US as the generosity of UI increases in recessions, but its fluctuations are usually relatively small. We therefore assume that $\left(1+\varepsilon^f\right) \cdot R_t$ remains constant at $(1+0.021) \times 0.58 = 0.59$.

We assume that the utility of consumption is log: $v(c) = \ln(c)$. This assumption implies a coefficient of relative risk aversion $\rho = 1$, consistent with labor supply behavior [Chetty, 2006*b*]. This coefficient of relative risk aversion is maybe on the low side of available estimates, but using log utility simplifies the analysis. With log utility, $v\left(c_t^e\right) - v\left(c_t^h\right) = \ln(c_t^e/c_t^h)$, where $c_t^h/c_t^e$ is the consumption drop upon unemployment. For food consumption $\varphi$, Gruber [1997] estimates $\left(\varphi^e - \varphi^h\right)/\varphi^e = 0.068$. As emphasized by Browning and Crossley [2001], total consumption is more elastic than food consumption to an income change. The estimates of Hamermesh [1982] imply that the elasticity of food consumption with respect to aggregate income for unem-

[21] A limitation of our measure of $\tau_t$ is that it is only valid as long as the share of recruitment done through recruiting firms is stable over the business cycle. In Online Appendix B, we construct an alternative measure of $\tau_t$ that does not rely on this assumption, but instead relies on the assumption that the cost of posting a vacancy is constant over the business cycle. This alternative measure is constructed using the number of vacancies and new hires computed by the BLS from JOLTS. This alternative measure is available from December 2000 to February 2014. We find that the two measures behave similarly over the business cycle (their correlation is 0.43).

ployed workers is 0.36.[22] Accordingly we expect that $\left(c^e - c^h\right)/c^e = \left[\left(\varphi^e - \varphi^h\right)/\varphi^e\right]/0.36 = 0.068/0.36 = 0.19$ and $c^h/c^e = 0.81$. Furthermore, Kroft and Notowidigdo [2011] show that the ratio $c^e/c^h$ is broadly constant over the business cycle. In sum, we consider that $\ln(c_t^e/c_t^h)$ remains constant at $\ln(1/0.81) = 0.21$.

We then measure $1/(\phi_t \cdot w_t)$. With $l \approx 1$ and log utility, we find that $1/(\phi_t \cdot w_t) \approx 1/(v'(c_t^e) \cdot w_t) \approx c_t^e/w_t$. Neglecting firms' profits that may be rebated to employed workers, $c_t^e/w_t$ is just 1 minus the payroll tax rate; the payroll tax does not vary over the business cycle so we consider that $1/(\phi_t \cdot w_t)$ remains constant at $1 - 0.0765 = 0.92$.

Finally, the term $z_t + k(e_t) + x(h_t)$ includes the nonpecuniary cost from unemployment, the cost from job search, and the cost from home production. Di Tella, MacCulloch and Oswald [2003] and Blanchflower and Oswald [2004] find that the nonpecuniary cost from unemployment is large: controlling for income and other personal characteristics, losing one's job is as costly as divorcing in terms of well-being.[23] Despite much research, a precise value for the nonpecuniary cost of unemployment remains unavailable. In the absence of a precise estimate for $z_t$, we assume that $k(e_t) + x(h_t) + z_t$ remains constant at 0.17; this calibration ensures that on average, the efficiency term is zero and the labor market is efficient.

Available evidence does not indicate that the term $\left(v\left(c_t^e\right) - v\left(c_t^h\right) + k(e_t) + x(h_t) + z_t\right)/(\phi_t \cdot w_t) + (1 + \varepsilon^f) \cdot R_t$ varies much over the business cycle; if anything, this term is perhaps slightly countercyclical. On the other hand, the term $[\eta/(1-\eta)] \cdot \tau_t/u_t$, displayed in Panel B of Figure 5, is subject to wide procyclical fluctuations over the business cycle. We conclude that the efficiency term is subject to wide countercyclical fluctuations over the business cycle. If the nonpecuniary cost of unemployment of 0.17, then the efficiency term is zero on average, implying

[22] This estimate includes food consumed at home and food away from home. Hamermesh [1982] estimates that for unemployed workers the permanent-income elasticity of food consumption at home is 0.24 while that of food consumption away from home is 0.82. He also finds that in the consumption basket of an unemployed worker, the share of food consumption at home is 0.164 while that of food consumption away from home is 0.041. Therefore the aggregate income elasticity of food consumption is $0.24 \times [0.164/(0.164 + 0.41)] + 0.82 \times [0.041/(0.164 + 0.41)] = 0.36$.

[23] In standard matching models of the labor market, it is typical to assume that unemployed workers enjoy some positive utility; however, this positive utility usually stands for UI benefits that are not modeled explicitly [for example, Shimer, 2005]. It is true that some scholars explicitly model the UI system and assume that in addition to the consumption utility afforded by UI benefits, unemployed workers enjoy nonpecuniary benefits from unemployment, usually referred to as "value from leisure". But this assumption has never been justified on empirical grounds; it is usually only introduced to increase the rigidity of Nash bargained wages and thus generate larger business cycle fluctuations.

that the efficiency term is positive in slumps and negative in booms. If the nonpecuniary cost of unemployment is different, we cannot delimit precisely periods when the efficiency term is positive and negative. Nevertheless, given the amplitude of the fluctuations in $[\eta/(1-\eta)]\cdot\tau_t/u_t$, the efficiency term was likely negative in the 1991–1994 and 2009–2013 periods and positive in the 1998–2001 period; the rest of the time, the efficiency term was likely close to zero.

## C. *Exploration of the Fluctuations of Optimal Unemployment Insurance*

We use the empirical evidence on the elasticity wedge and efficiency term to explore the fluctuations of optimal UI over the business cycle.

**Evaluation of a Small Reform Around the Current System.** Our optimal UI formula is useful to assess the desirability of a small reform around the current system. The statistics in the Baily-Chetty formula—microelasticity and coefficient of risk aversion—are well measured and are commonly used to estimate the Baily-Chetty rate [for example, Gruber, 1997]. If the current replacement rate is less than the right-hand-side term of the formula, increasing UI increases welfare, and conversely, if the current replacement rate is more than the right-hand-side term, decreasing UI increases welfare. Since the replacement rate of UI in the US is very close to the Baily-Chetty rate, we can assume that the evidence provided above about the signs of the elasticity wedge and efficiency term are valid with a replacement rate of UI equal to the Baily-Chetty rate.[24] This evidence indicates that the elasticity wedge is positive, and that the efficiency term is positive in slumps but negative in booms. Using the argument in Table 1, we infer that the optimal replacement rate of UI is more countercyclical than the Baily-Chetty rate.

We can also use the formula to get a sense of the amplitude of the cyclical fluctuations of optimal UI. With standard estimates of the microelasticity and risk aversion, a replacement rate of $R = 50\%$ roughly satisfies the Baily-Chetty formula [Chetty, 2006*a*]. Available evidence indicates that the elasticity wedge is positive, maybe around 0.3. Our measure of the efficiency term indicates that the between a boom and a slump, the efficiency term increases by 1.4. (We obtain the amplitude of 1.4 by comparing the through and the peak of the series $[\eta/(1-\eta)]\cdot$

[24]With the calibration in Table 3, the current US replacement rate of $R = 58\%$ exactly satisfies the Baily-Chetty formula. We establish this result in Online Appendix A.

$\tau_t/u_t$ in Panel B of Figure 5.) Hence, if the labor market tightness is efficient on average, the efficiency term increases by $1.4/2 = 0.7$ in slumps and falls by $1.4/2 = 0.7$ in booms. This means that the correction term maybe increases by $0.3 \times 0.7 = 0.2$ in slumps and falls by 0.2 in booms. Compared to a Baily-Chetty rate of 0.5, these fluctuations in the correction term are quantitatively significant; they require the optimal replacement rate of UI to depart markedly from the Baily-Chetty rate.

**Calibration and Simulation of the Job-Rationing Model.** To quantify more precisely the fluctuations of optimal UI over the business cycle, we calibrate and simulate a dynamic version of the job-rationing model of Section VI. We choose this model because it generates a positive elasticity wedge, consistent with available evidence.[25] Existing evidence does not suggest that the availability of self-insurance and the nonpecuniary cost of unemployment vary much over the business cycle. Hence, we keep the calibration and simulation simple by abstracting from self-insurance and setting the nonpecuniary cost of unemployment to zero.[26]

We represent the business cycle as a succession of steady states with different values for technology. Formally, the simulations provide a comparative steady-state analysis. This analysis provides a good approximation to a dynamic simulation because matching models of the labor market reach their steady state quickly.[27] We compute a collection of steady states spanning all the stages of the business cycle, from slumps with high unemployment to booms with low unemployment. We first compute a collection of steady states in which the replacement rate of UI remains constant at its average value of 58%. We then compute a collection of steady states in which the replacement rate of UI is at its optimal level given by formula (27).

We calibrate the model to US data, as summarized in Table 3. The calibration procedure is standard so we relegate it to Online Appendix A. Here we only explain how we calibrate the

[25]The aggregate-demand model also generates a positive elasticity wedge. We simulate the job-rationing model because it is more conventional than the aggregate-demand model.

[26]As more evidence becomes available, cyclical fluctuations in self-insurance and the nonpecuniary cost of unemployment could be incorporated into the simulation model to quantify more precisely the fluctuations of optimal UI over the business cycle.

[27]Shimer [2005] and Pissarides [2009] argue that in a standard matching model, the steady-state equilibrium with technology $a$ approximates well the equilibrium in a stochastic environment when the realization of technology is $a$. Their main reason is that the rates of inflow to and outflow from unemployment are large. Michaillat [2012] validates this approximation with simulations.

model to match empirical evidence on the microelasticity and elasticity wedge.

A large body of work has estimated the microelasticity $\varepsilon^m$.[28] The ideal experiment to estimate $\varepsilon^m$ is to offer higher or longer UI benefits to a randomly selected and small subset of jobseekers within a labor market and compare unemployment durations between treated and non-treated jobseekers. In practice, $\varepsilon^m$ is estimated by comparing individuals with different benefits in the same labor market at a given time, while controlling for individual characteristics. Most empirical studies do not estimate the microelasticity $\varepsilon^m$ of unemployment with respect to $\Delta v$ but estimate instead the microelasticity of unemployment with respect to the replacement rate, which we denote $\varepsilon^m_R$. In US administrative data from the 1980s, the classic study of Meyer [1990] finds an elasticity $\varepsilon^m_R = 0.6$ using state fixed-effects. In a larger US administrative dataset for the same years, and using a regression kink design to identify the elasticity, Landais [2015] finds an elasticity $\varepsilon^m_R = 0.3$. Overall, $\varepsilon^m_R = 0.5$ is a reasonable estimate. Using (12), we translate this estimate into an estimate $\varepsilon^m = 0.3$.

To obtain a search behavior consistent with the empirical evidence on the microelasticity, we assume a disutility of effort $k(e) = \omega_k \cdot e^{\kappa+1}/(\kappa+1) - \omega_k/(\kappa+1)$, and we set $\kappa = 3.1$ to match the microelasticity of $\varepsilon^m = 0.3$. We do not have direct evidence on the discouraged-worker elasticity, $\varepsilon^f$, but $\varepsilon^f$ is closely related to $\varepsilon^m$. We find that our estimate $\varepsilon^m = 0.3$ implies an estimate $\varepsilon^f = 0.021$.

To match the estimate of the elasticity wedge $1 - \varepsilon^M/\varepsilon^m = 0.3$, we calibrate the production-function parameter $\alpha$. This parameter determines the magnitude of the elasticity wedge and of the rat-race effect. We calibrate $\alpha$ to match the estimate of the elasticity wedge provided by Lalive, Landais and Zweimüller [2013]: $1 - \varepsilon^M/\varepsilon^m = 0.3$. Extending (31) to the dynamic environment without discounting yields

$$1 - \frac{\varepsilon^M}{\varepsilon^m} = \left(1 + \frac{\eta}{1-\eta} \cdot \frac{\alpha}{1-\alpha} \cdot \frac{1}{1+\varepsilon^f} \cdot \frac{\tau(\theta)}{u}\right)^{-1}.$$

We set $1 - \varepsilon^M/\varepsilon^m = 0.3$, $\eta = 0.7$, $\varepsilon^f = 0.021$, $\tau(\theta) = 2.2\%$, and $u = 6.6\%$ and obtain $\alpha = 0.75$.

Figure 6 displays the simulations of the job-rationing model. Each steady state is indexed by

[28] See Krueger and Meyer [2002] for a comprehensive survey.

Table 3: Parameter values in the simulation of the job-rationing model

| | Description | Value | Source |
|---|---|---|---|
| | *Panel A. Average values used for calibration* | | |
| $u$ | unemployment rate | 6.6% | CPS, 2000–2014 |
| $\theta$ | labor market tightness | 0.37 | JOLTS and CPS, 2000–2014 |
| $\tau(\theta)$ | recruiter-producer ratio | 2.2% | Villena Roldan [2010] and CES, 2000–2014 |
| $\varepsilon^m$ | microelasticity | 0.3 | literature |
| $e$ | search effort | 1 | normalization |
| $R$ | replacement rate | 58% | Chetty [2008] |
| | *Panel B. Calibrated parameters* | | |
| $\eta$ | unemployment-elasticity of matching | 0.7 | Petrongolo and Pissarides [2001] |
| $\rho$ | relative risk aversion | 1 | Chetty [2006*b*] |
| $\gamma$ | technology-elasticity of real wage | 0.5 | Haefke, Sonntag and van Rens [2008] |
| $s$ | monthly job-destruction rate | 3.5% | JOLTS, 2000–2014 |
| $\kappa$ | convexity of disutility of search | 3.1 | matches $\varepsilon^m = 0.3$ |
| $\alpha$ | marginal returns to labor | 0.75 | matches $1 - \varepsilon^M/\varepsilon^m = 0.3$ |
| $\omega_m$ | matching efficacy | 0.67 | matches average values |
| $r$ | recruiting cost | 0.84 | matches average values |
| $\omega_k$ | level of disutility of search | 0.38 | matches average values |
| $\omega$ | level of real wage | 0.75 | matches average values for $a = 1$ |

technology, $a \in [0.94, 1.06]$. Because of wage rigidity, the steady states with low $a$ have a high wage-technology ratio and therefore high unemployment: they represent slumps. Conversely, the steady states with high $a$ have a low wage-technology ratio and low unemployment: they represent booms. As showed in Panel A, unemployment falls from 12.9% to 4.6% when $a$ increases from 0.94 to 1.06 and UI remains constant. This numerical results implies that even a modest amount of wage rigidity, in line with the empirical findings of Haefke, Sonntag and van Rens [2008], generates realistic fluctuations in unemployment. Indeed, the elasticity of unemployment with respect to technology implied by the simulations is 9.5, larger than the elasticity of 4.2 measured in US data.[29]

It is not the unemployment rate that matters for optimal UI, but the efficiency term, depicted in Panel F. In a slump, the efficiency term is positive. The efficient term is 0 for $a = 1.02$ and an unemployment rate of 6.0%. In booms, the efficiency term is negative. In fact, the efficiency

[29] The elasticity is obtained by considering a small change in technology around the average. At $a = 1$, $u =$ 6.56%. At $a = 0.99$, $u = 7.18\%$. Hence the elasticity is $(1/6.56) \cdot (7.18 - 6.56)/(1 - 0.99) = 9.5$. Michaillat [2012] shows that the elasticity of unemployment with respect to technology in the US is 4.2.

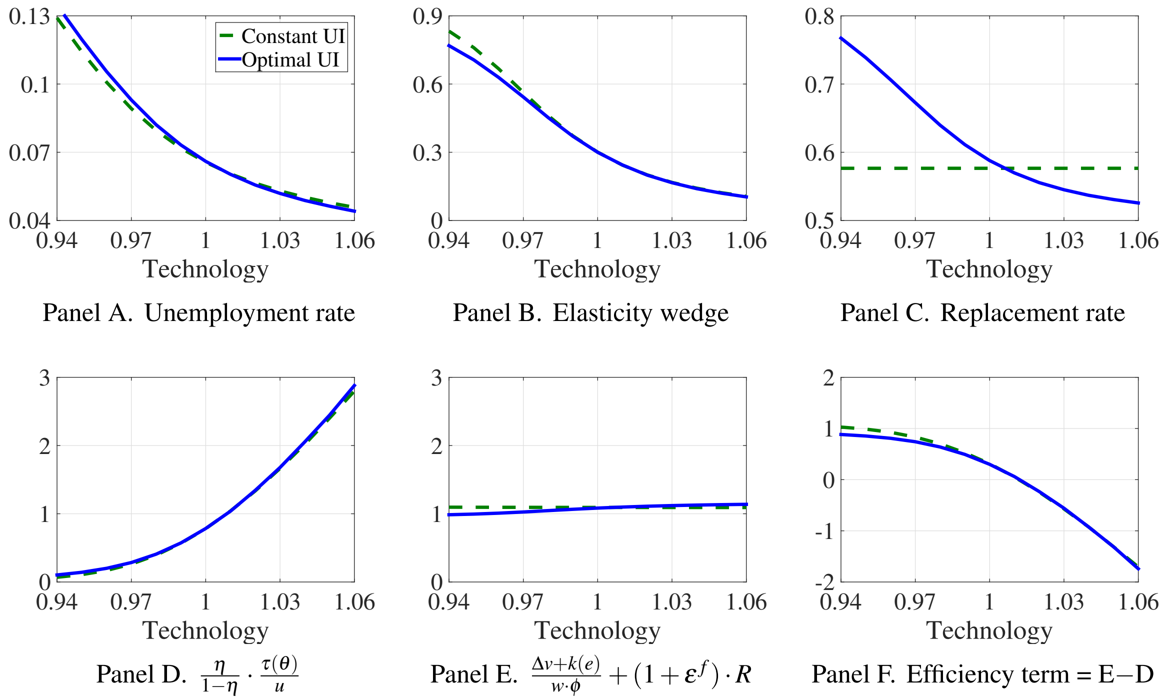



Figure 6: Optimal UI over the business cycle in the job-rationing model

term is constructed as the difference between $(\Delta v + k(e))/(w \cdot \phi) + (1+\varepsilon^f) \cdot R$, depicted in Panel E, and $[\eta/(1-\eta)] \cdot \tau(\theta)/u$, depicted in Panel D. The former is broadly constant at 1.1 for any value of technology; the latter increases steeply from 0.07 to 2.80 when technology increases from $a = 0.94$ to $a = 1.06$; hence the efficiency term decreases from 1.03 to -1.70 when technology increases from $a = 0.94$ to $a = 1.06$.

Panel B displays the elasticity wedge. The wedge is positive, which implies that the optimal replacement rate of UI should be below the Baily-Chetty rate in booms and above it in slumps. In fact, in an average situation with $a = 1$, the wedge is 0.3 and matches exactly the empirical evidence; this is due to our calibration of the production-function parameter, $\alpha$. The wedge is also countercyclical, consistent with the results of Proposition 8: from a slump with $a = 0.94$ to a boom with $a = 1.06$, the wedge falls from 0.83 to 0.11.

Panel C shows that the optimal replacement rate and the technology parameter driving business cycles are negatively correlated: the optimal replacement rate falls from 77% when $a = 0.94$ to 53% when $a = 1.06$. Thus, the optimal replacement rate is sharply countercyclical.

Of course, the unemployment rate responds to the adjustment of the replacement rate from its original level of 58% to its optimal level. In slumps, the optimal replacement rate is much higher than 58% so the unemployment rate increases above its original level: at $a = 0.94$ the unemployment rate increases by 0.6 percentage point from 12.9% to 13.5%. UI has little influence on unemployment in slumps because the macroelasticity is very low: for $a = 0.94$, $\varepsilon^M = 0.05$. In booms, the optimal replacement rate is slightly below 58% so the unemployment rate falls below its original level: at $a = 1.06$ the unemployment rate falls by 0.2 percentage point from 4.6% to 4.4%. UI has a strong influence on unemployment in booms because the macroelasticity is large: for $a = 1.06$, $\varepsilon^M = 0.27$.

To conclude, we would like to emphasize two points. First, diminishing marginal returns to labor are not necessary to obtain a countercyclical optimal UI. What is necessary is a positive elasticity wedge, which is obtained with any downward-sloping labor demand and any wage mechanism that does not respond too much to UI. Second, technology shocks are not necessary to obtain large unemployment fluctuations. In fact, when we simulate the aggregate-demand model, calibrated in the same fashion as the job-rationing model, we find the same quantitive results as those displayed in Figure 6. In the aggregate-demand model, however, marginal returns to labor are constant and unemployment fluctuations are driven by monetary shocks.

## VIII. Conclusion

This paper proposes a macroeconomic theory of optimal UI. The optimal replacement rate of UI is the sum of the conventional Baily-Chetty rate, which solves the trade-off between insurance and job-search incentives, and a correction term, which is positive when UI brings the labor market tightness closer to its efficient level. Therefore, the optimal replacement rate of UI is more generous than the Baily-Chetty rate if tightness is inefficiently low and UI raises tightness, or if tightness is inefficiently high and UI lowers tightness.

The paper also develops an empirical criterion to determine whether the labor market tightness is inefficiently high or low, and another empirical criterion to determine whether UI raises or lowers tightness. Empirical evidence suggests that tightness is inefficiently low in slumps and inefficiently high in booms, and that UI raises tightness. Our theory combined with this evidence indicates that the optimal replacement rate of UI is above the Baily-Chetty rate in slumps but below the Baily-Chetty rate in booms.

Finally, the paper simulates the job-rationing model of Michaillat [2012]. Unlike other matching models of the labor market, the job-rationing model is consistent with available empirical evidence because it predicts that UI raises tightness. In the simulations, the optimal replacement rate of UI is strongly countercyclical, increasing from below 55% to above 75% when the unemployment rate rises from 5% to 13%.

While available empirical evidence suggests that the optimal replacement rate of UI is countercyclical, more empirical evidence would be valuable to cement our conclusion about the cyclicality of the optimal replacement rate, and to quantify more precisely the fluctuations of the optimal replacement rate over the business cycle. Our theory provides guidance for future empirical work by defining the empirical statistics that are relevant for optimal UI. For instance, the theory implies that measuring by itself the macroelasticity of unemployment with respect to UI, as some macroeconomists do, is not directly relevant for the policy debate; it is only in conjunction with the microelasticity that the macroelasticity matters for the design of UI.

Our theory indicates that if UI has an influence on the labor market tightness, UI should be adjusted to bring tightness closer to its efficient level. Yet, our theory does not imply that UI is the best tool to stabilize the labor market and that other stabilization policies are superfluous. What the theory implies is that if the macroeconomic stabilization policies in place, such as monetary policy or government spending, are unable to stabilize the labor market perfectly, then UI can contribute to stabilizing the labor market. Of course, if the existing stabilization policies are able to stabilize the labor market perfectly, then the labor market tightness will always be at its efficient level, the correction term will be zero in our formula, and optimal UI will be given by the conventional Baily-Chetty formula. But if the existing stabilization policies do not work perfectly, then it is optimal to use UI to contribute to stabilizing the labor market.

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# Online Appendices

## Online Appendix A: Calibration

This appendix describes the calibration of the job-rationing model simulated in Section VII. All the derivations pertain to the steady state of the dynamic model without discounting. We first present the average values targeted in the calibration—some are normalizations and some are based on empirical evidence for the US. We then explain how we calibrate the parameters.

**Normalizations.** We normalize the average technology to $a = 1$. We normalize the average search effort to $e = 1$ and the disutility of effort such that $k(e = 1) = 0$. This normalization implies that on average, the costs of search while unemployed are of same magnitude as the costs of work while employed (which are not modeled).

**Average Values of Labor Market Variables.** We focus on the December 2000–February 2014 period. We compute the average value of key labor market variables in the data constructed by the BLS from the Job Openings and Labor Turnover Survey (JOLTS) and from the Current Population Survey (CPS). We take the average of the seasonally adjusted monthly total separation rate in all nonfarm industries from the JOLTS; we find that this average job-destruction rate is $s = 3.5\%$. We take the average of the seasonally adjusted monthly unemployment rate from the CPS; we find that this average unemployment rate is $u = 6.6\%$. We take the average of the seasonally adjusted monthly vacancy level in all nonfarm industries from the JOLTS, and divide it by the average of the seasonally adjusted monthly unemployment level from the CPS; we find that this average labor market tightness is $\theta = 0.37$ (average effort is normalized to 1).

**Average Value of the Recruiter-Producer Ratio.** We take the average over the December 2000–February 2014 period of the recruiter-producer ratio given by (32) and plotted in Panel A of Figure 5. We find that the average recruiter-producer ratio is $\tau(\theta) = 2.2\%$.

**Average Value of the Replacement Rate.** UI benefits replace between 50% and 70% of the pre-tax earnings of a worker. Following Chetty [2008] we set the benefit rate to 50%. Since earnings are subject to a 7.65% payroll tax, we set the average replacement rate to $R = 0.5 + 0.0765 = 58\%$.

**Calibrating Utility of Consumption.** We assume that the utility of consumption is log: $v(c) = \ln(c)$. This assumption implies a coefficient of relative risk aversion $\rho = 1$, consistent with labor supply behavior [Chetty, 2006*b*]. This coefficient of relative risk aversion is maybe on the low side of available estimates, but using log utility simplifies other part of the calibration. Naturally, the higher risk aversion, the more generous optimal UI.

**Measuring $\varepsilon^m$.** Using (12), we can translate the estimate $\varepsilon^m_R = 0.5$ obtained in the literature into an estimate of $\varepsilon^m$. With log utility, and since $c^e - c^u = (1-R) \cdot w$, we have $\Delta v/(v'(c^u) \cdot R \cdot w) = (1-R) \cdot \ln(c^e/c^u)/[R \cdot (c^e/c^u - 1)]$.

The next step is to express $c^e/c^u$ as a function of known quantities. The production function is $y(n) = n^\alpha$ so the firm's optimality condition, given by (2), implies that $y(n) = w \cdot l/\alpha$. We can therefore rewrite the government's budget constraint, given by (4), as $c^e - u \cdot (c^e - c^u) = w \cdot (1-u)/\alpha$, where $u = 1 - l$ is the unemployment rate. We divide this budget constraint by $\Delta c = c^e - c^u$. We get $(c^e/c^u)/(c^e/c^u - 1) = u + (1-u)/[(1-R)\cdot\alpha]$. After some algebra we obtain

$$\frac{c^e}{c^u} = \frac{1+\alpha\cdot(1-R)\cdot u/(1-u)}{1-\alpha\cdot(1-R)}, \tag{A1}$$

On average, $u = 6.6\%$ and $R = 58\%$. But $\alpha$ is unknown for the moment. However, for a broad range of $\alpha$, the value for $\varepsilon^m$ implied by $\varepsilon^m_R = 0.5$ does not change much. With $\alpha = 1$, $c^e/c^u = 1.79$, and $\varepsilon^m = 0.27$. With $\alpha = 0.66$, $c^e/c^u = 1.42$, and $\varepsilon^m = 0.31$. We therefore take $\varepsilon^m = 0.30$ as a calibration target. This is the value of $\varepsilon^m$ implied by $\varepsilon^m_R = 0.5$ and $\alpha = 0.75$ which is the value we use in our simulation.

**Linking $\varepsilon^f$ to $\varepsilon^m$.** Let $\varepsilon^e_\Delta \equiv (\Delta v/e)\cdot(\partial e^s/\partial \Delta v)$, $\kappa \equiv (e/k'(e))\cdot k''(e)$, and $L(x) \equiv x/(s+x)$. The elasticity of $L(x)$ is $1 - L(x)$. The effort supply $e^s(f, \Delta v)$ satisfies $k'(e^s) = (L(e^s \cdot f)/e^s) \cdot (\Delta v + k(e^s))$. Differentiating this condition with respect to $\Delta v$ yields

$$\kappa \cdot \varepsilon^e_\Delta = (1-l)\cdot\varepsilon^e_\Delta - \varepsilon^e_\Delta + \frac{\Delta v}{\Delta v + k(e)} + \varepsilon^e_\Delta \cdot \frac{e\cdot k'(e)}{\Delta v + k(e)}.$$

In equilibrium, $(e\cdot k'(e))/(\Delta v + k(e)) = l$. Therefore, $\varepsilon^e_\Delta = (1/\kappa)\cdot\Delta v/(\Delta v + k(e))$. Since the labor supply satisfies $l^s(f, \Delta v) = L(e^s(f,\Delta v)\cdot f)$, the elasticity of $l^s(\theta, \Delta v)$ with respect to $\Delta v$ is $(1-l)\cdot\varepsilon^e_\Delta$. By definition, $\varepsilon^m$ is $l/(1-l)$ times the elasticity of $l^s(\theta,\Delta v)$ with respect to $\Delta v$. Thus,

$$\varepsilon^m = \frac{1-u}{\kappa}\cdot\frac{\Delta v}{\Delta v + k(e)}. \tag{A2}$$

Similarly, differentiating the effort supply condition with respect to $f$ yields

$$\kappa\cdot\varepsilon^f = (1-l)\cdot(\varepsilon^f + 1) - \varepsilon^f + \varepsilon^f \cdot \frac{e\cdot k'(e)}{\Delta v + k(e)},$$

which implies that

$$\varepsilon^f = \frac{u}{\kappa}.$$

Combining this equation with (A2), we find that when $e = 1$ and thus $k(e) = 0$,

$$\varepsilon^f = \frac{u}{1-u}\cdot\varepsilon^m.$$

With $\varepsilon^m = 0.3$ and $u = 6.6\%$, we get $\varepsilon^f = 0.021$.

**Calibrating Matching Parameters.** We use a Cobb-Douglas matching function $m(e \cdot u, o) = \omega_m \cdot (e \cdot u)^{\eta} \cdot o^{1-\eta}$. We set $\eta = 0.7$, in the range of available estimates reported by Petrongolo and Pissarides [2001]. To calibrate $\omega_m$, we exploit the steady-state relationship $u \cdot e \cdot f(\theta) = s \cdot (1-u)$, which implies $\omega_m = s \cdot \theta^{\eta-1} \cdot (1-u)/(u \cdot e)$. With $s = 3.5\%$, $u = 6.6\%$, $e = 1$, and $\theta = 0.37$, we get $\omega_m = 0.67$.

To calibrate $r$ we exploit the steady-state relationship $\tau(\theta) = r \cdot s / [o_m \cdot \theta^{-\eta} - r \cdot s]$, which implies $r = o_m \cdot \theta^{-\eta} \cdot \tau(\theta) / [s \cdot (1 + \tau(\theta))]$. With $\omega_m = 0.67$, $s = 3.5\%$, $\theta = 0.37$, and $\tau(\theta) = 2.2\%$, we obtain $r = 0.84$.

**Calibrating $\alpha$ and Related Quantities.** As explained in Section VII, we set $\alpha = 0.75$. Using (A1), $u = 6.6\%$, $R = 58\%$ and $\alpha = 0.75$, we obtain $c^e/c^u = 1.50$ and $\Delta v = \ln(c^e/c^u) = 0.41$. With log utility and $k(e) = 0$, we have $(\Delta v + k(e))/(\phi \cdot w) = (1-R) \cdot \ln(c^e/c^u) \cdot (u + (1-u) \cdot c^e/c^u)/(c^e/c^u - 1)$. With $R = 58\%$, $u = 6.6\%$, and $c^e/c^u = 1.50$, we obtain $(\Delta v + k(e))/(\phi \cdot w) = 0.51$.

**Calibrating Disutility of Search.** We use a disutility of search $k(e) = \omega_k \cdot e^{\kappa+1}/(\kappa+1) - \omega_k/(\kappa+1)$. To calibrate $\kappa$, we use (A2). On average, $u = 6.6\%$, $k(e) = 0$, and $\varepsilon^m$=0.3 so $\kappa = 3.1$. To calibrate $\omega_k$, we exploit the steady-state relationship $k'(e) = [(1-u)/e] \cdot (\Delta v + k(e))$. This implies $\omega_k = (1-u) \cdot \Delta v$ when $e = 1$. With $u = 6.6\%$ and $\Delta v = 0.41$, we obtain $\omega_k = 0.38$.

**Calibrating the Wage Schedule.** To calibrate $\omega$, we exploit the steady-state relationship $\alpha \cdot n^{\alpha-1} = \omega \cdot a^{\gamma-1} \cdot (1 + \tau(\theta))$. With $a = 1$, $\tau(\theta) = 2.2\%$, $\alpha = 0.75$, and $n = (1-u)/(1+\tau(\theta)) = 0.915$, we obtain $\omega = 0.75$.

We calibrate $\gamma$ from microeconometric estimates of the elasticity for wages in newly created jobs—the elasticity that matters for job creation [Pissarides, 2009]. In panel data following production and supervisory workers from 1984 to 2006, Haefke, Sonntag and van Rens [2008] find that the elasticity of new hires' earnings with respect to productivity is 0.7 (Table 6, Panel A, Column 4). If the composition of the jobs accepted by workers improves in expansion, 0.7 is an upper bound on the elasticity of wages in newly created jobs. A lower bound is the elasticity of wages in existing jobs, estimated between 0.1 and 0.45 [Pissarides, 2009]. Hence we set $\gamma = 0.5$, in the range of plausible values.

**Solving the Baily-Chetty Formula.** With log utility the Baily-Chetty formula, given by (21), can be written as

$$\frac{R}{1-R} = (1-u) \cdot \frac{\Delta v(R)}{\varepsilon^m}$$

where

$$\Delta v(R) = \ln\left(\frac{c^e}{c^u}\right) = \ln\left(\frac{1 + \alpha \cdot (1-R) \cdot u/(1-u)}{1 - \alpha \cdot (1-R)}\right).$$

The expression from $\Delta v(R)$ comes from (A1). Setting $\varepsilon^m = 0.3$, $\alpha = 0.75$, and $u = 6.6\%$ and solving the Baily-Chetty formula for $R$, we obtain $R = 58\%$.

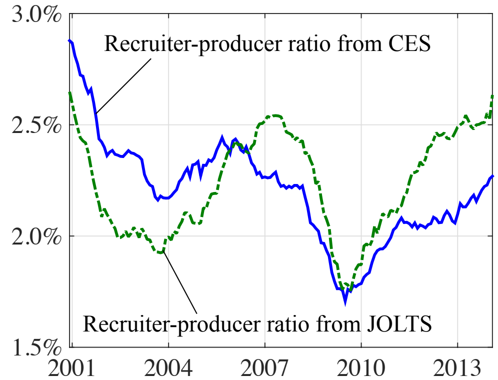


Figure A1: The recruiter-producer ratios constructed from CES data and from JOLTS

*Notes:* The time period is December 2000–February 2014. The recruiter-producer ratio depicted by the solid blue line is constructed from (32) using the number of workers in the recruiting industry computed by the BLS from CES data. The recruiter-producer ratio depicted by the dashed green line is constructed from (A3) using the number of vacancies and new hires computed by the BLS from JOLTS. Both time series are scaled to match the evidence on recruiting costs provided by Villena Roldan [2010]. The second time series is smoothed using a 5-point moving average to remove monthly volatility.

## Online Appendix B: Another Measure of the Recruiter-Producer Ratio

A limitation of the measure of the recruiter-producer ratio proposed in Section VII and displayed in Panel A of Figure 5 is that it is only valid as long as the share of recruitment done through recruiting firms is stable over the business cycle. In this appendix, we propose an alternative measure of the recruiter-producer ratio that does not rely on this assumption. This alternative measure is available from December 2000 to February 2014, which is the period when the data from the JOLTS are available.

The measure $\tau_t$ of the recruiter-producer ratio that we construct here relies on the assumption that the cost of posting a vacancy is constant over the business cycle. To construct $\tau_t$, we use the result that in the dynamic model, the recruiter-producer ratio satisfies

$$\tau_t = \frac{s \cdot r}{q_t - s \cdot r}, \tag{A3}$$

where $q_t$ is the rate at which vacancies are filled, $s$ is the job-destruction rate, and $r$ is the flow cost of posting a vacancy, assumed to be constant over time. We measure $q_t$ in the data constructed by the BLS from the JOLTS. Our measure is $q_t = m_t/v_t$, where $v_t$ is the seasonally adjusted monthly vacancy level in all nonfarm industries, and $m_t$ is the seasonally adjusted monthly number of hires in all nonfarm industries. Next, we set $s = 3.5\%$ as explained in Online Appendix A. Last, we set $r = 0.81$ such that the average value of $\tau_t$ is 2.2%, the same as the average value of the original recruiter-producer ratio between December 2000 and February 2014. Figure A1 plots the alternative measure $\tau_t$ and compares it with the original measure displayed in Panel A of Figure 5. We find that the two measures behave similarly over the business cycle: their correlation between December 2000 and February 2014 is 0.43.