Demand-driven knowledge clusters in a weightless economy

by

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ABSTRACT

That knowledge and technology matter for economic growth and performance is widely accepted. However, most formal analyses of knowledge-driven economies have focused on the supply side: productivity, and research and development. In modern high-tech economies where information and communications technologies figure prominently, just as important is the demand side of markets for goods akin to knowledge. This paper develops a model of knowledge cluster formation in a geographical space, driven by demand-side characteristics. Knowledge concentrations in space spontaneously emerge, even when physical distance and transportation costs are irrelevant. These clusters manifest in equilibrium as waves on a smooth, otherwise featureless, three-dimensional globe; they arise to resolve the tension between spatial spillover externalities and the costs of adapting to new sophisticated knowledge-products.

Keywords: codifiable knowledge, economics of superstars, emergence, Fourier analysis, geography, morphogenesis, tacit knowledge

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1 Introduction

That economists have historically identified Information and Communications Technology (ICT) as no more than just a special kind of *technology* has become more hindrance than help to economic analysis. In this identification, technology, accumulating due to progress in knowledge, is an input in the production function. Because it is just a special technology, ICT—if it does anything—affects productivity, at best.

But ICT is an example of a knowledge-intensive industry in two distinct ways. First, its *input* production processes demand high skills. Think of software design, cryptography, semiconductor engineering, or computer-human interface aesthetics.

Second, its *output* behaves like knowledge, regardless of whether an average scientist, mathematician, or engineer acknowledges it as such. The video game Tomb Raider has all the properties of knowledge (detailed below)—but in all likelihood its accumulation enters no production function in any significant way. No mathematician would recognize it as a theorem.

That we identify all knowledge-intensive activity with technology, and therefore productivity, hinders analysis because in modern economies, knowledge—or objects like knowledge—plays an increasingly significant role in consumption or the demand side, much as it

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has historically done on the production or supply side. Ignoring this side to knowledge as an economic good leaves out critical features of the modern knowledge-driven economy.

Economic growth has long held that technical progress accounts for much of the improvement in economic welfare of entire societies [Arrow, 1962, Romer, 1990, Solow, 1957]. Advances in knowledge are the most obvious forms of technical progress. Thus, it seems natural to ask for the contribution to productivity of any knowledge-intensive activity—whether it be research and development (R&D), blue-sky science, computer software and the Internet, or the New Economy (however that might be understood).

But other aspects to knowledge might matter even more for assessing its role in economic performance. Knowledge has peculiar properties as an economic good. It is *non-rival* or *infinitely expansible* [Arrow, 1962, Dasgupta, 1988]. However costly its first instance might be to create, subsequent copies—hosted on whatever medium—have trivial marginal cost. On the production side, this leads to where priority—the first to create, discover, or publish—is paramount, and winner-take-all outcomes are natural.

A second property to knowledge has, thus far, been much less acknowledged. Knowledge disrespects spatial distance, physical geography, national boundaries, and other natural barriers, i.e., it is *aspatial* or, perhaps more descriptive here, weightless. Provided the knowledge in question can be codified or digitized—written out as a sequence of 0s and 1s that can be subsequently decoded—it sees seamless and costless transmission across geographical space. Of course, the interpretation and use of the elsewhere-instantiated copy of this codifiable knowledge might have effectiveness that varies with the human capital or tacit knowledge embodied in the user. But this is conceptually distinct from the geography-blind nature of codifiable knowledge itself.[1]

Since at least Arrow [1962] and Machlup [1962] a large literature

[1] Jaffee, Trajtenberg, and Henderson [1993] document the importance of knowledge spillovers across geography, but do not explicitly distinguish codifiable and tacit knowledge in their empirical analysis.

in economics has emerged that analyzes knowledge's nonrivalry—for science, for research and development, for firm and industry dynamics, and for production. A partial list adding to references already mentioned includes Dasgupta and David [1994], Dasgupta and Maskin [1987], Dasgupta and Stiglitz [1980a,b], David [1992], Nordhaus [1969], and Romer [1990, 1993]. More narrowly, the literature on intellectual property rights (IPRs) studies how best to restrict the dissemination of knowledge. Yet, as Arrow [2001] shows through a rich set of evocative historical examples, one of the most significant aspects in economic development is not knowledge's over-dissemination, but instead the opposite, even in the absence of explicit IPRs. Knowledge—something economists have expended so much effort studying how to restrict—turns out, puzzlingly, to be one of the most difficult things to disseminate.

On a different front, that ICT is generally seen as a knowledge-intensive, technology industry has led also to studies on its productivity effects [e.g., Gordon, 2000]. Those studies conclude that ICT-related progress has fallen far short compared to changes wrought by key early 20th-century drivers of economic progress—among them electricity, the internal combustion engine, and chemicals. This obsession, accidental or otherwise, for relating knowledge (ICT and the New Economy) with productivity has led to an arguably disproportionate and unhealthy focus on certain questions: whether the natural rate of unemployment has shifted, whether inflation has permanently vanished, whether irrational exuberance moves stock markets, whether the New Economy is real, and whether desktop computers have improved productivity (the so-called productivity paradox, that "you see computers everywhere except in the productivity statistics").

This paper adopts a different perspective on knowledge as an economic good. It begins from a simple maintained hypothesis: The same flow of structural change that shifted economies from being agriculture-based to manufacturing to services, is now bringing about an ever greater role for knowledge in consumption. This *consumption* dimension constitutes the principal focus of the current paper, differentiating the analysis here from those above.[2] In analyzing the demand side, this paper also begins to address the question in Arrow [2001]—why knowledge, despite its nonrival and aspatial nature, turns out to be so hard to disseminate.

To add another example, more current, to those in Arrow [2001], take Open Source software. Although the Open Source community takes great pains to make its products available as widely as possible at zero or nominal sticker-price—trivial compared to work-alike commercial versions—the Open Source userbase remains relatively small. That this software is thought, rightly or otherwise, to be "hard to use" is, as far as I can see, the only constraint on its dissemination. That, obviously, is a demand-side property.

In economic notation, the perspective taken by this paper can be viewed as follows. In the traditional description, knowledge adds to the state of technology (or total factor productivity) A, which enters the economy-wide production function:

$$Y = F(K, N, A), \tag{1}$$

with Y aggregate output, K the stock of physical capital, and N the labor force employed. The economics of A can then be usefully viewed similarly as those of science and technology, R&D, patent races, and industrial organization. By contrast, this paper analyzes A in

$$U = U(C, A), \tag{2}$$

where U is the utility function describing agent preferences, and C is consumption of other commodities. The same properties for A in (1) appear for (2), but the economic incentives surrounding each will

[2] Quah [2001] had earlier provided a similar focus in discussing how aversion to using new technologies can lead to economic stagnation. He illustrated the point using as example the failed Industrial Revolution of 14th-century China, where the technological base more than matched 18th-century Western Europe but demand-side characteristics differed, allowing the latter to develop, but not the former.

differ.[3]

The analysis below combines the two key features of knowledge—non-rivalry and aspatiality—in a model designed to study how knowledge clusters might develop, emphasizing the demand side. In the model, cluster emergence occurs spontaneously (not randomly) across an explicit geographical space. Knowledge landscape dynamics are driven by demand-side incentives and characteristics. Demand-side characteristics can, in turn, be interpreted as tacit knowledge or human capital. The model incorporates a simple mechanism where codifiable and tacit knowledge are transformed, dynamically, one into the other.

2 The issues

It has long been noted that the most knowledge-intensive activities—universities, high-tech industry, Silicon Valley-wannabes, finance and consulting—cluster in spatial concentrations.

Many of these clusters are entirely explicable. Historical accident, government policy, location close by of complementary activities, largesse of a wealthy patron, physical access in sea, rail, or road transportation—and a host of other prosaic reasons—will explain many such observations. What is more difficult to understand is if the concentrations are *higher* than expected, given these obvious, observable causal factors.

What does concentration "higher than expected" mean? Since concentrations are distributional characteristics, I interpret this in the sense of the economics of superstars. Earlier, Sherwin Rosen [1981] had asked, Why might the distribution of income or earnings be more unequal than the underlying distribution of skills or attributes? In his work, what leads to this greater skewness in the outcomes distribution he called the economics of superstars. In the analysis here the

[3] Quah [2001] has A instead enter the consumer's budget constraint, interpreting different levels of A as harder (and thus costlier) to enjoy.

question is, Why might the spatial distribution of knowledge clusters be more concentrated than the underlying distribution of explanatory factors? Or, abusing somewhat the terminology in Kauffman [1993], do knowledge clusters have a spatial distribution that is *emergent*?

Traditional models of economic geography driven by increasing returns [Arthur, 1990, Fujita, Krugman, and Venables, 1999] trade off increasing-returns externalities against transportation costs, for moving products to distant consumers or intermediate inputs to other spatially-differentiated producers along a value chain.

Notable, however, about knowledge-products is their aspatiality. Knowledge-products endure zero costs in transportation. Analyses that rely on transportation costs cannot generate a credible knowledge profile across physical geography. As Krugman [1991] has emphasized, now-standard models of economic geography are not at all specifically designed for understanding technology- or knowledge-intensive activity. He makes this clear by describing geographic concentration in activities as diverse and low-tech as carpet manufacturing, jewelry production, the shoe industry, and rubber processing.

By contrast, the model we turn to in Section **3** below is specialized for analyzing knowledge dynamics. Although the mathematical tools it uses and the modelling ideas are general, the model has interpretation that is most sensible for the dissemination of knowledge and other similarly weightless commodities, such as in Internet-moderated economic activity [Quah, 2000].

The model produces equilibria that are knowledge waves in an explicit spatial geography. The law of motion in spatial distribution dynamics over a three-dimensional globe shows clusters or peaks along the dynamic transition path to long-run steady state. The model allows examining the forces that produce such clusters: Here, clusters emerge as the endogenous outcome of the tradeoff between spatial spillover externalities and the costs of adapting to new sophisticated knowledge-products.

The model addresses the question of emergence by situating its dynamics on a smooth, featureless three-dimensional globe. The underlying attributes have a uniform distribution—complete equality—which in turn manifests in a uniform spatial distribution being a long-run steady-state equilibrium. If this were all that the model produced, then emergence is absent. However, under appropriate conditions on the tradeoff described earlier, specific saddle-path stable transition dynamics carry equilibria that are multi-peaked, and therefore imply geographical clustering. Because the distribution of outcomes is more highly skewed than that of the underlying characteristics, the model also generates behavior similar to that Rosen [1981] called the economics of superstars.

Growth in this model occurs driven by demand-side characteristics. When consumers adapt easily to new technologies, the steady-state equilibrium growth rate is increased across all locations. Resistant consumers, on the other hand, reduce growth rates.

3 The model

Models of economic geography that theorize across different isolated locations—locate here or locate there—don't take into account how far apart the locations are or in what direction the locations sit relative to one another. Typically, those models also leave unspecified what happens *between* the distinct locations.

These remarks concern not just the technical issue of continuous- versus discrete-space modelling. Instead, they refer to how many models take as given only a small set of points for possible locations, ignoring all other possibilities. Taking the locations to be a discrete set of points and not filling in the empy space between them, many interesting economic questions cannot be addressed: Why is the spatial density of economic activity multi-peaked? Why do some locations transit from having intense economic activity to none at all? Why does the intensity of economic activity decline rapidly across space in some regions and only gradually in others?

Put differently, many models of economic geography have no geography in them. The current work overcomes this deficiency by beginning with an explicit description of a continuum geographical space. Because we wish to study the question of emergence, the underlying space is made as homogenous as possible. People are assumed to live uniformly distributed in that geographical space; they do not move. Their characteristics are endogenous, however, and so heterogeneities across people can arise and then dynamically evolve. It is this evolution that constitutes the central focus of the model.

3.1 Geography

Let $\mathbb{G}$ physical geography be a bounded and connected subset of a Hilbert space $(\mathbb{H}, \langle\cdot,\cdot\rangle)$. Denote the representative location $z \in \mathbb{G}$, and normalize the extent of $\mathbb{G}$ with respect to Lebesque measure so that $\int_{\mathbb{G}} dz = 1$. The inner product $\langle\cdot,\cdot\rangle$ induces a norm in the usual way,

$$|z - z'| = \langle z - z', z - z'\rangle^{1/2}.$$

Take geographical distance to be the metric implied by this norm. Denote the maximal distance on $\mathbb{G}$ by $\overline{d} = \sup_{z,z' \in \mathbb{G}} |z - z'|$.

Knowledge is $A_t(z) \in \mathbb{R}_+$, also interpreted as the *sophistication* of users at time t in location z. Write

$$A_t = \{A_t(z) \mid z \in \mathbb{G}\}$$

to denote the *profile* or *distribution* at time t of A across $\mathbb{G}$. Average knowledge is

$$\overline{A}_t = \int_{\mathbb{G}} A_t(z)\, dz.$$

The profile is *uniform* when

$$\forall z: \quad \overline{A}_t = A_t(z).$$

A uniform profile arises when knowledge usage is equalized everywhere. In this model, knowledge is intrinsically aspatial but whether its usage is uniform is an outcome determined in equilibrium, not assumed a priori.

As one example, Fig. 1 shows a flat physical geography $\mathbb{G}$, on top of which, at time t, knowledge profile A_t fluctuates in space.

The question is whether, as time proceeds, profile A_t converges to uniformity. Fig. 2 provides a different example, with geography $\mathbb{G}$ now the surface of a three-dimensional globe, so that A_t becomes a surface over $\mathbb{G}$. In both cases, we can take physical distance to be the ordinary distance in Euclidean space. If, as in Quah [2000], physical distance is irrelevant but timezones matter, then $\mathbb{G}$ in Fig. 2 is isomorphic to the equator.

For almost all the analysis to follow, the above is all that we need to assume for geography $\mathbb{G}$. However, for the explicit calculations on transition dynamics in Section **3.5**, we will specialize geography $\mathbb{G}$ to that given in Fig. 2:

Assumption G *Geography $\mathbb{G}$ is the surface of a three-dimensional globe.*

My hunch is that the discussion of transition dynamics can be generalized to any set over which one can perform Fourier analysis, but I have not yet been able to do this explicitly.

3.2 People

To focus on demand-side incentives and considerations, assume that as much A as demanded is forthcoming.

Consumers discount the future at constant rate $\rho > 0$. The representative consumer at time t in location z enjoys utility flow $U(A_t(z) \mid A_t)$, which depends on her sophistication $A_t(z)$ and A_t, the state of knowledge around the world at that time. This preference specification allows spatial spillovers and consumption externalities, or neighborhood effects, quite generally.

For the issues in this paper, it suffices to take the special case:

$$U(A_t(z) \mid A_t) = W_z(A_t) \times A_t(z) \tag{3}$$

where marginal utility is given by

$$W_z(A_t) = \int_{\mathbb{G}} K(z, z')A_t(z')\, dz', \tag{4}$$

with the spatial weighting function $K : \mathbb{G} \times \mathbb{G} \to \mathbb{R}_+$ such that for every given z, we have $K(z, \cdot)$ a probability density on $\mathbb{G}$, i.e.,

$$K(z, \cdot) \geq 0 \quad \text{and} \quad \int_{\mathbb{G}} K(z, z')\, dz' = 1.$$

To develop intuition on (4), consider some special cases of the weighting function K. If

$$\forall\ z \in \mathbb{G} : \quad K(z, z') = 1\ \forall z', \tag{5}$$

then spillovers are uniformly global: knowledge everywhere affects equally the enjoyment of consuming knowledge anywhere. On the other hand, if

$$K(z, z') = |z - z'|^{-1}, \tag{6}$$

then spillovers decline with geographical distance. As yet a different example, suppose $\overline{d} < \infty$ and we take for some $d \in (0, \overline{d})$

$$K(z, z') = \begin{cases} k_{\dim \mathbb{G}} \times (1 - |z - z'|^2) & \text{if } |z - z'|^2 \leq d < \overline{d} < \infty \\ 0 & \text{otherwise,} \end{cases} \tag{7}$$

(where $k_{\dim \mathbb{G}}$ is a positive normalizing constant depending on the dimension $\dim \mathbb{G}$), then spillovers extend only for finite distance d, after which the influence of neighboring knowledge vanishes. A closely related example is

$$K(z, z') = \begin{cases} k_{\dim \mathbb{G}} \times e^{([(z-z')^2 - d^2]^{-1})} & \text{if } |z - z'|^2 \leq d < \overline{d} \\ 0 & \text{otherwise,} \end{cases} \tag{8}$$

where differentiability holds at $|z - z'| = d$. Finally, if

$$K(z, z') = (2\pi)^{-\dim \mathbb{G}/2} \exp\left(-\frac{1}{2}|z - z'|^2\right), \tag{9}$$

(K is the multivariate Gaussian density) then, although spillover effects remain regardless of distance, the weighting function emphasizes neighborhoods that are closer together.

While it might help understanding to keep in mind one of these weighting function examples, nothing critical hinges on using any specific one—the analysis below is general. In particular, the clusters that emerge in equilibrium are not inherited directly from multimodality, local peaks, or otherwise in K. Moreover, while examples (6)–(9) show a monotone decline in weighting as geographical distance $|z - z'|$ increases, the conclusions below will apply even when the opposite holds, i.e., there can be increased weighting over portions of geographical space even as distance increases. It might be, for instance, that certain joint consumption activities are synchronized along different timezones (eight-hour separations, say), inducing multiple local modes in K—such aspatiality clarifies why K differs from physical distance.

However, an obvious instance where the analysis below fails is when K is a (Dirac delta) generalized function concentrating all mass on $z' = z$. Spillovers are then entirely absent. We need to rule out such degeneracies. Moreover, remember that, as far as possible, we want to refrain from inadvertently introducing heterogeneity into the specification of exogenous characteristics. Towards these ends, require that K satisfy two conditions:

Assumption R *First, for all $z \in \mathbb{G}$ there exists $\epsilon > 0$ such that spillovers occur outside the open neighborhood*

$$N_\epsilon(z) = \{z' \in \mathbb{G} : |z' - z| < \epsilon\},$$

i.e.,

$$\int_{\mathbb{G} \setminus N_\epsilon(z)} K(z, z')\, dz' > 0.$$

Second, K satisfies radial homogeneity,

$$\exists\ \varphi : \mathbb{H} \to \mathbb{R}_+ \quad \text{for which } K(z, z') = \varphi(z - z'). \tag{10}$$

Condition (10) says that consumer interaction is homogeneous but possibly asymmetrical across space. To understand this, consider the following example: Since London and Helsinki are 1800 km apart, with the latter 10 degrees north and 25 east of London, then that point similarly located relative to, say, San Francisco should bear the same consumption interaction spillovers to San Francisco, as Helsinki does to London. In (10) the representing function φ depends on $z - z'$ in general, not just geographical distance $|z - z'|$. Thus the impact of Helsinki on London can differ from that of London on Helsinki. (By contrast, examples (6)–(9) all show dependence on only distance.) Summarizing, assumption **R** describes how the analysis here models aspatiality: Unlike distance, K can be asymmetric and non-monotone in physical separation.

Consumers gain utility from consuming knowledge. However, enjoying particular levels of knowledge entails costs of training or retooling:

$$C(\dot{A}_t(z)) = \frac{1}{2}\zeta \times \dot{A}_t(z)^2, \quad \zeta > 0. \tag{11}$$

Retooling is quadratic in A's rate of change. The greater is coefficient ζ, the costlier or more difficult it is to retool, or the lower the ability to absorb new ideas. But how much absorbtion occurs is an economic decision, not a priori assumed. Because retooling costs vary with $\dot{A}$, not A's proportional rate of change, it is equally costly to retool whether the consumer begins at low or high levels of A.

(It is debatable what the appropriate specification is. Clearly, people already skilled in a particular subject find it easier to learn further advances in that area. On the other hand, equally obviously, many otherwise highly-skilled experts find learning to use computers, say, especially arduous—compared to how quickly six-year-olds pick up an understanding of computer and Internet usage.)

Equations (3) and (11) give a simple model of the relation between codifiable and tacit knowledge. Because, mediated through K, the profile A_t everywhere affects utility at z, relation (3) describes the aspatial nature of codifiable knowledge. On the other hand, codifiable knowledge everywhere can dynamically be transformed into tacit knowledge at z through (11). In turn, $A_t(z)$ is then viewed as codifiable knowledge by those at z' through its impact on them by the spillovers in (3).

Equilibrium below will turn out to hinge on a particular relation across absorption abilities and views of the future on the part of consumers. Express this in the following:

Assumption S *Consumer abilities and preferences display sufficient sluggishness*

$$4\zeta^{-1} < \rho^2. \tag{12}$$

Condition (12) says that relative to how much people discount the future, retooling costs must be sufficiently high. Alternatively, the condition specifies that given retooling costs ζ, people must value the present sufficiently highly relative to the future.

At each $z \in \mathbb{G}$, consumers accumulate knowledge $A(z)$ to solve the dynamic optimization problem:

$$\forall t \geq 0: \quad \sup_{\{A_s(z):s\geq t\}} \int_{s\geq t} \left[U(A_s(z) \mid A_s) - C(\dot{A}_s(z))\right] e^{-\rho s}\, ds \tag{13}$$

$$\text{s.t.} \begin{cases} \{A_t(z') : z' \neq z\} \\ \{A_s : s \geq t\}\,. \end{cases} \tag{14}$$

In words, at time t the consumer at z selects a plan for $\{A_s(z) : s \geq t\}$ that maximizes the present discounted value for utility flow net of retooling costs, taking as given the initial spatial profile across $\mathbb{G}$ and the dynamic plans of all other agents.

3.3 Equilibrium

I consider a rational expectations Nash equilibrium in dynamic plans solving (13)–(14) at each location z such that all plans are implemented and consistent with others' (and self) expectations.

Problem (13)–(14) has as first-order condition the Euler equation:

$$\begin{aligned} W_z(A_t) + \zeta \frac{d\dot{A}_t(z)}{dt} - \rho\zeta\dot{A}_t(z) &= 0 \\ \Longrightarrow \quad \dot{A}_t(z) = \zeta^{-1} \times \int_0^\infty W_z(A_{t+s}) e^{-\rho s}\, ds, & \qquad (15) \end{aligned}$$

where I have "solved stable roots backwards and unstable roots forwards" [Sargent, 1987, Ch. 9] to obtain the decision rule (15). This rule says that knowledge accumulation increases with the expected present discounted value of the stream of marginal utilities W. Because W at time t depends on the entire profile A_t, there is extensive interdependence in knowledge consumption across $\mathbb{G}$. The response in knowledge accumulation is higher, the lower is the retooling-costs coefficient ζ.

3.4 Steady state uniform equilibrium

Since the model has been set up with homogeneity everywhere, it is no surprise that a uniform profile provides a steady-state equilibrium.

Theorem 1 *Under assumption* ***S****, a uniform profile with $\overline{A}$ growing at the proportional rate*

$$g = \frac{\rho}{2} - \frac{(\rho^2 - 4\zeta^{-1})^{1/2}}{2} \in (0, \rho) \qquad (16)$$

is a steady-state equilibrium, with $dg/d\rho < 0$ and $dg/d\zeta < 0$. Provided the maximand (13) *remains bounded, this is the unique uniform profile steady-state equilibrium.*

Proof *At uniformity,*

$$W_z(A_{t+s}) = \int_{\mathbb{G}} K(z, z') A_{t+s}(z')\, dz' = \overline{A}_{t+s}.$$

Provided that $g < \rho$*, the RHS of* (15) *converges,*

$$\zeta^{-1}\int_0^\infty \overline{A}_{t+s}e^{-\rho s}\,ds = \zeta^{-1}\overline{A}_t\int_0^\infty e^{-(\rho-g)s}\,ds = [(\rho-g)\zeta]^{-1}\overline{A}_t.$$

Therefore,

$$\dot{\overline{A}}_t/\overline{A}_t = g = [(\rho-g)\zeta]^{-1},$$

which has roots in g *given by:*

$$g = \frac{\rho}{2} \pm \frac{(\rho^2-4\zeta^{-1})^{1/2}}{2} \tag{17}$$

Since $0 < 4\zeta^{-1} < \rho^2$*, both roots are real and in* $(0,\rho)$*. The smaller root* (16) *satisfies:*

$$\frac{dg}{d\rho} = \frac{1}{2}\left[1 - \frac{\rho}{(\rho^2-4\zeta^{-1})^{1/2}}\right] < 0$$
$$\frac{dg}{d\zeta} = -(\rho^2-4\zeta^{-1})^{-1/2}\times\zeta^{-2} < 0,$$

while the larger one has these signs reversed. Finally, the larger root in (17) *implies proportional growth rate* $\dot{\overline{A}}_t/\overline{A}_t$ *exceeding* $\rho/2$ *so that the objective function in* (13) *then does not converge.* *Q.E.D.*

The more consumers discount the future, the higher is ρ, so that knowledge accumulation is slower. The same outcome obtains as retooling costs rise: The higher is ζ, the slower is knowledge accumulation. In words, when consumers resist change, growth slows.

The subsequent discussion ignores the unbounded-utility outcome described in (17) of Theorem **1**. The comparative statics of the larger root in (17) are the opposite of the smaller root's, and are counter-intuitive.[4]

[4] Keely [2001] argues, however, that an unbounded utility solution is sensible and gives references from the psychology literature supporting this view.

3.5 Transition dynamics

Nothing guarantees uniqueness of the dynamic equilibrium outside of steady state. To continue the study, I consider only perturbations about the equilibrium identified in Theorem **1**, and analyze decision rules (15) that are Markov. By this I mean that at each $z \in \mathbb{G}$, there is some time-invariant functional M_z such that the expected present discounted value on the right of (15) can be taken to depend only on the *current* spatial profile:

$$\dot{A}_t(z) = M_z(A_t). \tag{18}$$

Even in the Markov simplification of equation (18), the evolution of A at z depends on the entire spatial profile $\{\, A(z') \,|\, z' \in \mathbb{G} \,\}$. This is a partial differential equation in the $1 + \dim \mathbb{G}$-dimensional space $(t, z) \in [0, \infty] \times \mathbb{G}$, and is difficult to analyze.

To make progress, detrend A by the exponential growth term e^{-gt}, using g from Theorem **1** to obtain

$$\dot{\widetilde{A}}_t(z) = \widetilde{M}_z(\widetilde{A}_t).$$

Then stack across locations z to obtain:

$$\dot{\widetilde{A}}_t = \widetilde{M}(\widetilde{A}_t), \tag{19}$$

where $\widetilde{M}$ is a operator whose section at z (for all $z \in \mathbb{G}$) is $\widetilde{M}_z$ in the detrended version of (18). While equation (19) resembles (18), we now have an ordinary differential equation in an infinite-dimensioned space of positive functions on $\mathbb{G}$.

To analyze perturbations about the uniform steady-state equilibrium identified in Theorem **1**, ask When will deviations from that steady state result in convergence back towards it? This is the same question that is asked of saddlepath stable transition about the steady state in a Cass-Koopmans growth model.

If (19) were linear and $\mathbb{G}$ were just a finite set of points, then $\widetilde{M}$ is a square matrix, with number of rows equal to the number of points in $\mathbb{G}$. Denoting the detrended steady-state level by $\overline{\widetilde{A}}$, (19) becomes

$$\dot{\widetilde{A}}_t = \widetilde{M} \times \left(\widetilde{A}_t - \overline{\widetilde{A}}\right)$$

Moreover, we can calculate an eigenvalue-eigenvector decomposition for the transition operator, now simply a matrix,

$$\widetilde{M} = \boldsymbol{\Phi} \boldsymbol{V} \boldsymbol{\Phi}^{-1}.$$

Premultiplying by $\boldsymbol{\Phi}^{-1}$ gives

$$\boldsymbol{\Phi}^{-1}\dot{\widetilde{A}}_t = \boldsymbol{V} \times \left(\boldsymbol{\Phi}^{-1}\widetilde{A}_t - \boldsymbol{\Phi}^{-1}\overline{\widetilde{A}}\right)$$
$$\Longrightarrow (\boldsymbol{\Phi}^{-1}\dot{\widetilde{A}}_t)_j = \nu_j \times \left((\boldsymbol{\Phi}^{-1}\widetilde{A}_t)_j - (\boldsymbol{\Phi}^{-1}\overline{\widetilde{A}})_j\right),$$

i.e., a collection of uncoupled, scalar ordinary differential equations. In that system, those equations where ν_j—the j-th eigenvalue or diagonal entry of $\boldsymbol{V}$—has negative real part will converge. Conversely, those where the real part of ν_j is positive will diverge. The system therefore has unstable components in the directions of eigenvectors corresponding to ν_j's with positive real parts. Convergence back to steady state occurs only when these unstable components are set to begin already at steady state. In other words, convergence occurs only for initial points $\widetilde{A}_0$ satisfying the condition:

$$\exists\ \tilde{q} : \quad \widetilde{A}_0 - \overline{\widetilde{A}} = \boldsymbol{\Phi} \times \begin{pmatrix} \tilde{q} \\ \underline{0} \end{pmatrix}.$$

While this reasoning is conceptually straightforward and well understood, its translation to (19) displays two difficulties. First, (19) is nonlinear, and so $\widetilde{M}$ evolves with $\widetilde{A}_t$. Similarly, the counterparts to $\boldsymbol{\Phi}$ and $\boldsymbol{V}$ will also be time-varying. Second, since $\widetilde{M}$ is infinite-dimensional, it is unclear what $\boldsymbol{\Phi}^{-1}$ means and how to calculate it.

The analysis proceeds by extending the reasoning in[5] Krugman and Venables [1997], Quah [2000], and Turing [1952]. Under assumptions **G** and **R**, the transition operator $\widetilde{M}$ is Toeplitz, with its spectra discrete, and its eigenfunctions sinusoidal wave forms. The spectra,

[5] Turing [1952] studied morphogonesis, a dynamic outcome from chemical reactions. Ioannides [2001] has applied ideas related to those here to analyze the dynamics of residential neighborhoods.

which parametrize the rates of convergence in time, reflect a tradeoff between spillovers K and retooling costs ζ. The counterpart of Cass-Koopmans convergence back to steady state is, in this spatial model, emergent as waves across space, with periodicities related to the dynamic rate of convergence described by the spectra of the transition operator.

To see this explicitly, assume **G** and **R**, and linearize equation (19) about $\overline{\widetilde{A}}$, to obtain for all $\widetilde{M}$ and all $\omega, \omega', \omega'' \in [-\pi, \pi) \times [-\pi, \pi)$,

$$\dot{\widetilde{A}}_t(z) = \int_{\mathbb{G}} \left[\mathfrak{L}_{\widetilde{M}}(z, z')(\widetilde{A}_t(z') - \overline{\widetilde{A}}(z')) \right] dz' \\ - \lambda_{\widetilde{M}} \times (\widetilde{A}_t(z) - \overline{\widetilde{A}}(z)), \tag{20}$$

with Frechet derivative $\mathfrak{L}_{\widetilde{M}} - \lambda_{\widetilde{M}} I$, and $\mathfrak{L}_{\widetilde{M}}$ Toeplitz, i.e.,

$$\mathfrak{L}_{\widetilde{M}}(e^{\omega' i}, e^{\omega i}) = \mathfrak{L}_{\widetilde{M}}(e^{(\omega' + \omega'')i}, e^{(\omega + \omega'')i}) \mod [-\pi, \pi) \times [-\pi, \pi).$$

Then,

Theorem 2 *Under assumptions **G** and **R**, for all $\mathfrak{L}_{\widetilde{M}}$ we have that its eigenfunctions are the complete orthonormal set of complex exponentials*

$$\left\{ (2\pi)^{-1/2} e^{i\omega j} : \omega \in (-\pi, \pi],\ j = -\infty, \ldots, +\infty \right\}$$

(independent of $\widetilde{A}_t$), and its spectrum is the (discrete) Fourier transform of any one of the sections $\mathfrak{L}_{\widetilde{M}}(z, \cdot)$ (varying with $\widetilde{A}_t$ but independent of z).

Proof *By direct calculation,*

$$\int \mathfrak{L}_{\widetilde{M}}(z', e^{i\omega}) e^{i\omega j} d\omega = \int \mathfrak{L}_{\widetilde{M}}(1, e^{(\omega - \omega')i}) e^{i\omega j} d\omega \\ = e^{i\omega' j} \int \mathfrak{L}_{\widetilde{M}}(1, e^{i\omega}) e^{i\omega j} d\omega,$$

where all integrals are over $\omega \in [-\pi, \pi) \times [-\pi, \pi)$. The last equation implies that for all integer j, $e^{i\omega j}$ is an eigenfunction, and the corresponding eigenvalue is the Fourier coefficient $\int \mathfrak{L}_{\widetilde{M}}(1, e^{i\omega})e^{i\omega j}\, d\omega$.

Q.E.D.

We can understand the economic forces driving the spatial and dynamic fluctuations from Fig. 3. This plots the real part of the spectrum described in Theorem **2**. Since the spectrum is discrete, the graph is defined only over the integer points of the horizontal axis. In (20) the spectrum of the linearized $\widetilde{M}$ comprises a contribution from $\mathfrak{L}_{\widetilde{M}}$ and a downward displacement due to the coefficient $\lambda_{\widetilde{M}}$. Fig. 3 shows these two components separately: The higher is $\lambda_{\widetilde{M}}$, the more of the spectrum of the linearized $\widetilde{M}$ has real parts negative, i.e., the higher the dimensionality of the stable manifold in the space of spatial profiles.

The coefficient $\lambda_{\widetilde{M}}$ depends in an intricate way on K and ζ.

(Provide explicit calculations and numerical examples here.)

Selecting initial conditions to zero out the unstable spectra means that certain sinusoidal components need to be omitted, for initial conditions that will converge to the uniform steady-state equilibrium. Explicitly, we need to select functions

$$q_t : \mathbb{G} \to \mathbb{R} \ni \int_{-\pi}^{+\pi} q_t(e^{i\omega})\, d\omega = 0 :$$
$$\widetilde{A}_t(e^{i\omega}) - \overline{\widetilde{A}}(e^{i\omega}) = (2\pi)^{-1/2} \sum_{j \in \mathbb{J}'} \tilde{q}_{t,j} e^{-i\omega j}.$$

The result from this is Fig. 4: Waves in space emerge as the Cass-Koopmans saddlepath stable initial conditions.

(Redraw a 3-d version of Fig. 4 over $\mathbb{G}$)

The discussion is usefully summarized as the following.

Theorem 3 *Under assumptions **G**, **R**, and **S**, all dynamically convergent time paths in the spatial distribution of knowledge are nontrivial, nondegenerate linear combinations of complex exponentials. A typical snapshot of an element of such timepaths is Fig. 4; convergence of the spatial distribution is towards a uniform profile growing at rate* (16).

In words, outside of steady state, clusters in space are the norm, not the exception.

4 Conclusion

This paper has developed a model of spatial knowledge dynamics that builds on two critical features of knowledge: Nonrivalry and aspatiality.

In contrast to most other studies of knowledge and technology, this paper has focused not on productivity and the supply side, but instead on consumption (or use) and the demand side. In modern high-tech economies where, say, information and communications technology figure prominently, just as important as technology's effect on productivity is the demand side of markets for goods akin to pure tokens of knowledge. Examples include computer software, education, entertainment, and other Internet-delivered goods.

The model generates equilibria in knowledge use that take the form of waves across space. Clusters in the spatial distributions emerge, even though the underlying exogenously-assumed economic characteristics that go into determining equilibrium have uniform, featureless distributions. In this model, demand-side characteristics determine both growth and geography simultaneously.

In focusing on the demand side, the model above hypothesizes there is an inexhaustible source of knowledge-products or ideas already extant. That pool only waits to be tapped. More realistically, of course, the economics of mining and implementing that putative source—the supply side—matters greatly. But ignoring the supply side in this way allows sharpening insights into what happens on the demand side. A complete analysis combines both, but that will need to be a follow-up part to this research.

Technically, the model here gives a different motivation for spatial dynamics than that in, say, Ioannides [2001], Krugman and Venables [1997], and Quah [2000]. The mathematical methods are, however, related. The model in this paper displays explicit optimizing dynamics under rational expectations, using techniques that apply for general functional forms.

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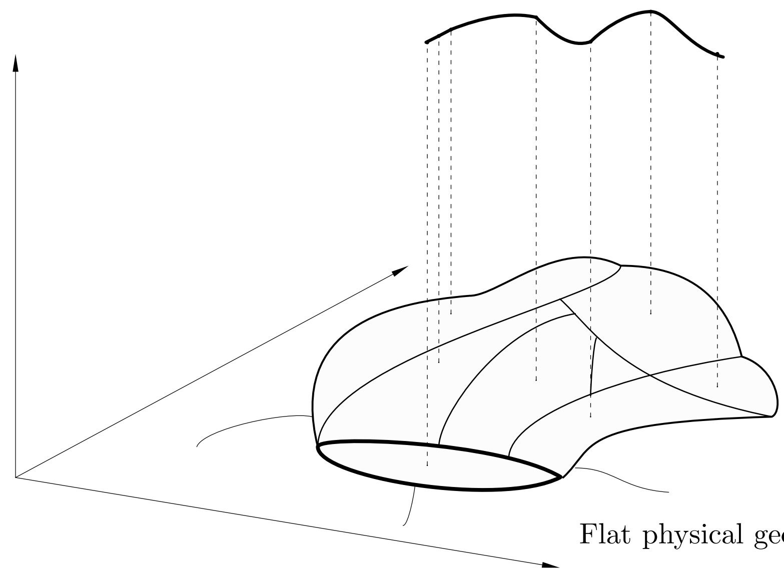

Fig. 1: Flat space fluctuations On a flat geography

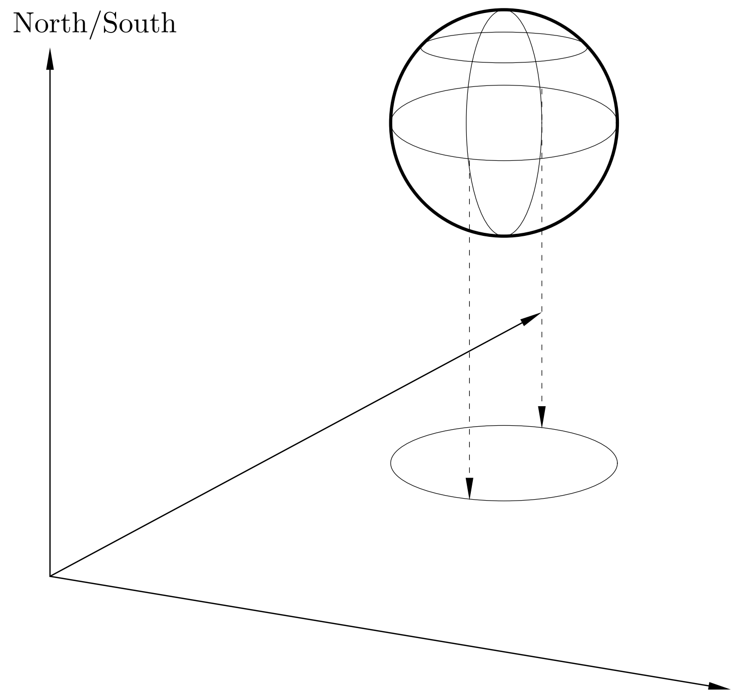

Fig. 2: Global fluctuations On 2-dimensional global geography, or projected down further to equatorial circle if physical distance doesn't matter but timezones do.

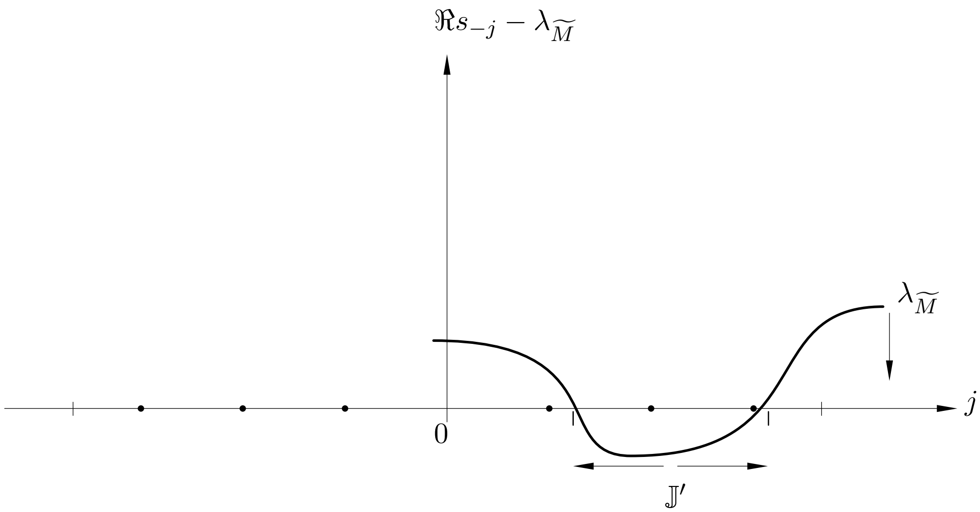

Fig. 3: Real part of the spectrum of $\mathfrak{L}_{\widetilde{M}}$ Displaced downwards by increasing $\lambda_{\widetilde{M}}$, the positive components that remain activate the associated complex exponentials in the invariant family of eigenfunctions.

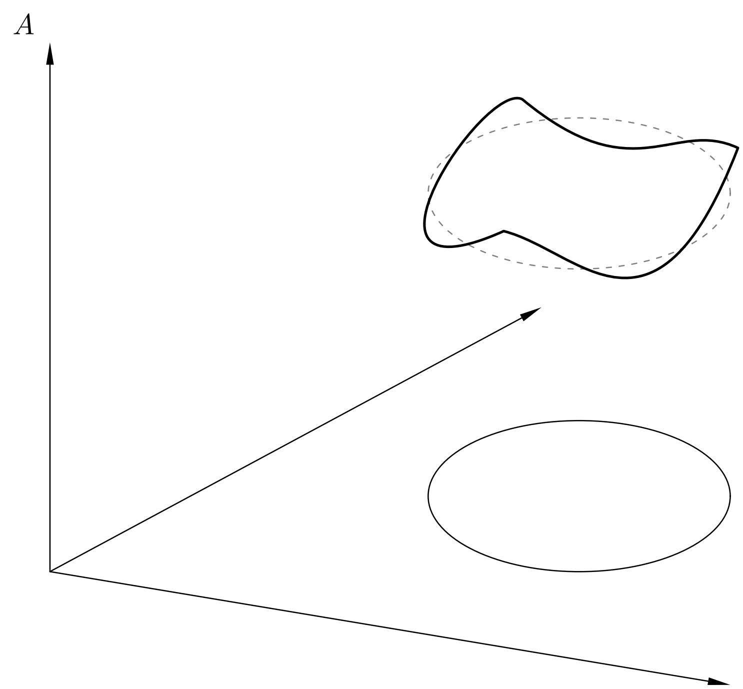

Fig. 4: Waves in space Local perturbations converge back to steady state only when they display cyclicalities in space