Life in unequal growing economies

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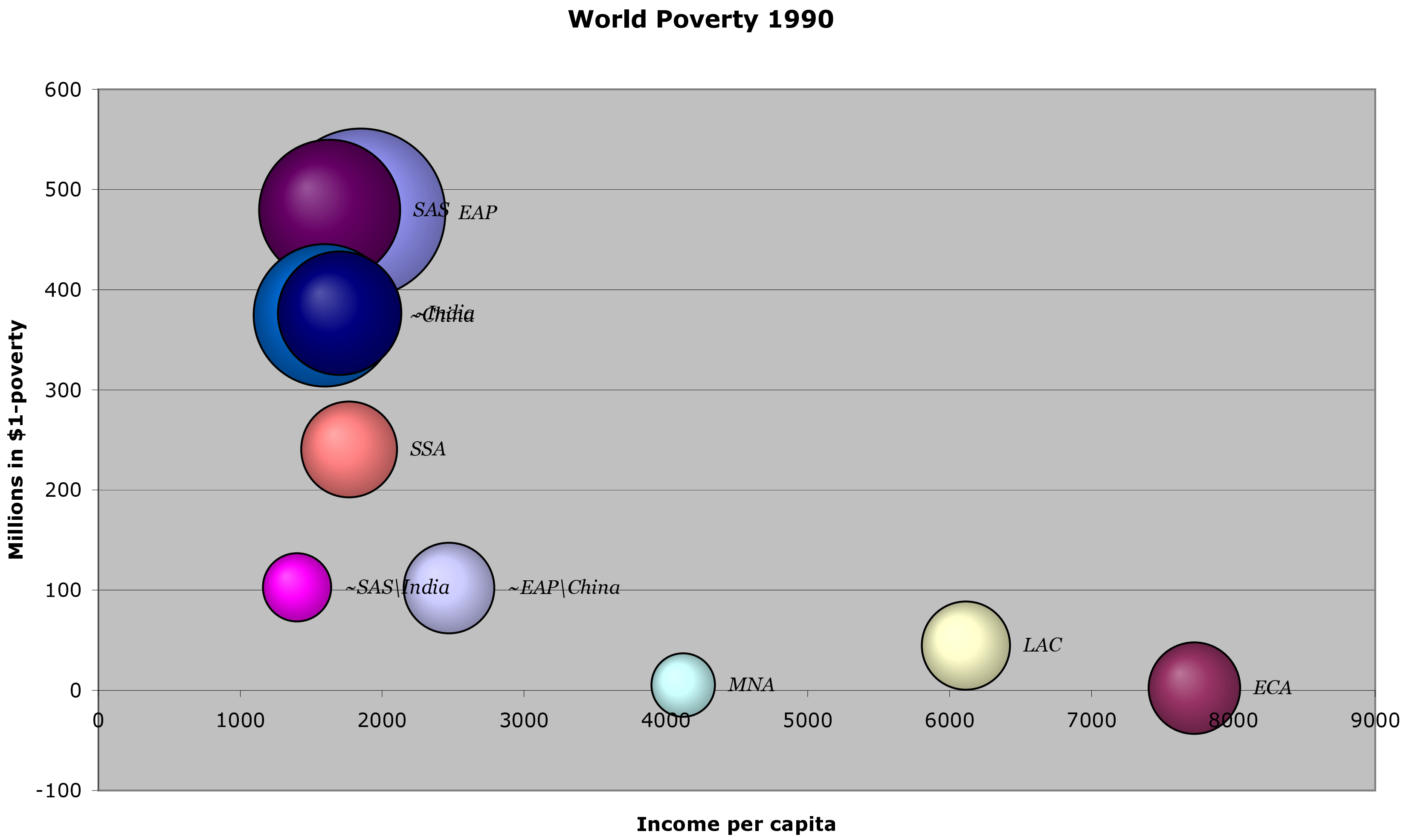
GROWTH AND INEQUALITY

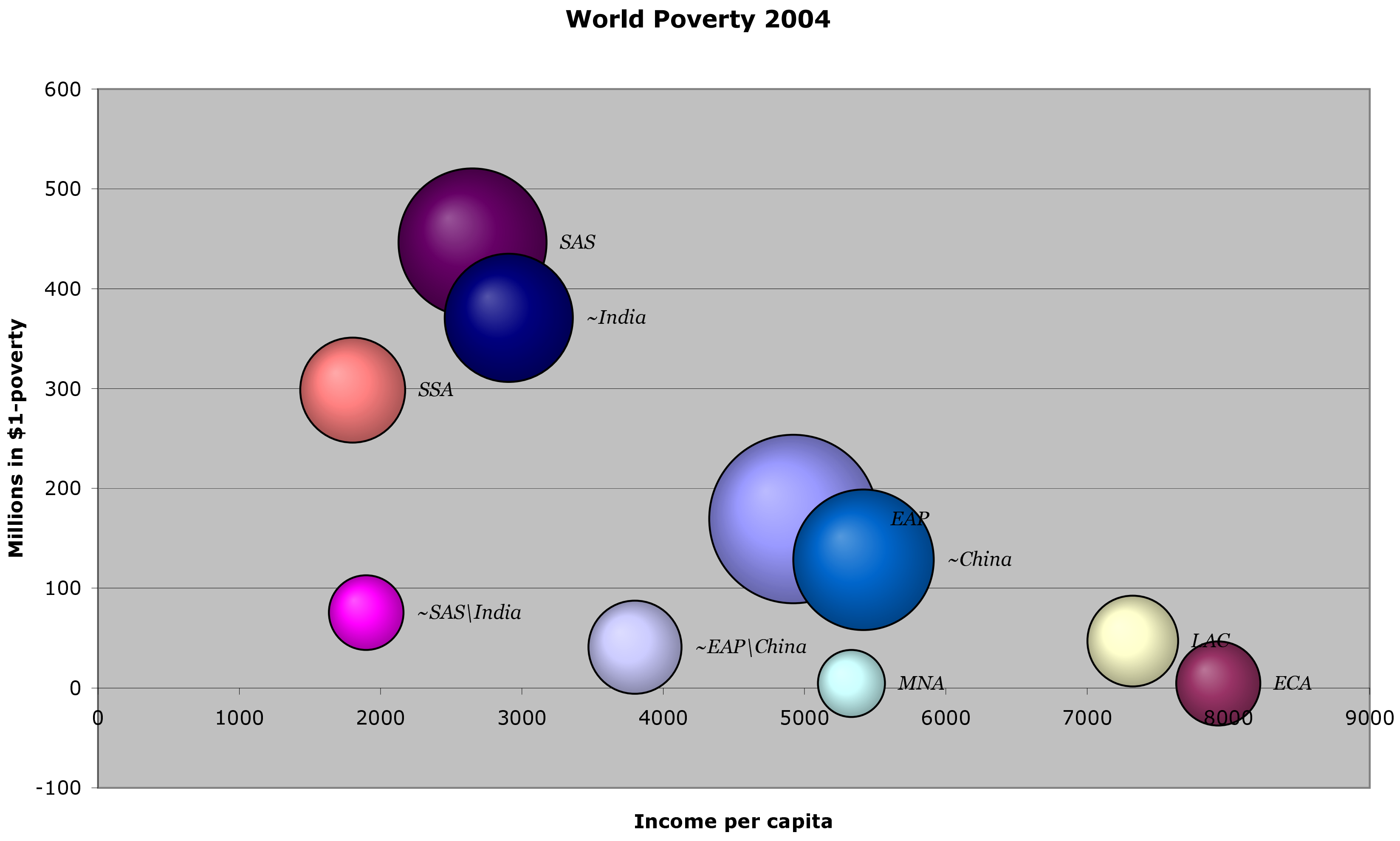
1. Causality
   - Aghion, Caroli, and Peñalosa 1999; Barro 2000; Banerjee and Duflo 2003; Benabou 1996; Galor and Zeira 1993; Persson and Tabellini 1994; . . .
2. Consequences
   - Chen and Ravallion 2004, 2007; Dollar and Kraay 2002; . . .
   - Córdoba and Verdier 2005
3. Global
   - Schultz 1998; Bourguignon and Morrisson 2002; Milanovic 2002, 2005; Sala-i-Martin 2006; Quah 2003; . . .

World growth and poverty

	1981	1990	1999	2004
GDP 10^{12} PPP$	24.4	33.1	43.3	52.2
per capita PPP$	5407.6	6291.5	7231.1	8198.5
World's $1-poor ($10^6$)	1470.3	1247.7	1108.6	969.5
China's $1-poor ($10^6$)	633.7	374.3	222.8	128.4
Remainder (10^6)	836.6	873.4	885.8	841.1

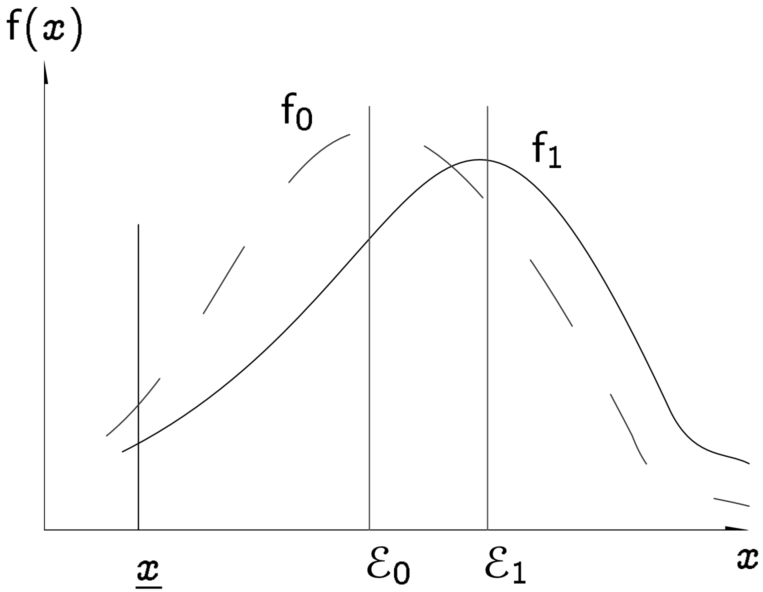
Table 1: **The importance of China** (PPP$ means constant 2000 international$)


World Poverty 1990
Millions in $1-poverty
Income per capita
600
500
400
300
200
100
0
-100
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
SAS
EAP
~India
~China
SSA
~SAS\India
~EAP\China
LAC
MNA
ECA


World Poverty 2004
Millions in $1-poverty
600
500
400
300
200
100
0
-100
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
Income per capita
SAS
~India
SSA
EAP
~China
~SAS\India
~EAP\China
MNA
LAC
ECA

ACCOUNTING

$\{X(t) : t \geq 0\} \iff$



Income and population in China and India, 1981–2004

	GDP per capita (PPP 2000 Intl$)			Population (10^6)	
	1981	2004	Annual growth	1981	2004
China	792.8	5418.5	8.36%	993.9	1296.2
India	1228.8	2907.1	3.74%	702.8	1079.7

Inequality in China and India, 1981–2004

	Gini coefficient (%)	
	1981	2004
China – Urban	15.0/16.1/16.7/18.5	32.9/34.0
China – Rural	23.9/23.9/25.1/25.0	33.4/34.0
China – All	29.1/	44.9/
India – Urban	34.1/33.3	34.7/37.6
India – Rural	30.1/30.1	26.3/30.5
India – All	31.4/	36.0/

$1-Poverty in China and India, 1981–2004

	$\underline{x} = 1;\ HC_{\underline{x}}\ (N_{\underline{x}}, 10^6)$	
	1981	2004
China	63.8 (633.7)	9.9 (128.4)
India	51.8 (363.7)	34.3 (370.7)

\$2-Poverty in China and India, 1981–2004

	$\underline{x} = 2;\ HC_{\underline{x}}\ (N_{\underline{x}}, 10^6)$	
	1981	2004
China	88.1 (875.8)	34.9 (452.2)
India	88.9 (624.9)	80.4 (867.6)

Analytical framework

$$U(\,[1+\Psi]\,C_A, \mathcal{Z}_A\,) = U(\,C_B, \mathcal{Z}_B\,).$$

- Welfare, not just inequality
- Dynamics
  - permanent, transitory
  - depth, duration
  - not just current inequality

Continuous time growth and inequality, 1

$$C_j(t) = e^{Z_j(t)} e^{\xi t}$$

$$W_j(t) = E_t \left[\int_t^\infty e^{-(s-t)\rho} U(C_j(s))\, ds \right], \qquad \rho > 0,$$

$$U(c) = \frac{c^{1-R} - 1}{1 - R}, \qquad R > 0.$$

Continuous time growth and inequality, 2

1. Z Brownian motion
2. Z (discrete) Markov chain
3. Z ...

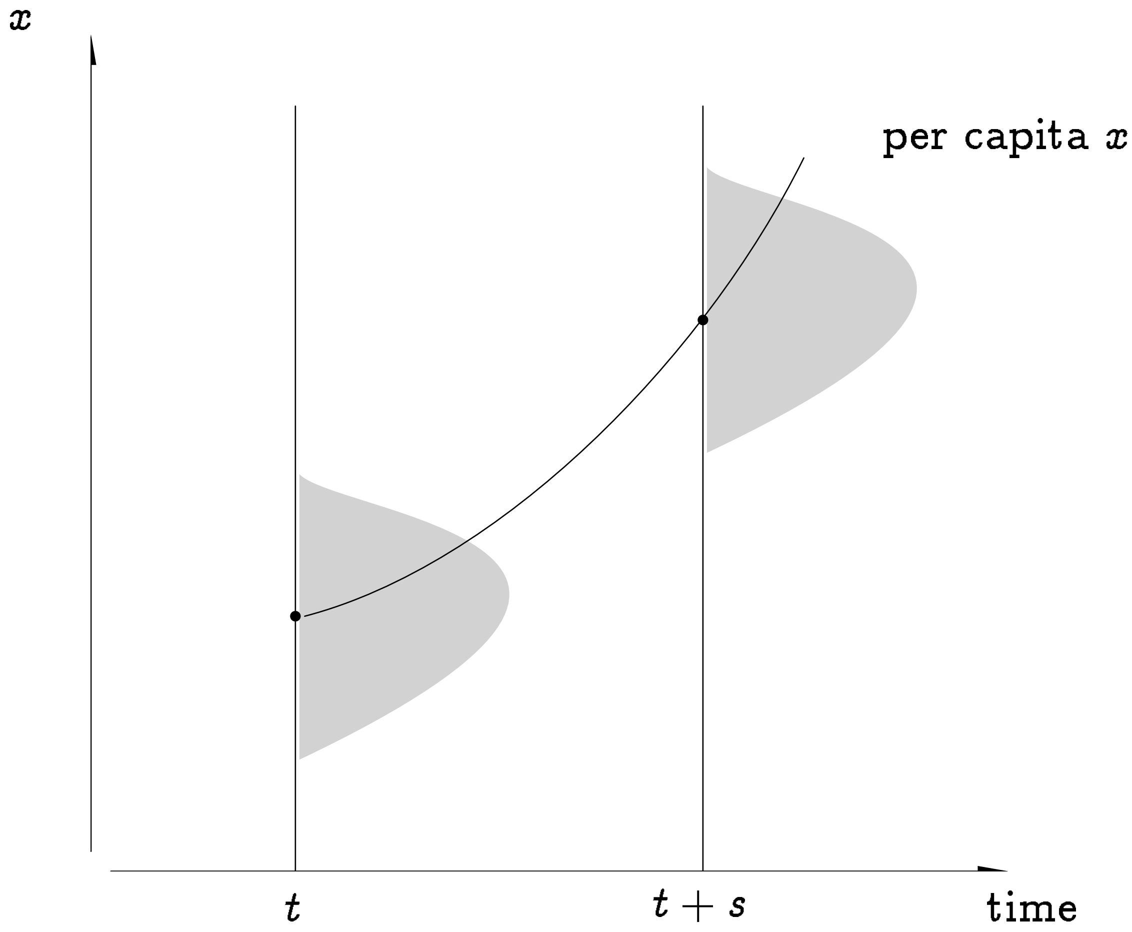


Figure 1: **Growth and inequality** Z fluctuations around a ξ growth path

Theorem 2.1 Suppose Z Markov, time-homogeneous ... then

$$W(0) = (\mathbf{R}_{\underset{\rho}{\sim}}\widetilde{U})(\log C(0)) + \text{constants}$$

Theorem 2.2 If further Z Brownian motion (σ^2) then

$$(\mathbf{R}_{\underset{\rho}{\sim}}\widetilde{U})(z) = \begin{cases} r(R,\rho,\xi,\sigma)^{-1} \times e^{(1-R)z} & \text{for } R \neq 1; \\ \rho^{-1} z & \text{otherwise.} \end{cases}$$

Theorem 2.3 Suppose Z Markov chain, with time-homogeneous transition probability matrices

$$P = \{\, P_t : t \geq 0 \,\}\,.$$

so that the infinitesimal generator of the semigroup P,

$$\mathbf{G} = \lim_{t \downarrow 0} \frac{P_t - I}{t}.$$

Then

$$(\mathbf{R}_{\widetilde{\rho}} \widetilde{U})(\overline{z}_m) = \left[\, (\widetilde{\rho} - \mathbf{G})^{-1} \overline{U} \,\right](m), \quad m = 0, 1, \ldots, M-1.$$

Discrete time growth and inequality, 1

$$W_j(t) = E_t\left[\sum_{s=0}^{\infty} \delta^s U(C_j(s))\right], \qquad \delta \in (0, 1),$$

$$C_j(t) = Z_j(t)\xi^t, \qquad \xi \geq 1,$$

$$Z_j(t) = \overline{z}_j \epsilon_j(t),$$

Discrete time growth and inequality, 2

$$\log \epsilon_j(t) = -\frac{1}{2}(1+\alpha)^{-1}\sigma_j^2 + \alpha \log \epsilon_j(t-1) + \nu_j(t),$$

with

$$|\alpha| < 1 \quad \text{and} \quad \nu_j(t) \sim \text{iid } \mathsf{N}(0, \sigma_j^2).$$

implying

Discrete time growth and inequality, 3

$$
\begin{aligned}
E\epsilon_j(t) &= \exp\left[E\log\epsilon_j(t) + \frac{1}{2}\operatorname{Var}\log\epsilon_j(t)\right] \\
&= \exp\left[-\frac{1}{2}(1-\alpha^2)^{-1}\sigma_j^2 + \frac{1}{2}(1-\alpha^2)^{-1}\sigma_j^2\right] = 1
\end{aligned}
$$

and

$$
\operatorname{Var}\epsilon_j(t) = \exp\left[(1-\alpha^2)^{-1}\sigma_j^2\right] - 1,
$$

increasing in σ_j^2.

Theorem 2.4: .... Otherwise

$$W_j(0; \epsilon_j(0), \xi, \overline{z}_j, \sigma_j^2, \alpha) = \ldots$$

$$\ldots \sum_{t=1}^{\infty} \delta^t \xi^{-(R-1)t} e^{D_1 \alpha^t} e^{-D_2 \alpha^{2t}}$$

[Completely messy — outside of log case with $R = 1$ (Lucas, 1987), or using unconditional expectation (Còrdoba and Verdier, 2005).]

Results

- Numerical

1. Growth ξ
2. Inequality

$$\mathcal{I}_G(t) = 2 \times \mathsf{F}_{\mathsf{N}(0,1)}\left(\left[\frac{1-\alpha^t}{1-\alpha^2}\,\sigma_j^2 \right]^{1/2} / \sqrt{2} \right) - 1$$

3. Mobility α

$\bar{\xi} = 1.02$		Compensating Ψ (%)			
ξ	R	1	2	5	10
1.005		107	58	26	14
1.01		62	32	14	7
1.03		−38	−19	−8	−4
1.04		−61	−32	−14	−8
1.05		−76	−42	−19	−10
1.06		−85	−49	−22	−12
1.07		−90	−54	−25	−14

Table 2: **Growth matters.** $\mathcal{I}_G = 0.32$, $\alpha = 0$, $\delta = 0.98$: For small rises in underlying growth rates . . .

		Compensating Ψ (%)					
R	$\mathcal{I}_G$	0.28	0.30	0.34	0.36	0.38	0.40
1		-4	-2	2	5	8	11
2		-8	-4	5	10	16	23
5		-19	-10	12	27	46	69
10		-34	-20	26	63	113	185

Table 3: **Distribution matters but, when R is low, not as much.** Base Var ϵ sets inequality $\mathcal{I}_G = 0.32$; growth $\xi = 1.02$; persistence $\alpha = 0$; and discount $\delta = 0.98$. To compensate for rises in inequality, permanent consumption levels have to increase some.

- For $R = 1$, growth multiplier $50\times$; Gini multiplier $1\times$. Changes in different directions as $R \nearrow$.

- Inequality more than business cycles (Krueger and Perri 2004 [6%]; Lucas 1987, 2003 [0.1%])

$$\Delta \log \overline{z} \doteq (R/2) \times \Delta \left(\sigma^2\right).$$

US business cycles, $\sigma = 0.013$
Inequality, $\sigma = 0.58(45\times)$

- China: $R = 2$ (adding: $+31$; combining: roughly the same)?
Or $R = 5$ (adding: -44, combining: roughly the same)

		Compensating Ψ (%)				
R	α	0.25	0.50	0.75	0.95	0.98
1		1	2	5	28	57
2		1	4	10	42	66
5		3	7	14	27	28
10		5	7	7	-8	-26

Table 4: **Mobility matters.** Initial $\epsilon(0)$ at 5th percentile of stationary distribution; base $\alpha = 0$; Var ϵ sets $\mathcal{I}_G = 0.32$. At a low initial level of consumption, to compensate for declines in mobility, permanent consumption levels have to rise (mostly). The relation is not monotone everywhere.

	Compensating Ψ (%)				
R α	0.25	0.50	0.75	0.95	0.98
1	0	0	-1	-4	-8
2	0	-1	-2	-10	-17
5	-1	-3	-7	-28	-42
10	-2	-5	-11	-42	-63

Table 5: **Mobility matters.** Initial $\epsilon(0)$ at 50th percentile of stationary distribution; base $\alpha = 0$; Var ϵ sets $\mathcal{I}_G = 0.32$. At an average initial level of consumption, to compensate for declines in mobility, permanent consumption levels have to fall, although not by as much as in Table 6, where initial consumption is yet higher.

		Compensating Ψ (%)				
R	α	0.25	0.50	0.75	0.95	0.98
1		0	−1	−4	−17	−28
2		−1	−2	−7	−26	−39
5		−2	−5	−12	−41	−59
10		−3	−6	−15	−50	−72

Table 6: **Mobility matters.** Initial $\epsilon(0)$ at 95th percentile of stationary distribution; base $\alpha = 0$; Var ϵ sets $\mathcal{I}_G = 0.32$. At a high initial level of consumption, to compensate for declines in mobility, permanent consumption levels have to fall a lot.

Mobility

1. $\epsilon(0)$ — 50%, 95%: Mobility soundly disliked.
2. $\epsilon(0)$ — 5%: Mobility favored, except at very high (R, α). Mobility has implications for uncertainty.
3. Mobility effects large: $\alpha = 0 \to 0.95$ at $R = 5$ comparable to $\mathcal{I}_G = 0.32 \to 0.36$ or $\xi = 1.02 \to 1.07$.

CONCLUSIONS

1. Inequality, mobility, growth: Dynamics, welfare
2. Poverty: Growth is unambiguously good
3. Welfare: Nuanced. Depends. China on the edge
4. Continuous time: Diffusion and jump processes; resolvent operator