Price stability in open economies*

Gianluca Benigno †
London School of Economics

Pierpaolo Benigno‡
New York University

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Abstract

This paper studies the theoretical conditions under which price stability is the optimal policy in a two-country open-economy model with imperfect competition and price stickiness. Special conditions on the levels of country-specific distortionary taxation and the intratemporal and intertemporal elasticities of substitution need to be satisfied. These restrictions apply to both cooperative and non-cooperative settings. Importantly, we show that cooperative and non-cooperative solutions do not coincide despite market completeness and producer currency pricing. We study the conditions under which quadratic approximations of single countries' welfare can be correctly evaluated by relying only on log-linear approximations of the equilibrium conditions.

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1 Introduction

There is wide consensus among policy-makers and students of monetary policy that price stability should be the main objective of a central bank. This goal will be desirable, *inter alia*, to the extent that it can induce an efficient allocation of resources

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† *Correspondence: Gianluca Benigno, Department of Economics and CEP, London School of Economics and Political Science, Houghton Street, London, WC2A 2AE, United Kingdom. E-mail: g.benigno@lse.ac.uk.*

‡ *Correspondence: Pierpaolo Benigno, Department of Economics, New York University, New York 10003, USA. E-mail: pierpaolo.benigno@nyu.edu*

across different uses and times. A growing theoretical literature on monetary policy evaluation has started to address the issue of optimal monetary policy in stochastic general-equilibrium models with monopolistic competition and price stickiness.[1]

In closed-economy models, the case for price stability is quite robust. Its desirability is associated with the possibility of reproducing the fluctuations that would arise in a flexible-price world. Under ex ante commitment and isoelastic preferences, a policy of price stability reproduces the flexible-price allocation, as shown in Goodfriend and King (2000). On the other hand, under discretion, the policy-maker has an incentive to inflate the economy and eliminate existing monopolistic distortions. When an appropriate taxation subsidy eliminates the 'inflation bias', price stability arises as an equilibrium reproducing the flexible-price allocation.

This paper considers the conditions under which a monetary authority would choose to implement the flexible-price allocation in an open-economy model. Our model has a simple structure: markets are complete, there are nominal rigidities in the form of one-period price contracts, and prices are set in the currency of the producer. The key difference from previous open-economy literature analysing optimal policy in a dynamic general equilibrium framework is that this paper considers a more general CES preference specification over domestic and foreign consumption goods. Hence, the intratemporal elasticity of substitution between home and foreign-produced goods may differ from the unitary value that has typically been assumed in the literature, as in Corsetti and Pesenti (2001b), Devereux and Engel (2000), and Obstfeld and Rogoff (2002).

We find that the conditions under which implementing the flexible-price allocation is optimal are much more restrictive than suggested by the previous literature. Even in a scenario whereby a single world social planner chooses policy optimally under full commitment, the policy-maker will generally only choose to implement the flexible-price allocation if shocks are perfectly correlated across countries, or the underlying structural distortions are equal. Otherwise, the world social planner has an incentive to manipulate the terms of trade in a welfare-improving manner, except under very particular preference specifications (which happen to include the unitary elasticity of

[1] Among others, closed economy models are described in Goodfriend and King (1997), King and Wolman (1998), Woodford (1996, 1999, 2000). Corsetti and Pesenti (2001b), Devereux and Engel (2000), Obstfeld and Rogoff (1998, 2002) and Sutherland (2002) consider open-economy models. This list is not exhaustive.

substitution assumed in the previous literature).

The conditions under which it would be optimal for two independent monetary policy-makers acting in an uncoordinated manner to implement the flexible price allocation are even more restrictive. It is no longer sufficient for the structural distortions to be equalised across countries. The reason is that in the Nash equilibrium, policy-makers face the externality that they can use sticky prices to manipulate the terms of trade to their own country's advantage. Only under the very restrictive preference specifications that erase this incentive does the flexible price allocation become optimal.

Thus, while the previous literature suggests a similar optimal policy prescription between an open economy and a closed economy- namely to implement the flexible price allocation- this prescription was derived based on a preference specification imposing an elasticity of unity between domestic and foreign goods. With more general preference specifications, even a policy-maker operating under commitment will typically have an incentive to depart from implementing the flexible-price allocation.

As in the closed economy case, policy-makers operating under discretion will not in general choose to implement the flexible-price allocation. The difference in the open economy case is that policy may have either an inflationary or deflationary bias, depending on underlying structural characteristics.[2] Only under special restrictions on the structural parameters, in which the bias to inflate (associated with monopolistic competition) exactly offsets the deflationary bias (to manipulate the terms of trade in one's favor), would policy-makers opt to implement the flexible price allocation. These conditions turn out to be of some independent interest: in particular, all linear terms drop in a quadratic approximation of the welfare function. It follows that the quadratic approximation of the welfare can be correctly evaluated by relying purely on log-linear approximations of the structural equilibrium conditions.[3]

The structure of this study is as follows: Section 2 presents the model emphasising the main assumptions; Section 3 studies the closed-economy limiting case; Section 4 discusses the conditions under which the flexible price allocation is efficient in our

[2] This result has been emphasized in perfect foresight models by Corsetti and Pesenti (2001a) and Tille (2001).

[3] Woodford (1999) analyzes such an issue in a closed-economy model. Kim and Kim (1999) discusses the problems associated with an open-economy analysis and proposes a different method.

open-economy framework; Section 5 discusses the strategic solutions and the conditions under which linear-quadratic models are appropriate in open-economy models. Section 6 concludes.

2 A two-country open economy model

The model belongs to a recent class of stochastic general equilibrium models with imperfect competition and price stickiness that have been used for positive and normative analysis. In this section we emphasise the main structure of the model and its crucial assumptions. We consider an open-economy model with two countries, home (H) and foreign (F). They produce a continuum of goods indexed on the intervals $[0,n)$ and $[n,1]$, respectively. In each country there is a continuum of economic agents, with a population size set equal to the range of produced goods: home and foreign households lie on the interval $[0,n)$ and $[n,1]$, respectively. Each agent is a monopolist in producing a single differentiated good. The preferences of a generic household j belonging to country H are given by

$$U_t^j = \mathrm{E}_t \left\{ \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[U(C_\tau^j) - V(y_\tau^j, z_\tau) \right] \right\},$$

where E_t is the expectation conditional on the information at time t; β is the intertemporal discount factor, with $0 < \beta < 1$. U is an increasing concave function in the consumption index C, while V is an increasing convex function of y. y^j denotes the production of the differentiated good produced by agent j, while z is a country-specific shock. The preferences of a generic household belonging to country F are identical, with the exception that variables specific to country F are denoted with a star. The consumption index C, which is common across countries, is defined as

$$C = \left[n^{\frac{1}{\theta}} C_H^{\frac{\theta-1}{\theta}} + (1-n)^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \ \theta > 0$$

where C_H and C_F are consumption bundles of the home and foreign-produced goods, respectively; θ denotes the intratemporal elasticity of substitution between C_H and C_F. The consumption bundles are defined as

$$C_H \equiv \left[\left(\frac{1}{n} \right)^{\frac{1}{\sigma}} \int_o^n c(h)^{\frac{\sigma-1}{\sigma}} dh \right]^{\frac{\sigma}{\sigma-1}}, \qquad C_F \equiv \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\sigma}} \int_n^1 c(f)^{\frac{\sigma-1}{\sigma}} df \right]^{\frac{\sigma}{\sigma-1}},$$

where $c(h)$ and $c(f)$ are consumptions of the generic differentiated goods produced in country H and F, respectively; σ is the elasticity of substitution across goods produced within a country, where $\sigma > 1$. The appropriate consumption-based price index that corresponds to the above specification of preferences is

$$P = \left[nP_H^{1-\theta} + (1-n)\left(P_F\right)^{1-\theta} \right]^{\frac{1}{1-\theta}},$$

with P_H and P_F given by

$$P_H = \left[\left(\frac{1}{n}\right) \int_o^n p(h)^{1-\sigma} dh \right]^{\frac{1}{1-\sigma}}, \qquad P_F = \left[\left(\frac{1}{1-n}\right) \int_n^1 p(f)^{1-\sigma} df \right]^{\frac{1}{1-\sigma}},$$

where $p(h)$ is the price in units of currency H of a generic differentiated good h produced in country H, while $p(f)$ is the price in units of currency H of a generic good f produced in country F.

The nominal exchange rate, S, is defined as the price of the foreign currency in terms of home currency. All goods are traded and the law of one price holds. Thus $p(h) = S \cdot p^*(h)$ and $p(f) = S \cdot p^*(f)$. Given the law of one price, and the fact that the consumption index C is common across countries, purchasing power parity holds, i.e. $P = SP^*$ and $P_H = SP_H^*$, $P_F = SP_F^*$.

Given the structure of preferences, demand for the generic goods h and f is given by

$$y^d(h) = \left[\frac{p(h)}{P_H}\right]^{-\sigma} \left[\frac{P_H}{P}\right]^{-\theta} C^W, \qquad y^d(f) = \left[\frac{p(f)}{P_F}\right]^{-\sigma} \left[\frac{P_F}{P}\right]^{-\theta} C^W \qquad (1)$$

where C^W is world consumption defined as $C^W \equiv nC + (1-n)\, C^*$.

We assume that markets are complete both at domestic and international levels. We allow agents to trade in a set of state contingent securities denominated in units of the common general consumption index. We also assume that at time -1 agents in both countries are committed to trade state contingent financial wealth to insure that their lifetime budget constraints are the same as of time 0. Given the producer-currency-pricing assumption and the fact that preferences are symmetric across countries, our complete-market assumption implies perfect consumption risk-sharing, i.e. $C = C^* = C^W$.

We do not model money explicitly, but we interpret this model as a cash-less limiting economy, in the spirit of Woodford (1998), in which the role of money balances in facilitating transactions is negligible. [4]

[4] We discuss more formally our cash-less limiting economy in the appendix. Our model can be

2.1 Flexible price allocation

Households act as monopolists in selling their differentiated goods. First, we focus on the flexible price allocation. A generic seller h that belongs to country H chooses her price $p(h)$ in order to maximise the function

$$\Psi_t = (1-\tau)\lambda_t p_t(h) y_t^d(h) - V(y_t^d(h), z_t) \tag{2}$$

where $y^d(h)$ is defined by (1), while τ is a country-specific proportional tax on firms' revenue; λ_t is the marginal utility of nominal income at time t, with $\lambda_t = U_C(C_t)/P_t$.[5] The optimal price-setting decision is identical across all sellers within a country. In the symmetric equilibrium, the price-setting conditions for country H and F imply

$$(1-\Phi)U_C(C_t)\frac{P_{H,t}}{P_t} = V_y\left(\left[\frac{P_{H,t}}{P_t}\right]^{-\theta} C_t, z_t\right), \tag{3}$$

$$(1-\Phi^*)U_C(C_t)\frac{P_{F,t}}{P_t} = V_y\left(\left[\frac{P_{F,t}}{P_t}\right]^{-\theta} C_t, z_t^*\right), \tag{4}$$

in all contingencies and at all times t. Equations (3) and (4), combined with the definition of the consumption-based price index P, determine the level of consumption and relative prices under the flexible price allocation. We have defined the variables Φ and Φ^* for country H and F, respectively, as the level of monopolistic distortions corrected by distortionary taxations, in the following way:

$$(1-\Phi) \equiv \frac{\sigma-1}{\sigma}(1-\tau), \qquad (1-\Phi^*) \equiv \frac{\sigma-1}{\sigma}(1-\tau^*),$$

where $\sigma/(\sigma-1)$ indicates the mark-up that arises because of the monopolistic-competition assumption. When $\Phi = 0$, the monopolistic distortions are completely eliminated by an appropriate taxation subsidy. An intuitive interpretation of equa-

interpreted as a limiting case in which the relative importance of the service flow from real money balances in the utility function goes to zero.

[5] These proportional taxes are rebated to the consumer through lump-sum transfers.

tions (3) and (4) follows from noting that real marginal costs are defined by

$$mc_t = \frac{\left(\frac{P_{H,t}}{P_t}\right)^{-1} V_y\left(\left[\frac{P_{H,t}}{P_t}\right]^{-\theta} C_t, z_t\right)}{U_C(C_t)},$$

$$mc_t^* = \frac{\left(\frac{P_{F,t}}{P_t}\right)^{-1} V_y\left(\left[\frac{P_{F,t}}{P_t}\right]^{-\theta} C_t, z_t^*\right)}{U_C(C_t)},$$

for countries H and F, respectively. In the flexible price allocation, real marginal costs are proportional to the level implied by the overall degree of monopolistic distortion. When $\Phi = \Phi^* = 0$ the resulting allocation reproduces the competitive one, since mark-ups are completely eliminated.

2.2 Welfare analysis

In this study, we assume that the monetary authorities are benevolent and maximise expected households' utility. The welfare criteria for the home and foreign policy-makers are defined as

$$W_t \equiv \mathrm{E}_t\left\{\sum_{\tau=t}^{\infty} \beta^{\tau-t} w_\tau\right\}, \qquad W_t^* \equiv \mathrm{E}_t\left\{\sum_{\tau=t}^{\infty} \beta^{\tau-t} w_\tau^*\right\}, \tag{5}$$

where w_t and w_t^* are the instantaneous average utility flows among all the households belonging to countries H and F, respectively:

$$w_\tau \equiv U(C_\tau) - \frac{\int_0^n V(y_\tau(h), z_\tau)dh}{n}, \qquad w_\tau^* \equiv U(C_\tau) - \frac{\int_{1-n}^1 V(y_\tau(f), z_\tau^*)df}{1-n}.$$

2.3 Preferences specification

We assume that $U(.)$ and $V(.)$ are isoelastic functions of the form

$$U(C_t) \equiv \frac{(C_t)^{1-\rho}}{1-\rho},$$

$$V(y_t^j, z_t) \equiv \frac{z_t(y_t^j)^\upsilon}{\upsilon} \text{ if } j \in H, \quad V(y_t^j, z_t^*) \equiv \frac{z_t^*(y_t^j)^\upsilon}{\upsilon} \text{ if } j \in F,$$

where ρ is the intertemporal elasticity of substitution in consumption with $\rho > 0$; while $\eta \equiv \upsilon - 1$, with $\upsilon \geq 1$, is the elasticity of labour supply. Goodfriend and

King (2001) analyse a closed-economy case within this class of preferences. In a two-country open economy model, Corsetti and Pesenti (2001b) assume $\rho = \upsilon = \theta = 1$; Devereux and Engel (2000) and Obstfeld and Rogoff (2002) assume $\upsilon = \theta = 1$. The latter work also includes non-tradable goods in the consumption index, which is a Cobb-Douglas index of tradable and non-tradable goods.

3 Commitment and discretion in a closed economy

Here we discuss the closed economy case, which can be obtained from the model presented in the previous section by letting n equal 1. The purpose of this section is to present familiar results related to the closed economy counterpart of our model and to introduce definitions and concepts useful for the open-economy analysis.

First, we consider the case in which all prices are set one period in advance or, as an alternative interpretation, with the information set of the previous period. The optimal pricing decision of a generic firm j in setting her price p_t^j at time t with the information set of time $t-1$ implies that

$$\mathrm{E}_{t-1}\left\{\left[(1-\Phi)\frac{U_C(C_t)}{P_t}p_t^j - V_y(y_t^j, z_t)\right] y_t^j\right\} = 0, \qquad \text{for all } j \tag{6}$$

where

$$y_t^j = \left(\frac{p_t^j}{P_t}\right)^{-\sigma} C_t.$$

An intuitive interpretation of condition (6) is that prices are set in order to keep average real marginal costs constant. All firms set the same price, $p_t^j = P_t$ and $y_t^j = Y_t = C_t$. We can then rewrite (6) as

$$\mathrm{E}_{t-1}\left\{\left[(1-\Phi)U_C(Y_t) - V_y(Y_t, z_t)\right] Y_t\right\} = 0, \tag{7}$$

at each time t. Under ex ante commitment, the policy-maker maximises the welfare function W_t as in (5), with the information set at time $t-1$, under the sequence of constraints (7) one for each period t onwards. In this closed-economy case, the utility flow w_τ is

$$w_\tau = U(Y_\tau) - V(Y_\tau, z_\tau).$$

With the aim of characterizing the optimal policy, we introduce the concept of *notional price*.[6] The notional price is defined as the price that a supplier would choose in principle, if it were free to choose a price in a certain period t independent of past and future prices. In fact p_t^N, the notional price for a generic period t, satisfies

$$(1-\Phi)U_C(Y_t)\frac{p_t^N}{P_t} = V_y\left(\left(\frac{p_t^N}{P_t}\right)^{-\sigma} Y_t, z_t\right). \tag{8}$$

In particular, with isoelastic functions, (8) implies that

$$\frac{Y_t}{Y_t^n} = \left(\frac{p_t^N}{P_t}\right)^{\frac{1+\sigma\eta}{\rho+\eta}}, \tag{9}$$

where Y_t^n represents the natural rate of output which would arise under flexible prices ($Y_t^n \equiv [(1-\Phi)z_t]^{\frac{1}{\rho+\eta}}$). At a generic time t, output can deviate from its natural rate if the notional price at time t differs from the average actual price for that period. A policy specified in terms of notional prices can determine the average actual price at each time t. Indeed, the result of substituting the expression for Y_t, derived from (9), into (7), is that prices P_t, which are preset at time $t-1$, only depend on the joint distribution of $\{p_t^N, Y_t^n\}$. Moreover, P_t is homogenous of degree 1 in p_t^N. Once P_t is determined, then the actual realization of p_t^N determines the actual level of output Y.[7]

In this context we can properly define price stability as follows:

Definition 1 *With prices all fixed one period in advance, a policy of zero notional inflation is defined as the equivalence between the notional price and the average actual price in all contingencies and at all times.*

Under a zero notional inflation policy, producers, if given the option, would not wish to alter their prices *ex post*, in any contingency. In this study, price stability refers to a policy of zero notional inflation. Using this definition, we are aware of important qualifications. In general, a policy of zero notional inflation does not imply zero inflation in terms of the 'actual' price index P. However, a policy of zero 'actual' inflation implies zero notional inflation.

[6] We are grateful to Mike Woodford for this suggestion.

[7] Using the consumers' Euler equation, one can retrieve the interest rate adjustment needed in order to control the notional price.

Proposition 2 *Within the class of preferences of Section 2.3, in a closed-economy model, price stability (i.e. a policy of zero notional inflation) is the optimal policy under ex ante commitment. This allocation coincides with the flexible-price allocation.*

A related proof can be found in Goodfriend and King (2001).[8] This result can be intuited from the observation that there are two distortions in this model, price stickiness and monopoly power. The latter produces an inefficient level of output. Under ex ante commitment, the monetary policy-maker binds itself not to 'inflate' the economy in a systematic way. The other remaining distortion is price stickiness which prevents efficient adjustment to the disturbances that affect the economy. A procyclical monetary policy can remove the sticky-price distortion by making production supply-determined, and then achieving the constrained-efficient equilibrium.[9] This equilibrium is constrained-efficient since output is inefficiently low due to the monopolistic distortions.

The optimal allocation can be implemented by setting the notional price equal to the actual average price in all contingencies and at all times. However, the optimal policy does not pin down the optimal rate of actual inflation. Indeed, in a model in which prices are all fixed one period in advance, positive expected inflation rate brings no explicit costs.

These results hold even when prices follow a partial adjustment rule *à la Calvo.* Sutherland (2001) has shown in a numerical analysis that long-run price stability turns out to be optimal. [10]

Under discretion, the policy-maker maximises welfare at a generic time t, subject to the incentive compatibility constraints given by (7) from periods $t+1$ onwards.

[8] In a model in which transaction services are not negligible, Adao *et al* (2001) have studied the conditions under which the flexible-price allocation is constrained efficient under a general class of utility functions and shocks. In their paper they focus on the commitment case in a closed economy framework.

[9] By applying an argument familiar to the theory of uniform optimal taxation, constant elasticities are necessary for mark-up constancy to be optimal. With time-varying elasticities and with public expenditure shocks, proposition 2 does not apply, as in Adao *et al* (2001).

[10] Sutherland (2001a) exploits second-order approximations to the structural equilibrium conditions, conditional on an appropriate policy rule in terms of money supply. King and Wolman (1999) reach a similar conclusion with contracts à la Taylor. Some qualifications should be added when the commitment is not taken from a timeless perspective. In this case, some transitional- dynamic issues arise due to temporary incentives to inflate the economy, as some prices are set before the commitment is taken.

The optimality condition at time t changes into

$$U_C(Y_t) = V_y(Y_t, z_t). \tag{10}$$

Once prices are fixed, a policy-maker that acts under discretion has an incentive to inflate and to push output towards a competitive level, thus creating an inflationary bias, as in the well-known Barro-Gordon argument. In general, the flexible-price allocation is not optimal. Our price stability argument applies only when the monopolistic distortions are completely offset by an appropriate taxation subsidy. This is indeed the only rational-expectation equilibrium with prices all fixed one period in advance. We note that this result is specific to the one-period price contracts because there are no costs of inflation. Under a staggered price-setting mechanism, a positive inflation rate is costly because it induces an inefficient distribution of production across firms, given that producers readjust their prices at different times. In this case, it is still true that the only equilibrium that implements the flexible-price allocation occurs when the monopolistic distortions are completely offset by a taxation subsidy and the equilibrium inflation rate is zero. Exactly around this point it is possible to evaluate a quadratic approximation of a utility-based welfare criterion by relying only on a log-linear approximation of the structural equilibrium conditions, as shown in Woodford (1999). On the other hand, with different levels of monopolistic distortions, rational-expectation equilibria exist and are characterized by a positive inflation bias.[11]

4 Price stability as an efficient equilibrium in open economies

We now analyse the open economy case. When prices are set one period in advance, the optimal price choice for period t maximises the expected value of (2) using information at time $t-1$, i.e. $\mathrm{E}_{t-1}\Psi_t$. The optimality condition requires that

$$\mathrm{E}_{t-1}\left\{\left[(1-\Phi)U_C(C_t)\frac{P_{H,t}}{P_t} - V_y\left(\left(\frac{P_{H,t}}{P_t}\right)^{-\theta} C_t, z_t\right)\right]\left(\frac{P_{H,t}}{P_t}\right)^{-\theta} C_t\right\} = 0, \tag{11}$$

[11] With Taylor's contract, Khan et al. (2001) show that there are multiple equilibria.

for country H, while

$$\mathrm{E}_{t-1}\left\{\left[(1-\Phi^*)U_C(C_t)\frac{P^*_{F,t}}{P^*_t}-V_y\left(\left(\frac{P^*_{F,t}}{P^*_t}\right)^{-\theta}C_t,z^*_t\right)\right]\left(\frac{P^*_{F,t}}{P^*_t}\right)^{-\theta}C_t\right\}=0 \qquad (12)$$

for country F.

First, we examine the conditions under which the flexible-price allocation is the constrained efficient policy. In particular we focus on the central planner's problem in maximising a weighted average of the expected utilities of home and foreign consumers

$$E_{t-1}\left\{nW_t+(1-n)W^*_t\right\}, \qquad (13)$$

where the weights are given by n and $1-n$. Another way to look at this issue is to ask whether price stability is the efficient policy. Following our closed-economy example, price stability is defined as the equalisation between notional producer price and the average actual producer price in a country. The notional producer price, $p^N_{H,t}$, is defined by

$$(1-\Phi)U_C(C_t)\frac{p^N_{H,t}}{P_t}=V_y\left(\left(\frac{p^N_{H,t}}{P_{H,t}}\right)^{-\sigma}\left(\frac{P_{H,t}}{P_t}\right)^{-\theta}C_t,z_t\right),$$

for country H, while $p^{*N}_{F,t}$, is defined by

$$(1-\Phi^*)U_C(C_t)\frac{p^{*N}_{F,t}}{P^*_t}=V_y\left(\left(\frac{p^{*N}_{F,t}}{P^*_{F,t}}\right)^{-\sigma}\left(\frac{P^*_{F,t}}{P^*_t}\right)^{-\theta}C_t,z^*_t\right),$$

for country F.

From the above conditions, one can observe that a policy of price stability in both countries implements the flexible-price allocation, as described by equations (3) and (4). Under an ex ante commitment solution, the efficient allocation is obtained by maximising (13) under the constraints given by (11), (12) and the price-index constraint

$$1=n\left(\frac{P_{H,t}}{P_t}\right)^{1-\theta}+(1-n)\left(\frac{P^*_{F,t}}{P^*_t}\right)^{1-\theta}, \qquad (14)$$

where we have used the law of one price and the assumption of symmetric preferences, i.e. $P^*_{F,t}/P^*_t=P_{F,t}/P_t$.

Proposition 3 *When shocks are symmetric, i.e.* $z_t = z_t^*$ *in all contingencies and at all times, price stability in both countries, i.e. the flexible price allocation, is always constrained efficient. When shocks are asymmetric, price stability in each country is always constrained efficient if* Φ=Φ^*; *otherwise it should be either* $\theta = 1$ *or* $\theta = \rho^{-1}$ *for any given* Φ *and* Φ^*.

Proof. Please refer to the Appendix ■

Even if the constrained-efficient equilibrium has been extensively studied in the literature, this proposition adds further insights into the conditions that have to be satisfied in order for the flexible-price allocation to be efficient. Our first result states that the flexible price allocation is the efficient response to common (i.e. symmetric) shocks. This result confirms previous findings (see Obstfeld and Rogoff, 2002).

New results emerge when we consider asymmetric shocks. In the producer-currency pricing model, Devereux and Engel (2000) and Obstfeld and Rogoff (2002) found that the flexible-price allocation is *always* efficient, independent of the degrees of monopolistic competition.[12] The common crucial assumption is the unitary intratemporal elasticity of substitution, θ.

By relaxing this assumption, we show that the flexible-price allocation is constrained efficient when the overall degrees of monopolistic distortions are equalised across countries. Otherwise, it is required that either $\theta = 1$ or $\theta = \rho^{-1}$. In such cases, price stability reproduces the flexible price allocation, and the exchange rate moves in order to accommodate asymmetric shocks.

As a first step, we explain why condition Φ=Φ^* is required for the flexible price allocation to be efficient.[13]

[12]However, Obstfeld and Rogoff (2002) have shown that their result breaks down when they assume an intertemporal elasticity of substitution in consumption, ρ, different from the unitary value. Since they assume non-tradable goods in the consumption index, $\rho \neq 1$ implies imperfect consumption risk-sharing at an international level. However, in the absence of non-tradable goods, their result holds even if $\rho \neq 1$. Corsetti and Pesenti (2001b) and Devereux and Engel (2000) have shown that with local currency pricing, the flexible-price allocation is not achievable in the centralised equilibrium.

[13]Note that not only do Φ and Φ^* matter, but also υ, the inverse of the elasticity of labor supply. When there are asymmetries in the elasticity of labor supply across countries, then Φ and Φ^* should be adjusted to characterize in an appropriate way the relevant conditions.

We observe that equations (11) and (12) can be written as

$$\mathrm{E}_{t-1}\{V(Y_{H,t}, z_t)\} = \frac{(1-\Phi)}{\upsilon}\mathrm{E}_{t-1}\left\{U_C(C_t)C_t\left(\frac{P_{H,t}}{P_t}\right)^{1-\theta}\right\}, \tag{15}$$

$$\mathrm{E}_{t-1}\{V\left(Y^*_{F,t}, z^*_t\right)\} = \frac{(1-\Phi^*)}{\upsilon}\mathrm{E}_{t-1}\left\{U_C(C_t)C_t\left(\frac{P^*_{F,t}}{P^*_t}\right)^{1-\theta}\right\}, \tag{16}$$

where we have used the isoelastic-function assumption and the fact that $Y_H = (P_{H,t}/P_t)^{-\theta}C_t$ and $Y^*_F = (P^*_{F,t}/P^*_t)^{-\theta}C_t$.

The crucial point of our intuition is the existence of a terms of trade externality that arises even from a cooperative perspective. This terms of trade externality is different in nature from the one analyzed by Corsetti and Pesenti (2001a). In their context, policy-makers can systematically affect the economy through unanticipated movements in the money supply. In our model, under commitment, the world social planner is not allowed to such systematic changes. However, uncertainty affects the expected value of variables even under commitment.[14] When $\Phi \neq \Phi^*$ and in the absence of shocks, the steady-state level of output is different across countries. There is then an additional distortion to cope with and the world social planner can indeed exploit the terms of trade volatility to correct for the different expected level of the disutility of producing goods across countries, as shown in (15) and (16). The flexible-price allocation is in general not efficient except when $\Phi = \Phi^*$, since this additional distortion disappears.

In the case $\Phi \neq \Phi^*$, it is required that either $\theta = 1$ or $\theta = \rho^{-1}$.

If $\theta = 1$, there is no role for the terms of trade in correcting the structural distortion since the ratio of the expected disutilities is constant

$$\frac{\mathrm{E}_{t-1}\{V(Y_{H,t}, z_t)\}}{\mathrm{E}_{t-1}\{V\left(Y^*_{F,t}, z^*_t\right)\}} = \frac{(1-\Phi)}{(1-\Phi^*)}.$$

When $\theta = \rho^{-1}$ domestic output is the only relevant variable for the analysis of the stabilization problem in each economy, for the two economies are insular.[15]

[14] As it has been emphasized by Henderson and Kim (1999) and Obstfeld and Rogoff (1998).

[15] Note that spillover effects from foreign shocks still exist on domestic consumption.

Conditions (11) and (12) can be written as

$$\mathrm{E}_{t-1}\{V\left(Y_{H,t}, z_t\right)\} = \frac{(1-\Phi)}{\upsilon}\mathrm{E}_{t-1}\left\{Y_{H,t}^{\frac{\theta-1}{\theta}}\right\},$$
$$\mathrm{E}_{t-1}\{V\left(Y_{F,t}^{*}, z_t^{*}\right)\} = \frac{(1-\Phi^{*})}{\upsilon}\mathrm{E}_{t-1}\left\{Y_{F,t}^{\frac{\theta-1}{\theta}}\right\}.$$

The utility with respect to the consumption index C becomes separable into C_H and C_F and then into Y_H and Y_F^* so that the social welfare function (13) boils down to

$$E_{t-1}\left\{n[U(Y_{H,t}) - V\left(Y_{H,t}, z_t\right)] + (1-n)[U(Y_{F,t}) - V\left(Y_{F,t}, z_t\right)]\right\}.$$

The social planner chooses Y_H and Y_F optimally. Furthermore, with isoelastic preferences, the real marginal costs in each country become proportional to the respective output gaps, and the home and foreign output gaps can be controlled directly by the deviations of the respective notional producer price with respect to the average actual price.

When the conditions stated in proposition 3 are not met, in general, the efficient equilibrium might require variable mark-ups at a country level. In these cases, the optimal allocation under sticky prices improves upon the flexible price allocation, which is still feasible but not longer optimal. In general, a policy of state-contingent notional-price inflation is optimal although we do not quantify its dimension here.

Our result is related to the one obtained by Adao *et al.* (2001) in a closed-economy framework. They characterise the conditions under which the flexible price allocation is optimal and show that in general the optimal policy under sticky prices dominates the flexible price one. In our open economy framework, departures from price stability arise even without assuming transaction frictions, public expenditure shocks or more general preferences.

5 Price stability as a Nash equilibrium

5.1 Commitment

In this section, we analyze the strategic interaction between the two policy-makers. Our objective is to characterise the conditions under which price stability can be implemented in a decentralised setting. An underlying important question is to see whether there are gains from cooperation.

To this end, it is crucial to specify the strategy space of each policy-maker. We assume that each policy-maker can set her policy in terms of the ratio of the notional price with respect to the average actual price. So country H controls the ratio $p_{H,t}^{N}/P_{H,t}$ while country F controls $p_{F,t}^{*N}/P_{F,t}^{*}$.

At a first pass, and similarly to Devereux and Engel (2000) and Obstfeld and Rogoff (2002), we analyse the case in which both policy-makers commit to the chosen policy. Conditions (11) and (12) act as incentive compatibility constraints.

Proposition 4 *Within the class of preferences of Section 2.3, when shocks are symmetric, i.e.* $z_t = z_t^*$ *in all contingencies and at all times, price stability is always a Nash equilibrium, under ex ante commitment solution. When shocks are asymmetric, the strategy of price stability in both countries is a Nash equilibrium under ex ante commitment if either* $\theta = 1$ *or* $\theta = \rho^{-1}$ *for any given* Φ *and* Φ^*.

Proof. Please refer to the appendix. ■

This proposition revises the findings of Obstfeld and Rogoff (2002). It is no longer true that there is a one-to-one correspondence between the conditions under which the flexible price allocation is efficient and those under which it can be implemented as a Nash equilibrium under ex ante commitment –within our class of strategies. Comparing propositions (3) and (4), we note that the conditions that characterize price stability in a decentralized setting are a subset of those that hold in a centralized setting. The scenario in which $\Phi = \Phi^*$ is, therefore, no longer sufficient to implement the flexible-price allocation.

The intuition for this result is simple: even if $\Phi = \Phi^*$, each policy-maker tries to exploit the terms of trade volatility to reduce her expected disutility of producing goods, when possible, without internalizing the negative externality on the other country as shown in equations (15) and (16). As in the case analyzed before, when $\theta = 1$ the terms of trade channel is ineffective while when $\theta = \rho^{-1}$ the economies are insular with respect to terms of trade movements. In these cases, there is mutual agreement between the two policy-makers on stabilizing the economy at the flexible price allocation. Furthermore, when $\theta = \rho^{-1}$, the resulting Nash equilibrium is in dominant strategies, within the class of strategies assumed. This depends on the fact that the country-specific output gap can be controlled directly by the strategy of the respective policy-maker without any link to the strategy of the other policy-maker.[16]

[16] With more general preference specifications (i.e. ρ and ν that differ across countries), the results

Our results point towards the conclusion that Nash and cooperative solutions do not coincide even when financial and goods markets become more integrated. In general, as we have discussed, the incentives to internalise externalities differ whether one looks at the problem from a centralised or a decentralised perspective. Only under special 'knife-edge' conditions can self-oriented policy rules implement the flexible price allocation. There are then gains from cooperation even under complete markets.[17]

The issue of characterizing possible equilibria when these special conditions are not satisfied is an important topic for future research. When $\theta \neq 1$, a closed-form solution based on log-normal shocks is no longer available and there is a need to rely on approximations of the welfare and of the structural equilibrium condition in order to characterize the optimal policy functions. Steps in this direction have been taken by Sutherland (2002).

As a further difference with respect to the previous literature, the strategies that have been discussed in this study are specified in terms of notional producer prices, without any reference to shocks affecting the economy, whether they originate from the Home or Foreign economy. On the other hand, it should be noted that there are different strategies that can implement the efficient allocation as a Nash equilibrium, which, however, might require each monetary authority to respond to the shocks of the other country. In their analysis, Devereux and Engel (2000) and Obstfeld and Rogoff (2002) use money rules that react to domestic and foreign shocks, while Corsetti and Pesenti (2001b) adopt strategies in terms of nominal spending.

Devereux and Engel (2000) and Corsetti and Pesenti (2001b) have further shown that with local currency pricing there are gains from cooperation. In a model with wage stickyness and imperfect pass-through, Obstfeld (2002) has proposed an interesting case in which both the flexible-wage allocation is efficient and feasible and there are no gains from cooperation. Thus, full pass-through is neither a necessary condition to guarantee the replication of the allocation without wage and price frictions.

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apply with some qualifications. It then follows that condition $\theta = 1$ is still valid for any ρ, ρ^* and ν, ν^* while it should be the case that $\theta\rho = \theta\rho^* = 1$ for any ν, ν^*.

[17] These results go through in a model with Calvo's style price-setting behavior as shown in Sutherland (2001). He further quantifies that the gains from coordination are not small when the intratemporal elasticity of substitution is non-unitary.

5.2 Discretion

Policy-makers that act under discretion re-optimise in each period, taking as given the constraint implied by the optimal price-setting choice. As in the closed-economy model, in general, the allocation that results from the strategic game in which policy-makers act with discretion is not the flexible-price allocation. Unlike the closed economy case, the open economy discretionary equilibrium that enforces the flexible-price allocation involves a positive degree of monopolistic distortion.[18]

Proposition 5 *Within the class of preferences of Section 2.3, when* $\theta = 1$, *the strategy of price stability is a time-consistent Nash equilibrium if and only if* $\Phi = \overline{\Phi}$ *and* $\Phi^* = \overline{\Phi}^*$, *with*

$$\overline{\Phi} = \frac{(1-n)n^{-1\frac{(\rho+\eta)}{(1+\eta)}}}{1+(1-n)n^{-1\frac{(\rho+\eta)}{(1+\eta)}}} \qquad \overline{\Phi}^* = \frac{(1-n)^{-1}n^{\frac{(\rho+\eta)}{(1+\eta)}}}{1+(1-n)^{-1}n^{\frac{(\rho+\eta)}{(1+\eta)}}}.$$

In the case $\theta = \rho^{-1}$, *the strategy of price stability is a time-consistent Nash equilibrium if and only if* $\Phi = \overline{\overline{\Phi}}$ *and* $\Phi^* = \overline{\overline{\Phi}}^*$ *with*

$$\overline{\overline{\Phi}} = 1 - n \qquad \overline{\overline{\Phi}}^* = n.$$

Under these assumptions, this result holds even under a Calvo-style price-setting mechanism.

Proof. Please refer to the appendix. ■

A simple intuition explains why the flexible-price equilibrium can only be supported at a positive level of monopolistic competition. As in the closed economy case, monopolistic competition is associated with an inflationary-biased policy. However, in an open-economy framework, as shown by Corsetti and Pesenti (2001a) and Tille (2001), each policy-maker also faces a deflationary-biased policy, due to the incentive to manipulate the terms of trade in her favour. Each policy-maker would normally try to generate a surprise deflation. Indeed, a contractionary monetary policy in a country decreases consumption and causes the exchange rate, and then the terms of trade, to appreciate. Thus through the expenditure-switching effect, production decreases within the country and increases abroad. The reduction in utility

[18] With prices fixed one period in advance and consistent with Betts and Devereux (2000), there is no finite discretionary equilibrium inflation rate for other parameter values.

that comes from the decrease in consumption could potentially be more than offset by the reduction in the disutility of producing goods. This deflationary bias can also be welfare-improving. There is a point, characterized by positive monopolistic distortions, at which the inflationary and deflationary incentives balance exactly. This point is a function of the various elasticities of substitution and, most importantly, of the economic size of a country.[19]

When $\theta = \rho^{-1}$, price stability is a dominant strategy. In this case, if $\Phi \neq \overline{\overline{\Phi}}$ the policy-maker in country H has an incentive to inflate or to deflate depending on Φ being above or below, respectively, $\overline{\overline{\Phi}}$. If $\theta = 1$, however, the inflationary or deflationary-bias argument applies, provided the equilibrium strategy of the policy-maker of country F is taken as given.

It is worth stressing that other strategies might exist that implement the flexible-price allocation as a Nash equilibrium, in a discretionary way. These strategies (e.g. money rules) do not necessarily require the same degree of monopolistic distortions as in our proposition. Nor do they imply that conditions (3) and (4) can be taken as given in the strategic game. [20]

As in the closed-economy Calvo-style model, the conditions that characterize price stability as a discretionary equilibrium are exactly the ones that allow a quadratic approximation of the welfare of the single countries to be evaluated by relying only on a log-linear approximation to the structural equilibrium conditions.

First, we discuss the case in which $\theta = \rho^{-1}$, assuming that prices are set according to a partial adjustment rule *à la Calvo*.[21] As we have underlined in the previous section, when $\theta = \rho^{-1}$ the country-specific real marginal costs are proportional to

[19]It is worth noting that the conditions on Φ and Φ^* are similar to the conditions that characterise the absence of the incentive to strategically use terms of trade in the perfect foresight models of Corsetti and Pesenti (2001a), Tille (2001) and Benigno (2001a). However, in these frameworks the set of strategies comprise only unexpected and exogenous movements in the money supply.

[20]With log-utility in consumption and linear disutility in labour, Corsetti and Pesenti (2001b) have shown, along these lines, that a strategy expressed in terms of nominal spending can implement the flexible price allocation in a discretionary equilibrium in dominant strategies. With local currency pricing, they have further shown that a policy-maker acting under discretion would not find it optimal to replicate the flexible price allocation, since this implies excessive variation of the exchange rate.

Betts and Devereux (2000) examine the problem of international monetary cooperation in a local currency-pricing model with perfect foresight in which policymakers act under discretion.

[21]Details can be found in the appendix, as well as in Benigno and Benigno (2001a,b).

their respective output gap. When Φ and Φ^* are equal to $\overline{\overline{\Phi}}$ and $\overline{\overline{\Phi}}^*$ respectively, the quadratic approximation of the welfare of each country can be written as

$$W_t = -\Lambda \mathrm{E}_t \left\{ \sum_{\tau=t}^{\infty} \beta^{\tau-t} [\varphi y_{H,\tau}^2 + \pi_{H,\tau}^2] \right\} \qquad W_t^* = -\Lambda^* \mathrm{E}_t \left\{ \sum_{\tau=t}^{\infty} \beta^{\tau-t} [\psi^* y_{F,\tau}^2 + \pi_{F,\tau}^{*2}] \right\},$$

where y_j and π_j are the country's j output gap and inflation rate; Λ, Λ^*, φ and φ^* depend on the structural parameters of the model. A log-linear approximation of the Calvo-style price-setting mechanism delivers two aggregate supply (AS) equations of the form

$$\pi_{H,t} = k y_{H,t} + \beta \mathrm{E}_t \pi_{H,t+1} \qquad \pi_{F,t}^* = k^* y_{F,t}^* + \beta \mathrm{E}_t \pi_{F,t+1}^*$$

where k and k^* are functions of the structural parameters of the model. Under the assumption $\theta = \rho^{-1}$, each country can control its own output gap by specifying a path for the inflation rate. Moreover, there is no trade-off between stabilising the output gap and the producer inflation rate in each country. It is then the case that a strategy of zero producer inflation is a Nash equilibrium in dominant strategies. The conditions on Φ and Φ^* are required to eliminate any first-order incentive to inflate or deflate, which arises independently of the stabilisation problem.

If the assumption $\theta = \rho^{-1}$ is not satisfied, we cannot completely separate the maximisation problems of the two policy-makers.

In general, as shown in Benigno and Benigno (2001b), there is no direct relation between the output gap and the producer inflation rate. The two AS equations can be written as

$$\pi_{H,t} = k_H[(1-n)(1+\eta\theta)q_t + (\rho+\eta)y_t^W] + \beta E_t \pi_{H,t+1}, \tag{17}$$

$$\pi_{F,t}^* = k_F[-n(1+\eta\theta)q_t + (\rho+\eta)y_t^W] + \beta E_t \pi_{F,t+1}^*, \tag{18}$$

where q is the deviation of the terms of trade from the flexible-price allocation and y^W is the world output gap; η, k_H and k_F are parameters of the model. However, under the assumption $\theta = 1$, further interesting results can be obtained. In this case, *given the strategy of a zero inflation rate in one country*, e.g. country F, we can write a proportional relation between the terms of trade gap and the output gap, by using equation (18):

$$q_t = \frac{(\rho + \eta)}{n(1+\eta)} y_t^W. \tag{19}$$

Equation (19) holds as an *exact* condition, under the hypothesis of isoelastic preferences and the assumption $\theta = 1$. Given the strategy of zero producer inflation in country F, we can write the AS equation of country H as

$$\pi_{H,t} = k_H(1-n)n^{-1}[(\rho + \eta)y_t^W] + \beta E_t \pi_{H,t+1}. \tag{20}$$

Under such conditions, it is also possible to write the approximation of the welfare of country H as

$$W_t = -\Sigma \mathrm{E}_t \left\{ \sum_{\tau=t}^{\infty} \beta^{\tau-t}[\lambda_W (y_\tau^W)^2 + \pi_{H,\tau}^2] \right\}, \tag{21}$$

where Σ and λ_W are functions of the structural parameters of the model. The result that (19) holds in an exact form is crucial, combined with the assumption that $\Phi = \overline{\Phi}$. In fact, were (19) holding in a first-order approximation, then second-order terms would be crucial in evaluating a second-order expansion of the utility function. A second-order expansion of the structural equilibrium conditions would be needed. Instead, under the above assumptions, (21) implies that, given the zero producer inflation strategy in country F, there is no trade-off between stabilising the Home producer inflation and the world output gap. A strategy of zero producer inflation in the home country maximises (21) under (20). Then, under the conditions stated above, the strategy of zero-producer inflation in both countries is a Nash equilibrium.

Along these lines, Clarida et al (2001) have shown that the stabilization problem in a small open-economy can be reconducted to be isomorphic to the closed-economy case previously analyzed in Clarida et al (1999). Although appealing, all these results hinge on special assumptions. Beside the analysis of equilibria with price stability, strategic and stabilization problems in open economies can be solved by exploiting other tools, e.g. second-order approximations to the structural equilibrium conditions. Progress in this direction has been made by Sims (2000) and Sutherland (2001).

6 Conclusion

In this paper we have analysed the conditions under which price stability arises as an equilibrium outcome in open economies. We show that the flexible price allocation is not efficient unless special conditions are met. In general, the degrees of monopolistic distortion need to be equalised across countries. Otherwise, special values for the intratemporal elasticity of substitution are required.

The analysis of non-cooperative solutions suggests that price stability, in the special sense employed in this paper, is unlikely to emerge as an equilibrium, even in the restricted cases where it is efficient. Price stability as a Nash equilibrium under ex ante commitment relies on a subset of the conditions under which price stability is efficient. In particular, the scenario in which the degree of monopolistic distortions is equalised across countries is not sufficient in implementing price stability as a decentralised equilibrium. Under discretion, there is even less scope for an equilibrium to exist. Only one finite discretionary equilibrium exists, supported by a specific value of monopolistic distortions. At this value, inflationary and deflationary policy biases offset each other conditionally on the equilibrium strategy of the other policy-maker.

By focusing on price stability, the important lesson our study provides is that non-cooperative Nash equilibria only coincide with cooperative ones under *special* circumstances. There are, of course, other ways to achieve optimal allocation. Given our focus, one conclusion is that gains from international cooperation may be possible, even if markets are complete and producer currency pricing holds.

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Appendix

Cashless economy In this appendix, we discuss the meaning of a cashless-limiting economy that we use in the model. We consider a generic utility function for the representative household of country H that also includes the utility derived from real money balance in an additive way:

$$U_t^j = \mathrm{E}_t \left\{ \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[U(C_\tau) + \chi L(M_\tau/P_\tau) - V(Y_{H,\tau}, z_\tau) \right] \right\},$$

where χ indicates the importance of the utility derived from the liquidity service of holding money with respect to the other terms in the utility function; M denotes the money holding, while $L(.)$ is an increasing concave function of the real money balances which displays satiation at a determined level of real money balance. We interpret a cashless-limiting economy as the case in which χ goes to zero. To counteract this interpretation, we show that each country's monetary policy-maker can control her notional price level by moving the money supply; and that, as χ goes to zero, the utility derived from the real money balances decreases with respect to the other terms in the utility function.

Recalling the equations that implicitly define the notional price levels in both countries

$$(1-\Phi) U_C(C_t) \frac{p_{H,t}^N}{P_t} = V_y \left(\left(\frac{p_{H,t}^N}{P_{H,t}} \right)^{-\sigma} \left(\frac{P_{H,t}}{P_t} \right)^{-\theta} C_t, z_t \right),$$

$$(1-\Phi^*) U_C(C_t) \frac{p_{F,t}^{*N}}{P_t^*} = V_y \left(\left(\frac{p_{F,t}^{*N}}{P_{F,t}^*} \right)^{-\sigma} \left(\frac{P_{F,t}^*}{P_t^*} \right)^{-\theta} C_t, z_t^* \right),$$

and the restriction on the relative prices implied by the consumption-based price indexes

$$1 = n \left(\frac{P_{H,t}}{P_t} \right)^{1-\theta} + (1-n) \left(\frac{P_{F,t}^*}{P_t^*} \right)^{1-\theta},$$

we can write

$$\begin{aligned} C_t &= F_{1,t} \left(\frac{p_{H,t}^N}{P_{H,t}}, \frac{p_{F,t}^{*N}}{P_{F,t}^*}, z_t, z_t^* \right), \\ \frac{P_{H,t}}{P_t} &= F_{2,t} \left(\frac{p_{H,t}^N}{P_{H,t}}, \frac{p_{F,t}^{*N}}{P_{F,t}^*}, z_t, z_t^* \right), \\ \frac{P_{F,t}^*}{P_t^*} &= F_{3,t} \left(\frac{p_{H,t}^N}{P_{H,t}}, \frac{p_{F,t}^{*N}}{P_{F,t}^*}, z_t, z_t^* \right). \end{aligned}$$

Substituting the above conditions into the equation that defines the optimal price for country H we obtain

$$\mathrm{E}_{t-1}\left\{\left[(1-\Phi)U_C(\digamma_{1,t}(.))\digamma_{2,t}(.)-V_y\left(\digamma_{2,t}(.)^{-\theta}\digamma_{1,t}(.),z_t\right)\right]\digamma_{2,t}(.)^{-\theta}\digamma_{1,t}(.)\right\}=0.$$

Once the strategy of the other policy-maker in terms of notional price with respect to the respective average actual price is taken as given, the average actual price in country H for time t is a function of the joint distribution of the notional prices expected for time t and the shocks z and z^*, with the information set of time $t-1$. Moreover $P_{H,t}$ is a homogeneous function of degree 1 in $p_{H,t}^N$.

Considering the Euler equation in the home economy

$$\frac{U_C(C_t)}{P_{H,t}}\frac{P_{H,t}}{P_t}=\beta(1+i_t)\mathrm{E}_t\left\{\frac{U_C(C_{t+1})}{P_{H,t+1}}\frac{P_{H,t+1}}{P_{t+1}}\right\}$$

we can write it as

$$\frac{U_C(\digamma_{1,t}(.))}{P_{H,t}}\digamma_{2,t}(.)=\beta(1+i_t)\mathrm{E}_t\left\{\frac{U_C(\digamma_{1,t}(.))}{P_{H,t+1}}\digamma_{2,t}(.)\right\} \tag{A.1}$$

where one can see how the interest rate should be brought about to obtain the desired path of the notional price, once the strategy of the other policy-maker is taken as given. We can also derive the money demand equation associated with the utility function above as

$$\frac{\chi L_M(M_t/P_t)}{U_C(C_t)}=\frac{i_t}{1+i_t}$$

which can be rewritten in a more familiar form as

$$\frac{M_t}{P_t}=\chi\Gamma(C_t,i_t)$$

where Γ is an increasing function of C and decreasing in i. The above equation can be also rewritten in the form

$$\frac{M_t}{P_{H,t}}=\frac{P_t}{P_{H,t}}\chi\Gamma(C_t,i_t), \tag{A.2}$$

in which we can substitute for C_t, $P_{H,t}/P_t$ and i_t the respective functions of the home notional price, the strategy of the policy-maker of country F and the shocks z and z^*. By using (A.2) at time t and for the subsequent periods, we can derive the path of money supply needed in order to control the home notional price, for the given strategy of the other policy-maker. Once the policy-maker has chosen a desired path of notional prices, as χ goes to zero the path of money needed in order to sustain the desired path of notional prices varies. The level of money decreases in all periods. However, the paths of C and Y_H do not change. It follows that as χ goes to zero, the utility derived from the liquidity services given by money greatly decreases with respect to the other terms in the utility function for the desired path of notional prices. This is the interpretation we refer to when, in the text, we neglect the additional terms given by the utility derived from real money balances.

Proof of Propositions In what follows we define:

$$V(H_t) \equiv V\left(\left(\frac{P_{H,t}}{P_t}\right)^{-\theta} C_t, z_t\right) \qquad V(F_t) \equiv V\left(\left(\frac{P_{F,t}}{P_t}\right)^{-\theta} C_t, z_t^*\right),$$

$$\Pi_{H,t} \equiv \frac{P_{H,t}}{P_t} \qquad \Pi_{F,t} \equiv \frac{P_{F,t}}{P_t} = \frac{P^*_{F,t}}{P^*_t}.$$

Proof of Proposition 3.

In the efficient allocation, the central planner maximizes the welfare

$$\mathrm{E}_{t-1}\left\{\sum_{\tau=t}^{\infty} \beta^{\tau-t}\left[U(C_\tau) - nV(H_t) - (1-n)V(F_t)\right]\right\}, \tag{A.3}$$

under the constraints

$$\mathrm{E}_{t-1}\left\{\left[(1-\Phi)U_C(C_t)\Pi_{H,t} - V_y(H_t)\right]\Pi_{H,t}^{-\theta}C_t\right\} = 0, \tag{A.4}$$

$$\mathrm{E}_{t-1}\left\{\left[(1-\Phi^*)U_C(C_t)\Pi_{F,t} - V_y(F_t)\right]\Pi_{F,t}^{-\theta}C_t\right\} = 0, \tag{A.5}$$

for each time t and the constraints

$$1 = n\Pi_{H,t}^{1-\theta} + (1-n)\Pi_{F,t}^{1-\theta}, \tag{A.6}$$

for each contingencies at each time t. Since there are no intertemporal linkages, we can simplify the analysis and look at the Lagrangian problem at a generic time t. We denote with $n \cdot \Gamma$ the Lagrangian multiplier associated with the constraint (A.4); $(1-n)\cdot\Upsilon$ is the Lagrangian multiplier associated with the constraint (A.5), while μ_t is the state-contingent Lagrangian multiplier associated with the constraint (A.6).

Taking the first-order condition with respect to C at a generic contingency at time t, we obtain

$$\begin{aligned} 0 = U_C(C_t) - n\Pi_{H,t}^{-\theta}V_y(H_t) - (1-n)\Pi_{F,t}^{-\theta}V_y(F_t) - n(1-\Phi)\Gamma U_C(C_t)\Pi_{H,t}^{1-\theta} + \\ n\Gamma\Pi_{H,t}^{-\theta}V_y(H_t) - n(1-\Phi)\Gamma U_{CC}(C_t)\Pi_{H,t}^{1-\theta}C_t + n\Gamma\Pi_{H,t}^{-\theta}V_{yy}(H_t)\Pi_{H,t}^{-\theta}C_t + \\ -(1-n)(1-\Phi^*)\Upsilon U_C(C_t)\Pi_{F,t}^{1-\theta} + (1-n)\Upsilon\Pi_{F,t}^{-\theta}V_y(F_t) + \\ -(1-n)(1-\Phi^*)\Upsilon U_{CC}(C_t)\Pi_{F,t}^{1-\theta}C_t + (1-n)\Upsilon\Pi_{F,t}^{-\theta}V_{yy}(F_t)\Pi_{F,t}^{-\theta}C_t, \end{aligned} \tag{A.7}$$

where V_y is the derivative of function V with respect to the first argument.

Taking the first-order condition with respect to Π_H at a generic contingency at time t, we obtain

$$\begin{aligned} 0 = \theta V_y(H_t)\Pi_{H,t}^{-1}C_t - (1-\theta)(1-\Phi)\Gamma U_C(C_t)C_t + \\ -\theta\Gamma V_y(H_t)\Pi_{H,t}^{-1}C_t + -\theta\Gamma V_{yy}(H_t)C_t\Pi_{H,t}^{-\theta-1}C_t - \mu_t(1-\theta). \end{aligned} \tag{A.8}$$

Taking the derivative with respect to Π_F at a generic contingency at time t, we obtain

$$\begin{aligned} 0 &= \theta V_y(F_t)\Pi_{F,t}^{-1}C_t - (1-\theta)(1-\Phi^*)\Upsilon U_C(C_t)C_t + \\ &-\theta\Upsilon V_y(F_t)\Pi_{F,t}^{-1}C_t + -\theta\Upsilon V_{yy}(F_t)C_t\Pi_{F,t}^{-\theta-1}C_t - \mu_t(1-\theta). \end{aligned} \tag{A.9}$$

Combining conditions (A.8) and (A.9), we get

$$\begin{aligned} & V_y(H_t)\Pi_{H,t}^{-1}[\theta - \theta\Gamma(1+\eta)] - (1-\Phi)(1-\theta)\Gamma U_C(C_t) \\ &= V_y(F_t)\Pi_{F,t}^{-1}[\theta - \theta\Upsilon(1+\eta)] - (1-\Phi^*)(1-\theta)\Upsilon U_C(C_t), \end{aligned} \tag{A.10}$$

while condition (A.7) can be written

$$\begin{aligned} & U_C(C_t)[1 - n(1-\Phi)(1-\rho)\Gamma\Pi_{H,t}^{1-\theta} - (1-n)(1-\Phi^*)(1-\rho)\Upsilon\Pi_{F,t}^{1-\theta}] \\ &= n\Pi_{H,t}^{-\theta}V_y(H_t)[1-\Gamma(1+\eta)] + (1-n)\Pi_{F,t}^{-\theta}V_y(F_t)[1-\Upsilon(1+\eta)]. \end{aligned} \tag{A.11}$$

We can then rewrite conditions (A.10) and (A.11) as

$$\begin{aligned} & \{1 - (1-\Phi)(1-\rho)\Upsilon + n\Theta\Pi_{H,t}^{1-\theta}\}U_C(C_t)\Pi_{F,t} \\ &\quad = [1-\Upsilon(1+\eta)]V_y(F_t) \end{aligned} \tag{A.12}$$

$$\begin{aligned} & \{1 - (1-\Phi^*)(1-\rho)\Gamma - (1-n)\Theta\Pi_{F,t}^{1-\theta}\}U_C(C_t)\Pi_{H,t} \\ &\quad = [1-\Gamma(1+\eta)]V_y(H_t). \end{aligned} \tag{A.13}$$

where

$$\Theta \equiv [(1-\rho) + (1-\theta)\theta^{-1}][(1-\Phi)\Upsilon - (1-\Phi^*)\Gamma].$$

Taking the ratio of (A.12) and (A.13), using the assumption of isoelastic preferences and combining with equation (A.6), it can be shown that if the shocks are symmetric, i.e. $z_t = z_t^*$ at all times and contingencies, then $\Pi_{H,t}$ and $\Pi_{F,t}$ are time-invariant. Actually, there is at least a solution in which $\Pi_{H,t} = \Pi_{F,t} = 1$. The time-invariant property of $\Pi_{H,t}$ and $\Pi_{F,t}$ implies in (A.12) and (A.13) that the flexible-price allocation is the optimal response to symmetric shocks. When the shocks are asymmetric, the flexible price allocation is optimal under certain conditions. Either $(1-\Phi)\Upsilon = (1-\Phi^*)\Gamma$, which is only possible if $\Phi = \Phi^*$, or $\theta = 1$ for any Φ, Φ^* or $\theta = \rho^{-1}$ for any Φ, Φ^*.

Proof of Proposition 4

Under ex ante commitment, we show that, given the strategy of price stability for the foreign policy-maker, the optimal strategy for the home policy-maker is price stability when appropriate conditions are satisfied. Under ex ante commitment, the home policy-maker maximises domestic agents' expected utility

$$\mathrm{E}_{t-1}\left\{\sum_{\tau=t}^{\infty}\beta^{\tau-t}\left[U(C_{\tau})-V\left(H_{\tau}\right)\right]\right\}, \tag{A.14}$$

under the sequence of incentive compatibility constraints given by the price-setting condition in the home country

$$\mathrm{E}_{t-1}\left\{\left[(1-\Phi)U_C(C_t)\Pi_{H,t}-V_y\left(H_t\right)\right]\Pi_{H,t}^{-\theta}C_t\right\}=0, \tag{A.15}$$

one for each date t, taking into account that the price-stability strategy of the foreign policy-maker implies

$$(1-\Phi^{*})U_C(C_t)\Pi_{F,t}=V_y\left(F_t\right), \tag{A.16}$$

in all contingencies and at all times and the usual constraint on price indexes

$$n\Pi_{H,t}^{1-\theta}+(1-n)\Pi_{F,t}^{1-\theta}=1, \tag{A.17}$$

in all contingencies and at all times. Since there are no intertemporal linkages, we can focus on the optimal condition at a generic time t. First, we analyse the Ramsey problem in which it is possible to choose freely C_t, $\Pi_{H,t}$, $\Pi_{F,t}$. Γ is the Lagrangian multiplier associated with the constraint (A.15), λ_t is the state contingent Lagrangian multiplier associated with the constraint (A.16) and μ_t is the lagrangian multiplier associated with the constraint (A.17).

Taking the first-order condition with respect to C_t, we obtain

$$\begin{aligned}0&=U_C(C_t)-\Pi_{H,t}^{-\theta}V_y(H_t)-\Gamma(1-\Phi)U_{CC}(C_t)C_t\Pi_{H,t}^{1-\theta}+\\ &\Gamma\Pi_{H,t}^{-\theta}V_{yy}(H_t)\Pi_{H,t}^{-\theta}C_t-\Gamma(1-\Phi)U_C(C_t)\Pi_{H,t}^{1-\theta}+\Gamma V_y(H_t)\Pi_{H,t}^{-\theta}+\\ &-\lambda_t(1-\Phi^{*})U_{CC}(C_t)\Pi_{F,t}+\lambda_t V_{yy}(F_t)\Pi_{F,t}^{1-\theta}.\end{aligned} \tag{A.18}$$

Taking the derivative with respect to $\Pi_{H,t}$ we obtain

$$\begin{aligned}0&=\theta\Pi_{H,t}^{-\theta-1}C_tV_y(H_t)-\Gamma(1-\Phi)(1-\theta)U_C(C_t)C_t\Pi_{H,t}^{-\theta}+\\ &-\theta\Gamma V_{yy}(H_t)C_t\Pi_{H,t}^{-\theta}C_t\Pi_{H,t}^{-\theta-1}-\theta\Gamma V_y(H_t)C_t\Pi_{H,t}^{-\theta-1}-(1-\theta)n\mu_t\Pi_{H,t}^{-\theta}.\end{aligned} \tag{A.19}$$

Taking the derivative with respect to $\Pi_{F,t}$ we obtain

$$\lambda_t(1-\Phi^{*})U_C(C_t)+\lambda_t\theta V_{yy}(F_t)C_t\Pi_{F,t}^{-\theta-1}+(1-n)(1-\theta)\mu_t\Pi_{F,t}^{-\theta}=0. \tag{A.20}$$

We can combine conditions (A.19) and (A.20), obtaining

$$\begin{aligned}&(\theta-\theta\Gamma-\theta\Gamma\eta)V_y(H_t)C_t\Pi_{H,t}^{-1}-\Gamma(1-\Phi)(1-\theta)U_C(C_t)C_t=\\ &-\frac{n}{1-n}(1+\theta\eta)\lambda_t\Pi_{F,t}^{\theta-1}V_y(F_t),\end{aligned} \tag{A.21}$$

where η is the inverse of the elasticity of substitution in the disutility of providing the goods. We can instead rewrite (A.18) as

$$[1+\Gamma(1-\Phi)(\rho-1)\Pi_{H,t}^{1-\theta}]U_C(C_t) = \Pi_{H,t}^{-\theta}V_y(H_t)[1-(1+\eta)\Gamma] - \lambda_t C_t^{-1}V_y(F_t)[\rho+\eta]. \tag{A.22}$$

Combining equations (A.21) and (A.22), we finally obtain

$$\frac{U_C(C_t)\Pi_{H,t}}{V_y(H_t)} = \frac{(1-\Gamma(1+\eta))[\Pi_{H,t}^{1-\theta} + \frac{1-n}{n}\theta\frac{\rho+\eta}{1+\theta\eta}\Pi_{F,t}^{1-\theta}]}{1+\Gamma(1-\Phi)[(\rho-1)\Pi_{H,t}^{1-\theta} + \frac{1-n}{n}\frac{\rho+\eta}{1+\theta\eta}(1-\theta)\Pi_{F,t}^{1-\theta}]}, \tag{A.23}$$

A similar condition can be obtained for the other country. Combining conditions (A.16), (A.17) and (A.23) and using the assumption of isoelastic preferences, it is possible to show that when the shocks are symmetric, i.e. $z_t = z_t^*$, then $\Pi_{H,t}$ and $\Pi_{F,t}$ are time-invariant. There is at least one time-invariant solution in which $\Pi_{H,t}$=$\Pi_{F,t} = 1$. It then follows that the Nash-equilibrium response to the shocks, when each country strategy is that of price stability, coincides with the response that arises under flexible price. If the shocks are asymmetric, it should be noted that the flexible price allocation in the home country implies that

$$(1-\Phi)U_C(C_t)\Pi_{H,t} = V_y(H_t). \tag{A.24}$$

Comparing condition (A.23) with (A.24), it can be shown that they coincide when either $\theta = 1$ or $\theta = \rho^{-1}$. Under these conditions, given that (A.24) is implied by a strategy of price stability in the home country, then price stability is a Nash equilibrium in a solution with ex ante commitment.

Proof of Proposition 5

Prices fixed one period in advance

Under discretion, we show that given the strategy of price stability for the foreign policy-maker, the optimal strategy for the home policy-maker is price stability when the appropriate conditions are satisfied.

In the discretionary equilibrium, at time t the domestic policy-maker re-optimises without taking into account the constraint (A.15). Once prices are fixed, the policy-maker maximises the utility at a generic time t

$$U(C_t) - V(H_t) + E_t\left\{\sum_{\tau=t+1}^{\infty}\beta^{\tau-t}\left[U(C_\tau) - V(H_\tau)\right]\right\}$$

under the constraints (A.16) and (A.17) in all contingencies and at each time t. Again we focus on a generic time t. λ_t is the state contingent Lagrangian multiplier associated with the constraint (A.16) and μ_t is the state-contingent Lagrangian multiplier associated with the constraint (A.17).

Taking the derivative of the Lagrangian with respect to C_t we obtain

$$U_C(C_t) = V_y(H_t)\Pi_{H,t}^{-\theta} + \lambda_t(1-\Phi^*)U_{CC}(C_t)\Pi_{F,t} - \lambda_t V_{yy}(F_t)\Pi_{F,t}^{-\theta}, \tag{A.25}$$

Taking the derivative with respect to $\Pi_{H,t}$ we obtain

$$\theta \Pi_{H,t}^{-\theta-1} C_t V_y(H_t) - (1-\theta)\mu_t n \Pi_{H,t}^{-\theta} = 0, \tag{A.26}$$

Taking the derivative with respect to $\Pi_{F,t}$ we obtain

$$-\lambda_t(1-\Phi^*)U_C(C_t) - \theta\lambda_t V_{yy}(F_t) C_t \Pi_{F,t}^{-\theta-1} - (1-n)\mu_t(1-\theta)\Pi_{F,t}^{-\theta} = 0. \tag{A.27}$$

Combining conditions (A.26) and (A.27), we get

$$\lambda V_y(F_t)\Pi_{F,t}^{-1}(1+\theta\eta) = -\frac{1-n}{n}\theta\Pi_{H,t}^{-1} C_t V_y(H_t)\Pi_{F,t}^{-\theta},$$

that can be used in (A.25) to obtain

$$U_C(C_t)\Pi_{H,t} = V_y(H_t)\left[\Pi_{H,t}^{1-\theta} + \frac{1-n}{n}\theta\left(\frac{\rho+\eta}{1+\theta\eta}\right)\Pi_{F,t}^{1-\theta}\right]. \tag{A.28}$$

Now, the flexible price condition would instead require

$$(1-\Phi)U_C(C_t)\Pi_{H,t} = V_y(H_t). \tag{A.29}$$

Comparing conditions (A.28) and (A.29), one can see that they will coincide, if $\theta = 1$ when Φ is equal to $\overline{\Phi}$ where

$$\overline{\Phi} = \frac{(1-n)n^{-1}\frac{(\rho+\eta)}{(1+\eta)}}{1+(1-n)n^{-1}\frac{(\rho+\eta)}{(1+\eta)}}.$$

Instead, when $\theta\rho = 1$, Φ should be such that

$$\overline{\overline{\Phi}} = 1-n.$$

Now a strategy of price stability, i.e. notional prices equal to the average actual price in all contingencies, implies (A.29). It is then the optimal strategy given the price-stability strategy of the policy-maker in countries F. Doing the same steps for country F, one can see that the following conditions are required. When $\theta = 1$, Φ^* should be equal to $\overline{\Phi}^*$ where

$$\overline{\Phi}^* = \frac{(1-n)^{-1}n\frac{(\rho+\eta)}{(1+\eta)}}{1+(1-n)^{-1}n\frac{(\rho+\eta)}{(1+\eta)}},$$

instead, when $\theta\rho = 1$,Φ^* should be such that

$$\overline{\overline{\Phi}}^* = n.$$

Calvo-style price-setting mechanism

We now show that this proposition can be extended under a more general price-setting mechanism. We consider a partial adjustment mechanism *á la* Calvo, in which each seller faces a fixed probability $1-\alpha$ of changing its price at a certain date t independently of the time that has elapsed since its last adjustment. In this case the optimal pricing decision of a home firm that is able to change its price $\widetilde{p}_t(h)$ at a generic time t is

$$\mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\beta)^k \left\{ \left[(1-\Phi) U_C(C_{t+k}) \frac{\widetilde{p}_t(h)}{P_{H,t+k}} \left(\frac{P_{H,t+k}}{P_{t+k}} \right) - V_y(\widetilde{y}^d_{t,t+k}(h), z_{t+k}) \right] \widetilde{y}^d_{t,t+k}(h) \right\} = 0, \tag{A.30}$$

where

$$\widetilde{y}^d_{t,t+k}(h) = \left(\frac{\widetilde{p}_t(h)}{P_{H,t+k}} \right)^{-\sigma} \left(\frac{P_{H,t+k}}{P_{t+k}} \right)^{-\theta} C_{t+k}.$$

is the total demand for the domestic firm which produces the good h, conditional on $\widetilde{p}_t(h)$ being applied at period $t+k$. Note that condition (A.30) holds in all contingencies and at all times t. We specify the strategy space in terms of the actual inflation rate, showing that the strategy of zero actual inflation in both countries is a Nash equilibrium if the above conditions on Φ and Φ^* hold. If country F adopts a strategy of zero producer inflation, it follows that

$$(1-\Phi^*) U_C(C_t) = \frac{P_{F,t}}{P_t} V_y \left(\left[\frac{P_{F,t}}{P_t} \right]^{-\theta} C_t, z_t^F \right). \tag{A.31}$$

in all states of nature at date t. In a discretionary equilibrium, the home policy-maker chooses the sequence $\{\Pi_\tau\}_{\tau=t}^{\infty}$ with $\Pi_t = P_{H,t}/P_{H,t-1}$ in order to maximise the welfare criterion

$$\begin{aligned} W_t &\equiv \mathrm{E}_t \left\{ \sum_{\tau=t}^{\infty} \beta^{\tau-t} w_\tau \right\}, \\ w_\tau &= U(C_\tau) - \frac{\int_0^n V(y_\tau(h), z_\tau) dh}{n} \end{aligned}$$

under the constraints given by (A.31) and

$$P_{H,t}^{1-\sigma} = \alpha P_{H,t-1}^{1-\sigma} + (1-\alpha^H) p_t(h)^{1-\sigma}, \tag{A.32}$$

which represents the state equation for the price index P_H under Calvo's model. We identify this maximization problem as problem (A). First, we consider the general problem, problem (B) in which the home policy-maker can freely control the sequences $\{\Pi_\tau, C_\tau, \Pi_{H,\tau}, \Pi_{F,\tau}\}_{\tau=t}^{\infty}$. By enlarging the set of controls to all the variables involved in problem (B), it is possible to obtain its first best. Moreover, the maximum value of the welfare attainable in problem (B) is always at least as good as the maximum value in problem (A), because the latter is nested in the former. Given the convexity of the disutility function in supplying labour and the fact that $\sigma > 1$, for any path of C, Π_H and Π_F, a necessary condition for a plan in the problem (B) to be optimal is to avoid dispersion of prices across the goods produced in the same country, $\Pi_t = 1$ at all time t. It follows that, in problem (B), it is optimal to stabilise the producer price level. Instead, the sequences of consumption and relative price satisfy the same conditions as in the problem with prices fixed one period in advance. These conditions can be arranged to obtain

$$U_C(C_t)\Pi_{H,t} = V_y(H_t)\left[\Pi_{H,t}^{1-\theta} + \frac{1-n}{n}\theta\left(\frac{\rho+\eta}{1+\theta\eta}\right)\Pi_{F,t}^{1-\theta}\right]. \tag{A.33}$$

which again requires the same restriction on Φ in order to be satisfied by the condition

$$(1-\Phi)U_C(C_t)\Pi_{H,t} = V_y(H_t)$$

Looking back at the problem (A), the strategy of zero producer inflation can replicate the optimal path of problem (B), if either $\theta = 1$ or $\theta = \rho^{-1}$ under the appropriate restrictions on Φ. It further satisfies the constraints (A.30) at all dates t. It is then the optimal strategy in problem (A). The strategy of zero inflation rate is then a time-consistent Nash equilibrium.

Welfare Approximations

First we show the proposition for the case in which $\theta = 1$. As shown in Benigno (2001b), the second-order approximation of the utility flows in the welfare functions (5) can be written as

$$\begin{aligned} w_t &= U_C\overline{C}[\widehat{C}_t + \frac{1}{2}(1-\rho)\widehat{C}_t^2 - (1-\Phi)\cdot\widehat{Y}_{H,t} - \frac{(1-\Phi)}{2}\cdot[\widehat{Y}_{H,t}]^2 + \\ &\quad -(1-\Phi)\frac{\eta}{2}\cdot[\widehat{Y}_{H,t}]^2 - \frac{(1-\Phi)}{2}(\sigma^{-1}+\eta)\cdot \mathrm{var}_h\widehat{y}_t(h) + \\ &\quad (1-\Phi)\eta\cdot\widehat{Y}_{H,t}\overline{Y}_t] + \text{t.i.p.} + o(\|\xi\|^3), \end{aligned} \tag{A.34}$$

for country H, while

$$\begin{aligned} w_t^* &= U_C\overline{C}[\widehat{C}_t + \frac{1}{2}(1-\rho)\widehat{C}_t^2 - (1-\Phi^*)\cdot\widehat{Y}_{F,t} - \frac{(1-\Phi^*)}{2}\cdot[\widehat{Y}_{F,t}]^2 + \\ &\quad -(1-\Phi^*)\frac{\eta}{2}\cdot[\widehat{Y}_{F,t}]^2 - \frac{(1-\Phi^*)}{2}(\sigma^{-1}+\eta)\cdot \mathrm{var}_f\widehat{y}_t(f) + \\ &\quad +(1-\Phi^*)\eta\cdot\widehat{Y}_{F,t}\overline{Y}_t^*] + \text{t.i.p.} + o(\|\xi\|^3), \end{aligned} \tag{A.35}$$

for country F. We have defined as an hat variable the log deviation of a variable from the steady state value; $Y_{H,t}$, $Y_{F,t}$, $y_t(h)$ and $y_t(f)$ are defined as

$$\begin{array}{cc} Y_{H,t} = T_t^{1-n}C_t, & Y_{F,t} = T_t^{-n}C_t, \\ y_t(h) = \left(\frac{p(h)}{P_{H,t}}\right)^{-\sigma} T_t^{1-n}C_t, & y_t(f) = \left(\frac{p(f)}{P_{F,t}}\right)^{-\sigma} T_t^{-n}C_t, \end{array}$$

where $T \equiv P_F/P_H$. Moreover var is the operator variance, t.i.p. includes terms that are independent of the policy and $o(\|\xi\|^3)$ includes terms that are of an order higher than the second in the bound $\|\xi\|$ on the amplitude of the shocks considered in the approximation. Furthermore we have defined $V_{yz}(z_t - \overline{z}) \equiv -V_{yy}\overline{Y}_{H,t}\overline{Y}_t$ and $V_{yz}(z_t^* - \overline{z}) \equiv -V_{yy}\overline{Y}_{F,t}\overline{Y}_t^*$. $\overline{C}$ is the steady-state level of consumption.

We show that, given that one country is following a strategy of zero producer inflation, then the strategy of zero producer inflation is also optimal for the other policy-maker and vice versa. If the policy-maker in country F is following the strategy of zero producer inflation, then, with isoelastic preferences, condition (A.16) can be written as

$$(1-\Phi^*)C_t^{-\rho} = T_t^{-n}(T_t^{-n}C_t)^{\eta}z_t^*,$$

at each date t, which in a log-linear *exact* form implies that

$$\widehat{T}_t = \frac{(\rho+\eta)}{n(1+\eta)}\widehat{C}_t - \frac{\eta}{n(1+\eta)}\overline{Y}_t^*. \tag{A.36}$$

Condition (A.36) in (A.34), combined with the value of

$$\Phi = \overline{\Phi} = \frac{D}{1+D} \qquad \text{with} \qquad D \equiv (1-n)n^{-1}\frac{(\rho+\eta)}{(1+\eta)},$$

implies that the linear term $\widehat{C}_t - (1-\overline{\Phi})\cdot\widehat{Y}_{H,t}$ disappears.

Furthermore we can write

$$\begin{aligned}
(\widehat{Y}_{H,t})^2 &= (1+D)^2\cdot\widehat{C}_t^2 - 2(1+D)\cdot\frac{1-n}{n}\frac{\eta}{1+\eta}\cdot\widehat{C}_t\overline{Y}_t^* + \text{t.i.p},\\
\widehat{Y}_{H,t}\overline{Y}_t &= (1+D)\cdot\widehat{C}_t\overline{Y}_t + \text{t.i.p.}
\end{aligned}$$

From which we can simplify w_t to

$$\begin{aligned}
w_t &= U_C\overline{C}[\frac{1}{2}(1-\rho)\widehat{C}_t^2 - (1+\eta)\frac{(1-\overline{\Phi})}{2}\cdot(1+D)^2\cdot\widehat{C}_t^2 + \\
&\quad +(1-\overline{\Phi})\frac{(1-n)}{n}\eta\cdot(1+D)\cdot\widehat{C}_t\overline{Y}_t^* + \\
&\quad +(1-\overline{\Phi})\eta\cdot(1+D)\cdot\widehat{C}_t\overline{Y}_t + \\
&\quad -\frac{(1-\overline{\Phi})}{2}(\sigma^{-1}+\eta)\cdot\text{var}_h\widehat{y}_t(h)] + \text{t.i.p.} + o(\|\xi\|^3),
\end{aligned}$$

Noting that $(1-\overline{\Phi})\cdot(1+D)=1$, we can further simplify the equation to

$$\begin{aligned}
w_t &= U_C\overline{C}[-\frac{(\rho+\eta)}{2n}\widehat{C}_t^2 + \frac{\eta}{n}[n\overline{Y}_t + (1-n)\overline{Y}_t^*]\cdot\widehat{C}_t \\
&\quad -\frac{(1-\overline{\Phi})}{2}(\sigma^{-1}+\eta^H)\cdot\text{var}_h\widehat{y}_t(h)] + \text{t.i.p.} + o(\|\xi\|^3),
\end{aligned}$$

and to

$$w_t = U_C\overline{C}[-\frac{1}{2n}(\rho+\eta)(\widehat{C}_t - \widetilde{C}_t)^2 - \frac{(1-\overline{\Phi})}{2}(\sigma^{-1}+\eta^H)\cdot \text{var}_h\widehat{y}_t(h)] + \text{t.i.p.} + o(\|\xi\|^3),$$

where we have used the definition of $\widetilde{C}$

$$\widetilde{C} \equiv \frac{\eta}{\rho+\eta}(n\overline{Y}_t + (1-n)\overline{Y}^*_t).$$

Following Woodford (1999) in deriving the term $\text{var}_h\widehat{y}_t(h)$, we can write the welfare criterion W as

$$W_t = -\Sigma \text{E}_t\left\{\sum_{\tau=t}^{\infty}\beta^{\tau-t}[\lambda_W(y^W_\tau)^2 + \pi^2_{H,\tau}]\right\}, \tag{A.37}$$

which corresponds to equation (21) in the main text. Σ and λ_W are functions of the structural parameters of the model. One can further show that under condition (A.36), the AS equation for country H can be written as

$$\pi_{H,t} = k_H(1-n)n^{-1}[(\rho+\eta)y^W_t] + \beta E_t\pi_{H,t+1}. \tag{A.38}$$

Given the zero producer inflation strategy of the foreign policy-maker, the optimal policy for the home policy-maker is to stabilize its producer price inflation at all dates t, if $\Phi = \overline{\Phi}$. The other side of the construction of the Nash equilibrium follows specularly.

Here we outline the proof for the case in which $\theta = \rho^{-1}$. Note that here we can write

$$U(C_t) = \frac{C_t^{1-\rho}}{1-\rho} = n^{\frac{1}{\theta}}\frac{C_{H,t}^{1-\rho}}{1-\rho} + (1-n)^{\frac{1}{\theta}}\frac{C_{F,t}^{1-\rho}}{1-\rho}.$$

It should be remembered that

$$C_{H,t} = n\left(\frac{P_{H,t}}{P_t}\right)^{-\theta}C$$

which can be rewritten as

$$C_{H,t} = nY_{H,t},$$

using the definition of Y_H. We can then write the utility flow for country H as

$$w_t = U(C_t) - \frac{\int_0^n V(y_t(h), z_t)dh}{n} = n\frac{Y_{H,t}^{1-\rho}}{1-\rho} + (1-n)\frac{Y_{F,t}^{1-\rho}}{1-\rho} - \frac{\int_0^n V(y_t(h), z_t)dh}{n},$$

where

$$y_t(h) = \left(\frac{p_t(h)}{P_{H,t}}\right)^{-\sigma} Y_{H,t}.$$

We can then decompose w_t into

$$w_t = \left[nU(Y_{H,t}) - \frac{\int_0^n V(y_t(h), z_t)\,dh}{n}\right] + (1-n)U(Y_{F,t}). \tag{A.39}$$

Note that the terms in square brackets can be expanded following Woodford (1999a) into

$$W = -\Lambda \mathrm{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t [\varphi y_{H,t}^2 + \pi_{H,t}^2] \right\} + \text{t.i.p.} + o(\|\xi\|^3) \tag{A.40}$$

where the other terms in (A.39), of order lower than the third, can be collapsed in t.i.p for the appropriate class of strategies (including the equilibrium class). In fact, with the specification of the strategy space in terms of actual GDP inflation, the Home policy-maker cannot control Y_F, but can control directly Y_H (under the assumption $\theta\rho = 1$). Note that in deriving (A.40), it should be assumed that $\overline{\overline{\Phi}} = 1 - n$. Indeed in the terms in the square bracket, the utility of $Y_{H,t}$ is weighted by n. The expansion for country F follows specularly.