Designing Targeting Rules for International Monetary Policy Cooperation*

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Abstract

This study analyzes a two-country dynamic general equilibrium model with nominal rigidities, monopolistic competition and producer currency pricing. A quadratic approximation to the utility of the consumers is derived and assumed as the policy objective function of the policymakers.

It is shown that only under special conditions there are no gains from cooperation and moreover that the paths of the exchange rate and prices in the constrained-efficient solution depend on the kind of disturbance that affects the economy. It might be the case either for fixed or floating exchange rates. Despite this result, simple targeting rules that involve only targets for the growth of output and for both domestic GDP and CPI inflation rates can replicate the cooperative allocation.

Keywords: monetary policy cooperation, sticky prices, welfare analysis, targeting rules, inflation target.

JEL classification: E52, F41, F42.

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> "*The national economies that make up the world economy have become increasingly interdependent. Monetary policy in each country affects economic welfare both at home and abroad: the policymaker in each country generates externalities for the policymakers in the other countries. Therefore, the policymaker in each country must take account of the actions of policymakers in other countries.*"[1]

The previous quotation outlines the basic idea behind the literature on international monetary policy cooperation in the 80's and 90's. The existence of externalities, whether positive or negative, is the source of a need of international monetary cooperation when countries do not internalize the effects of their actions on other countries.

In this study, we depart from the previous literature, discussed among others in Canzoneri and Gray (1985), Canzoneri and Henderson (1991) and Persson and Tabellini (1995), by considering a two-country model in which both the structure of the economy and the welfare criteria of the policymakers are derived from microfoundations. We revisit the scope for international monetary policy cooperation in a world in which goods and capital markets are perfectly integrated and where the disturbances that affect the economies originate from productivity, public expenditure and mark-up shocks.

We are not the first to address this issue in a microfounded model.[2] However, our contribution to the literature is to use a linear-quadratic solution method, as discussed in Benigno and Woodford (2004), to allow a direct comparison of the objective functions of the policymakers and the structure of the economies with the ones that were assumed in the previous literature. In a two-country open economy model, we derive quadratic objective functions for policymakers that maximize the utility of the agents that live in their own country. These objectives are not

[1] Canzoneri and Henderson (1991), pg. 1.

[2] Our approach follows recent contributions in the open-macro literature which have studied the analysis of international monetary cooperation with microfounded models and utility-based welfare criteria, as Benigno and Benigno (2003), Corsetti and Pesenti (2001), Devereux and Engel (2003), Obstfeld and Rogoff (2002), Sutherland (2002a, 2002b), Tille (2003). However, differently from these analyses, we characterize a dynamic model in which prices are sticky and staggered following the Calvo (1983) model and we allow for a more general structure of the economy, in terms of preferences and shocks. With the use of numerical methods, Kollman (2003), Tchakarov (2003) and Sutherland (2001) have evaluated optimal monetary policies in two-country dynamic general equilibrium models.

only quadratic in domestic output gap and producer inflation but contain other targets for the terms of trade as well as for foreign output gap and foreign producer inflation.

We then analyze the cooperative and non-cooperative allocation. First, our analysis shows that it is not possible to give a conclusive prescription on which exchange-rate regime can enforce cooperation except for saying that it should be contingent on the kind of disturbance that hits the economies. As in Devereux and Engel (2003) and Obstfeld and Rogoff (2002), the exchange rate should float in order to accommodate asymmetric productivity shocks mirroring Friedman's prescription for flexible exchange rates.[3] Monetary policymakers are then left with the role of pursuing the domestic goal of price stability. On the opposite, when the economy is hit by other shocks, as for example mark-up disturbances, the optimal cooperative outcome might imply a managed or sometimes a fixed exchange rate regime. Instead, prices and outputs should move to accommodate the shock. Second, in general our model suggests that there exists gains from cooperation.[4] The driving force steams from the same terms-of-trade externality that was central in the previous literature. However, the existence of gains from cooperation can be intuited by studying the interaction between the terms-of-trade externality and the model's distortions given by rigidity in prices and monopoly power in the goods market.

At a first sight, these results would suggest that the task of designing institutions that can implement the cooperative solution is a difficult one, since it would require to specify some control of the exchange rate conditional on the type of disturbance that occurs. Despite this initial premise, we show that it is still possible to design simple monetary institutions that can implement the optimal cooperative outcome. We appeal to the concept of targeting rules proposed by Svensson (2002, 2003, 2004) which represent Euler equations that characterize the optimizing behavior of central banks. In our context, they are constructed using first-order conditions of the optimal cooperative allocation following the principles of Giannoni and Woodford (2002). Our targeting rules can be written as a combi-

[3] Friedman (1953).

[4] Indeed, our model has more general preference specifications and structure of the shocks than Devereux and Engel (2003) and Obstfeld and Rogoff (2002). However, in limiting cases, we nest their results.

nation of only domestic targets, both GDP and CPI inflation rates and domestic output, with no explicit reference to the exchange rate.

Our results qualify the general message of Obstfeld and Rogoff (2002). Policymakers that act in their self interest do not generally achieve outcomes that are optimal from a global perspective. However it still possible that monetary institutions can be designed with self-oriented targets that maintain enough flexibility to accommodate also external objectives.

The paper is structured as it follows. Section 1 presents the structure of the model. Section 2 presents the cooperative and non-cooperative problems. Section 3 solves the problems in a log-linear approximation to the solution under the special condition that the monopolistic distortions are completely neutralized. Section 4 presents the general case. Section 5 concludes.

1 Structure of the Model

We consider a world economy populated by a measure one of households. The population on the segment $[0, n)$ belongs to the Home country (H) while the one on the segment $[n, 1]$ belongs to the Foreign country (F). Each individual maximizes the following utility function:

$$U_t^j = \mathrm{E}_t \left\{ \sum_{T=t}^{\infty} \beta^{T-t} \left[U(C_T^j) - V(y_T(j), \xi_T^i) \right] \right\},$$

where the index j denotes a variable that is specific to household j and the index i denotes a variable specific to the country H or F in which j resides. To clarify the notation that follows i will be replaced by an asterisk when referring to country F and will be suppressed when referring to country H; E_t denotes the expectation conditional on the information set at date t and β is the intertemporal discount factor, with $0 < \beta < 1$. Households get utility from consumption and disutility from producing goods. The function $U(\cdot)$ is increasing and concave in the consumption index C which is defined as a Dixit-Stiglitz aggregator of home and foreign bundles of goods as

$$C^j \equiv \left[n^{\frac{1}{\theta}} (C_H^j)^{\frac{\theta-1}{\theta}} + (1-n)^{\frac{1}{\theta}} (C_F^j)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}},$$

where C_H^j and C_F^j are consumption sub-indexes of the continuum of differentiated goods produced respectively in country H and F

$$C_H^j \equiv \left[\left(\frac{1}{n} \right)^{\frac{1}{\sigma}} \int_o^n c^j(h)^{\frac{\sigma-1}{\sigma}} dh \right]^{\frac{\sigma}{\sigma-1}}, \qquad C_F^j \equiv \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\sigma}} \int_n^1 c^j(f)^{\frac{\sigma-1}{\sigma}} df \right]^{\frac{\sigma}{\sigma-1}},$$

where $\sigma > 1$ is the elasticity of substitution across goods produced within a country and θ is the elasticity of substitution between the bundles C_H and C_F. It is assumed that there is a continuum of goods produced in country H and F on the respective segments $[0, n)$ and $[n, 1]$. All the goods are traded across borders with no trade frictions. The appropriate consumption-based price indices expressed in units of the currency of the respective country i are defined as

$$P^i \equiv \left[n(P_H^i)^{1-\theta} + (1-n)\,(P_F^i)^{1-\theta} \right]^{\frac{1}{1-\theta}}, \tag{1.1}$$

with

$$P_H^i \equiv \left[\left(\frac{1}{n} \right) \int_o^n p^i(h)^{1-\sigma} dh \right]^{\frac{1}{1-\sigma}}, \qquad P_F^i \equiv \left[\left(\frac{1}{1-n} \right) \int_n^1 p^i(f)^{1-\sigma} df \right]^{\frac{1}{1-\sigma}},$$

where $p^i(h)$ and $p^i(f)$ are prices in units of domestic currency of the home-produced and foreign-produced goods, respectively. Prices are set in the currency of the producer and the law of one price holds: $p(h) = Sp^*(h)$ and $p(f) = Sp^*(f)$, where S is the nominal exchange rate (the price of foreign currency in terms of domestic currency). Given these assumptions and the structure of preferences, purchasing power parity holds, i.e. $P = SP^*$. Moreover relative prices are independent of the currency of denomination, which means that when writing relative-price variables we can suppress the index i.

Finally $V(\cdot)$ is an increasing convex function of household j's supply of one of the differentiated good $y(j)$ produced in its country and ξ^i denotes a generic vector of shocks (to be specified in the analysis that follows) which are specific to country i. The total demands of the generic good h, produced in country H, and of the good f, produced in country F, are respectively:

$$y(h) = \left(\frac{p(h)}{P_H} \right)^{-\sigma} \left[\left(\frac{P_H}{P} \right)^{-\theta} C^W + G \right], \qquad y(f) = \left(\frac{p(f)}{P_F} \right)^{-\sigma} \left[\left(\frac{P_F}{P} \right)^{-\theta} C^W + G^* \right], \tag{1.2}$$

where $C^W \equiv \int_0^1 C^j dj$ is aggregate consumption in the world economy and G and G^* are country-specific government purchase shocks.

We assume that asset markets are complete both at domestic and international levels. Households can trade in a set of nominal state-contingent securities denominated in the currency of the home country and they all inherit at time 0 initial state-contingent wealth such that their lifetime budget constraints are identical. This complete-market assumption implies that consumption risk is perfectly pooled among households within a country and across countries. It follows that

$$C_t^j = C_t \quad \text{for all } j, \tag{1.3}$$

at all times and across all states of nature. Equation (1.3) is derived from the set of optimality conditions that characterize the optimal allocation of wealth among the state-contingent securities, having used the assumption on the initial level of wealth and the fact that purchasing power parity holds.[5] At each time t, there is one of these conditions for each of the states of nature at time $t+1$. The set of optimality conditions of the households' behavior is completed by the appropriate transversality conditions.

In the analysis that follows, we assume that preferences are isoelastic of the form

$$U(C_t^j) \equiv \frac{(C_t^j)^{1-\rho}}{1-\rho}, \qquad\qquad V(y_t(j), \xi_t^i) \equiv (a_t^i)^{-\eta} \frac{(y_t(j))^{1+\eta}}{1+\eta},$$

where a_t^i can be interpreted as a country-specific productivity shock. Here ρ is the inverse of the intertemporal elasticity of substitution in consumption, with $\rho > 0$, and η is the inverse of the elasticity of goods production,with $\eta \geq 0$.

Each household is also a monopolistic producer of one of the differentiated goods. As a monopolistic producer the home household sets its price $p(h)$taking as

[5] We do not report these conditions here since they will not be used in the analysis that follows. Nor we report the standard stochastic Euler equations, which are implied by these conditions and which are used to price the risk-free nominal interest rates. Indeed they will be needed only to determine the optimal path of the interest rates in a residual way, once the optimal paths of inflation, relative prices and consumption are determined. Our model can be interpreted as a cashless limiting model (as in Woodford, 2003, chapter 2), in which the remuneration of money is equal to that of a one-period-maturity risk-free nominal bond in a way that the opportunity cost of holding money is zero.

given P, P_H, P_F and C. The price-setting behavior is modelled following a mechanism *á la* Calvo (1983) according to which each seller has the opportunity to change its price with a given probability $1-\alpha$. We allow for different α^i across countries. When a household in the home country has the opportunity to set a new price in period t, it does so in order to maximize the expected discounted value of its net profits. The price setting decision at time t determines the net profits at time T with $T > t$ only in states of nature in which the seller does not change the price from $t+1$ to T inclusive: this occurs with probability α^{T-t}. The objective function is then[6]

$$\mathrm{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\frac{U_C(C_T)}{P_T}(1-\tau_T)\tilde{p}_t(h)\tilde{y}_{t,T}(h) - V(\tilde{y}_{t,T}(h), \xi_T) \right],$$

where after-tax revenues are converted in units of utility through the marginal utility of nominal income, $U_C(C_T)/P_T$, which is the same for all households belonging to a country because of the complete-market assumption; τ_t denotes a time-varying tax on sales;[7] $\tilde{p}_t(h)$ denotes the price of the good h chosen at date t in the producer currency and $\tilde{y}_{t,T}(h)$ is the total demand of good h at time T conditional on the fact that the price $\tilde{p}_t(h)$ has not changed,

$$\tilde{y}_{t,T}(h) = \left(\frac{\tilde{p}_t(h)}{P_{H,T}} \right)^{-\sigma} \left[\left(\frac{P_{H,T}}{P_T} \right)^{-\theta} C_T + G_T \right]. \tag{1.4}$$

The optimal choice of $\tilde{p}_t(h)$ is

$$\tilde{p}_t(h) = \frac{\mathrm{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} V_y(\tilde{y}_{t,T}(h), \xi_T)\tilde{y}_{t,T}(h)}{\mathrm{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \frac{1}{\mu_T} \frac{U_C(C_T)}{P_T} \tilde{y}_{t,T}(h)}, \tag{1.5}$$

where $1/\mu_t$ has been defined as

$$\frac{1}{\mu_t} \equiv \frac{(1-\tau_t)(\sigma-1)}{\sigma}.$$

[6] All households within a country that can modify their price at a certain time face the same discounted value of the streams of current and future marginal costs under the assumption that the new price is maintained. Thus they will set the same price.

[7] We introduce a time-varying tax on sales to obtain inefficient fluctuations in the marginal rate of substitution between consumption and goods production. We could have obtained the same outcome by introducing heterogenous labor market in each industry and having a time-varying monopoly power of wage setters as in Clarida et al. (2002) and Woodford (2003). Giannoni (2000) obtains the same outcome by using a time-varying elasticity of substitution σ. To complete the characterization of the model, we assume that there are lump-sum taxes that can be levied in a way that the government's intertemporal budget constraint is satisfied.

In particular μ_t can be interpreted as the inefficient wedge in the marginal rate of substitution between consumption and goods production when prices are flexible. In what follows we will refer to fluctuations in this wedge as mark-up shocks. Given Calvo's mechanism, the evolution of the price index P_H is described by the following law of motion

$$P_{H,t}^{1-\sigma} = \alpha P_{H,t-1}^{1-\sigma} + (1-\alpha)\tilde{p}_t(h)^{1-\sigma}. \tag{1.6}$$

Similar conditions hold for the producers in country F, with the appropriate modifications.

2 Objectives and strategies in the cooperative and non-cooperative solutions

In this section, we first specify the objective functions, the constraints and the strategy space of the policymakers for the cooperative and non-cooperative solutions and then present the solution method for our problems. We assume that the policy objective for each monetary policymaker is the maximization of the sum of the expected utilities of the consumers that belong to its country. In the home country, it corresponds to maximizing

$$W = \mathrm{E}_{t_0}\left\{\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left[U(C_t) - \frac{1}{n}\int_0^n V(y_t(h), \xi_t)dh\right]\right\}$$

which can be rewritten using the assumption of isoelastic preferences as

$$W = \mathrm{E}_{t_0}\left\{\sum_{t=0}^{\infty}\beta^{t-t_0}[U(C_t) - V(Y_{H,t}, \xi_t)\Delta_t]\right\} \tag{2.7}$$

where $Y_{H,t}$ is defined by

$$Y_{H,t} \equiv \left(\frac{P_{H,t}}{P_t}\right)^{-\theta} C_t + G_t \tag{2.8}$$

and Δ_t is an index of price dispersion defined as

$$\Delta_t \equiv \frac{1}{n}\int_0^n \left(\frac{p_t(h)}{P_{H,t}}\right)^{-\sigma(1+\eta)} dh \geq 1.$$

When prices are set according to a partial adjustment rule *á la* Calvo, the index Δ_t evolves according to the following law of motion

$$\Delta_t = \alpha \Delta_{t-1} \Pi_{H,t}^{\sigma(1+\eta)} + (1-\alpha) \left(\frac{1 - \alpha \Pi_{H,t}^{\sigma-1}}{1-\alpha} \right)^{-\frac{\sigma(1+\eta)}{1-\sigma}}, \tag{2.9}$$

where we have used (1.6) and defined $\Pi_{H,t} \equiv P_{H,t}/P_{H,t-1}$.

In the same way, we can write the objective for the foreign policymaker as

$$W^* = \mathrm{E}_{t_0} \left\{ \sum_{t=0}^{\infty} \beta^{t-t_0} [U(C_t) - V(Y_{F,t}^*, \xi_t^*) \Delta_t^*] \right\}, \tag{2.10}$$

where now

$$Y_{F,t}^* = \left(\frac{P_{F,t}}{P_t} \right)^{-\theta} C_t + G_t^*, \tag{2.11}$$

and

$$\Delta_t^* = \alpha^* \Delta_{t-1}^* (\Pi_{F,t}^*)^{\sigma(1+\eta)} + (1-\alpha^*) \left(\frac{1 - \alpha^* (\Pi_{F,t}^*)^{\sigma-1}}{1-\alpha^*} \right)^{-\frac{\sigma(1+\eta)}{1-\sigma}}, \tag{2.12}$$

and we have defined $\Pi_{F,t}^* \equiv P_{F,t}^*/P_{F,t-1}^*$. The economy is characterized by the aggregate supply relationships that determine the link between consumption, output and prices. Given the assumption of isoelastic preferences and the law of motion (1.6), we can express (1.5) in the following form

$$\frac{1 - \alpha \Pi_{H,t}^{\sigma-1}}{1-\alpha} = \left(\frac{F_t}{K_t} \right)^{\frac{\sigma-1}{1+\sigma\eta}} \tag{2.13}$$

where we have defined

$$K_t \equiv E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} a_T^{-\eta} Y_{H,T}^{1+\eta} \left(\frac{P_{H,T}}{P_{H,t}} \right)^{\sigma(1+\eta)}, \tag{2.14}$$

$$F_t \equiv E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \mu_T^{-1} C_T^{-\rho} Y_{H,T} \left(\frac{P_{H,T}}{P_T} \right) \left(\frac{P_{H,T}}{P_{H,t}} \right)^{\sigma-1}. \tag{2.15}$$

In the foreign country we obtain that

$$\frac{1 - \alpha^* (\Pi_{F,t}^*)^{\sigma-1}}{1-\alpha^*} = \left(\frac{F_t^*}{K_t^*} \right)^{\frac{\sigma-1}{1+\sigma\eta}}, \tag{2.16}$$

where we have defined

$$K_t^* \equiv E_t \sum_{T=t}^{\infty} (\alpha^* \beta)^{T-t} (a_T^*)^{-\eta} (Y_{F,T}^*)^{1+\eta} \left(\frac{P_{F,T}^*}{P_{F,t}^*} \right)^{\sigma(1+\eta)}, \tag{2.17}$$

$$F_t^* \equiv E_t \sum_{T=t}^{\infty} (\alpha^* \beta)^{T-t} (\mu_T^*)^{-1} C_T^{-\rho} Y_{F,T}^* \left(\frac{P_{F,T}}{P_T} \right) \left(\frac{P_{F,T}^*}{P_{F,t}^*} \right)^{\sigma-1}. \tag{2.18}$$

Finally, using (1.1) we can write the link between relative prices P_H/P and P_F/P as

$$n \left(\frac{P_{H,t}}{P_t} \right)^{1-\theta} + (1-n) \left(\frac{P_{F,t}}{P_t} \right)^{1-\theta} = 1. \tag{2.19}$$

Having specified the policy objectives and the constraints that policymakers face, we now characterize the cooperative and non-cooperative problems. In the cooperative allocation, countries agree to maximize aggregate welfare W^C defined as a weighted sum of the utilities of the consumers belonging to the world economy:

$$W^C \equiv E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [U(C_t) - nV(Y_{H,t}, \xi_t)\Delta_t - (1-n)V(Y_{F,t}^*, \xi_t^*)\Delta_t^*]. \tag{2.20}$$

The optimal cooperative allocation is the sequence $\{x_t\}$, where $x_t \equiv (C_t,Y_{H,t},Y_{F,t}, \Delta_t, \Delta_t^*, F_t, F_t^*, K_t, K_t^*, P_{H,t}/P_t, P_{F,t}/P_t, \Pi_{H,t}, \Pi_{F,t}^*)$, that maximizes (2.20) under the constraints (2.8), (2.9) and (2.11)–(2.19), given the sequences of shocks $\{\xi_t, \xi_t^*\}$ and the initial conditions $\Delta_{t_0-1}, \Delta_{t_0-1}^*$.[8]

In order to characterize the non-cooperative allocation, we need to specify the strategic game. We assume that each policymaker's strategy is specified in terms of each country's GDP inflation rate as a function of the sequence of shocks. In particular, the home and foreign policymakers set $\Pi_{H,t}$ and $\Pi_{F,t}^*$, respectively, as $\Pi_{H,t} = f_t(\{\xi_t, \xi_t^*\})$ and $\Pi_{F,t}^* = f_t^*(\{\xi_t, \xi_t^*\})$ where $f_t(\cdot)$ and $f_t^*(\cdot)$ are time-varying functions. In particular we have specified that each producer inflation rate can be a time-varying function of all past, present and future shocks that are in the model.

We assume that policymakers commit at time t_0 to their respective sequence of strategies $\{f_t(\cdot)\}$ and $\{f_t^*(\cdot)\}$ for each $t \geq t_0$ to maximize their objective functions. In a Nash game each policymaker maximizes its own utility by choosing a sequence

[8] The vectors ξ_t and ξ_t^* include a_t, G_t, μ_t and a_t^*, G_t^*, μ_t^*, respectively.

of GDP inflation rates taking as given the equilibrium sequence of GDP inflation rates of the other policymaker. An implication of our specification of the strategy space is that we can express the home policymaker's optimization problem as maximizing (2.7) by choosing the sequences $\{C_t, Y_{H,t}, Y_{F,t}, \Delta_t, \Delta_t^*, F_t, F_t^*, K_t, K_t^*, P_{H,t}/P_t, P_{F,t}/P_t, \Pi_{H,t}\}$ under the constraints (2.8), (2.9) and (2.11)–(2.19), given the sequences of shocks $\{\xi_t, \xi_t^*\}$, the initial conditions $\Delta_{t_0-1}, \Delta_{t_0-1}^*$ and given the sequence of foreign GDP inflation rates $\{\Pi_{F,t}^*\}$ implied by the strategy of the other policymaker. A similar problem can be written for the foreign policymaker in maximizing (2.10).

Since the cooperative and non-cooperative problems cannot be solved in a closed-form solution, we apply an approximation method based on a generalization of Taylor's approximation theorem.[9] In order to apply this method we need first to solve the cooperative and non-cooperative allocations for the case in which the sequence of shocks $\{\xi_t, \xi_t^*\}$ is constant (in what follows we will refer to these problems as 'deterministic').[10] Indeed, the approximation method requires that the existence of a well-defined steady-states as a solution for the cooperative and non-cooperative deterministic problems. Moreover, since the approximation for the stochastic problems (cooperative and non-cooperative) will be taken around their respective deterministic steady states, it is necessary that the cooperative and non-cooperative steady states coincide so that it is possible to compare the cooperative and non-cooperative allocations in a meaningful way.

In general, there are two issues that make the analysis incompatible with these two requirements. First, as noted by Kydland and Prescott (1980), given that the structural constraints (2.14), (2.15), (2.17), (2.18) are forward looking, the optimization problems are not fully recursive, so that the solutions to the deterministic problems are not necessarily stationary. Second, cooperative and non-cooperative deterministic solutions can differ because of the existence of a contractionary bias in the non-cooperative solution that does not arise in the cooperative solution, as discussed in Corsetti and Pesenti (2001) and Benigno (2002). We show that we can handle both issues by introducing a stronger form of commitment— a commitment from a 'timeless perspective' as in Woodford (1999).

[9] The generalization is to Banach spaces which are the ones that apply to our context as discussed in Woodford (1986).

[10] In this deterministic solution, without losing generality, we are assuming $a_t = a_t^* = \bar{a} > 0$, $\mu_t = \mu_t^* = \bar{\mu} \geq 1$, $G_t = G_t^* = \bar{G} \geq 0$.

In the deterministic cooperative problem, we thus assume that policymakers face additional constraints on the variables $F_{t_0}, F^*_{t_0}, K_{t_0}, K^*_{t_0}$ of the form $F_{t_0} = \bar{F}, F^*_{t_0} = \bar{F}^*, K_{t_0} = \bar{K}, K^*_{t_0} = \bar{K}^*$ such that $\bar{F}/\bar{K} = 1$, $\bar{F}^*/\bar{K}^* = 1$. In Appendix A, we show that given these additional constraints and initial conditions $\Delta_{t_0-1} = \Delta^*_{t_0-1} = 1$, there exist a stationary sequence for the set of variables x_t such that $x_t = \bar{x}$, where in particular $F_t = \bar{F}$, $F^*_t = \bar{F}^*$, $K_t = \bar{K}$, $K^*_t = \bar{K}^*$, $\Delta_t = \Delta^*_t = P_{H,t}/P_t = P_{F,t}/P_t = \Pi_{H,t} = \Pi^*_{F,t} = 1$ at all times. In the deterministic non-cooperative problem, we assume that the home policymaker solves the above specified problem with the additional constraint $F_{t_0} = \bar{F}, K_{t_0} = \bar{K}$ such that $\bar{F}/\bar{K} = 1$, while the foreign policymaker solves its problem with the additional constraints $F^*_{t_0} = \bar{F}, K^*_{t_0} = \bar{K}^*$ such that $\bar{F}^*/\bar{K}^* = 1$. We show that in this deterministic non-cooperative game, a candidate for the Nash equilibrium is the combination of strategies $\Pi_{H,t} = \Pi^*_{F,t} = 1$ at all times. It follows that the cooperative and non-cooperative steady states coincide and that there are no gains from cooperation.[11]

Our final objective is the to study the approximation for the stochastic problems around the above well-defined deterministic steady state. Indeed, around this steady state, we consider bounded deviations of the initial conditions Δ_{t_0-1}, $\Delta^*_{t_0-1}$, F_{t_0}, $F^*_{t_0}$, K_{t_0}, $K^*_{t_0}$ as well as of the sequences of shocks $\{\xi_t, \xi^*_t\}$ from their steady-state values and study how the sequence $\{x_t\}$ departs from its steady-state value (in a bounded way) both in the cooperative and non-cooperative allocations. We aim to analyze how, given these small perturbations to the deterministic problems, stabilization policies might differ depending on the nature of the strategic interaction (i.e. cooperative versus non-cooperative).

3 Cooperative and non-cooperative allocations: a special case

We are interested in studying a local approximation to the solution of the above non-linear stochastic problems. In particular we aim to characterize a first-order approximation around the deterministic steady state described in the previous section. As shown in Benigno and Woodford (2004), this log-linear solution can be

[11]This statement will be better qualified in the next section when we consider the possibility for policymakers to react to sunspot shocks.

obtained by solving a linear-quadratic problem. The characterization of the non-linear stochastic problem in a linear-quadratic framework allows us to compare our objective functions and structural equations to the ones adopted in the previous approach to international monetary policy coordination as in Canzoneri and Henderson (1991).

As we described in details in appendix B, our linear-quadratic representation is obtained by taking a second-order approximation to the objective functions and to the structural constraints. We then use the second-order approximation to the structural constraints to substitute out the first-order terms in the approximation of the objective function with only second-order terms. In this way, we show that we can obtain an objective function which is purely quadratic and that can be correctly evaluated by only a log-linear approximation to the structural constraints. The solution to this problem (i.e. the linear-quadratic one) gives us a log-linear approximation to the solution of the non-linear problems described in the previous section.

3.1 Cooperative solution

To get more intuition, we consider the simple case in which the steady-state monopolistic distortions are offset by appropriate taxation subsidies, i.e. $\bar{\mu} = 1$. First, we focus on the characterization of the optimal cooperative solution. In this case the quadratic approximation of the objective function for the cooperative problem assumes the following form (see Appendix B):

$$L^W = \frac{1}{2} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [n\lambda_y^w (\hat{Y}_{H,t} - \tilde{Y}_{H,t}^w)^2 + (1-n)\lambda_y^w (\hat{Y}_{F,t}^* - \tilde{Y}_{F,t}^w)^2 +$$

$$+ n(1-n) s_c \theta \psi \lambda_y^w (\hat{T}_t - \tilde{T}_t^w)^2 + \frac{\sigma n s_c^2}{\kappa} \lambda_y^w \pi_{H,t}^2 + \frac{\sigma(1-n) s_c^2}{\kappa^*} \lambda_y^w \pi_{F,t}^{*2}], \qquad (3.21)$$

where s_c represents the steady-state share of consumption over output, $s_c = \bar{C}/\bar{Y}$, $\lambda_y^w = (s_c\eta + \rho)s_c^{-2} > 0$ and we have defined $\psi \equiv (1-\rho\theta)/(\rho s_c^{-1} + \eta)$ and $\kappa^i \equiv (1-\alpha^i)(1-\alpha^i\beta)(\rho s_c^{-1} + \eta)/[\alpha^i(1+\sigma\eta)]$. Variables with the hat denotes log-deviations of the respective variable from the steady state while we have defined $\pi_{H,t} \equiv \ln \Pi_{H,t}$, $\pi_{F,t}^* \equiv \ln \Pi_{F,t}^*$ and $T_t = P_{F,t}/P_{H,t}$. The variables $\tilde{Y}_{H,t}^w$, $\tilde{Y}_{F,t}^w$, $\tilde{T}_t^w$ are the following functions of the shocks[12]

$$\tilde{Y}^w_{H,t} \equiv s_c \tilde{C}_t + (1-n)\theta s_c \tilde{T}^w_t + \hat{G}_t,$$

$$\tilde{Y}^w_{F,t} \equiv s_c \tilde{C}_t - n\theta s_c \tilde{T}^w_t + \hat{G}^*_t,$$

$$\tilde{T}^w_t = \frac{\eta}{(1+\theta s_c \eta)}[\hat{a}_{R,t} - \hat{G}_{R,t}],$$

where $\tilde{C}_t$ is given by

$$\tilde{C}_t \equiv \frac{\eta}{(s_c\eta + \rho)}\left(\hat{a}_{W,t} - \hat{G}_{W,t}\right).$$

The variables $\tilde{Y}^w_{H,t}$, $\tilde{Y}^w_{F,t}$, $\tilde{T}^w_t$ can be interpreted as the desired targets that policymakers wish to achieve in a cooperative agreement for domestic output, foreign output and the terms of trade respectively. When $\bar{\mu} = 1$, these targets coincide with the flexible-price allocation that would arise when there are no mark-up shocks.

The quadratic loss function (3.21) can be evaluated by only a log-linear approximation to the structural constraints (2.8), (2.11) and (2.13)–(2.19). In particular by log-linearizing (2.13)–(2.15) and (2.16)–(2.18) using (2.19) and the definition $T \equiv P_F/P_H$, we obtain the two aggregate supply relationships:

$$\pi_{H,t} = \kappa[(\hat{Y}_{H,t} - \tilde{Y}^w_{H,t}) + (1-n)\psi(\hat{T}_t - \tilde{T}^w_t) + u_t] + \beta E_t \pi_{H,t+1}, \tag{3.22}$$

$$\pi^*_{F,t} = \kappa^*[(\hat{Y}^*_{F,t} - \tilde{Y}^w_{F,t}) - n\psi(\hat{T}_t - \tilde{T}^w_t) + u^*_t] + \beta E_t \pi^*_{F,t+1}, \tag{3.23}$$

for country H and F, respectively. By log-linearizing (2.8) and (2.11) we obtain a relation between terms of trade and output difference across countries as

$$(\hat{T}_t - \tilde{T}^w_t) = \theta^{-1} s_c^{-1}[(\hat{Y}_{H,t} - \tilde{Y}^w_{H,t}) - (\hat{Y}^*_{F,t} - \tilde{Y}^w_{F,t})]. \tag{3.24}$$

In particular, in equations (3.22) and (3.23) u_t and u^*_t are just proportional to the mark-up shocks

$$u_t = \frac{\hat{\mu}_t}{(s_c\eta + \rho)}, \qquad u^*_t = \frac{\hat{\mu}^*_t}{(s_c\eta + \rho)}. \tag{3.25}$$

[12] We have defined $\hat{a}_{W,t} \equiv n\hat{a}_t + (1-n)\hat{a}^*_t$, $\hat{G}_{W,t} \equiv n\hat{G}_t + (1-n)\hat{G}^*_t$, $\hat{a}_{R,t} \equiv \hat{a}_t - \hat{a}^*_t$, $\hat{G}_{R,t} \equiv \hat{G}_t - \hat{G}^*_t$ with $\hat{G}_t \equiv (G_t - \bar{G})/\bar{Y}$ and $\hat{G}^*_t \equiv (G^*_t - \bar{G})/\bar{Y}$.

By specifying a path for $\pi_{H,t}$ and $\pi^*_{F,t}$, the variables $\hat{Y}_{H,t}$, $\hat{Y}^*_{F,t}$ and $\hat{T}_t$ can be determined by (3.22)-(3.24) and this is all that is needed to evaluate (3.21). Finally we need to consider the constraints implied by the 'timeless perspective' commitment on the initial conditions π_{H,t_0} and π^*_{F,t_0} given by $\pi_{H,t_0} = \bar{\pi}_{H,t_0}$ and $\pi^*_{F,t_0} = \bar{\pi}^*_{F,t_0}$.[13]

As in the closed-economy model of Galí and Gertler (1999) and Sbordone (2002), in the AS equations (3.22) and (3.23) GDP inflation rates depend on the present discounted value of the aggregate real marginal costs. In general, in open economies, real marginal costs are not only proportional to the output gap but they also depend on relative prices, namely the terms of trade. (see Svensson, 2000.) This dependence captures the expenditure-switching effect; only in the special case in which $\rho\theta = 1$ the terms of trade channel disappears. Equations (3.22) and (3.23) replace the traditional expectations-augmented Phillips-curve in the models of Canzoneri and Henderson (1991) and Persson and Tabellini (1995, 1996). Equation (3.24) captures the relation between terms of trade and output differential across countries. This relation is also familiar to the previous literature.

The restriction $\rho\theta = 1$ implies also that the terms of trade is no longer a target in the loss function (3.21), since $\psi = 0$. In general, however, terms-of-trade volatility with respect to its desired target can be a cost or a benefit depending on the sign of the term ψ. Since ψ can be negative, it might seem that our minimization problem is not necessarily well behaved since the loss function might not be convex. However, when $\bar{\mu} = 1$, monopolistic distortions are completely neutralized in a way that the steady state is efficient from a cooperative perspective. Thus, any fluctuation around this steady state is costly.[14]

The optimal stabilization problem can be characterized in a simple way in the absence of mark-up disturbances, i.e. $\hat{\mu}_t = \hat{\mu}^*_t = 0$. In this case, by inspecting (3.22)–(3.24), it can be easily seen that $\pi_{H,t} = \pi^*_{F,t} = 0$ minimize completely the

[13] In particular π_{H,t_0} and π^*_{F,t_0} represent the log-linear approximation to initial commitments to the variables F_{t_0}/K_{t_0} and $F^*_{t_0}/K^*_{t_0}$. Here $\bar{\pi}_{H,t_0}$ and $\bar{\pi}^*_{F,t_0}$ are functions of predetermined and exogenous variables that will be self-consistent in the equilibrium in the sense that they will be the same functions that will result in equilibrium at later dates. (see Woodford, 2003, chapter 7)

[14] A formal proof can be done in the following way. Rewrite the loss function in terms of the targets $\hat{C}_t - \tilde{C}^w_t$, $\hat{T}_t - \tilde{T}^w_t$, $\pi_{H,t}$, $\pi^*_{F,t}$, and note that all the coefficients are now positive. The loss function is then convex. Indeed in the same way the aggregate supply equations can be written in terms of the above four variables. These two aggregate supply equations are all that is needed to evaluate this modified loss function.

loss function and achieve the first-best. In particular the optimal cooperative allocation replicates the flexible-price outcome. This is not a surprise, since both the steady-state is efficient and there is no trade-off because of further inefficiencies due to the shocks. Indeed the existence of mark-up shocks would create an inefficient wedge in the marginal rate of substitution between goods consumption and production. Absent this friction it is then optimal to stabilize the remaining shocks as in the flexible-price allocation. At the same time setting $\pi_{H,t} = \pi^*_{F,t} = 0$ eliminates the price-dispersion distortion due to staggered pricing mechanism implicit in the Calvo's adjustment rule. In the optimal cooperative allocation (absent mark-up shocks), since producer prices are fixed, the nominal exchange rate moves to accommodate asymmetric shocks in the following way

$$\ln S_t/\bar{S} = \frac{\eta}{1+\theta\eta s_c}[\hat{a}_t - \hat{a}^*_t - (\hat{G}_t - \hat{G}^*_t)]. \tag{3.26}$$

When the home country has a favorable productivity shock, the home currency depreciates so that the demand for the home-produced goods can increase. The same effect follows a decrease in home government purchases. These findings support the Friedman's argument for having floating exchange rate regime. Moreover, to enforce the optimal cooperative solution is sufficient that each country stabilizes its own producer inflation rate. As in Obstfeld and Rogoff (2002), self-oriented policymakers can achieve the first best.

On the other hand, when the economy is subject to mark-up fluctuations, then monetary policymakers have a role in stabilizing those inefficiencies and move away from the flexible price allocation. To study the optimal cooperative allocation in this more general case, we write the following Lagrangian:

$$\begin{aligned}
\mathcal{L} = E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\lambda_y^w[\frac{1}{2}ny^2_{H,t} + \frac{1}{2}(1-n)y^{*2}_{F,t} + \frac{1}{2}n(1-n)s_c\theta\psi q_t^2 + \frac{1}{2}\frac{\sigma n s_c^2}{\kappa}\pi^2_{H,t} + \\
+\frac{1}{2}\frac{\sigma(1-n)s_c^2}{\kappa^*}\pi^{*2}_{F,t}] + n\varphi_{1,t}[\kappa^{-1}\pi_{H,t} - y_{H,t} - (1-n)\psi q_t - \beta\kappa^{-1}\pi_{H,t+1}] + \\
+(1-n)\varphi_{2,t}[\kappa^{*-1}\pi^*_{F,t} - y^*_{F,t} + n\psi q_t - \beta\kappa^{*-1}\pi^*_{F,t+1}] + n(1-n)\varphi_{3,t}[q_t + \\
-\theta^{-1}s_c^{-1}y_{H,t} + \theta^{-1}s_c^{-1}y^*_{F,t}] - n\varphi_{1,t_0-1}\kappa^{-1}\pi_{H,t_0} - (1-n)\varphi_{2,t_0-1}\kappa^{*-1}\pi^*_{F,t_0},
\end{aligned}$$

where we have defined $y_{H,t} \equiv (\hat{Y}_{H,t} - \tilde{Y}^w_{H,t})$, $y^*_{F,t} \equiv (\hat{Y}^*_{F,t} - \tilde{Y}^w_{F,t})$ and $q_t \equiv (\hat{T}_t - \tilde{T}^w_t)$ and we have appropriately normalized the Lagrangian multiplier in a way to obtain time-invariant first-order conditions. The first-order condition with respect to $y_{H,t}$, $y^*_{F,t}$ and q_t are

$$\lambda_y^w y_{H,t} = \varphi_{1,t} + (1-n)\theta^{-1}s_c^{-1}\varphi_{3,t}, \tag{3.27}$$

$$\lambda_y^w y^*_{F,t} = \varphi_{2,t} - n\theta^{-1}s_c^{-1}\varphi_{3,t}, \tag{3.28}$$

$$\lambda_y^w s_c \theta \psi q_t = \psi\varphi_{1,t} - \psi\varphi_{2,t} - \varphi_{3,t}, \tag{3.29}$$

for each $t \geq t_0$, while the ones with respect to $\pi_{H,t}$ and $\pi^*_{F,t}$ are

$$\lambda_y^w \sigma s_c^2 \pi_{H,t} = -(\varphi_{1,t} - \varphi_{1,t-1}), \tag{3.30}$$

$$\lambda_y^w \sigma s_c^2 \pi^*_{F,t} = -(\varphi_{2,t} - \varphi_{2,t-1}), \tag{3.31}$$

for each $t \geq t_0$.

We show in appendix A that equations (3.27)–(3.31), combined with the structural equations (3.22)–(3.24) and the initial conditions φ_{1,t_0-1} and φ_{2,t_0-1} determine the equilibrium path of outputs, inflation rates, and terms of trade along with the Lagrangian multipliers. When $\bar{\mu} = 1$, we show (see Appendix B) that the nominal exchange rate follows

$$\ln S_t/\bar{S} = \left(\frac{1}{\sigma} - \frac{1}{\theta s_c}\right)\frac{s_c^2}{(\rho + \eta s_c)}(\varphi_{1,t} - \varphi_{2,t}) + \frac{\eta}{1 + \theta s_c \eta}[\hat{a}_t - \hat{a}^*_t - (\hat{G}_t - \hat{G}^*_t)].$$

We note here that when $\theta s_c = \sigma$ and there are only mark-up shocks, then the optimal cooperative solution would require to fix the exchange rate. Otherwise, when there are no mark-up shocks then $\varphi_{1,t} = \varphi_{2,t} = 0$ at each time and the exchange rate will follow (3.26).

In order to study the optimal transmission mechanism of shocks, we calibrate a quarterly model for countries with equal size, i.e. $n = 1/2$. Following Rotemberg and Woodford (1997), we set $\beta = 0.99$ and $\eta = 0.47$. We assume $\alpha = 0.66$ and $\alpha^* = 0.75$ implying an average length of price contracts equal to 3 and 4, respectively. We then assume that the elasticity σ across goods produced within a country is 10, while $\bar{\mu}$ in this case is equal to 1. The steady-state level of consumption over output is calibrated to $s_c = 0.8$. Finally, the risk aversion coefficient ρ is usually assumed to be in a range between 1 and 5, and we use 3, while following Obstfeld and Rogoff (2000), the intratemporal elasticity of substitution θ is in the range 3 to 6 and we choose 4.5. An important implication of this

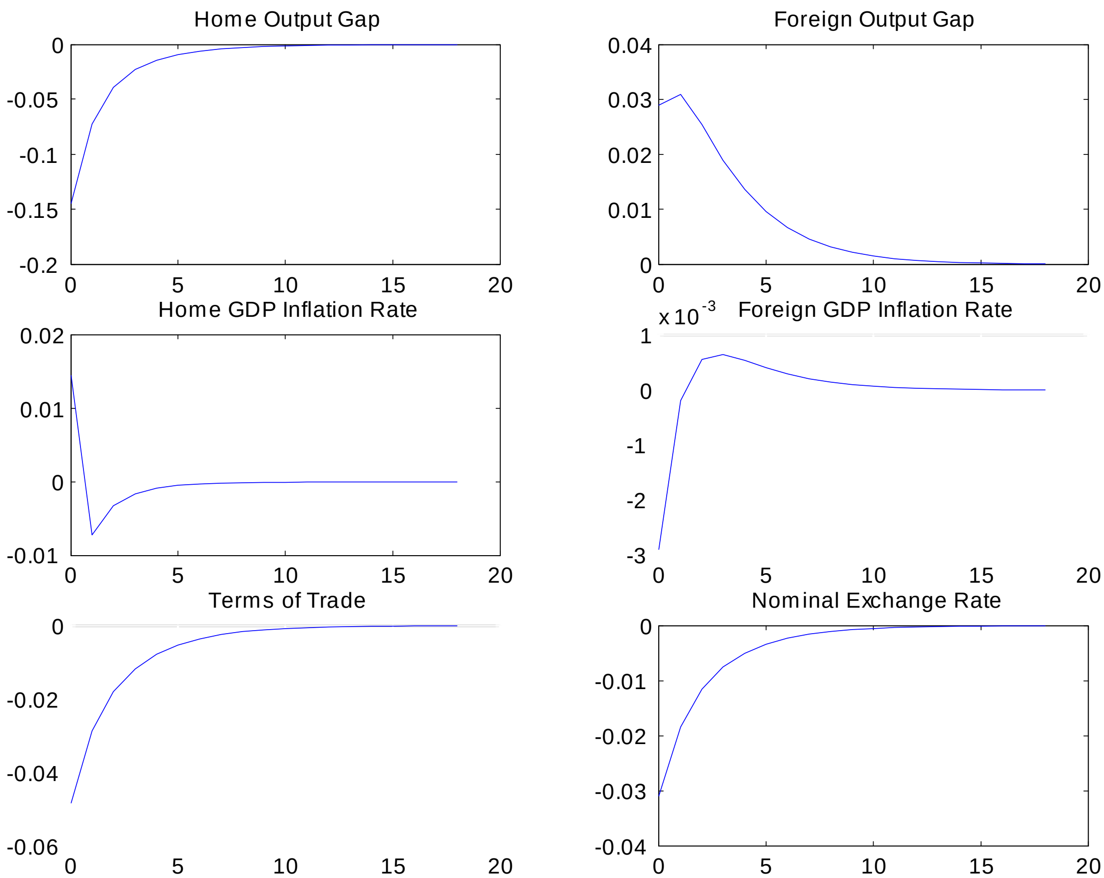

Figure 1: Figure 1: Impulse responses of home and foreign outputs, home and foreign GDP inflation rates, terms of trade and exchange rate to a home mark-up shock.

calibration is that $\theta > \frac{1}{\rho}$, i.e. the intratemporal elasticity of substitution is higher than the intertemporal elasticity of substitution, which means that the home and foreign bundles of goods are substitute in the utility.[15]

Figure 1 presents the impulse responses following a positive one-time mark-up shock in the domestic economy. In a similar way to the closed-economy model of Clarida et al. (1999) and Woodford (2003, ch. 3), a mark-up shock in the home country is absorbed by a temporary fall in the home output gap and by an initial jump in home GDP inflation rate. After the shock, the output gap

[15] Two goods are substitute in the utility when the marginal utility of one good decreases as the consumption of the other good increases.

converges back to the initial steady state and the price level converges as well to the initial level through periods of deflation. The fall in the output worsen the home country terms of trade ($\hat{T}$ decreases). The key insight to understand the optimal transmission mechanism of the mark-up shock across countries is the link between foreign real marginal costs and the terms of trade. When goods are substitute in utility, an improvement in the foreign terms of trade ($\hat{T}$ decreases) reduces the real marginal costs for foreign producers, acting as a negative mark-up shock for them. Producer prices fall and the output gap rises. Home and foreign output gaps and the two GDP inflation rates commove in a negative way following the shock. Under a different parametrization, $\theta < \frac{1}{\rho}$, the co-movement would be positive, while no spillover effect would occur if $\theta = \frac{1}{\rho}$. In the calibrated example the exchange rate appreciates but moves less than in the case the economy is hit by a productivity shock. Most interesting, following any kind of stationary shock, the optimal cooperative solution requires both prices and exchange rate to revert back to their initial values.

3.2 Non-cooperative allocation

We now analyze the approximation to the non-cooperative stochastic problem to study the existence of gains from cooperation. Moreover our framework allows us to discuss the form of policymakers' objective functions when they do not cooperate. As before, we initially focus on the case in which $\bar{\mu} = 1$. As detailed in the appendix B, the loss function for the policymaker in the home country is

$$L = \frac{1}{2} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [\lambda_{y_h} (\hat{Y}_{H,t} - \tilde{Y}^h_{H,t})^2 + \lambda_{y_f} (\hat{Y}^*_{F,t} - \tilde{Y}^h_{F,t})^2 + \lambda_q (\hat{T}_t - \tilde{T}^h_t)^2 + \lambda_{\pi_h} \pi^2_{H,t} + \lambda_{\pi_f} \pi^{*2}_{F,t}] \tag{3.32}$$

while for the policymaker in the foreign country is

$$L^* = \frac{1}{2} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [\lambda^*_{y_h} (\hat{Y}_{H,t} - \tilde{Y}^f_{H,t})^2 + \lambda^*_{y_f} (\hat{Y}^*_{F,t} - \tilde{Y}^f_{F,t})^2 + \lambda^*_q (\hat{T}_t - \tilde{T}^f_t)^2 + \lambda^*_{\pi_h} \pi^2_{H,t} + \lambda^*_{\pi_f} \pi^{*2}_{F,t}] \tag{3.33}$$

where $\lambda^i_{y_h}$, $\lambda^i_{y_f}$, λ^i_q, $\lambda^i_{\pi_h}$, $\lambda^i_{\pi_f}$ are parameters, defined in the appendix A, and $\tilde{Y}^h_{H,t}$, $\tilde{Y}^h_{F,t}$, $\tilde{T}^h_t$, $\tilde{Y}^f_{H,t}$, $\tilde{Y}^f_{F,t}$, $\tilde{T}^f_t$ are combinations of the shocks of the model, defined as well in the appendix A, and have the interpretation of desired targets for the respective variables. We note here that in general these targets might be different from the ones implied by the cooperative loss function.

Our approach shows that a quadratic representation of a welfare-based loss function has a different form compared to the quadratic objective functions that have been previously assumed in the literature on international monetary policy cooperation. In those papers, the loss functions of the policymakers were quadratic in the deviations of output (or unemployment) with respect to a 'desired' level and in the CPI inflation rate, as in Canzoneri and Gray (1985) and Canzoneri and Henderson (1991). Other studies, as Persson and Tabellini (1995, 1996), have included also a concern for terms of trade stabilization. In our model the loss functions of country H and F present the same target variables but with country-specific weights and 'desired' targets. In particular each policymaker should be concerned about quadratic deviations of both domestic and foreign outputs, domestic and foreign GDP inflation rates and of the terms of trade from their country-specific desired targets. These expressions differ sharply from their closed-economy counterpart, as in Rotemberg and Woodford (1997), Woodford (2003, ch. 6) and Benigno and Woodford (2004). In these studies the loss function is usually quadratic in the inflation rate and in the deviation of output with respect to a desired target. This should be less surprising result once we observe that the objective function captures the distortions existing in the economy and that the two countries are interdependent both in the consumption and in the production of goods.[16]

In the linear-quadratic approximation to the stochastic non-cooperative problem, policymaker in country H minimizes (3.32) under the constraints given by (3.22)-(3.24) by choosing the sequence of its own GDP inflation rates $\{\pi_{H,t}\}_{t=t_0}^{\infty}$ taking as given the strategy of the other policymaker which implies a given sequence of foreign GDP inflation rates $\{\pi_{F,t}^{*}\}_{t=t_0}^{\infty}$ and given the initial conditions implied by the additional timeless-perspective constraints on π_{H,t_0}.

Before computing the non-cooperative allocation, a first important issue to address is whether the second-order conditions of the above minimization problem are really satisfied. As shown in Benigno and Woodford (2004) this question

[16] In a static model, with prices all fixed one-period in advance, Sutherland (2002b) has shown that home and foreign utility-based welfare criteria depend on foreign and domestic outputs as well as on the nominal exchange rate.

is closely related to the issue of studying whether purely random policy can be welfare improving. Although we are still assuming $\bar{\mu} = 1$, it might happen that the steady-state around which the approximation is taken does not satisfy second-order conditions for being the equilibrium outcome of a Nash game even if it is the efficient steady state from the cooperative level. The intuition is that when each policymaker maximizes the welfare of its own country taking as given the strategy of the other policymaker, it would have incentives to maintain higher monopolistic distortions, to worsen the terms of trade and shift the burden of production to the other country, as discussed in Corsetti and Pesenti (2002). In our context, since we analyze a commitment solutions, policymakers cannot engage in surprise inflation or deflation; however they might still want to affect randomly the economy and improve their own welfare. To study this possibility, we assume that we start with some equilibrium process $\{\pi_{H,t},\ \pi^*_{F,t},\ \hat{Y}_{H,t},\ \hat{Y}^*_{F,t},\ \hat{T}_t\}$ consistent with the constraints (3.22)-(3.24) and we consider the possibility of perturbing this equilibrium by adding terms that depend on a sunspot realization υ_t at some date $t > t_0$. The variable υ_t is assumed to have conditional expectation zero at date t_0 and variance σ^2_υ and to be distributed independently of the processes $\{\pi_{H,t},\ \pi^*_{F,t},\ \hat{Y}_{H,t},\ \hat{Y}^*_{F,t},\ \hat{T}_t\}$ and of all the fundamentals shocks. Given the strategy of the foreign policymaker and the implied sequences of $\{\pi^*_{F,t}\}$, we assume that the sunspot shock adds a contribution $\varphi_j \upsilon_j$ to $\pi_{H,t+j}$ for each $j \geq 0$, where $\{\varphi_j\}$ is an arbitrary sequence of coefficients. This shock adds a contribution $\varsigma_1 \kappa^{-1}(\varphi_j - \beta\varphi_j)\upsilon_t$, $\varsigma_2 \kappa^{-1}(\varphi_j - \beta\varphi_j)\upsilon_t$, $\varsigma_3 \kappa^{-1}(\varphi_j - \beta\varphi_j)\upsilon_t$ to $\hat{Y}_{H,t+j}$ and $\hat{Y}_{F,t+j}$ and $\hat{T}_{t+j}$ consistently with (3.22)-(3.24), where $\varsigma_1 \equiv (1 + n\psi\theta^{-1}s_c^{-1})/(1 + \psi\theta^{-1}s_c^{-1})$, $\varsigma_2 \equiv n\psi\theta^{-1}s_c^{-1}/(1 + \psi\theta^{-1}s_c^{-1})$, $\varsigma_3 \equiv \theta^{-1}s_c^{-1}/(1 + \psi\theta^{-1}s_c^{-1})$. The contribution to the loss function will be then

$$\beta^t \sigma^2_\upsilon \sum_{j=0}^{\infty} \beta^j \left[\lambda_{\pi_h} \varphi_j^2 + \lambda_y \left(\frac{\varphi_j - \beta\varphi_{j+1}}{\kappa} \right)^2 \right]$$

where $\lambda_y \equiv \lambda_{y_h}\varsigma_1^2 + \lambda_{y_f}\varsigma_2^2 + \lambda_q \varsigma_3^2$. Randomization will be locally undesirable if and only if this expression is positive in the case of all possible non-zero bounded sequences $\{\varphi_j\}$. Randomization of policy is welfare decreasing if and only if: λ_{π_h} and λ_y are not both equal to zero and either (i) $\lambda_y \geq 0$ and $\lambda_{\pi_h} + (1-\beta^{1/2})^2\kappa^{-2}\lambda_y \geq 0$, holds, or (ii) $\lambda_y \leq 0$ and $\lambda_{\pi_h} + (1+\beta^{1/2})^2\kappa^{-2}\lambda_y \geq 0$, holds.

In a closed economy case, when preferences are isoelastic, the conditions for randomization to be welfare improving rely on implausible parameters, i.e. high share of government purchase in steady-state output (see Benigno and Woodford, 2004). In our two-country open-economy model, these conditions are less restrictive. In particular a critical parameter is the intratemporal elasticity of substitution θ. When θ increases λ_q becomes negative and pushes λ_y to be as well negative. Moreover λ_{π_h} becomes negative, too. It then follows that condition (i) is satisfied only for low values of θ while conditions (ii) is in general never satisfied. In the above calibrated example, leaving θ as a free parameter, we find that condition (i) is satisfied for θ less than 5. Similar procedure for the other country delivers a close result. This finding points toward the conclusion that policymakers have an incentive to create volatility in excess of 'fundamental' disturbances to improve their own welfare. Indeed, there is a terms-of-trade externality and each policymaker has an incentive to use its volatility to increase its own expected consumption and decrease its own expected disutility of production. It is not a surprise that the elasticity of output toward terms-of-trade movements is the critical parameters and that for high values of this elasticity the incentives are higher. Absent any 'fundamental' disturbance, when randomization is welfare improving, there is no Nash equilibrium in which producer inflation rates are zero in the deterministic non-cooperative problem. Our approximation method cannot be used in this case, except for recognizing the existence of this incentive for policymakers.

When conditions (i) or (ii) hold, we can analyze the equilibrium response to fundamental shocks in the case countries do not cooperate in a Nash equilibrium. The minimization problem for the home policymaker can be represented by the following Lagrangian

$$\mathcal{L} = E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [\frac{1}{2}\lambda_{y_h}(y_{H,t}^h)^2 + \frac{1}{2}\lambda_{y_f}(y_{F,t}^h)^2 + \frac{1}{2}\lambda_q^h q_t^2 + \frac{1}{2}\lambda_{\pi_h}\pi_{H,t}^2 +$$
$$+\frac{1}{2}\lambda_{\pi_f}\pi_{F,t}^{*2}] + \phi_{1,t}[\kappa^{-1}\pi_{H,t} - y_{H,t}^h - (1-n)\psi q_t^h - \beta\kappa^{-1}\pi_{H,t+1}] +$$
$$+\phi_{2,t}[\kappa^{*-1}\pi_{F,t}^* - y_{F,t}^{h*} + n\psi q_t^h - \beta\kappa^{*-1}\pi_{F,t+1}^*] + n(1-n)\phi_{3,t}[q_t +$$
$$-\theta^{-1}s_c^{-1}y_{H,t}^h + \theta^{-1}s_c^{-1}y_{F,t}^{h*}] - \phi_{1,t_0-1}\kappa^{-1}\pi_{H,t_0}$$

where we have defined $y_{H,t}^h \equiv (\hat{Y}_{H,t} - \tilde{Y}_{H,t}^h)$, $y_{F,t}^{*h} \equiv (\hat{Y}_{F,t}^* - \tilde{Y}_{F,t}^h)$ and $q_t^h \equiv (\hat{T}_t - \tilde{T}_t^h)$ and we have appropriately normalized the Lagrangian multiplier in a way to obtain time-invariant first-order conditions. The first-order condition with respect to $y_{H,t}$, $y^*_{F,t}$, q_t and $\pi_{H,t}$ are

$$\lambda_{y_h} y^h_{H,t} = \phi_{1,t} + \theta^{-1} s_c^{-1} \phi_{3,t},$$

$$\lambda_{y_f} y^{h*}_{F,t} = \phi_{2,t} - \theta^{-1} s_c^{-1} \phi_{3,t},$$

$$\lambda_{y_q} q^h_t = (1-n)\psi\varphi_{1,t} - \psi\varphi_{2,t} - \varphi_{3,t},$$

$$\lambda_{y_h} \pi_{H,t} = -\kappa^{-1}(\varphi_{1,t} - \varphi_{1,t-1}),$$

for each $t \geq 0$. In a similar way, we can characterize the first-order conditions for the optimization problem of the foreign policymaker. The set of all the first-order conditions and the structural constraints determine the non-cooperative allocation.

There are some special cases in which cooperative and non-cooperative allocations coincide. In particular it is necessary that there are no mark-up shocks, $\hat{\mu}_t = \hat{\mu}^*_t = 0$ at all times. In the previous section, we found that absent mark-up shocks the flexible-price allocation is the optimal cooperative policy, when $\bar{\mu} = 1$. However even under this restriction and differently from previous findings (see Obstfeld and Rogoff, 2002), there can be gains from cooperation. Indeed, each policymaker has an incentive to use the terms of trade in its favor by shifting the burden of production to the other country. Since the steady-state is efficient and randomization of policy is welfare decreasing, it is not possible to engineer surprise terms-of-trade movements. However, a committed policymaker can appropriately stabilize the shocks and increase its country's expected utility of consumption while reducing the expected disutility of producing goods. As in Henderson and Kim (1999) and Obstfeld and Rogoff (1998), the ability to precommit does not prevent this possibility because the expected values of variables depend on the expected value of first-order and second-order terms.[17] Indeed, an appropriate stabilization of the shocks affect the second moments of variables and then the expected level.

There are no gains from cooperation under two special cases. One simple case is when $L = L^* = L^W$. This occurs when at the same time $\theta = 1$, i.e. preferences are Cobb-Douglas across home and foreign produced goods, $s_c = 1$, i.e. there is no steady-state public expenditure, and when there are only productivity shocks,

[17] Loosely speaking, in a commitment equilibrium the expected value of first-order terms is equal to zero, while terms of order higher than the second are not relevant in a second-order approximation.

$\hat{a}_t$ and $\hat{a}_t^*$. We retrieve here the Obstfeld and Rogoff (2002) case. As discussed in Benigno and Benigno (2003), there is too much risk-sharing under these parametric restrictions. Indeed, the expected disutilities of producing goods are equalized across countries along with the marginal utilities of consumption. In this case the terms of trade is ineffective in stabilizing shocks for its own country's utility, since the disutility of goods production is tied across countries. In a numerical example, Sutherland (2001, 2002b) and Tchakarov (2002) have quantified as important the gains from cooperation in the case that θ differs from 1.

Having observed that the nature of the negative externality lies directly in the use of the terms of trade, we can look at other cases of absence of gains from cooperation by focusing on the particular case in which the terms of trade channel is not effective. The previous literature on international monetary policy cooperation (see Sachs, 1988) has related the gains of cooperation to a parameter of interdependence that measures the importance of the terms of trade in the transmission mechanism across countries. In our context, the terms of trade interdependence is determined by the parameter ψ. When the intratemporal and intertemporal elasticities of substitution are equal, i.e. $\theta = 1/\rho$, then $\psi = 0$ and each policymaker can control its own output by manoeuvring its own GDP inflation rate. However, differently from the previous literature, this case does not necessarily imply the absence of gains from cooperation. Indeed, as clarified in Canzoneri et al. (2002), the case in which $\theta = 1/\rho$ describes economies that are independent of the terms of trade only in goods production, but they are still interrelated in goods consumption.

Using the definition of the coefficients and variables given in the appendix A when $\theta = 1/\rho$, the cooperative loss function (3.21) simplifies to a quadratic form that displays only GDP inflation and output targets, since $\psi = 0$, while the loss functions for each country simplify to

$$L = \frac{1}{2} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [\lambda_{y_h} (\hat{Y}_{H,t} - \tilde{\tilde{Y}}_{H,t}^{h})^2 + \lambda_{\pi_h} \pi_{H,t}^2] + \text{t.o.c.}$$

for country H and

$$L^* = \frac{1}{2} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [\lambda_{y_f}^* (\hat{Y}_{F,t}^* - \tilde{\tilde{Y}}_{F,t}^{f})^2 + \lambda_{\pi_f}^* \pi_{F,t}^{*2}] + \text{t.o.c.}$$

for country F, where t.o.c. denotes terms that are out of the control of the policymaker and include foreign GDP inflation and output. Note that the targets of the loss function that can be controlled by the policymakers mirror the ones that can be found in closed-economy models, as in Woodford (2003, ch. 6), since the objective function collapses to a standard quadratic function in an appropriately defined domestic output gap and GDP inflation. However, this result does not imply that cooperative and non cooperative solutions will necessarily coincide, since there are still spillover effects on consumption. Indeed the central planner weighs each country disutility of goods production less than what the single country does, since it recognizes that production in the country is absorbed by consumption in both economies, while the single country weighs more its disutility of goods production since it does not internalize the consumption and the utility of consumption of the other country. Only when the desired targets between the pairs of loss functions L, L^W and L^*, L^W coincide, i.e. $\tilde{\tilde{Y}}_{H,t}^{h} = \tilde{Y}_{H,t}^{w}$ and $\tilde{\tilde{Y}}_{F,t}^{f} = \tilde{Y}_{F,t}^{w}$, then the cooperative and non-cooperative equilibria coincide and there are no gains from cooperation. In appendix B, we show that this happens when $s_c = 1$ along with $\theta = 1/\rho$ and there are only productivity shocks.

The analysis of symmetric shocks is also an interesting source of comparisons with the previous literature. Models in the fashion of Canzoneri and Henderson (1991) found that the gains from cooperation were arising even with symmetric disturbances. Obstfeld and Rogoff (2002) instead show that with symmetric productivity shocks there are no gains from cooperation. Here, we find that this result holds for symmetric productivity shocks, provided $s_c = 1$. Otherwise, with other kind of disturbances, as for example mark-up and public expenditure shocks, or with $s_c < 1$, there are still gains from cooperation even when shocks are global.[18]

In general, the model analyzed here shows that the conditions under which there are no gains from cooperation are very restrictive. Although we do not quantify the magnitude of the gains from cooperation, it is worth mentioning that public expenditure shocks and mark-up shocks have been found to be important driving factors of the business cycle, as in Galí et al. (2003). Moreover, some

[18] Sutherland (2002b) shows that even symmetric productivity shocks may imply gains from cooperation. His framework is different from ours: indeed, he considers a structure in which contingent claims market open after policymakers have chosen their policy strategies.

simple numerical examples of Canzoneri et al. (2001) and Tchakarov (2002) have shown that this class of models which rely on microfounded loss functions can produce larger gains from cooperation than the previous literature did in the 80's and 90's.

3.3 Targeting rules for international monetary policy cooperation

In the previous sections, we have considered policymakers that maximize the utility of the consumers in their respective countries. However, a policymaker that shares the preferences of the consumers or society does not internalize the negative externality that it may impose on other countries so that in general the cooperative and non-cooperative allocation diverge. How is then possible to design institutions, as central banks, with the 'right incentives'? There are several examples in the literature in which this issue is solved by delegating a new objective function to an independent agent, a central bank, as shown in the contributions of Rogoff (1985), Persson and Tabellini (1993, 1995, 1996), Walsh (1995), Svensson (1997), and Jensen (2000, 2002). As discussed in Svensson (2002, 2003), designing institutions by imposing a commitment to a loss function can be interpreted as a 'general targeting rule', which is a general operational objective.

Here we follow an alternative and perhaps more direct way to design institutions through the assignment of 'specific targeting rules' (as proposed in Svensson, 2003) that each policymaker should follow. These specific targeting rules represent Euler equations derived from the behavior of optimizing central banks.

Our goal is to design targeting rules that are optimal from the cooperative perspective. To this end we follow the method proposed by Giannoni and Woodford (2002). In a linear-quadratic model they show that optimal targeting rules can be obtained by eliminating the Lagrangian multipliers from the first-order conditions of the optimal policy problem. Targeting rules built on this principle present some desirable characteristics. First, by ensuring that these targeting rules hold at all times, a determinate rational expectations equilibrium can be achieved and this equilibrium coincides with the optimal policy from a timeless perspective. Second, these targeting rules are optimal regardless of the statistical properties of the exogenous shocks. They depend on the shocks insofar as the targets specified in the loss function depend on them.

To derive the desirable targeting rules we use the first-order conditions (3.27) to (3.31). First, we take a weighted average with weights n and $(1-n)$ of (3.27) and (3.28), obtaining

$$\lambda_y^w[ny_{H,t} + (1-n)y^*_{F,t}] = n\varphi_{1,t} + (1-n)\varphi_{2,t}. \tag{3.34}$$

We take the difference of (3.27) and (3.28) and combine it with (3.29), obtaining

$$\lambda_y^w(y_{H,t} - y^*_{F,t}) = (\varphi_{1,t} - \varphi_{2,t}), \tag{3.35}$$

where we have used the fact that $y_{H,t} - y^*_{F,t} = \theta s_c q_t$. By using (3.34) and (3.35), we can obtain

$$\lambda_y^w y_{H,t} = \varphi_{1,t},$$

$$\lambda_y^w y^*_{F,t} = \varphi_{2,t},$$

which combined with (3.30) and (3.31) yields the following relation

$$\sigma s_c^2 \pi_{H,t} + \Delta y_{H,t} = 0, \tag{3.36}$$

$$\sigma s_c^2 \pi^*_{F,t} + \Delta y^*_{F,t} = 0. \tag{3.37}$$

In particular, targeting rule (3.36) should be assigned to the monetary policymaker in country H and (3.37) to the policymaker in country F. In appendix A, we show that by committing to these rules a determinate rational expectations equilibrium can be achieved that implements the optimal cooperative solution from a 'timeless perspective'.

The above rules present some other interesting characteristics. For both policymakers, they involve the same set of target variables –GDP inflation and output– with the same combination of weights in the overall targeting rules. However, each of these target variables enters into the targeting rule in deviation from 'desired' target that can be country-specific. For both countries, the 'desired' target for GDP inflation is zero, while the 'desired' targets for output are in general different from zero and country-specific. Under our special case in which $\bar{\mu} = 1$, these rules are similar to closed-economy targeting rules (see Giannoni and Woodford

(2003)). Most important, even if there are gains from cooperation and the optimal behavior of the exchange rate depends on the kind of shock, we found that the optimal targeting rules do not involve any target to the exchange rate but only to domestic variables.

4 General Case

Having gained some intuition, we now move to the general case in which the monopolistic distortions are not necessarily neutralized, i.e. $\bar{\mu} > 1$. Under this more general assumption the cooperative loss function can be written as

$$L^W = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t [n\lambda_y^w (\hat{Y}_{H,t} - \tilde{Y}_{H,t}^w)^2 + (1-n)\lambda_y^w (\hat{Y}_{F,t}^* - \tilde{Y}_{F,t}^w)^2 + \\ + n(1-n)\lambda_q^w (\hat{T}_t - \tilde{T}_t^w)^2 + n\lambda_{\pi_h}^w \pi_{H,t}^2 + (1-n)\lambda_{\pi_f}^w \pi_{F,t}^{*2}], \qquad (4.38)$$

where the parameters λ_y^w, λ_q^w, $\lambda_{\pi_h}^w$, $\lambda_{\pi_f}^w$ and the variables $\tilde{Y}_{H,t}^w$, $\tilde{Y}_{F,t}^w$, $\tilde{T}_t^w$ are defined in appendix B. Here the variables $\tilde{Y}_{H,t}^w$, $\tilde{Y}_{F,t}^w$, $\tilde{T}_t^w$ represent the desired targets for the respective variables when countries cooperate. The countries' loss function are instead of the same form as (3.32) and (3.33) where now the parameters and targets that are different function of 'deep' parameters and shocks, as detailed in appendix B. We now investigate whether previous results hold in this more general case. We indeed find that the conditions under which the flexible-price allocation is optimal under cooperation are much more restrictive. In appendix B we prove the following proposition.

Proposition 1. Under cooperation, the flexible-price allocation is constrained-optimal when there are no mark-up shocks and either (i) $\bar{\mu} = 1$ or (ii) there are no government purchases, i.e. $s_c = 1$ and $\hat{G}_t = \hat{G}_t^* = 0$

When the steady-state is not efficient, but constrained efficient, further requirements are needed. Indeed there should be no government purchases. The intuition is similar to previous analysis. In a cooperative agreement, policymakers aim to commit to policies that raise the expected level of consumption and output in both countries, since they are inefficiently low. In general, when $\bar{\mu} > 1$, i.e. the steady-state level of output is inefficiently low, stabilization policies can be used to increase the expected level of output. There is a tension between replicating the flexible-price response to the shocks and stabilizing the shocks appropriately to increase the expected level of output.[19]

On the other hand the existence of gains from cooperation does not depend on the presence of steady-state distortion:

PROPOSITION 2. There are no gains from cooperation when there are no-mark shocks and government purchases, i.e. $\hat{\mu}_t = \hat{\mu}_t^* = \hat{G}_t = \hat{G}_t^* = 0$ and $s_c = 1$, and either (i) $\theta = 1$ or (ii) $\theta\rho = 1$.

We now investigate whether the optimal targeting rules change under this more general case. Using the same method described in the previous section, it is possible to show that the Lagrangian problem of minimizing (4.38) under the constraints (3.22)–(3.24) and the initial conditions on π_{H,t_0} and π^*_{F,t_0} yields to the following targeting rules

$$(\kappa\lambda^w_{\pi_h} + \gamma)\pi_{H,t} + \lambda^w_y \Delta y_{H,t} - \gamma(\pi_t - \tilde{\pi}_t) = 0, \tag{4.39}$$

$$(\kappa^*\lambda^w_{\pi_f} + \gamma)\pi^*_{F,t} + \lambda^w_y \Delta y^*_{F,t} - \gamma(\pi^*_t - \tilde{\pi}^*_t) = 0, \tag{4.40}$$

where we have used the fact that $\lambda^w_q = \theta s_c^{-1}\psi[s_c^2\lambda^w_y - \bar{\mu}^{-1}(\bar{\mu}-1)(1-s_c)s_c\eta(s_c\eta + \rho)^{-1}]$ and defined γ as $\gamma \equiv \psi\bar{\mu}^{-1}s_c^{-1}\eta(\bar{\mu}-1)(1-s_c)(s_c\eta+\theta^{-1})^{-1}$where $\tilde{\pi}_t \equiv (1-n)(\tilde{T}^*_t - \tilde{T}^*_{t-1})$ and $\tilde{\pi}^*_t \equiv -n(\tilde{T}^*_t - \tilde{T}^*_{t-1})$.

The above rules present some other interesting characteristics with respect to the previous case. They involve an additional target in terms of CPI inflation rates in contrast with previous findings when $\bar{\mu} = 1$. For both policymakers, they involve the same set of target variables –GDP, CPI inflation and output– with the same combination of weights in the overall targeting rules. As in the previous special case, each of these target variables enters into the targeting rule in deviation from a 'desired' target which is instead country specific. For both countries, the 'desired' target for GDP inflation is zero, while the 'desired' targets for output and CPI inflation are in general different from zero and country specific. Most interesting,

[19] This is not the case when there are productivity shocks since preferences are 'isoelastic' so that stabilization policies are not effective in increasing the expected level of output.

the 'desired' targets for CPI inflation rates move in the opposite direction when comparing the two countries and implicitly define a desired path for the exchange rate such that $\ln S_t/\bar{S} = \tilde{T}_t^*$.

Differently from the closed-economy counterpart, our targeting rules in the general case should include a distinction between GDP and CPI inflation rates. Indeed, in the basic model by Giannoni and Woodford (2003), with only sticky prices and monopolistic competition, the optimal targeting rule is expressed as a combination of inflation rate and output growth with respect to a desired target. In their framework there is no distinction between GDP and CPI inflation rates.

The extent to which the CPI target is relevant depends on the parameters of the model and not on the kind of disturbance that affects the economy. The dependence on CPI target disappears when either steady-state monopolistic distortions are completely offset ($\bar{\mu} = 1$), as shown in the previous section, or the two economies are independent, $\rho\theta = 1$, or when in the steady-state government purchase is equal to zero, $s_c = 1$. Interestingly, the case $\theta = 1$ does not appear as a case that excludes CPI from the target and in general the conditions that define the absence of gains from coordination do not necessarily coincide with the conditions that exclude CPI inflation from the target.

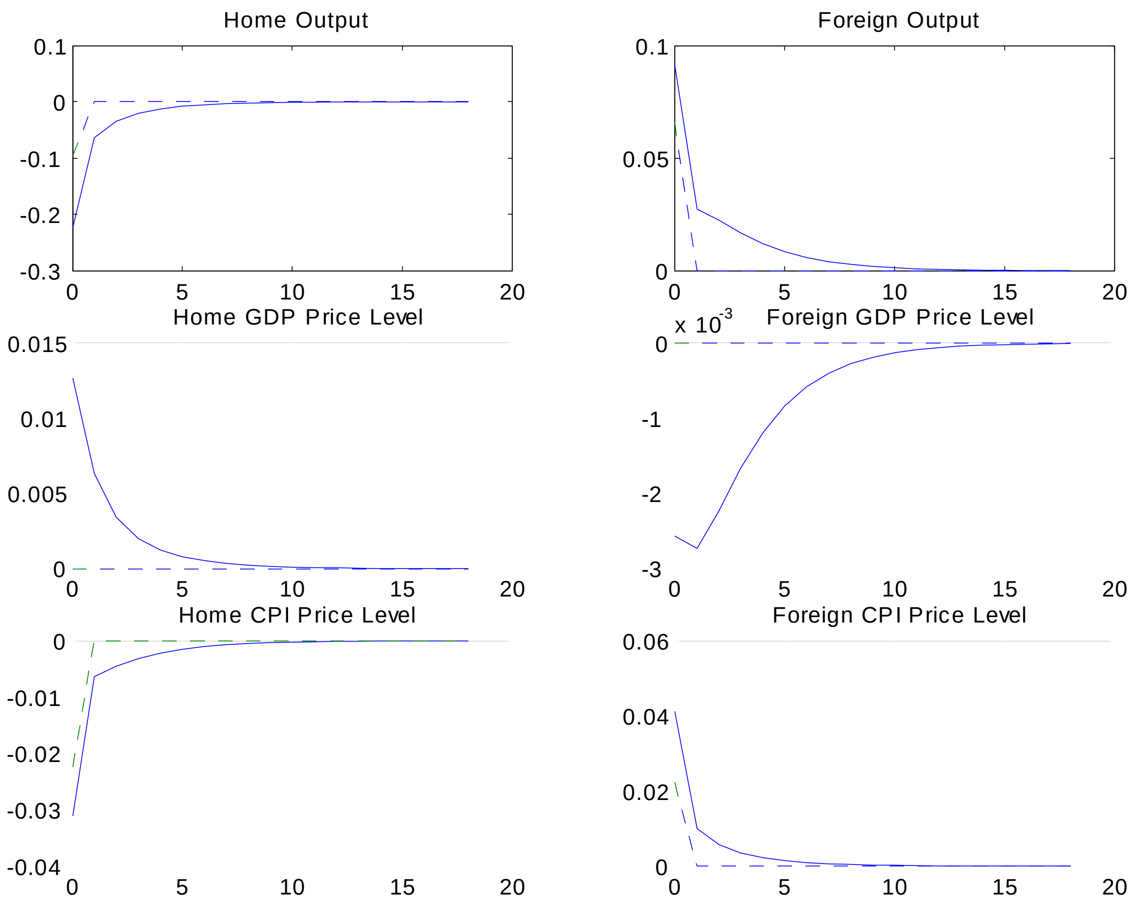

Figure2: Impulse responses of the target variables and 'desired' targets (dash lines) in the targeting rules following a home mark-up shock.

Figure 2 shows the impulse response functions following a home mark-up shock. In this figure we keep the same parametrization as in Figure 1, except for the value of the steady state monopolistic distortions since we now assume $\bar{\mu} = 1.38$ with a steady state tax $\bar{\tau} = 0.2$. This figure features the flexibility of adopting such targeting rules: the behavior of each target variable indeed does not necessarily coincides with their desired level. Focusing on the Home economy we observe that, following a positive domestic mark-up shock, the desired value for the GDP price level should not move while its actual GDP price level increases and smoothly converges towards its long-run desired level. On the other hand the desired output and CPI price level should fall in the period the shock occurs and converge back to the initial value once the shock disappears. Instead, the actual values for the

CPI price level and output fall more and converge slowly towards the initial steady state with respect to their desired values.

The implementation of these targeting rules can be in principle solved as in Persson and Tabellini (1995) and Jensen (2000). Central banks are assumed to be risk neutral and their objective function is designed to be the loss function of the country plus a penalty determined by a contract which is written in terms of observable variables.[20] Given these modified loss functions, the two central banks, acting in a non-cooperative equilibrium, implement the cooperative outcome. In our context, maintaining the assumption that central banks are risk neutral, we can design contracts of the form $\delta_0^i - \delta_1^i(\Lambda_t^i)^2$ for $i = H, F$ and given parameters δ_0^i and δ_1^i where Λ_t^i is defined as the country i targeting rule, e.g. $\Lambda_t^H \equiv (\kappa\lambda_{\pi_h}^w + \gamma)\pi_{H,t} + \lambda_y^w \Delta y_{H,t} - \gamma(\pi_t - \tilde{\pi}_t)$. Given these contracts, central banks are forced to follow the targeting rules. However, the restriction written in the contract is not stronger than the one implied by adjusting the objective functions of the central banks using contracts, in the fashion of Persson and Tabellini (1995) and Jensen (2000). As in these approaches, central banks maintain the flexibility implicit in the targeting rules and moreover the flexibility in choosing their instrument to meet their objectives. However, both our approach and theirs do not really solve the delegation problem and shift the cooperation problem at the delegation stage.[21]

5 Conclusions

We have shown that in a two-country general equilibrium model characterized by goods and financial markets integration, the efficient paths of the exchange rate

[20] In Persson and Tabellini (1995, 1996), the contract is restricted to be linear in observed variables but the optimal contract need to have state-contingent parameters. Jensen (2000) shows that by assigning quadratic contracts in observed variables the contract can be made non-state contingent.

[21] Indeed, each country has no incentive to assign to its central bank that type of contract even in the case that the other country is behaving in that way. This is discussed extensively in Bilbiie (2002). One solution would be to follow the folk theorem in delegation games of Fershtman et al. (1991), as in Persson and Tabellini (1995), and condition the parameters of the contracts, δ_0^i and δ_1^i, to the possible outcomes as in "take-it-or-leave-it" offers. However, this set of state-contingent non-linear contracts will be highly unrealistic since part of them is contingent on the payoffs, which can be difficult to observe. The other solution, as in Persson and Tabellini (1995), is to consider a delegation to a common supranational institutions endowed with the cooperative loss function with the task to design appropriately contracts for the single central banks.

and prices depend on the source of the disturbance that hits the economy. The interaction between the existing distortions and source of disturbance generates in general gains from cooperation so that policymakers that maximize their own welfare behave inefficiently in the non-cooperative allocation. This lack of coordination can be amended by assigning simple targeting rules to each policymaker so that the optimal cooperative outcome can be achieved. We have shown that surprisingly these rules depend only on domestic variables despite full goods and capital market integration.

Further research should investigate the robustness of these findings for economies in which asset markets are incomplete and when consumer prices are less responsive to exchange rate changes as in Devereux and Engel (2003) and possibly also the interdependence between monetary and fiscal policies that we have neglected here.

Another important open issue is the enforcement of the proposed targeting rules. We have briefly addressed this issue acknowledging that, as in previous contributions in the literature, the cooperation problem is simply shift at the delegation stage to a supranational authority.

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A Appendix

Deterministic Solution

In this appendix, we show the existence of a stationary solution for the deterministic version of the cooperative and non-cooperative problems. In the deterministic problem the exogenous shocks a_t, G_t, μ_t, a_t^*, G_t^*, μ_t^* take constant values such that $a_t = a_t^* = \bar{a} > 0$, $\mu_t = \mu_t^* = \bar{\mu} \geq 1$, $G_t = G_t^* = \bar{G} \geq 0$. First, we start from the cooperative problem and we show that, given initial conditions $\Delta_{t_0-1} = \Delta_{t_0-1}^* = 1$ and initial constraints $F_{t_0} = \bar{F}, F_{t_0}^* = \bar{F}^*, K_{t_0} = \bar{K}, K_{t_0}^* = \bar{K}^*$ such that $\bar{F}/\bar{K} = 1$, $\bar{F}^*/\bar{K}^* = 1$, there exist a stationary sequence for the set of variables x_t such that $x_t = \bar{x}$, where in particular $F_t = \bar{F}$, $F_t^* = \bar{F}^*$, $K_t = \bar{K}$, $K_t^* = \bar{K}^*$, $\Delta_t = \Delta_t^* = P_{H,t}/P_t = P_{F,t}/P_t = \Pi_{H,t} = \Pi_{F,t}^* = 1$ at all times. In the cooperative deterministic solution, countries agree to maximize

$$W^C \equiv \sum_{t=t_0}^{\infty} \beta^{t-t_0}[U(C_t) - nV(Y_{H,t}, \bar{\xi})\Delta_t - (1-n)V(Y_{F,t}^*, \bar{\xi}^*)\Delta_t^*],$$

under the following constraints

$$Y_{H,t} = \left(\frac{P_{H,t}}{P_t}\right)^{-\theta} C_t + \bar{G}, \tag{A.1}$$

$$\Delta_t = \alpha\Delta_{t-1}\Pi_{H,t}^{\sigma(1+\eta)} + (1-\alpha)p(\Pi_{H,t})^{-\frac{\sigma(1+\eta)}{1-\sigma}}, \tag{A.2}$$

$$p(\Pi_{H,t})^{\frac{1+\sigma\eta}{\sigma-1}} = \left(\frac{F_t}{K_t}\right), \tag{A.3}$$

$$K_t = \bar{a}^{-\eta}Y_{H,t}^{1+\eta} + \alpha\beta K_{t+1}\Pi_{H,t+1}^{\sigma(1+\eta)}, \tag{A.4}$$

$$F_t = \bar{\mu}^{-1}C_t^{-\rho}Y_{H,t}\left(\frac{P_{H,t}}{P_t}\right) + \alpha\beta\Pi_{H,t+1}^{\sigma-1}F_{t+1}, \tag{A.5}$$

$$Y_{F,t}^* = \left(\frac{P_{F,t}}{P_t}\right)^{-\theta} C_t + \bar{G}, \tag{A.6}$$

$$\Delta_t^* = \alpha^*\Delta_{t-1}^*(\Pi_{F,t}^*)^{\sigma(1+\eta)} + (1-\alpha^*)p(\Pi_{F,t}^*)^{-\frac{\sigma(1+\eta)}{1-\sigma}}, \tag{A.7}$$

$$p(\Pi_{F,t}^*)^{\frac{1+\sigma\eta}{\sigma-1}} = \left(\frac{F_t^*}{K_t^*}\right), \tag{A.8}$$

$$K_t^* = \bar{a}^{-\eta}(Y_{F,t}^*)^{1+\eta} + \alpha^*\beta K_{t+1}^*(\Pi_{F,t+1}^*)^{\sigma(1+\eta)}, \quad \text{(A.9)}$$

$$F_t^* = \bar{\mu}^{-1}C_t^{-\rho}Y_{F,t}^*\left(\frac{P_{F,t}}{P_t}\right) + \alpha^*\beta F_{t+1}^*(\Pi_{F,t+1}^*)^{\sigma-1}, \quad \text{(A.10)}$$

$$n\left(\frac{P_{H,t}}{P_t}\right)^{1-\theta} + (1-n)\left(\frac{P_{F,t}}{P_t}\right)^{1-\theta} = 1, \quad \text{(A.11)}$$

where we have defined $p(\Pi_{H,t}) \equiv \frac{1-\alpha\Pi_{H,t}^{\sigma-1}}{1-\alpha}$ and $p(\Pi_{F,t}^*) \equiv \frac{1-\alpha^*(\Pi_{F,t}^*)^{\sigma-1}}{1-\alpha^*}$. In what follows, we attach Lagrange multipliers $\phi_{1,t}$, $\phi_{2,t}$, $\phi_{3,t}$, $\phi_{4,t}$, $\phi_{5,t}$, $\phi_{6,t}$, $\phi_{7,t}$, $\phi_{8,t}$, $\phi_{9,t}$, $\phi_{10,t}$, $\phi_{11,t}$ to the constraints (A.1)–(A.11) and we also add multilipliers dated t_0 to the initial constraints $K_{t_0} = \bar{K}$, $F_{t_0} = \bar{F}$, $K_{t_0}^* = \bar{K}^*$, $F_{t_0}^* = \bar{F}^*$. The latter multipliers are normalized in such a way that the resulting first-order conditions take the same form at date t_0 as at all future dates. The first-order condition of the optimal cooperative problem with respect to C_t yields to

$$\begin{aligned} 0 \;=\; & U_c(C_t) - \left(\frac{P_{H,t}}{P_t}\right)^{-\theta}\phi_{1,t} + \rho\bar{\mu}^{-1}C_t^{-\rho-1}Y_{H,t}\left(\frac{P_{H,t}}{P_t}\right)\phi_{5,t} - \left(\frac{P_{F,t}}{P_t}\right)^{-\theta}\phi_{6,t} + \\ & +\rho\bar{\mu}^{-1}C_t^{-\rho-1}Y_{F,t}^*\left(\frac{P_{F,t}}{P_t}\right)\phi_{10,t}; \end{aligned}$$

the one with respect to $Y_{H,t}$ yields to

$$-nV_y(Y_{H,t},\bar{\xi})\Delta_t + \phi_{1,t} - (1+\eta)\bar{a}^{-\eta}Y_{H,t}^{\eta}\phi_{4,t} - \bar{\mu}^{-1}C_t^{-\rho}\left(\frac{P_{H,t}}{P_t}\right)\phi_{5,t} = 0;$$

the one with respect to Δ_t yields to

$$-nV(Y_{H,t},\bar{\xi}) + \phi_{2,t} - \alpha\beta\phi_{2,t+1}\Pi_{H,t+1}^{\sigma(1+\eta)} = 0;$$

the one with respect to F_t yields to

$$-\left(\frac{1}{K_t}\right)\phi_{3,t} + \phi_{5,t} - \alpha\Pi_{H,t}^{\sigma-1}\phi_{5,t-1} = 0;$$

the one with respect to K_t yields to

$$\left(\frac{F_t}{K_t^2}\right)\phi_{3,t} + \phi_{4,t} - \alpha\Pi_{H,t}^{\sigma(1+\eta)}\phi_{4,t-1} = 0;$$

the one with respect to $\Pi_{H,t}$ yields to

$$\begin{aligned} 0 \;=\; & -\alpha\sigma(1+\eta)\Delta_{t-1}\Pi_{H,t}^{\sigma(1+\eta)-1}\phi_{2,t} + \alpha\sigma(1+\eta)p(\Pi_{H,t})^{-\frac{\sigma(1+\eta)}{1-\sigma}-1}\Pi_{H,t}^{\sigma-2}\phi_{2,t} \\ & -(1+\sigma\eta)\frac{\alpha}{1-\alpha}\Pi_{H,t}^{\sigma-2}\phi_{3,t} - \sigma(1+\eta)\alpha K_t\Pi_{H,t}^{\sigma(1+\eta)-1}\phi_{4,t-1} - (\sigma-1)\alpha\Pi_{H,t}^{\sigma-2}F_t\phi_{5,t-1}; \end{aligned}$$

the one with respect to $\frac{P_{H,t}}{P_t}$ yields to

$$\theta \left(\frac{P_{H,t}}{P_t}\right)^{-\theta-1} C_t\phi_{1,t} - \bar{\mu}^{-1}C_t^{-\rho}Y_{H,t}\phi_{5,t} + (1-\theta)n\left(\frac{P_{H,t}}{P_t}\right)^{-\theta}\phi_{11,t} = 0;$$

the one with respect to $Y^*_{F,t}$ yields to

$$-(1-n)V_y(Y^*_{F,t},\bar{\xi})\Delta^*_t + \phi_{6,t} - (1+\eta)\bar{a}^{-\eta}(Y^*_{F,t})^{\eta}\phi_{9,t} - \bar{\mu}^{-1}C_t^{-\rho}\left(\frac{P_{F,t}}{P_t}\right)\phi_{10,t} = 0;$$

the one with respect to Δ^*_t yields to

$$-(1-n)V(Y^*_{F,t},\bar{\xi}) + \phi_{7,t} - \alpha^*\beta\phi_{7,t+1}(\Pi^*_{F,t+1})^{\sigma(1+\eta)} = 0;$$

the one with respect to F^*_t yields to

$$-\left(\frac{1}{K^*_t}\right)\phi_{8,t} + \phi_{10,t} - \alpha^*(\Pi^*_{F,t})^{\sigma-1}\phi_{10,t-1} = 0;$$

the one with respect to K^*_t yields to

$$\left(\frac{F^*_t}{(K^*_t)^2}\right)\phi_{8,t} + \phi_{9,t} - \alpha^*(\Pi^*_{F,t})^{\sigma(1+\eta)}\phi_{9,t-1} = 0;$$

the one with respect to $\Pi^*_{F,t}$ yields to

$$\begin{aligned} 0 \;=\; & -\alpha^*\sigma(1+\eta)\Delta^*_{t-1}(\Pi^*_{F,t})^{\sigma(1+\eta)-1}\phi_{7,t} + \alpha^*\sigma(1+\eta)p(\Pi^*_{F,t})^{-\frac{\sigma(1+\eta)}{1-\sigma}-1}(\Pi^*_{F,t})^{\sigma-2}\phi_{7,t} \\ & -(1+\sigma\eta)\frac{\alpha^*}{1-\alpha^*}(\Pi^*_{F,t})^{\sigma-2}p(\Pi^*_{F,t})^{\frac{1+\sigma\eta}{\sigma-1}-1}\phi_{8,t} - \sigma(1+\eta)\alpha^*K^*_t(\Pi^*_{F,t})^{\sigma(1+\eta)-1}\phi_{9,t-1} \\ & +(\sigma-1)\alpha^*(\Pi^*_{F,t})^{\sigma-2}F^*_t\phi_{10,t-1}; \end{aligned}$$

the one with respect to $\frac{P_{F,t}}{P_t}$ yields to

$$\theta\left(\frac{P_{F,t}}{P_t}\right)^{-\theta-1} C_t\phi_{6,t} - (\bar{\mu}^*)^{-1}C_t^{-\rho}Y^*_{F,t}\phi_{10,t} + (1-\theta)(1-n)\left(\frac{P_{F,t}}{P_t}\right)^{-\theta}\phi_{11,t} = 0.$$

We search for a steady-state solution of the above first-order conditions in which $\Delta_t = \Delta^*_t = P_{H,t}/P_t = P_{F,t}/P_t = \Pi_{H,t} = \Pi^*_{F,t} = 1$ and $C_t = \bar{C}$, $Y_{H,t} = Y_{F,t} = \bar{Y}_H = \bar{Y}_F = \bar{Y}$ and $F_t = K_t = F^*_t = K^*_t = \bar{F} = \bar{K} = \bar{F}^* = \bar{K}^*$. In this steady-state solution, the Lagrange multipliers take also constant values. In this steady-state equations (A.1)–(A.11) imply

$$\bar{a}^{-\eta}(\bar{C}+\bar{G})^{\eta} = \bar{\mu}^{-1}\bar{C}^{-\rho} \tag{A.12}$$

$$\bar{Y}_H = \bar{Y}_F = \bar{Y} = \bar{C} + \bar{G}$$

$$\bar{F} = \bar{K} = \bar{F}^* = \bar{K}^* = \bar{\mu}^{-1}\bar{C}^{-\rho}\bar{Y},$$

where (A.12) can be solved for the steady-state value of $\bar{C}$. In this steady-state the above first-order conditions imply that

$$U_c(\bar{C}) - V_y(\bar{Y}, \bar{\xi}) = \bar{\mu}^{-1}\bar{C}^{-\rho}[\eta + \rho(1 + s_G)](\bar{\phi}_4 + \bar{\phi}_9)$$

$$n\theta\bar{C}V_y(\bar{Y}, \bar{\xi}) = -\bar{\mu}^{-1}\bar{C}^{1-\rho}[(1 + s_G) + \theta\eta]\bar{\phi}_4 - n(1-\theta)\bar{\phi}_{11}$$

$$(1-n)\theta\bar{C}V_y(\bar{Y}, \bar{\xi}) = -\bar{\mu}^{-1}\bar{C}^{1-\rho}[(1 + s_G) + \theta\eta]\bar{\phi}_9 - (1-n)(1-\theta)\bar{\phi}_{11}$$

$$nV(\bar{Y}, \bar{\xi}) = \bar{\phi}_2(1 - \alpha\beta)$$

$$(1-n)V(\bar{Y}, \bar{\xi}) = \bar{\phi}_7(1 - \alpha^*\beta)$$

$$\bar{\phi}_5 = -\bar{\phi}_4 = (1-\alpha)^{-1}\bar{K}^{-1}\bar{\phi}_3$$

$$\bar{\phi}_{10} = -\bar{\phi}_9 = (1-\alpha^*)^{-1}\bar{K}^{-1}\bar{\phi}_8$$

$$\bar{\phi}_1 = nV_y(\bar{Y}, \bar{\xi}) + \eta\bar{\mu}^{-1}\bar{C}^{-\rho}\bar{\phi}_4$$

$$\bar{\phi}_6 = (1-n)V_y(\bar{Y}, \bar{\xi}) + \eta\bar{\mu}^{-1}\bar{C}^{-\rho}\bar{\phi}_9$$

which can be uniquely solve for the values of the lagrange multipliers, given $\bar{C}, \bar{Y}$, $\bar{K}$ and the shocks. The conjectured solution exists.

We now focus on the non-cooperative deterministic solution and we show that $\Pi_{H,t} = \Pi^*_{F,t} = 1$ is a Nash solution. For the home country, we show the existence of stationary solution in which $\Pi_{H,t} = 1$ at all times given the strategy $\Pi^*_{F,t} = 1$ at all times of the other policymaker, and given initial conditions $\Delta_{t_0-1} = 1$ and initial constraints $F_{t_0} = \bar{F}, K_{t_0} = \bar{K}$ such that $\bar{F}/\bar{K} = 1$. In particular we note that $\Pi^*_{F,t} = 1$ at all times implies that $K^*_t = F^*_t$ and $(\bar{a})^{-\eta}(Y^*_{F,t})^{1-\eta} = (\bar{\mu})^{-1}C_t^{-\rho}Y^*_{F,t}\left(\frac{P_{F,t}}{P_t}\right)$. The optimization problem for the home policymaker can be written as

$$W \equiv \sum_{t=t_0}^{\infty} \beta^{t-t_0}[U(C_t) - V(Y_{H,t}, \bar{\xi})\Delta_t],$$

under the following constraints

$$Y_{H,t} = \left(\frac{P_{H,t}}{P_t}\right)^{-\theta} C_t + \bar{G}, \tag{A.13}$$

$$\Delta_t = \alpha\Delta_{t-1}\Pi_{H,t}^{\sigma(1+\eta)} + (1-\alpha)p(\Pi_{H,t})^{-\frac{\sigma(1+\eta)}{1-\sigma}}, \tag{A.14}$$

$$p(\Pi_{H,t})^{\frac{1+\sigma\eta}{\sigma-1}} = \left(\frac{F_t}{K_t}\right), \tag{A.15}$$

$$K_t = \bar{a}^{-\eta}Y_{H,t}^{1+\eta} + \alpha\beta K_{t+1}\Pi_{H,t+1}^{\sigma(1+\eta)}, \tag{A.16}$$

$$F_t = \bar{\mu}^{-1}C_t^{-\rho}Y_{H,t}\left(\frac{P_{H,t}}{P_t}\right) + \alpha\beta\Pi_{H,t+1}^{\sigma-1}F_{t+1}, \tag{A.17}$$

$$Y_{F,t}^* = \left(\frac{P_{F,t}}{P_t}\right)^{-\theta}C_t + \bar{G}^*, \tag{A.18}$$

$$(\bar{a})^{-\eta}(Y_{F,t}^*)^{1+\eta} = (\bar{\mu})^{-1}C_t^{-\rho}Y_{F,t}^*\left(\frac{P_{F,t}}{P_t}\right), \tag{A.19}$$

$$n\left(\frac{P_{H,t}}{P_t}\right)^{1-\theta} + (1-n)\left(\frac{P_{F,t}}{P_t}\right)^{1-\theta} = 1, \tag{A.20}$$

In what follows, we attach Lagrange multipliers $\vartheta_{1,t}$, $\vartheta_{2,t}$, $\vartheta_{3,t}$, $\vartheta_{4,t}$, $\vartheta_{5,t}$, $\vartheta_{6,t}$, $\vartheta_{7,t}$, $\vartheta_{8,t}$ to the constraints (A.13)-(A.20) and we also add multilipliers dated t_0 to the initial constraints $K_{t_0} = \bar{K}$, $F_{t_0} = \bar{F}$. The latter multipliers are normalized in such a way that the resulting first-order conditions take the same form at date t_0 as at all future dates. The first-order conditions of the optimal problem with respect to C_t yields to

$$\begin{aligned} 0 \;=\; & U_c(C_t) - \left(\frac{P_{H,t}}{P_t}\right)^{-\theta}\vartheta_{1,t} + \rho\bar{\mu}^{-1}C_t^{-\rho-1}Y_{H,t}\left(\frac{P_{H,t}}{P_t}\right)\vartheta_{5,t} - \left(\frac{P_{F,t}}{P_t}\right)^{-\theta}\vartheta_{6,t} + \\ & +\rho(\bar{\mu}^*)^{-1}C_t^{-\rho-1}Y_{F,t}^*\left(\frac{P_{F,t}}{P_t}\right)\vartheta_{7,t}; \end{aligned}$$

the one with respect to $Y_{H,t}$ yields to

$$-V_y(Y_{H,t},\bar{\xi})\Delta_t + \vartheta_{1,t} - (1+\eta)\bar{a}^{-\eta}Y_{H,t}^{\eta}\vartheta_{4,t} - \bar{\mu}^{-1}C_t^{-\rho}\left(\frac{P_{H,t}}{P_t}\right)\vartheta_{5,t} = 0;$$

the one with respect to Δ_t yields to

$$-V(Y_{H,t},\bar{\xi}) + \vartheta_{2,t} - \alpha\beta\vartheta_{2,t+1}\Pi_{H,t+1}^{\sigma(1+\eta)} = 0;$$

the one with respect to F_t yields to

$$-\left(\frac{1}{K_t}\right)\vartheta_{3,t} + \vartheta_{5,t} - \alpha\Pi_{H,t}^{\sigma-1}\vartheta_{5,t-1} = 0;$$

the one with respect to K_t yields to

$$\left(\frac{F_t}{K_t^2}\right)\vartheta_{3,t}+\vartheta_{4,t}-\alpha\Pi_{H,t}^{\sigma(1+\eta)}\vartheta_{4,t-1}=0;$$

the one with respect to $\Pi_{H,t}$ yields to

$$\begin{aligned}0 &= -\alpha\sigma(1+\eta)\Delta_{t-1}\Pi_{H,t}^{\sigma(1+\eta)-1}\vartheta_{2,t}+\alpha\sigma(1+\eta)p(\Pi_{H,t})^{-\frac{\sigma(1+\eta)}{1-\sigma}-1}\Pi_{H,t}^{\sigma-2}\vartheta_{2,t}\\ &\quad -(1+\sigma\eta)\frac{\alpha}{1-\alpha}\Pi_{H,t}^{\sigma-2}\vartheta_{3,t}-\sigma(1+\eta)\alpha K_t\Pi_{H,t}^{\sigma(1+\eta)-1}\vartheta_{4,t-1}-(\sigma-1)\alpha\Pi_{H,t}^{\sigma-2}F_t\vartheta_{5,t-1};\end{aligned}$$

the one with respect to $\frac{P_{H,t}}{P_t}$ yields to

$$\theta\left(\frac{P_{H,t}}{P_t}\right)^{-\theta-1}C_t\vartheta_{1,t}-\bar{\mu}^{-1}C_t^{-\rho}Y_{H,t}\vartheta_{5,t}+(1-\theta)n\left(\frac{P_{H,t}}{P_t}\right)^{-\theta}\vartheta_{8,t}=0;$$

the one with respect to $Y_{F,t}^*$ yields to

$$\vartheta_{6,t}-(\bar{\mu}^*)^{-1}C_t^{-\rho}\left(\frac{P_{F,t}}{P_t}\right)\vartheta_{7,t}+(1+\eta)(\bar{a}^*)^{-\eta}(Y_{F,t}^*)^{\eta}\vartheta_{7,t}=0;$$

the one with respect to $\frac{P_{F,t}}{P_t}$ yields to

$$\theta\left(\frac{P_{F,t}}{P_t}\right)^{-\theta-1}C_t\vartheta_{6,t}-(\bar{\mu}^*)^{-1}C_t^{-\rho}Y_{F,t}^*\vartheta_{7,t}+(1-\theta)(1-n)\left(\frac{P_{F,t}}{P_t}\right)^{-\theta}\vartheta_{8,t}=0.$$

We search for a steady-state solution of the above first-order conditions in which $\Delta_t=P_{H,t}/P_t=P_{F,t}/P_t=\Pi_{H,t}=1$ and $C_t=\bar{C}$, $Y_{H,t}=Y_{F,t}=\bar{Y}_H=\bar{Y}_F=\bar{Y}$ and $F_t=K_t=\bar{F}=\bar{K}$. In this steady-state solution, the Lagrange multipliers take also constant values. In this steady-state equations (A.13)–(A.20) imply

$$\bar{a}^{-\eta}(\bar{C}+\bar{G})^{\eta}=\bar{\mu}^{-1}\bar{C}^{-\rho} \tag{A.21}$$

$$\bar{Y}_H=\bar{Y}_F=\bar{Y}=\bar{C}+\bar{G}$$

$$\bar{F}=\bar{K}=\bar{\mu}^{-1}\bar{C}^{-\rho}\bar{Y},$$

where (A.21) can be solved for the steady-state value of $\bar{C}$. In this steady-state the above first-order conditions imply that

$$U_c(\bar{C})-V_y(\bar{Y},\bar{\xi})=\bar{\mu}^{-1}\bar{C}^{-\rho}[\eta+\rho(1+s_G)](\bar{\vartheta}_4-\bar{\vartheta}_7)$$

$$\theta\bar{C}V_y(\bar{Y},\bar{\xi}) = -\bar{\mu}^{-1}\bar{C}^{1-\rho}[(1+s_G)+\theta\eta]\bar{\vartheta}_4 - n(1-\theta)\bar{\vartheta}_8$$

$$(1-n)(1-\theta)\bar{\vartheta}_8 = \bar{\mu}^{-1}\bar{C}^{1-\rho}[(1+s_G)+\theta\eta]\bar{\vartheta}_7$$

$$V(\bar{Y},\bar{\xi}) = \bar{\vartheta}_2(1-\alpha\beta)$$

$$\bar{\vartheta}_5 = -\bar{\vartheta}_4 = (1-\alpha)^{-1}\bar{K}^{-1}\bar{\vartheta}_3$$

$$\bar{\vartheta}_1 = V_y(\bar{Y},\bar{\xi}) + \eta\bar{\mu}^{-1}\bar{C}^{-\rho}\bar{\vartheta}_4$$

$$\bar{\vartheta}_6 = -\eta\bar{\mu}^{-1}\bar{C}^{-\rho}\bar{\vartheta}_7$$

which can be uniquely solve for the values of the lagrange multipliers, given $\bar{C}, \bar{Y}$, $\bar{K}$ and the shocks. We note that setting $\Pi_{H,t} = 1$ at all dates, given the initial condition $\Delta_{t_0} = 1$ and $F_t = K_t = \bar{F} = \bar{K}$, and given the strategy of the other policymaker $\Pi_{F,t} = 1$ implements uniquely the above steady-state allocation. The same argument can be made for the other country. It follows that $\Pi_{H,t} = \Pi^*_{F,t} = 1$ at all dates is a Nash solution for the deterministic non-cooperative problem.s

Loss Functions Coefficients in Section 3.2

In what follows we report the coefficients and the target variables for the loss functions of each single country for the special case in which $\bar{\mu} = 1$ as in section 3.2. We have that $\lambda^i_{y_h}$, $\lambda^i_{y_f}$, λ^i_q, $\lambda^i_{\pi_h}$, $\lambda^i_{\pi_f}$ are parameters,that assumes the following form:

$$\lambda_{y_h} = \lambda^w_y\left[1-(1-n)a\right] + (1-n)b, \quad \lambda^*_{y_h} = n\left[\lambda^w_y a - b\right]$$

$$\lambda_{y_f} = (1-n)\left[\lambda^w_y a - b\right], \quad \lambda^*_{y_f} = \lambda^w_y\left[1-na\right] + nb$$

$$\lambda_q = (1-\theta\rho)\theta(1-n)\left[na + c(1-n)\right]$$

$$\lambda^*_q = (1-\theta\rho)\theta n\left[(1-n)a + nc\right]$$

$$\lambda_{\pi_h} = \frac{\sigma}{ks_c}\left[na + c\right], \quad \lambda^*_{\pi_h} = \frac{n\sigma}{ks_c}a$$

$$\lambda_{\pi_f} = \frac{(1-n)\sigma}{k^*s_c}a, \quad \lambda^*_{\pi_f} = \frac{\sigma}{k^*s_c}\left[(1-n)a + c\right]$$

While the target variables in this case are given by:

$$\tilde{Y}^h_{H,t} \equiv (\lambda_{y_h})^{-1}\left[\begin{array}{c} \frac{\eta(n+(1-n)c)}{s_c}\hat{a}_{W,t} + \frac{\eta(1-n)(nd+(1-n)c)}{s_c}\hat{a}_{R,t} \\ +\left[\frac{(1-n)a}{s_c}\right]\left(\hat{\mu}_{W,t} + (1-n)\hat{\mu}_{R,t}\right) + \frac{1}{\lambda_{y_h}}\left[n\lambda^w_y c_7 - (1-n)\theta^{-1}s_c^{-2}h_7\right]\hat{G}_t \\ +\frac{1}{\lambda_{y_h}}\left[n\lambda^w_y c_8 - (1-n)\theta^{-1}s_c^{-2}h_8\right]\hat{G}^*_t \end{array}\right]$$

$$\tilde{Y}^h_{F,t} \equiv (\lambda_{y_f})^{-1} \left[\begin{array}{c} \frac{\eta(1-n)a}{s_c}\hat{a}_{W,t} - \frac{(1-n)\eta n[e-a]}{s_c}\hat{a}_{R,t} \\ -\left[\frac{(1-n)a}{s_c}\right]\left(\hat{\mu}_{W,t} - n\hat{\mu}_{R,t}\right) + \frac{1-n}{\lambda_{y_f}}\left[\lambda^w_y d_7 + \theta^{-1}s_c^{-2}k_7\right]\hat{G}_t + \\ +\frac{1-n}{\lambda_{y_f}}\left[\lambda^w_y d_8 + \theta^{-1}s_c^{-2}k_8\right]\hat{G}^*_t \end{array} \right]$$

$$\tilde{T}^h_t = (\lambda_q)^{-1}\left[\eta\theta e\hat{a}_{R,t} + \theta e\left((1-n)(1-2n)-\eta\right)\hat{G}_{R,t}\right]$$

$$\tilde{Y}^f_{H,t} \equiv (\lambda^*_{y_h})^{-1} \left[\begin{array}{c} \frac{\eta n a}{s_c}\hat{a}_{W,t} + \frac{\eta n(1-n)(a-e)}{s_c}\hat{a}_{R,t} \\ -\frac{an}{s_c}\left(\hat{\mu}_{W,t} + (1-n)\,\hat{\mu}_{R,t}\right) + \frac{n}{\lambda^*_{y_h}}\left[\lambda^w_y c_7 + \theta^{-1}s_c^{-2}h_7\right]\hat{G}_t + \\ +\frac{n}{\lambda^*_{y_h}}\left[\lambda^w_y c_8 + \theta^{-1}s_c^{-2}h_8\right]\hat{G}^*_t \end{array} \right]$$

$$\tilde{Y}^f_{F,t} \equiv (\lambda^*_{y_f})^{-1} \left[\begin{array}{c} \frac{\eta((1-n)+nc)}{s_c}\hat{a}_{W,t} - \frac{\eta n((1-n)d+nc)}{s_c}\hat{a}_{R,t} \\ +\frac{an}{s_c}\left(\hat{\mu}_{W,t} - n\hat{\mu}_{R,t}\right) + \frac{1}{\lambda^*_{y_f}}\left[(1-n)\lambda^w_y d_7 - n\theta^{-1}s_c^{-2}k_7\right]\hat{G}_t + \\ \frac{1}{\lambda^*_{y_f}}\left[(1-n)\lambda^w_y d_8 - n\theta^{-1}s_c^{-2}k_8\right]\hat{G}^*_t \end{array} \right]$$

$$\tilde{T}^f_t = (\lambda^*_q)^{-1}\left[\eta\theta e\hat{a}_{R,t} - \theta e\left(n(1-2n)+\eta\right)\hat{G}_{R,t}\right]$$

where

$$a \equiv \frac{[s_c\theta(1+\eta)]}{(1+\eta s_c\theta)}, b \equiv \frac{(1-s_c)}{s_c^2(1+\eta s_c\theta)}, c \equiv \frac{(1-\theta s_c)}{(1+\eta s_c\theta)}, d \equiv \frac{\theta(s_c\eta+\rho)}{(1+\eta s_c\theta)}, e \equiv \frac{(1-\theta\rho)}{(1+\eta s_c\theta)}$$

and the parameters $c_7, c_8, d_7, d_8, h_7, h_8, k_7$ and k_8 are defined in appendix B.

Proof of determinacy of the optimal cooperative solution

We show that the first-order conditions (3.27)–(3.31) combined with the constraints (3.22)–(3.24) and the initial conditions $\varphi_{1,-1}$ and $\varphi_{2,-1}$ yield to a determinate equilibrium. First we use (3.27)–(3.31) and (3.24) to write (3.22) and (3.23) in terms of only the lagrangian multipliers and the shocks as it follows

$$E_t\varphi_{1,t+1} = \left(1 + \frac{1}{\beta} + \frac{\vartheta_1\xi\kappa}{\beta}\right)\varphi_{1,t} + \frac{(1-n)\vartheta_2\xi\kappa}{\beta}\varphi_{2,t} - \frac{1}{\beta}\varphi_{1,t-1} + \frac{\xi\kappa}{\beta}u_t \quad \text{(A.22)}$$

$$E_t\varphi_{2,t+1} = \left(1 + \frac{1}{\beta} + \frac{\vartheta_3\xi\kappa^*}{\beta}\right)\varphi_{2,t} + \frac{n\vartheta_2\xi\kappa^*}{\beta}\varphi_{1,t} - \frac{1}{\beta}\varphi_{2,t-1} + \frac{\xi\kappa^*}{\beta}u^*_t \quad \text{(A.23)}$$

where

$$\vartheta_1 \equiv \frac{ns_c^2}{s_c\eta+\rho} + \frac{(1-n)(\theta s_c+\psi)^2}{\theta\left(s_c\eta\theta+1\right)},$$

$$\vartheta_2 \equiv \frac{s_c^2}{s_c\eta+\rho} - \frac{(\theta s_c+\psi)^2}{\theta\left(s_c\eta\theta+1\right)},$$

$$\vartheta_3 \equiv \frac{(1-n)\, s_c^2}{s_c \eta + \rho} + \frac{n(\theta s_c + \psi)^2}{\theta\,(s_c \eta \theta + 1)},$$

$$\xi \equiv \frac{\sigma}{s_c}$$

where we used $\lambda_y = \frac{s_c \eta + \rho}{s_c^2}$. In this case, given that $\bar{\mu} = 1$, we have that $\xi > 0$, $\vartheta_1 > 0$ and $\vartheta_3 > 0$. We can write (A.22) and (A.23) in the following form

$$E_t z_{t+1} = \begin{bmatrix} A_1 & A_2 \\ A_3 & 0 \end{bmatrix} z_t + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} \epsilon_t \tag{A.24}$$

where $z_t' \equiv [\varphi_t \ \varphi_{t-1}]$ and $\varphi_t \equiv [\varphi_{1,t} \ \varphi_{2,t}]$; $\epsilon_t' \equiv [u_t \ u_t^*]$, A_j with $j = 1, 2, 3$, and B_1 are two by two matrices. In particular

$$A \equiv \begin{bmatrix} A_1 & A_2 \\ A_3 & 0 \end{bmatrix}$$

$$A_1 \equiv \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad A_2 \equiv \begin{bmatrix} -\beta^{-1} & 0 \\ 0 & -\beta^{-1} \end{bmatrix} \qquad A_3 \equiv \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

with

$$\begin{aligned} a_{11} &\equiv \left(1 + \frac{1}{\beta} + \frac{\vartheta_1 \xi \kappa}{\beta}\right) > 0 \\ a_{12} &\equiv \frac{(1-n)\vartheta_2 \xi \kappa}{\beta} \\ a_{21} &\equiv \frac{n \vartheta_2 \xi \kappa^*}{\beta} \\ a_{22} &\equiv \left(1 + \frac{1}{\beta} + \frac{\vartheta_3 \xi \kappa^*}{\beta}\right) > 0 \end{aligned}$$

and B_1 is a block-diagonal matrix with elements $\frac{\xi\kappa}{\beta}$, $\frac{\xi\kappa^*}{\beta}$. In order to study determinacy, we need to inspect the roots of the characteristic polynomial associated with the matrix A which is

$$P(\psi) = \psi^4 - (a_{11} + a_{22})\psi^3 + (a_{11}a_{22} - a_{21}a_{12} + 2\beta^{-1})\psi^2 - (a_{11} + a_{22})\beta^{-1}\psi + \beta^{-2}.$$

First we note that

$$\psi_1 \psi_2 \psi_3 \psi_4 = \beta^{-2}, \tag{A.25}$$

$$\psi_1 + \psi_2 + \psi_3 + \psi_4 = a_{11} + a_{22} > 2(1 + \beta^{-1}); \tag{A.26}$$

moreover if $P(\psi)=0$ then $P(\psi^{-1}\beta^{-1})=0$ so that we can further conclude that

$$\psi_1\psi_2=\beta^{-1} \qquad\qquad \psi_3\psi_4=\beta^{-1}. \tag{A.27}$$

Moreover, by Descartes sign rule all the roots are positive. We note that

$$\begin{aligned} P(1) &= (1+\beta^{-1})^2-(1+\beta^{-1})(a_{11}+a_{22})+a_{11}a_{22}-a_{21}a_{12} \\ &= \frac{\sigma s_c}{(s_c\eta+\rho)\,\theta\,(s_c\eta\theta+1)}>0 \end{aligned}$$

$$P(0)=\beta^{-2}>0$$

The fact that all the roots are positive and that $P(1)>0, P(0)>0$ imply that there are either 0 or 2 real or complex roots or 4 complex roots within the unit circle. Conditions (A.26) and (A.27) exclude the first and latter possibilities. From conditions (A.27), we can further conclude that the two roots are within the unit circle. The unique and stable solution of the system is obtained with the following steps. Let V the two by four matrix of left eigenvectors associated with the unstable roots. By pre-multiplying the system (A.24) with V we obtain

$$E_t k_{t+1}=\Lambda k_t+VB\epsilon_t \tag{A.28}$$

where Λ is a two by two diagonal matrix of the unstable eigenvalues on the diagonal and $k_t\equiv Vz_t$. The unique and stable solution to (A.28) is given by

$$k_t=-\sum_{j=0}^{\infty}\Lambda^{-j}VBE_t\epsilon_{t+j}$$

which implies that

$$\varphi_t=-V_1^{-1}V_2\varphi_{t-1}-V_1^{-1}\sum_{j=0}^{\infty}\Lambda^{-j}VBE_t\epsilon_{t+j} \tag{A.29}$$

where V_1 and V_2 are such that $V=[V_1\ V_2]$. Equation (A.29) characterizes the optimal path of the vector φ_t given initial condition φ_{-1}; the paths for y_H, y_F^*, π_H, π_F^*, q_t can be derived using the conditions (3.27)–(3.31).

Proof of determinacy of the solution implemented by the targeting rules.

We now show that the targeting rules (3.36) and (3.37), combined with the constraints (3.22) to (3.24) yield to a determinate equilibrium that coincides with the optimal cooperative solution. We follow here an argument similar to Woodford (2003, ch. 6). Let us define $\varphi_{1,t}$ and $\varphi_{2,t}$ for all $t \geq -1$ as

$$\varphi_{1,t} \equiv \lambda_y^w y_{H,t}, \tag{A.30}$$

$$\varphi_{2,t} \equiv \lambda_y^w y_{F,t}^*, \tag{A.31}$$

from which it follows that

$$\lambda_y^w \sigma s_c^2 \pi_{H,t} = -(\varphi_{1,t} - \varphi_{1,t-1}), \tag{A.32}$$

$$\lambda_y^w \sigma s_c^2 \pi_{F,t}^* = -(\varphi_{2,t} - \varphi_{2,t-1}). \tag{A.33}$$

Using (3.24) and (A.30)-(A.33), we can then retrieve the system of equations (A.22) and (A.23) which yields to a determinate equilibrium given the initial conditions

$$\varphi_{1,-1} \equiv \lambda_y^w y_{H,-1},$$

$$\varphi_{2,-1} \equiv \lambda_y^w y_{F,-1}^*.$$

Indeed the lagrangian multiplier $\varphi_{1,-1}$ and $\varphi_{2,-1}$ measure the commitment to expectations taken in periods before time 0. The timeless perspective optimal policy is the one that assigns a particular value to the commitment to expectations prior to period 0 such that the resulting optimal policy is time invariant.

B Appendix

Appendix B is available under the homepage of the Authors.