Endogenous Parties: Cooperative and Non-cooperative Analysis

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1 Introduction

In Levy (2001) I attempt to analyse whether political parties are effective. That is, whether they change the political outcome relative to the case in which they do not exist and candidates can only run independently. The main result shows that parties are not effective when the policy space is one dimensional but may become effective when the policy space has more than one dimension. To derive this result, restrictions are imposed on how parties can form and which array of parties is defined as stable. This paper shows the robustness of the results in Levy (2001) to the introduction of different stability concepts for endogenous party formation.

In the model, society is formed of \( N \) groups, with cardinality \( n \). The size of group \( i \) as a share of the population is \( p_i \); \( \sum_{i=1}^{n} p_i = 1 \). There is a feasible policy set \( Q \subseteq \mathbb{R}^k \): Each group of voters, \( i \); has a preference ordering \( \succ_i \) on \( Q \) represented by a utility function \( u_i(\phi) \) which is continuous and concave. I will focus on preferences which are single-peaked. The ideal policy of group \( i \) is denoted for simplicity by \( \phi_i \); These groups form the set of voters.

The set of players is composed of a representative from each group, i.e., there are \( N \) (finite) players, with player \( i \) having the same ideological preferences as group \( i \); \( u_i(\phi) \):

In the first phase of the analysis, which I introduce in the next section, the players are organized in coalitions/parties. These coalitions choose platforms on which the voters vote sincerely. The platform that receives the highest number of votes is implemented. In this phase, we analyse the possible equilibria in the platform game for any coalition structure.

In the second phase, we determine which are the equilibria and the coalition structures that are stable. This is introduced in section 3, by
using different stability concepts. The aim is to compare the stable political outcomes in the presence of parties, and in the absence of parties, i.e., when coalitions are composed of one player only. I show that both cooperative and non-cooperative concepts rationalize the result about multidimensionality of the policy space as a necessary condition for parties being effective.

2 The Platform Game

There are \( N \) players organized in a coalition structure \( \frac{1}{N} \) which is a partition on the set \( N \). The set of all possible partitions is denoted \( \mathcal{P} \). A coalition \( S \) is a non-empty subset of players. For a partition \( \frac{1}{N} \), denote by \( \frac{1}{N_k} \) the reduction of \( \frac{1}{N} \) when \( k \) players leave the partition. That is, when the other \( N - k \) players do not change their positions and stay in their (possibly shrunk) coalitions.

Denote by \( Q_S \) the Pareto set of coalition \( S \):

**Definition 1:**
\[
Q_S = \{ q_j \mid \text{s.t. } \forall i \in S; u_i(q) \geq u_i(q_j) \text{ with at least one strict inequality} \}
\]

Given the above, define \( k^* = \dim(Q_N) \) as the effective number of dimensions of conflict.

The platform game has one strategic stage, in which each coalition \( S \) chooses, simultaneously, an action \( q_S \), \( q_S \in Q_S \). The notation \( \emptyset \) means that the coalition offers no policy in the elections, i.e., it chooses not to run.

Denote by \( q(\frac{1}{N}) \) a vector of policy platforms (which may include the null platform \( \emptyset \), offered by some coalitions) where each platform is chosen by a coalition \( \frac{1}{N} \). The vector \( q(\frac{1}{N}) \) is the outcome of the platform game.

Given the policy vector \( q(\frac{1}{N}) \), elections are held and each voter votes for the policy that she likes most. Voters who are indifferent between several winning policies, use a fair mixing device. If no policy is offered, a default status quo \( d \) is implemented. I assume that \( u_i(d) = 1 \) for all \( i \), so at least one coalition chooses to run in equilibrium. Denote by \( V_S(q(\frac{1}{N})) \) the share of votes that a platform \( q_S \) receives when \( q(\frac{1}{N}) \) is offered, \( q_S, q(\frac{1}{N}) \in Q_S \).

**Definition 2:**
Given \( q(\frac{1}{N}) \); if \( q(\frac{1}{N}) \neq \emptyset \); \( d \) is the elections' outcome. Otherwise, let \( W(q(\frac{1}{N})) = \max_{q_S} V_S(q(\frac{1}{N})) \text{ s.t. } q_S \in Q_S \).

The expected outcome of the elections is
\[
\frac{1}{W(q(\frac{1}{N}))} \sum_{q_S} w(q(\frac{1}{N})) q_S,
\]
where \( w(q(\frac{1}{N})) = \max_{q_S} V_S(q(\frac{1}{N})) \). The expected utility of the players from the platform game is their expected utility from the elections' outcome, that is, for all \( i \in N \),
\[
U_i(q(\frac{1}{N})) = \frac{1}{W(q(\frac{1}{N}))} \sum_{q_S} w(q(\frac{1}{N})) u_i(q_S) \text{ if } q(\frac{1}{N}) \neq \emptyset \text{ and } u_i(d) \text{ otherwise}.
\]

Let \( \xi_S \) be the set of probability distributions over \( Q_S \).
(mixed) strategy for a coalition $S$ is $\pm 2 \in S$: A set of strategies for each coalition $S$ in the partition $\frac{1}{4}$ is $f_{\pm S}^{\pm \frac{1}{2}}$: Given a mixed strategy, denote the expected utility of player $i$ by $U_i(f_{\pm S}^{\pm \frac{1}{2}})$. Let $\pm S$ denote the strategies taken by all coalitions but $S$.

Definition 3: Equilibrium in the platform game is a collection $f_{\pm S}^{\pm \frac{1}{2}}$ such that for all $S$ there exist no $\pm_0^0 2 \in S$; $\pm_0^0 \notin \pm S$ s.t. for all $i \notin S$; $U_i(\pm_0^0; \pm S)$, $U_i(\pm S; \pm S)$ with at least one strict inequality.

The first proposition assures the existence of equilibria.

Proposition 1 For all $\frac{1}{4} 2 \in S$, there exists an equilibrium $\pm \frac{1}{4}$:

The analysis from now on will focus on pure-strategy equilibria, whenever they exist. I also add a tie breaking rule that if a coalition is indifferent between running and not running, it chooses not to run.

3 Endogenous coalitions

The analysis of the platform model have fixed the possible utilities that a player may acquire from each partition. In particular, each partition is characterized by an equilibrium (or a set of equilibria) and the utility from such an equilibrium, $U_i(\pm \frac{1}{4})$; is the utility that a player $i$ may accrue from a partition $\frac{1}{4}$.

In the second phase of the analysis, we are looking for stable partitions and their respective equilibria, i.e., $(\frac{1}{4} \pm \frac{1}{4})$. Note that $\pm \frac{1}{4}$ is defined as an equilibrium in the platform game given $\frac{1}{4}$ and not just a strategy vector. It may be that some partitions are stable for one equilibrium but not for another. If a partition is not stable it means that there is no equilibrium in the platform game that can support it. The analysis of stability pins down therefore not only the stable party structures but also the stable political outcomes. In particular, I am interested in comparing the political outcome in stable partitions in which there exist at least one coalition $S^0$ with $|S^0| > 1$; to the partition denoted by $\frac{1}{2}^0$; in which $|S^0| = 1$ for all $S \neq \frac{1}{2}^0$.

Note that a feature of this model, is that the utility for each player from any partition depends not only on which coalition he belongs to (as in the characteristic function), and not only on the structure of other coalitions (as in the partition function, analysed by Ray and Vohra (1997,1999), Yi (1997), and Bloch (1996) among others), but also on the actions of the different coalitions due to multiplicity of equilibria.

To pin down stable coalition structures, we can use either cooperative analysis or non cooperative solutions. When we discuss cooperative solutions, we start from a specified partition and examine whether a coalition would prefer to deviate. If no coalition wishes to deviate, a partition is considered stable. Two problems are raised. First, how do other players
react to the initial deviation? Do they act to maximize or minimize the payo× of the deviating coalition? Second, when a new partition is constructed by the deviators, which equilibrium is played? The rst problem is common and was discussed in length in the cooperative game theory literature. The second question applies only to situations in which there exist multiple equilibria for each partition structure. Beliefs can be pessimistic (the worst equilibrium, from the point of view of the deviators, will be played) or optimistic (the best equilibrium will be played). When we discuss non cooperative solutions, similar problems arise. In particular, the multiplicity of equilibria makes non cooperative solutions, such as bargaining games, especially problematic. A player who oxes to other players to join together in a coalition, cannot also suggest an equilibrium platform, since this depends on the actions of other coalitions, not only on which other coalitions form. We then have to assume that for any partition the equilibrium that is played is xed in advance, which forces us to analyse too many games.

In Levy (2001), I use a stability concept derived by Ray and Vohra (1997). In this concept, cooperative in nature, players start from some coalition structure (a partition), and are allowed to fragment coalitions. The players are not allowed to form new coalitions. The deviations can be unilateral or multi-lateral, i.e., coalitions can deviate as well. The deviators take into account future deviations, both by members of their own coalitions and by members of other coalitions. Credible threats are deviations to ner partitions which are stable themselves. This concept allows for optimistic beliefs in terms of the equilibrium that will be played once a deviation occurs. On the other hand it restricts the possible deviations (only fragmentation) but takes into account further deviations by other players. I then show the following results:

Proposition 2 (Levy (2001)): When \( k^= 1 \); for all distributions of preferences with a median voter, parties are not e×ective. That is, the outcome in any stable party structure coincides with an outcome in \( \forall^0 \) and it is the median voter’s expected ideal policy. When \( k^> 1 \), parties may be e×ective.

Proposition 3 (Levy (2001)): When \( k^> 1 \), even if a median voter (a Condorcet winner) exists in \( \forall^0 \), parties may still be e×ective. In particular, when the Condorcet winner is a unique equilibrium in \( \forall^0 \); then parties are e×ective if \( k^= N - 1 \).

In this paper, I analyse the same problem using other stability concepts in order to examine the robustness of these results. Given the above discussion, there are many stability concepts and game forms that we can consider. I restrict the analysis for obvious reasons, and consider the core and stable sets among the cooperative solutions and a membership
game, as a non cooperative solution.

3.1 Definitions

3.1.1 Cooperative stability concepts

Before we introduce the cooperative concepts, we need one more definition.

Definition 4: A partition \( \frac{1}{4}(\mathcal{K}) \) is induced from \( \frac{1}{4} \) by a coalition \( \mathcal{K} \) if \( \mathcal{K} \subseteq \frac{1}{4}(\mathcal{K}) \) and \( \frac{1}{4}_{\mathcal{K}} \supseteq \frac{1}{4}(\mathcal{K}) \):

That is, a group of players \( \mathcal{K} \) deviates from \( \frac{1}{4} \) by forming its own coalition, while other players do not change their position. The group of deviators can be composed of members from different original coalitions. Thus, this definition allows deviators to form new parties.

The Core \( \left( \frac{1}{4}; \frac{1}{4}(\mathcal{K}) \right) \) is core-stable if there does not exist a coalition \( \mathcal{K} \) and some \( \frac{1}{4}(\mathcal{K}) \); for which \( u_i(\cdot) > u_i(\cdot) \) for all \( i \in 2 \mathcal{K} \).

Stable sets A set \( Z \) with a typical element \( \mathcal{K}; \frac{1}{4}(\mathcal{K}) \) is stable, if

(i) For any \( \left( \frac{1}{4}; \frac{1}{4}(\mathcal{K}) \right) \supseteq Z \); there exists a coalition \( \mathcal{K} \) and \( \frac{1}{4}(\mathcal{K}) \); for which \( u_i(\cdot) > u_i(\cdot) \); \( 8i \in 2 \mathcal{K} \),

(ii) For any \( \left( \frac{1}{4}; \frac{1}{4}(\mathcal{K}) \right) \supseteq Z \); there exists no coalition \( \mathcal{K} \) and \( \frac{1}{4}(\mathcal{K}) \); for which \( u_i(\cdot) > u_i(\cdot) \); \( 8i \in 2 \mathcal{K} \).

3.1.2 Non-cooperative games of coalition formation

A membership game Each player \( i; \) announces a coalition \( S \) which includes \( i; \) \( fS_{i=1}^{N} \) \( \ \ S \) is a strategy vector. Define \( \frac{1}{4}(S) \) as the outcome of the game. A coalition \( S \supseteq \frac{1}{4}(S) \) if and only if \( S_i = S \) for all \( i \in 2 S \). Otherwise, \( i \supseteq \frac{1}{4}(S) \): That is, all players that do not belong to coalitions can only run independently. Then, a strategy vector \( S^* \) and an equilibrium \( \cdot \frac{1}{4}(S^*) \) are Nash-stable if for no \( i \in 2 N \); there exists an announcement \( S^*_i \) that satisfies \( u_i(\cdot) > u_i(\cdot) \) for some \( \frac{1}{4}(S^*_i; S^*_{i-1}) \).

3.2 Results

Proposition 4 Parties are not effective in the one dimensional policy space under the core, stable sets, or a membership game. In the multidimensional policy space, the core may yield empty predictions. However, parties may be effective in the multidimensional policy space under stable sets and a membership game.

Proof: Case 1: one dimension policy space:

Define \( m \) as the politician who represents the median voter. Consider \( \cdot m \) the core. Assume a partition \( \frac{1}{4} \) and a political outcome which is different than \( m \); for example biased to the right. Then \( K = m \).

To be more precise, this concept is termed the \( \pm \) core by Hart and Kurz (1983).
the coalition of $m$ and all leftist players, can deviate to $\frac{1}{4}(K)$. In the new partition (actually, disregarding how other members on the right are organized), there exists an equilibrium in which the deviating coalition offers $m$ and wins the elections. This is a profitable deviation. Moreover, a coalition structure in which $m$ is the political outcome is core-stable, since no other coalition can deviate and improve pay-offs for all its members. No coalition whose members come from one side of the median can win against the median, and no coalition whose members come from both sides of the median can improve utility for all its members.

We now analyse stable sets. We will show that there exist a unique stable set, which is composed of all $(\frac{1}{4}, \pm \frac{1}{4})$ for which $\pm \frac{1}{4}$ yields $m$ as the political outcome. Consider a partition in which $m$ is the political outcome. As in the argument for the core, there is no other partition and outcome that can win against it. Hence, all the partition in which $m$ is the political outcome must be in the stable set. None of them wins against the other. Consider any partition in which the political outcome is biased away from the median, for example to the right. Then, as in the argument for the core, all left members along with $m$ can deviate to the structure in which they compose one coalition and in equilibrium $m$ is offered. This structure must be in the stable set by the first argument and hence any structure with a biased outcome cannot be in the stable set.

Finally, think of the membership game. If $m$ is a coalition member and the outcome is biased away from $m$; then he has a profitable deviation of announcing $S_m = m$ and yielding a partition in which $m$ is the outcome when he runs unopposed. What if $m$ is not a member of a coalition, and the political outcome is biased away from $m$? It can only happen if there exist coalition members on both sides of the median. Otherwise, if coalitions exist only on one side of the median, then $m$ must win the elections. But, if coalition members exist on both sides of the median and the outcome is biased towards one side, it must be that at least one coalition member prefers to dissolve his coalition. This creates a partition in which there exists an equilibrium in which $m$ runs by himself and wins the election.

Case 2: multi-dimensional policy space:

Consider the following example. There are 3 players, $a$, $b$ and $c$. These players represent 3 groups. Let $p_i < \frac{1}{2}$ for all $i \in \{a, b, c\}$. The groups/politicians have the following preferences: $u_i(x; y) = i \circ(i_i x \cdot y)^2_1$ for $i \in \{a, b, c\}$ and $\circ = :5$. Let $(a_x; a_y) = (0; 0)$; $(b_x; b_y) = (1; 0)$; $(c_x; c_y) = (1; 1)$. Restrict the policy set to be the triangle formed from their ideal points. If the partition is $ajbc$; generically $b$ wins
the elections. If it is abc; ab win the election with a policy \((x; 0); \) for \(x \in (0; 1]: \) If it is bca; then bc win the elections with \((1; y); \) for \(y \in [0; 1]: \) If the partition is acjb; then ac win with a policy \(\bar{x} (x; x); \) for \(x \in [1 - \rho; 1]: \) If the partition is abjc; all feasible policies can be an equilibrium.

We rst analyse the core. From any policy outcome in the triangle, two players can form a coalition and improve their utility. Thus, the core is empty.\(^2\) Next, we look at the following candidate for a stable set: \(\{acjb; f abjc; f a jb cg; \} \) with the following policies respectively: \((x; x);\) for \(x \in (0; 1]: \) (z; 0) and \((1; y)\) for \((1; y) \Rightarrow (x; x);\) (z; 0) \(\Rightarrow (1; y)\) and \((z; 0) \Rightarrow (x; x);\) for \(x \in (1 - \rho; 1]: \) Any other policy implemented by ab in abjc; is beaten by either a cooperation of bc or of ac and so on. Any policy implemented by abc is beaten by at least two coalitions if not all possible three. None of these partitions is beaten by any partition in the set. This set constitutes therefore a stable set in which parties are effective.

Finally, we examine the membership game. If a and c both announce ac; and b announces b; then any of the equilibria of the partition acjb is Nash-stable. If a or c deviates, they b wins and ac yield a (weakly) better outcome for both. Any other structure of parties is not stable. In fact, b has a weakly dominant strategy of announcing b; and staying by himself. By doing so, he can ensure not to be in a coalition with a and c or both. Hence, the only possible stable structures are abjc or acjb; as in Levy (2001).\(^2\)

Proposition 5 When \(k^u > 1,\) even if a median voter (a Condorcet winner) exists in \(\frac{1}{2}p,\) parties may still be effective when we use a membership game or stable sets. In particular, when we use membership games, if the Condorcet winner is a unique equilibrium in \(\frac{1}{2}p;\) then parties are effective if \(k^u = N - 1: \)

Proof: For the rst assertion of the proposition, it is su cient to use the example in the proof of Proposition 4. Consider the second assertion and the membership game. Assume that in \(\frac{1}{2}p\) there is a unique equilibrium which identi.\(\)es with one of the groups, i: Then consider the structure in which N ni form a coalition, while i announces Si = i and stays by himself. The coalition offers a policy \(\bar{p} = Q_{N \backslash i} \setminus Q_{N \backslash i}(i);\) where \(Q_{N \backslash i}(i)\) denotes all q such that \(u_j (q) > u_j (i)\) for all \(j \in N \backslash i:\)

To see that such a policy exists, we have to show that i does not belong to the Pareto set of N ni: This is assured when \(k^u = N - 1\) (see Lemma 2 in the proof of Proposition 4 in Levy (2001)).

\(^2\)If we choose pessimistic beliefs regarding the equilibrium that is played given a deviation, then the core may yield stable predictions. For example, acjb with its respective equilibria become core-stable. For c or a it can get worse if they cooperate with b independently or together. It is also better for both than breaking the structure.
Since such a policy exists, all such policies constitute a Nash-stable structure. If any of these players deviates, then no coalitions are formed in equilibrium. Hence, the outcome is $i$: But $q^0$ is preferred to $i$ by all coalition members, a contradiction.

4 Conclusion

In this paper I showed that the result that multidimensionality is a necessary condition for effectiveness of parties is robust to both cooperative and non-cooperative solution of the coalition formation game. When the policy space is one dimension, all the solution concepts that were analysed showed that parties are not effective. In the multidimensional policy space, non-cooperative solutions can provide a prediction that parties are effective, due to the restrictions on player's actions. Cooperative notions may be empty. However, when deviations are restricted (as in stable sets or in Ray and Vohra (1997)), some cooperative solutions do yield stable coalition structures in which parties affect the political outcome. In particular, cooperative and non-cooperative solutions can yield the same outcomes, as is the case for the membership game and the cooperative notion analysed in Levy (2001).
References


