

How to Sell a (Bankrupt) Company*

FRANCESCA CORNELLI[†] AND LEONARDO FELLI[‡]

[†]London Business School and CEPR, London, UK and

[‡]London School of Economics and CEPR, London, UK

ABSTRACT

This paper suggests a way to sell a company that maximizes the proceeds from the sale. The key to this proposal is the option left to the seller to retain a fraction of the shares of the company. Indeed, by retaining the minority stake, the seller can transfer the control of the company while reducing to a minimum the rents that the sale of the company leaves in the hands of the buyer. We then focus on two main applications of this idea: bankruptcy procedures and carve-outs.

I. INTRODUCTION

This paper suggests a way to sell a company that maximizes the proceeds from the sale. Situations where the owners of a company want to sell abound in real life. The owners may, for example, want to cash out of their investments, the company may be in financial distress, or simply the company may have a higher value in someone else's hands. One way to sell a company is of course to list it on a stock exchange by doing an initial public offering (IPO), but this is not the only way to proceed. In reality, we observe companies sold in many other ways. The key question is of course what distinguishes selling a company from selling any other asset. We focus here on three main reasons. First, there is of course a lot of uncertainty on the value of a company; moreover, the value itself may depend on who the buyer is. Second, the owner does not have to sell the entire company but can choose to sell only a fraction, this is of course not necessarily possible in the case of the sale of any asset. Third, in the case of a company, there exists a minimum fraction of the ownership, which will guarantee control of the firm (the identity of the owner of this fraction determines the firm's value).

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The key to our proposed way to sell a company is a very simple point: it is not necessarily optimal to sell the entire ownership of a company. Instead, it might be optimal to retain an equity stake in the firm. This is because it is possible to transfer the control of the company in the hands of the individual that maximizes its value without transferring all the shares in his hands.

The intuition is as follows. One of the major sources of complexity when selling a company is the difficulty in evaluating what will be the value of the company in different hands. Potential buyers value the company differently because they may have different plans for the future or because of synergies with their other businesses. The seller will in general not know in advance how much these buyers are prepared to pay and will need to rely on the competition among buyers to identify the individual who is willing to pay more for the company.¹ However, if the company has different values in the hands of different individuals, competition among buyers is not perfect and the buyer is able to obtain the company for a price lower than its value. In many situations, the value attached to a company may differ so much among potential buyers that the price may end up being substantially lower. Our proposal aims at reducing this rent, which is left to the buyer, increasing the returns from the sale. By transferring control and retaining an equity stake in the company, the seller can make sure that at least on this equity stake he captures the full value of the company and minimizes the rents left in the hands of the buyer. In other words, by auctioning off only a fraction of the company, the seller reduces the differences among potential buyers, increasing competition and reducing the buyer's rents.

We show that when control does not entail any private benefits, it is always optimal for the seller to sell the minimum stake necessary to transfer control. In other words, it is optimal to separate completely the voting rights from the cash flow rights of the company: the seller should sell all the voting rights and possibly retain all the cash flow rights. When the control of the firm entails some private benefits, it may no longer be optimal to sell only the minimum control stake. Private benefits of control, in fact, create a trade-off between *ex post* and *ex ante* efficiency since the bidder who is willing to pay the most for the minimum control stake of the company might not be the one who maximizes the company's value. It might still be optimal for the seller to retain part of the equity stake of the firm, but not necessarily the minimum stake necessary to transfer control. In other words, the seller does not want to separate completely the voting rights from the cash flow rights of the company. Bundling these rights together but retaining as much as possible of the cash flow rights of the firm allows the creditors to maximize their returns while selling to the buyer in whose hands the company's value is highest. The optimal mechanism is then an auction of the lowest control stake that renders this buyer also the individual with the highest willingness to pay for the company.

1 Of course, if the owner knew the value of the company in the hands of potential buyers, he could make a take-it-or-leave-it offer to the buyer who is willing to pay more for the company and in this way capture all the increase in the company value the buyer will generate.

In most of our analysis, the choice of the selling procedure, which maximizes the seller's revenues, does not imply a trade-off between *ex post* and *ex ante* efficiency. Indeed, the mechanism we propose also allocates the company in the hands of those who maximize its value (and in the case of private benefits, it is optimal for the seller to adjust the fraction sold so that this result is still true). However, the seller also has the option to further increase their proceeds by introducing a reservation price. This introduces a trade-off between *ex ante* and *ex post* efficiency since a reservation price entails a loss in *ex post* efficiency. We show that reducing the fraction of the equity auctioned off reduces the *ex post* inefficiency associated with the reservation price. In other words, when the seller uses a reservation price, reducing the control stake auctioned off improves both *ex ante* and *ex post* efficiency.

The mechanism we propose carries important implications for several situations. A clear example is the case of a bankrupt firm, when the creditors have taken control of the firm and are trying to find a buyer for it.² Companies facing financial distress often get sold in order to repay creditors or to give control to parties who may be able to fix the problem that caused the financial distress to begin with. Most countries' bankruptcy procedures allow for the sale of the company, either as a piecemeal or as a whole. As Cimirzi et al. (2011) point out, the 2008 financial crisis was followed by a global downturn, which led to an increase of insolvencies and has shifted politicians' attention to bankruptcy laws. Our paper has a clear recommendation in this situation.

The recent financial crisis has also led to the sale of several distressed banks. The case of banks is particularly relevant for our analysis because banks' assets are especially hard to value in a crisis and because banks are clearly more valuable if sold as a going concern. James (1991) studies bank failures and shows that 'there is a significant going concern value that is preserved if the failed bank is sold to another bank.' On the Federal Deposit Insurance Corporation website, there is a long list of banks recently sold to new owners. The most well-known case is the one of Lehman Brothers Holdings, which is expected to conduct an auction to sell Aurora Bank (formerly known as Lehman Brothers Bank), worth around \$850 million, as the bankrupt holding company sells off its pieces to pay off the creditors.³

Another interesting application of our result is a company decision to spin off (carve-out) one of its divisions. Also in this case, the selling party (the mother company) has an interest in retaining an equity stake in the spin-off company. Therefore, the IPO should apply only to a fraction of the equity of the division while the remaining shares should be retained by the mother company or possibly sold in the market after the spin-off.

2 See, for example, the cases of Sunbeam-Oster (HBS # 5-293-046) and Marvel Entertainment Group (HBS # 5-298-028).

3 Beyond the most famous cases, there are networks such as mergenetwork.com or commercialbankforsale.com where one can find numerous examples of small US commercial banks up for sale.

There has been a lot of talk recently about the possibility that, under the new regulatory requirements, US and European banks may need to sell assets to raise capital.⁴ These sales are exactly the spin-offs discussed above if the assets sold are companies or business entities, where one can separate control rights and cash flow rights. For example, the Royal Bank of Scotland is planning to sell its insurance business. Similarly, Lloyds TSB is expected to complete the sale of 632 of its branches by the end of 2013 to comply with the European Union state-aid rules.

An interesting example is the one of Citigroup that joined a group of banks – including Goldman Sachs and Bank of America that sold holdings in Asia investments to boost capital and meet regulatory requirements for risk buffers – in the sale of their participation in Housing Development Finance Corp., India's largest mortgage lender. Interestingly, and in line with the predictions of this paper, Citigroup retained a 9.9% stake in this company.

The rest of the paper is structured in the following way. We review the related literature in Section II. Section III presents the main results of the paper under the assumption that potential buyers cannot trade among themselves their acquired stake in the company. In Section IV, we consider three extensions: the possibility that by acquiring a larger fraction of the company shares the buyer's incentives will be more in line with the interests of the minority shareholders, the possibility of a reservation price and of *ex post* trading. Section V shows how our framework applies particularly well to the cases of bankruptcy and carve-outs. Section VI concludes.

II. RELATED LITERATURE

The papers most closely related to ours are concerned either with the transfer of control (Bebchuk 1994; Zingales 1995) or with auctions with contingent payments (Hansen 1985; McAfee and McMillan 1986, 1987; Samuelson 1987; Riley 1988).

We consider first the literature on the transfer of control. Zingales (1995) is the closest paper to ours. It analyzes how the owner of a firm can extract the highest possible surplus from a raider. Zingales shows that the incumbent may want to sell the minority stake of the firm on the stock market before facing the raider, in order to free ride on any increase in the value of the firm induced by the transfer of control. The main difference with our analysis lies in the fact that Zingales focuses on the case in which only one raider is planning to take over the firm, while we consider the case where there is competition among potential buyers for the company.

In Zingales (1995), the incumbent, if he owns the entire company when bargaining with a unique potential buyer, will not be able to extract any additional surplus from the raider by selling only the control stake of the firm.

4 See, for example, the Bloomberg report, November 4, 2011 on the sale of Asian assets of the *Financial Times* on October 6, 2011 'Goldman predicts a fire sale of USD assets by French banks'.

In fact, when the incumbent bargains with the raider, the reservation price that makes him indifferent between selling or not the firm will adjust. As a result, the amount of surplus the incumbent will be able to extract is the same whatever stake of the company is sold. However, this is not true if the incumbent has transformed the minority stake of the firm in cash in advance by selling it on the stock market. Therefore, in Zingales (1995), the only way in which the incumbent will be able to maximize the rent he extracts from the raider, even in the absence of private benefits from control, is by selling the minority stake of the firm on the stock market in advance.

In our analysis, this is not true. Indeed, the presence of competition among potential buyers for the firm prevents the reservation value of the incumbent from adjusting when selling only the control stake. Therefore, it is strictly optimal for the creditors to retain the minority stake of the firm so as to extract the highest surplus from the potential buyers.⁵

The other paper on the transfer of control that is relevant for our analysis is Bebchuk (1994). This paper analyzes the efficiency properties of different procedures for the sale of control of a company in the presence of private benefits from control. Bebchuk shows that a procedure that does not give any say to the minority shareholders of the company (market rule) may result in inefficient transfers of control, while a procedure that does give a veto power to minority shareholders (equal opportunity rule) may prevent efficient transfers of control.

In Bebchuk (1994), the critical condition that yields (*ex post*) inefficiencies in the transfer of control is whether the private benefits of the seller and the buyer of the company are positive or negatively correlated with the benefits that are shared by the minority shareholders. The equivalent condition in our analysis (Section III below) is whether the private benefits of potential buyers are positively or negatively correlated with the public or transferable benefits associated with their shareholding. The main difference with our analysis is that, since we focus on a unique owner free riding will not occur, hence the transfer of control will always be *ex post* efficient. However, the correlation between private and public benefits will determine the proportion of shares, in excess of the minimum necessary to transfer the control, that creditors will decide to auction off.

We are certainly not the first to argue that a contingent payment – such as royalty fees or a minority stake in the ownership of a company auctioned off – is a tool that allows the seller to extract a higher surplus in an auction with both private or common values [see Section 7 of McAfee and McMillan (1987) for a survey of the literature]. Indeed, the literature on auctions with contingent payments shows that by making the bids contingent on an *ex post* signal of the bidders' valuation, such as royalty fees, it is possible for the auctioneer to extract an amount of surplus higher than the one he would be able to extract otherwise, both in private values (Hansen 1985; Samuelson 1987) and common values

5 Notice that our result holds also in the case in which there is only one potential buyer if the incumbent does not know the buyer's willingness to pay. In this case it is optimal to use the number of shares sold as a screening device (Cornelli and Li 1997).

auctions (McAfee and McMillan 1986; Riley 1988). As discussed in McAfee and McMillan (1987), this observation leads to the obvious question why the royalty fees are not set at 100%. The argument used in this literature to justify royalty fees lower than 100% is the presence of some form of moral hazard that affects the value of the good in the hands of the bidder.⁶

This paper studies instead the case where the item auctioned off is a company. In such a case, there is a natural way to structure the auction with contingent payments, given that it is possible to transfer the control of the company (the transfer that affects the value of the object auctioned off) without at the same time transferring the whole ownership. This result applies both when the auction is a private values auction (Section III below) or when the buyers' valuations have a common value component (Section IV below). Moreover, in the presence of private benefits from control (Section III below), an endogenous bound arises on the percentage of contingent payments that is optimal in equilibrium. In other words, we provide an alternative answer to the question why the royalty fee are not equal to 100%. We show that it is not optimal anymore to sell only the minimum stake entailing the transfer of control and the size of the optimal stake is endogenously determined by the private benefits from control and their correlation with the market value of the company.

Finally, few recent papers have discussed the role of auctions in bankruptcy. Baird (1986) and Aghion et al. (1992) argue that in a world without cash or credit constraints (like the one we are analyzing), auctions are an efficient bankruptcy procedure, distributional issues notwithstanding. We do not disagree with this point. However, we argue that an auction achieves *ex post* efficiency (since it allocates the firm's control optimally) but does not necessarily maximize the creditors' proceeds, if the creditors are required to auction off the entire company, as it usually happens in bankruptcy procedures. In other words, modifying the procedure so as to allow the creditors to auction off only the control stake of the firm may increase creditors' revenues.

III. HOW TO SELL THE COMPANY

In this section, we study how to optimally allocate the control of a company. We assume that the control goes to the shareholder who owns a fraction $\underline{\alpha}$ of the shares, where $0 < \underline{\alpha} < 1$.⁷ We denote V_i the firm's market value, transferable and public, if the control is in the hands of individual i . Thus, the firm has a different value depending on who has the control, reflecting the fact that each potential buyer has different skills or complementarities (or, in the case of

6 Some of the literature on bankruptcy advocates the use of noncash auctions that are equivalent to auctions with contingent payments. However, these papers propose these auctions mainly to overcome the limits on bids that arise when bidders are credit constrained (Aghion et al. 1992; Rhodes-Kropf and Vishnathan 2000).

7 The fraction $\underline{\alpha}$ can take different values depending on the voting structure or other characteristics of the company. We take $\underline{\alpha}$ to be exogenous in the paper and discuss in the conclusions what is the optimal level of $\underline{\alpha}$.

bankruptcy, a different restructuring plan). For the same reason, we also assume that the values V_i are independent across individuals (private values).⁸

The different values of the company in the hands of different buyers also entail different private benefits of control since different ways to run the company yield different private benefits. We denote B_i the benefits that an individual i may extract if he has control. Therefore, an individual i who owns a fraction of the company's shares $\alpha \geq \underline{\alpha}$ has a payoff equal to

$$\alpha V_i + B_i.$$

Individual i enjoys a fraction of the value of the firm equal to the fraction of the shares he owns and in addition, obtains private benefits from having control.

We analyze two possible scenarios. First, we briefly describe the case with two buyers and only two possible values. This simple case conveys most of the intuition of our results in a simple framework. We then move to the case of N potential buyers.

A. Two buyers, two values

There are two potential buyers, labeled 1 and 2. Each buyer has a specific plan on how to run the company if in control and the firm, under his control, has value V_1 and V_2 , respectively. Without loss of generality, let us assume that $V_1 < V_2$, that is, buyer 2 is the one who maximizes the value V_i of the firm. In addition, control of the firm by buyer i yields a private benefit B_i . Depending on the relation between public and private values, two cases may arise.

i. Case 1

The buyer with the highest payoff from buying a fraction $\alpha \geq \underline{\alpha}$ of the shares is also the buyer who maximizes the value of the firm.

Since we assumed that $V_1 < V_2$, in this case, the following inequality holds:

$$\underline{\alpha} V_2 + B_2 > \underline{\alpha} V_1 + B_1. \quad (1)$$

Clearly, if this inequality holds for $\underline{\alpha}$, it will hold for any α larger than $\underline{\alpha}$; therefore, buyer 2 has the highest payoff for any fraction $\alpha \geq \underline{\alpha}$. This case arises when there is no conflict between private benefits and public values (i.e., $B_2 \geq B_1$: a buyer who is more efficient at maximizing the value of the company is also better at extracting private benefits from control) or such conflict is limited. Since there is no strong conflict between public and private benefits, the only relevant issue is how to extract as much surplus as possible from the winner of the auction.⁹

It is easy to see that in this case, it is never optimal to sell the entire company. If the entire company is auctioned off, the unique equilibrium of the auction is

8 In Section IV, we show that the ability to resell the stake in the firm after the auction may create a common value component. Our results go through even in this setting.

9 Case 1 includes the special case $B_1 = B_2 = 0$, corresponding to the case where control of the firm does not yield any private benefit.

such that buyer 2 obtains the firm at the price $V_1 + B_1$.¹⁰ If instead only $\underline{\alpha}$ shares are auctioned off, the equilibrium price of the auction is $\underline{\alpha}V_1 + B_1$ (the maximum willingness to pay of buyer 1 for the control stake of the firm). The total payoff to the seller is

$$\underline{\alpha}V_1 + (1 - \underline{\alpha})V_2 + B_1. \quad (2)$$

Since the revenues in equation (2) exceed the revenues from auctioning the entire firm, it is optimal to auction off the minimum control stake of the firm, $\underline{\alpha}$. Notice also, that if we write the revenues in equation (2) as a function of a generic α , these revenues are monotonic decreasing in α : it is optimal to decrease the stake sold to the level $\underline{\alpha}$.

ii. Case 2

The buyer with the highest payoff from buying a fraction $\underline{\alpha}$ of the shares is not the buyer who maximizes the value of the firm.

In other words, the following inequality holds:

$$\underline{\alpha}V_2 + B_2 < \underline{\alpha}V_1 + B_1. \quad (3)$$

In this case, there is a strong enough conflict between public values and private benefits, so that selling the minimum stake $\underline{\alpha}$ does not maximize the auction revenues. In fact, the revenues from selling $\underline{\alpha}$ would be $\underline{\alpha}V_2 + B_2 + (1 - \underline{\alpha})V_1$. Given that $V_1 < V_2$, if we increase slightly the stake sold, assuming the winner of the auction does not change, revenues increase.

In order to maximize their revenues, a seller should auction off a percentage of shares μ ($\underline{\alpha} < \mu \leq 1$) such that

$$\mu V_2 + B_2 = \mu V_1 + B_1 \quad (4)$$

if such percentage exists. If $V_2 + B_2 \leq V_1 + B_1$, such fraction μ does not exist: in such case, we set $\mu = 1$, that is, the seller should auction off the entire firm.

If there exists a μ , which satisfies equation (4), bidder 2 will win the auction for a stake μ of the firm. The revenues will then be

$$\mu V_1 + B_1 + (1 - \mu)V_2 = V_2 + B_2$$

which are higher than the revenues from auctioning off either a fraction $\underline{\alpha}$ or the entire firm. Notice that in this case, the seller extracts the entire surplus from the winning bidder by auctioning off a percentage of the shares of the firm that is strictly bigger than the minimum control stake $\underline{\alpha}$ but strictly smaller than

10 When solving this simple auction game, we restrict attention to a *cautious equilibrium*. The basic idea behind this equilibrium concept is that no buyer should be willing to make a bid that would leave the buyer in the case he wins worse off relative to the equilibrium if he refused to participate in the auction. The dynamic version of the same equilibrium notion has been used in the analysis of Bergemann and Välimäki (1996) and Felli and Harris (1996). In our static environment, a cautious equilibrium is equivalent to an equilibrium in weakly dominant strategies.

100%. In other words, the seller can use the fact that one bidder has a higher private value to get the bidder with the higher public value to increase his bid. That is, the seller can take advantage of the differences in private and public benefits to extract all the surplus.

Thus Case 2 shows that even when there is a conflict between private and public benefits, it may still be optimal not to sell the entire company. The only difference is that now the fraction sold μ is larger than the minimum control stake: if a fraction lower than μ were sold, the firm control would not be efficiently allocated and the value of the minority stake would not be maximized.

If there does not exist a μ , which satisfies equation (4), the entire firm is auctioned off and bidder 1 obtains the firm paying $V_2 + B_2$. Since $V_2 > V_1$, the total payoff $\mu V_2 + (1 - \mu)V_1 + B_2$ would be lower. This is the only case in which it is strictly optimal to auction off the entire firm. This is because in this case, the benefits of control are very high, so that extracting these benefits is the best way to maximize revenues.

To summarize, the presence of private benefits of controls introduces a trade-off. The public component of the firm value in the hands of potential buyers requires the seller to reduce the fraction of the equity sold to the minimum necessary to transfer control. However, the presence of private benefits from control may induce the owner to sell more than this minimum fraction, in order to make sure that the firm is allocated efficiently. The higher are the private benefits of control (relative to the market value of the firm), the higher the fraction of the equity which should be sold.

B. The general case

Assume, now, that there are N potential buyers of the firm, each with a V_i and B_i , which are private information. For tractability, we restrict our analysis to the case where there exists a linear relation between private benefits from control and public or transferable values of the company:¹¹

$$B_i = \bar{B} + \beta V_i \quad (5)$$

where β parametrizes the correlation between public values V_i and private benefits B_i . When $\beta > 0$, there is no potential conflict between the two, while when $\beta < 0$, there is a potential conflict, which is more pronounced, the higher is the absolute value of β . The literature on private benefits of control usually assumes that private benefits of control and value of the firm are in a negative relation: The private benefits of control represent a diversion of money (or of time and effort by the manager/owner) away from the firm. Consequently, if a buyer enjoys a high degree of private benefits, the public value of the firm is likely to suffer from the diversion, as a consequence.

Thus, a buyer i who obtains $\alpha \geq \underline{\alpha}$ shares has a payoff

11 This assumption allows us to analyze the problem without addressing the issue of the multidimensionality of the adverse selection faced by the creditors in this setting.

$$\alpha V_i + B_i = \bar{B} + (\alpha + \beta)V_i \tag{6}$$

It is common knowledge that each V_i is drawn independently from the same distribution function $F(\cdot)$ over the interval $[0, \bar{V}]$, with density $f(\cdot)$. If $V = (V_i)_{i \in N}$, and $V_{-i} = (V_j)_{j \in N, j \neq i}$, we can define $G(V) \equiv [F(V_i)]^N$ and $G_{-i}(V_{-i}) \equiv [F(V_j)]^{N-1}$ with corresponding densities $g(V)$ and $g_{-i}(V_{-i})$. We can now characterize the optimal mechanism to sell the company and, in particular, the stake of the company that should be optimally sold.¹²

By Revelation Principle, we can restrict attention to the direct revelation mechanisms where the buyers simultaneously announce their valuation \tilde{V}_i to the seller and in equilibrium report the truth $\tilde{V}_i = V_i$. Prior to this announcement, the seller chooses the mechanism $\{p_i(\tilde{V}), t_i(\tilde{V}), \alpha\}$, where $p_i(\tilde{V})$ is the probability that buyer i gets control; $t_i(\tilde{V})$ is the amount he has to pay and α is the stake of the company sold. We identify a Bayesian Nash equilibrium of this mechanism under the condition that buyers truthfully reveal their own valuations.

If the firm has value V_i under buyer i 's control, his expected payoff when declaring \tilde{V}_i is the value of his equity stake minus the payment:

$$U_i(V_i, \tilde{V}_i) \equiv \int_{V_{-i}} \{[\bar{B} + (\alpha + \beta)V_i] p_i(\tilde{V}_i, V_{-i}) - t_i(\tilde{V}_i, V_{-i})\} g_{-i}(V_{-i}) dV_{-i}. \tag{7}$$

The seller's revenues are then the total payments from the buyers plus the expected value of the minority stake retained by the seller:

$$\int_V \left[\sum_i t_i(V) + \sum_i (1 - \alpha)V_i p_i(V) \right] g(V) dV. \tag{8}$$

The seller chooses α , p_i and t_i so as to maximize the revenues in equation (8) subject to the incentive compatibility constraints (that guarantee that each buyer will declare his true value V_i in equilibrium):

$$U_i(V_i, V_i) \geq U_i(V_i, \tilde{V}_i), \forall \tilde{V}_i \in [0, \bar{V}], \forall i \in N, \forall V_i \in [0, \bar{V}] \tag{9}$$

the individual rationality constraints (that guarantee that each buyer is willing to participate)

$$U_i(V_i, V_i) \geq 0, \forall i \in N, \forall V_i \in [0, \bar{V}] \tag{10}$$

and the two feasibility constraints

$$\sum_i p_i(V) \leq 1 \tag{11}$$

$$\underline{\alpha} \leq \alpha \leq 1. \tag{12}$$

12 Cornelli and Li (1997) show, in a different context, that the seller could actually do even better by not committing to a given number of shares to be sold, but by making α contingent on the bids.

The incentive compatibility constraints in equation (9) can clearly be rewritten as a maximization problem. As standard in the optimal auction literature (Myerson 1981), we can then use this formulation of the incentive compatibility constraints (see Appendix A1) to write the objective function in equation (8) as

$$\int_V \left\{ \sum_i \left[\bar{B} + (1 + \beta)V_i - (\alpha + \beta) \frac{1 - F(V_i)}{f(V_i)} \right] p_i(V) \right\} g(V) dV \quad (13)$$

We are now in a position to characterize the optimal selling procedure.

Proposition 1: Assume $F(V)$ has a monotonic increasing hazard rate. The company optimal selling procedure depends on β , the potential trade-off between public values and private benefits of control.

- (A) If the trade-off between public values and private benefits is non-existent or not too pronounced ($-\beta < \underline{\alpha}$), the optimal mechanism is an auction of $\underline{\alpha}$ shares of the company. The control will be allocated to the most efficient buyer.
- (B) If the trade-off between public values and private benefits is more pronounced ($-\beta > \underline{\alpha}$), we can distinguish two cases:
 - (a) If the trade-off is high but not exceedingly so, the optimal mechanism is an auction of β shares of the company. The control will be allocated to the most efficient buyer.
 - (b) If the trade-off is instead extreme (β very negative), the optimal mechanism is an auction of $\underline{\alpha}$ shares of the company. In this case, however, the control is allocated to an inefficient buyer.

Proof: Define $R_i(V_i, \alpha) \equiv \bar{B} + (1 + \beta)V_i - (\alpha + \beta) \frac{1 - F(V_i)}{f(V_i)}$ to be the kernel of the integral in equation (13). In case (A), the objective function in equation (13) is monotonic decreasing in α . It is therefore optimal to minimize α and set it equal to $\underline{\alpha}$. In this case, the utility function of bidder i is increasing in V_i , so the highest bidder will win the auction. Appendix A1 shows that the second-order conditions are satisfied in this case. In case (B), the objective function is still monotonic decreasing in α ; however, the second-order conditions are violated if we set $\alpha = \underline{\alpha}$. Therefore, there are two cases, depending on whether $(\partial R_i(V_i, \underline{\alpha}) / \partial V_i)$ is positive or negative. From $(\partial^2 R_i(V_i, \alpha) / \partial V_i \partial \beta) > 0$, $R_i(V_i, \underline{\alpha})$ is more likely to be monotonic decreasing in V_i the lower is β (the more negative is β).

If $R_i(V_i, \underline{\alpha})$ is monotonic increasing in V_i , then the optimal solution is to set $\alpha = \beta$; this is the minimum α that guarantees that the second-order conditions are satisfied (as shown in Appendix A1). In this case, the utility function of bidder i is increasing in V_i and thus the bidder with the highest V_i will acquire control. Alternatively, if $R_i(V_i, \underline{\alpha})$ is monotonic decreasing in V_i , it is optimal to choose the lowest $\alpha = \underline{\alpha}$ provided that the choice is to allocate the firm to the

bidder that announces the lowest V_i . Again, Appendix A1 shows that the second-order conditions are satisfied QED.

The intuition of Proposition 1 is the following. When there are benefits of control but $\beta > 0$, there is no trade-off between public values and private benefits. The buyer with the highest benefits from control is also the one who maximizes the value of the company. Thus, the presence of the benefits of control does not introduce any trade-off and we can safely ignore them. Case (A) covers all cases where β is positive or equal to 0 ($\beta = 0$ essentially corresponds to the case of no private benefits of control).¹³ This is a standard auction, and the only question is what fraction of the shares to sell. We know that in such case, the 'object' being sold (the control of the company) will go to the buyer with the highest value (in this case, the highest V_i) and that the bidder will pay strictly less than her private value (in order to satisfy the incentive compatibility constraint). The minority stake remaining in the hands of the seller, however, will have the value V_i and therefore the seller appropriates all the increase in value of the shares remaining in his/her hands. The seller has then the incentive to keep as many shares as possible, compatibly with transferring the control in the hands of the individual who maximizes the value of the shares. Selling $\underline{\alpha}$ shares accomplishes exactly that.

When instead, there is a trade-off between public values and private benefits, the buyer with the highest payoff from holding the minimum control stake $\underline{\alpha}$ is not necessarily the buyer who maximizes the value of the shares, that is, the buyer with the highest V_i . This is exactly the distinction we draw in the case with two buyers. If β is negative, but less in absolute value than $\underline{\alpha}$, it means that the trade-off between public and private values does not matter. The buyer with the highest personal value from buying $\underline{\alpha}$ shares is still the buyer with the highest V_i , and thus, it is still optimal to sell only $\underline{\alpha}$ shares. In the linear case we consider, this case is captured by the parameter $-\beta < \underline{\alpha}$.

Conversely, when $-\beta > \underline{\alpha}$ – case (B) in Proposition 2 – not only there is a trade-off between public values and private benefits, but also this trade-off does matter as β is large in absolute value. In this case, it is still true that the seller wants to sell the minimum possible stake, but if he sells only $\underline{\alpha}$ shares he is going to attract the buyer with the lowest public value V_i . As a result, he will retain $(1 - \underline{\alpha})$ shares with lower value than they could have. To be able to sell to the buyer with the highest V_i (and thus maximize the values of the minority stake), the seller has to increase the number of shares sold. The trade-off is thus clear. On one side, the seller wants to sell as few shares as possible, on the other side because of the private benefits of control, in so doing it minimizes the value of the shares remaining in his hands. To avoid selling it to an inefficient buyer,

13 To be precise, we need to assume also $\bar{B} = 0$ for private benefits not to exist. However, even if $\bar{B} > 0$, these benefits would be identical for every potential buyer and thus would not affect the optimal selling procedure. The seller can simply increase the buyers' payment by the flat fee \bar{B} .

he has to increase the number of shares sold. He has two possible choices: if the trade-off is not too strong, he does not have to increase too much the number of shares sold, and therefore, he will set $\alpha = \beta$ and in this way maximize the value of the minority stake. If instead the trade-off is extreme, then it is not worth for the seller to try to sell the control to the efficient buyer. It is then optimal to sell the minimum possible fraction $\underline{\alpha}$. This case is exemplified by the extreme case of $\beta < -1$: even if the seller sold the entire firm, he would still attract the least efficient buyer. The benefits of control completely offset any increase in the value of the shares and therefore the individual with the highest benefits of control is always the one willing to bid the most. In this case, the seller might as well resign himself to the fact that the shares will not have a high value *ex post* but, by selling the minimum control stake, he can at least extract as much surplus as possible.

IV. EXTENSIONS

In this section, we address three issues we have ignored in the analysis above. First, we consider the possibility that a larger fraction of the shares allocated to the buyer aligns his incentives with the interests of the minority shareholders, and thus increase the public value of the company. Second, we consider the possibility of the seller imposing a reservation price. Third, we allow buyers to trade their share holding after the auction.

A. Alignment of incentives

So far, we assumed that potential buyers were endowed with an innovation plan or a set of skills that generate the values B_i and V_i , independently of the fraction α of the company they acquire (as long as this fraction provides them with control $\alpha \geq \underline{\alpha}$). However, it is reasonable to assume (Burkart et al. 1998) that the fraction α of the firm has two effects on the incentives of the party in control. On the one hand, it guarantees that control cannot be challenged and hence allows the party in control to extract the benefits associated with it: entrenchment effect. On the other hand, the incentives of the party in control will be more in line with the interests of the minority shareholders the higher is the share α of the firm owned: alignment effect. For example, the party in control may invest more effort into maximizing the public value V_i at the expense of the extraction of the private benefits B_i . Burkart et al. (1998) focus on this second effect. If the second effect is dominant, then one may argue that minimizing the fraction of the shares sold is not any more optimal. In what follows, we will look at how the results are modified when we introduce this effect (we will ignore the entrenchment effect since it would just strengthen our results).

Consider the general model and focus on the case $\beta < 0$. Define V_i the value of the company in the absence of any effort, on the part of the buyer, into diverting money toward private benefits of control. For any given fraction α of

the company, the buyer will divert from V_i ; in other words, the actual value value of the company is $\hat{V}_i = V_i - \phi(\alpha)$, where $\phi(\alpha) > 0$, $\phi' < 0$ and $\phi'' < 0$.¹⁴ The corresponding private benefits of control are then

$$B_i = \bar{B} + \beta \hat{V}_i = \bar{B} + \beta(V_i - \phi(\alpha)) \quad (14)$$

where $\beta < 0$. The function ϕ thus captures the buyer's choice between public values and private benefits: the smaller is the fraction of the shares the buyer owns, the more he will choose to steal away from the company and increase his own private benefits. The total payoff to buyer i from purchasing a fraction α of the company is¹⁵

$$\alpha \hat{V}_i + B_i = \bar{B} + (\alpha + \beta)(V_i - \phi(\alpha)). \quad (15)$$

Given this new payoff, we can proceed as in Appendix A1, and derive the seller's objective function:

$$\int_V \left\{ \sum_i \left[\bar{B} - (\alpha + \beta)\phi(\alpha) + (1 + \beta)V_i - (\alpha + \beta) \frac{1 - F(V_i)}{f(V_i)} \right] p_i(V) \right\} g(V) dV \quad (16)$$

As in the case considered above, the optimal selling procedure still allows for cases where it is optimal for the seller not to auction off the entire company. We once again identify different scenarios depending on the magnitude of the trade-off between public values and private benefits. However, the seller will want to sell the entire company for a larger set of parameter values.

Proposition 2: Assume $F(V)$ has a monotonic increasing hazard rate. The company optimal selling procedure depends on β , the potential trade-off between public values and private benefits of control.

- (A) If the trade-off between public values and private benefits is nonexistent or not too pronounced ($-\beta < \underline{\alpha}$), the optimal mechanism is an auction of either $\underline{\alpha}$ shares or of the entire company. The control will be allocated to the most efficient buyer.
- (B) If the trade-off between public values and private benefits is more pronounced ($-\beta > \underline{\alpha}$), we can distinguish two cases:
 - (a) If the trade-off is high but not exceedingly so, the optimal mechanism is an auction of β shares of the company. The control will be allocated to the most efficient buyer.
 - (b) If the trade-off is instead extreme (β very negative), the optimal mechanism is an auction of $\underline{\alpha}$ shares of the company. In this case, however, the control is allocated to an inefficient buyer.

Proof: As in the case of Proposition 1, we define

14 The function ϕ could also be convex with no major changes in the analysis.

15 A complete model of the buyer's problem might require an explicit cost of the diversion effort by the buyer and hence of α . However, adding this cost only strengthens the seller's incentives to minimize the stake α sold. We thus abstract from this cost in the analysis.

$$\hat{R}_i(V_i, \alpha) \equiv \bar{B} - (\alpha + \beta)\phi(\alpha) + (1 + \beta)V_i - (\alpha + \beta)\frac{1 - F(V_i)}{f(V_i)}$$

the kernel of the integral in equation (16) above. Consider now the solution to the unconstrained maximization of equation (16). We are interested in whether the solution is such that $\alpha = \underline{\alpha}$, that is, the objective function is monotonic decreasing in α . Notice that

$$\frac{\partial \hat{R}_i}{\partial \alpha} = -(\alpha + \beta)\phi'(\alpha) - \phi(\alpha) - \frac{1 - F(V_i)}{f(V_i)}. \quad (17)$$

The last two terms are negative, but the first term can be positive or negative, depending on the sign of $\alpha + \beta$. We can therefore distinguish three cases. In case (A), we have $\underline{\alpha} + \beta > 0$ and therefore the derivative in equation (17) could be positive. If the derivative in equation (17) is still negative, then the unconstrained solution is $\underline{\alpha}$. Since the second-order conditions of the incentive compatibility problem are satisfied, this is the solution of the problem. If the derivative in equation (17) is positive, then $\underline{\alpha}$ is not any more the unconstrained solution. If $\phi'' < 0$ then $(\partial^2 \hat{R}_i / \partial \alpha^2) > 0$ and the solution is $\alpha = 1$.

In case (B), we have $\underline{\alpha} + \beta < 0$ and therefore, the derivative in equation (17) is negative. In this case, the objective function is monotonic decreasing in α and the unconstrained solution is $\underline{\alpha}$. However, in this case, the second-order conditions are violated if we set $\alpha = \underline{\alpha}$. Therefore, the solution is exactly as in case (B) of Proposition 1 QED.

B. Reservation price

Notice that the optimal selling mechanism derived above is *ex post* efficient since the firm is allocated in the hands of the buyer that maximizes its social value, interpreted as the sum of the public value and private benefits from control. However, this is due to the fact that we ignored the possibility for the seller to impose a reservation price. In the corollary below, we introduce this possibility.

Corollary 1: It is optimal for the creditors to sell the company to buyer *i* only if $V_i \geq V^*$, where V^* is defined so that

$$R_i(V^*, \alpha) = 0.$$

Proof: It is easy to see that if $V_i < V^*$ then $R_i(V_i, \alpha) < 0$ and it is therefore optimal to set $p_i(V_i, V_{-i}) = 0$ QED.

First, notice that, in contrast to the standard auction where private benefits are absent, there does not necessarily exist a $V^* > 0$, that is, it is not necessarily optimal to impose a reservation price. Second, the reservation price introduces a trade-off between *ex ante* and *ex post* efficiency. Setting a reservation

price increases the creditors' expected revenues, but it introduces an *ex post* inefficiency. This inefficiency arises when the buyer with the highest willingness to pay has a valuation V_i lower than V^* (or, in terms of the auction, his bid is below the reservation price). In this case, the firm will not be sold, although its value is maximized in the hands of that buyer.¹⁶

An important observation, however, is that the inefficiency introduced by imposing an optimal reservation price is reduced if the seller does not auction off the entire company:

$$\frac{dV^*}{d\alpha} = -\frac{\partial R/\partial \alpha}{\partial R/\partial V_i}.$$

Since we are assuming that $F(V_i)$ has a monotonic increasing hazard rate $h(V_i) = f(V_i)/[1 - F(V_i)]$, then $(\partial R/\partial V_i) > 0$. Therefore, if $(\partial R/\partial \alpha) < 0$ then V^* decreases when α decreases. This is the case under Proposition 1. Under Proposition 2, we saw, in the proof, that when $(\partial \hat{R}/\partial \alpha) > 0$, it is anyway optimal to sell the entire company. In other words, whenever it is optimal to sell less than the entire company, doing so decreases the optimal reservation price and therefore the *ex post* inefficiency. Reducing the fraction of equity sold increases both *ex ante* and *ex post* efficiency.

C. Trading among bidders

One possible objection to the procedure suggested above is that the result relies on the fact that we do not allow the buyers to trade the (control stake of the) firm, once it is in their hands. One might argue that if we allow the buyers to trade stakes of the firm between themselves, the value of the firm would be the same for all the bidders. Therefore, selling a control stake would be equivalent to selling the entire firm.

We now show that our result holds even if we allow buyers to trade stakes of the firm among themselves. In other words, it is still optimal for the creditors to retain the minority stake of the firm and to sell only the control stake. The intuition is that, when reselling the company, a bidder will be able to capture only part of the value of the company in the hands of the buyer depending on his bargaining power. Therefore, the value of the option to resell in general does not reflect the full increase in the value of the company due to the transfer of control. However, by retaining a minority stake, the buyers can guarantee themselves the full increase in value of the company at least on the minority stake they retain.

16 We are assuming that the firms have no value if it remains in the hands of the seller. It is possible to assume that the firm has a value also in the hands of the seller and this introduces an additional reason for introducing a reservation price (that does not increase *ex post* inefficiency). All the results of the paper hold in this case.

For simplicity, we prove the result only for the case of no private benefits of control. We first look at the simple case of two buyers and two values and then we generalize it to the case of N buyers with imperfect and asymmetric information.

i. The perfect information case with trading

Assume that buyer 1, after acquiring the control stake of the company, can resell it to buyer 2. Assume that in the first period, the seller auctions off a control stake (or the entire firm); while in the second period, buyers may re-trade this stake between each other.

We start from the second period in which buyers trade between each other. This stage takes the form of a bilateral trade between the bidder who got the firm in the first period (say bidder 1) and the bidder that can maximize the *ex post* value of the firm (bidder 2) – as long as these two bidders are not the same individual, of course.¹⁷

Whatever fraction of the firm is auctioned off in the first period, in the second period, it is a weakly optimal strategy for bidder 1 to trade only the control stake of the firm α and retain the minority stake for herself (the same intuition that we derived in Section III above holds also here). As a consequence, we can restrict attention to the case in which the investor who won the auction is going to sell only the fraction α of the company.

To keep the model of bilateral trade as simple as possible, we make the assumption that with probability ψ , bidder 1 makes a take-it-or-leave-it offer to bidder 2, and with the complementary probability $(1 - \psi)$ bidder 2 makes a take-it-or-leave-it offer to bidder 1.

In order to solve the game, we have to determine the reservation price of both parties in period 2. The highest price bidder 2 is willing to pay for the control stake is αV_2 (his entire surplus from obtaining the control stake α). The lowest price bidder 1 is willing to accept for the control stake of the firm is slightly more complex. It is the price that makes him indifferent between selling the control stake of the firm or retaining it for himself. If only the control stake of the firm is auctioned off in period one, then this reservation price is αV_1 . If instead the entire firm is auctioned off in period one, then the price for the control stake of the company αV is such that $\alpha V + (1 - \alpha)V_2 = V_1$.¹⁸

Consider first the case in which the entire firm is auctioned off in period one. The price bidder 1 is able to obtain in period two for the control stake of the firm is

$$\alpha[\psi V_2 + (1 - \psi)V] \tag{18}$$

which yields a total revenue to the seller equal to

17 Ausubel and Cramton (1999) provide a general analysis of an auction where *ex-post* resale achieves Pareto efficiency.

18 For simplicity, we assume that $V_1 > (1 - \alpha)V_2$. The whole analysis can be easily adjusted to account for the case in which the above inequality does not hold.

$$\Pi^* = (1 - \underline{\alpha})V_2 + \underline{\alpha}[\psi V_2 + (1 - \psi)V] = \psi V_2 + (1 - \psi)V_1. \quad (19)$$

Equation (19) identifies the highest willingness to pay of bidder 1 in the auction in period one and, hence, the equilibrium winning bid. In other words, equation (19) specifies the total returns to the seller when he auctions off the entire firm in period one.¹⁹

Consider now the case in which the seller auctions off only the control stake of the firm in period one. The price bidder 1 is able to obtain in period two is:

$$\underline{\alpha}[\psi V_2 + (1 - \psi)V_1] \quad (20)$$

This will be the equilibrium winning bid in the auction of the control stake in period one. Hence, the total returns to the seller are

$$\Pi^{**} = (1 - \underline{\alpha})V_2 + \underline{\alpha}[\psi V_2 + (1 - \psi)V_1] \quad (21)$$

Clearly the returns to the seller are greater when only the control stake of the firm is auctioned off in period one ($\Pi^{**} > \Pi^*$).

The intuition behind this result is simple. By auctioning off only a control stake of the firm, the seller can guarantee himself a share of the future value of the firm $(1 - \underline{\alpha})V_2$ that is not going to be affected by the future trade (hence, the bargaining power) between bidders.

A separate issue concerns the case in which the bidder with the higher valuation for the firm is not present at the auction but is available only later on.²⁰

Assume that after the auction an individual, labelled 3, with valuation $V_3 > V_2$ will want to buy the firm and assume no discounting. Assume that this information is known to all the parties in period 1. If the seller has not yet sold the firm when buyer 3 appears then the seller can bargain with this buyer and his proceeds are:

$$\psi V_3 + (1 - \psi)\underline{V} \quad (22)$$

where \underline{V} is the value of the firm when kept in the hands of the seller. As in equation (21), it does not matter in this bargaining whether the seller sells the entire firm to buyer 3 or only the control stake.

19 Equation (19) shows that it does not matter whether bidder 1 trades the entire firm or only its control stake in period two. He is in fact indifferent. The reason is that the reservation value in the bargaining between bidder 1 and 2 in period two differs in these two cases so as to leave the seller with exactly the same surplus.

20 This is not so unusual, for example, in the cases of bankruptcy of large firms, where it is not easy to find immediately the best possible buyers. Sometimes, delays in Chapter 11 have been justified by the need to look around for the best buyer. We therefore ask whether it may be optimal for the creditors to hold on to the company, waiting for the individual in whose hands the value of the firm is highest to materialize. We show that, even with no discounting, creditors are strictly better off by allocating the control stake of the firm immediately. The reason is that the bidders are able to internalize the possibility to resell the firm and at the auction stage the competition among potential buyers provides the seller with the opportunity to extract a higher surplus from them.

Assume instead that the seller auctions off the control stake of the firm in period 1 to bidders 1 and 2 and let the winner of this auction bargain with buyer 3 later on. Then the value bidder $i = 1, 2$ expects from the sale of the firm is

$$\psi V_3 + (1 - \psi)V_i \quad (23)$$

The winning bid is then $[\psi V_3 + (1 - \psi)V_1]$ and the revenues from the auction are

$$(1 - \underline{\alpha})V_3 + \underline{\alpha}[\psi V_3 + (1 - \psi)V_1] \quad (24)$$

Notice that even if $V_1 = \underline{V}$, the revenues in equation (24) are higher than the revenues in equation (22).

ii. The private information case with trading

We now proceed to consider the case of N potential buyers that have private information about the value of the firm under their control. To simplify the analysis, we assume that after the shares are sold, all V_i s are common knowledge. In other words, there is imperfect information only during the period-one auction. This is admittedly a strong assumption, but it allows us to focus on the issue of the optimal selling procedure by the original owner of the firm, the key point of the paper, and avoid issues of multiplicity of equilibria that would arise if there were asymmetric information at the bargaining stage.²¹

Assume that creditors have sold α shares to a buyer i with valuation V_i . This value could be the highest possible for the firm or there may exist an individual j whose valuation is higher than V_i . Consider the second case ($V_i < V_j$). As in the previous section, individual i will sell only the minimum control stake to buyer j . The price individual i is able to obtain from buyer j is

$$\underline{\alpha}[\psi V_j + (1 - \psi)V]$$

where the lowest price i is willing to accept for the sale of the control stake of the firm $\underline{\alpha}V$ is now $\underline{\alpha}V = V_i - (\alpha - \underline{\alpha})V_j$. The resulting total revenue to i is then

$$\underline{\alpha}[\psi V_j + (1 - \psi)V_i].$$

If instead all the potential buyers have a valuation lower than V_i , the shares are not sold to anyone else.

21 Haile (1999) analyzes auctions with resale where imperfect information remains also in the resale market. It shows that if there are no new participants in the resale market, then the equilibrium of the auction without resale is also an equilibrium of the same auction followed by resale; in such case therefore, our result still holds. If instead new buyers, who were not at the auction, are present in the resale market, then there exist signaling equilibria. Even in this case, however, the resale market will add a common value component but the bids still depend on the bidders' private information and our result remains true. Zheng (2000) also looks at auctions with resale and identifies the optimal auction. Our result remains true also in that set-up.

Define $V_{-i}^- \equiv \{V_j \in (0, V_i), \forall j \neq i\}$, the set of vectors of firms' values V_j such that all values are strictly lower than V_i and V_{-i}^+ its complement. If all the values V_j are lower than V_i , there will be no trading in the second period, if instead at least one V_j is higher than V_i , then there will be trading. Then

$$U_i(V_i, \tilde{V}_i) \equiv \int_{V_{-i}^-} [\alpha V_i p_i(\tilde{V}_i, V_{-i}) - t_i(\tilde{V}_i, V_{-i})] g_{-i}(V_{-i}) dV_{-i} + \int_{V_{-i}^+} \{\alpha[\psi V_j^{max} + (1 - \psi)V_i] p_i(\tilde{V}_i, V_{-i}) - t_i(\tilde{V}_i, V_{-i})\} g_{-i}(V_{-i}) dV_{-i} \quad (25)$$

where V_j^{max} is the highest value in the vector V_{-i}^+ .

We can now rewrite the seller's objective function (see Appendix A2) as

$$\int_0^{\bar{V}} \left\{ \int_{V_{-i}^-} \sum_i \left[V_i - \alpha \frac{1 - F(V_i)}{f(V_i)} \right] p_i(V) dG_{-i}(V_{-i}) + \int_{V_{-i}^+} \sum_i \left[V_i + \psi \alpha (V_j^{max} - V_i) - (1 - \psi) \alpha \frac{1 - F(V_i)}{f(V_i)} \right] p_i(V) dG_{-i}(V_{-i}) \right\} dF(V_i) \quad (26)$$

The intuition behind this expression is quite simple and it is the same one that applies in the case of perfect information: even when the willingness of a bidder is affected by the option to resale, a higher V_i allows the buyer to extract a higher payment, in proportion $1 - \psi$, while only a fraction ψ of the highest value is extracted. We now have all the elements to prove that auctioning off the minimum stake α that transfers control is optimal.

Proposition 3: If $F(V)$ has a monotonic increasing hazard rate, the optimal selling procedure when bidders can trade their shares of the company after these shares are allocated is an auction where only the minimum control stake α is auctioned off.

Proof: Since $F(V)$ has an increasing hazard rate, it is optimal to set $p_i(V) = 1$ for $V_i = V_j^{max}$. Then, the objective function in equation (26) is monotonic decreasing in α_i . It is therefore optimal to minimize α_i . Moreover, a constant $\alpha_i(V) = \alpha$ satisfies the second-order conditions of the incentive compatibility constraint as in case (A) of Proposition 1 QED.

The intuition of what is happening here is simple. The buyer with the highest valuation (V_j^{max}) obtains the control stake of the firm, but the payment is determined by the second highest willingness to pay. However, only the fraction $1 - \psi$ extracted by the buyer is relevant for the payment, and that fraction is decreasing in α .

Notice that also in this case, it is optimal to impose a reservation price and not to serve a buyer with valuation $V_i < V^*$ (where V^* is defined as in the previous case); therefore, the same analysis applies.

V. TWO APPLICATIONS: BANKRUPTCY AND CARVE-OUTS

So far, we have discussed the optimal selling mechanism of a company in the abstract. In this section, we consider two specific applications of such a mecha-

nism. These are bankruptcy, where the control of a firm in financial distress is reallocated by means of restructuring, and carve-outs, where a company sells shares of a subsidiary. Our purpose is twofold. First, we provide evidence that indeed there are real-world instances where the seller chooses to sell less than the entire company. This evidence is provided in the context of IPOs and carve-outs. Second, we provide a normative message. While bankruptcy laws in most countries does not allow creditors to sell less than the entire company, we do believe that the efficiency of the bankruptcy procedure, as well as its speed, could be enhanced if creditors are given the option to retain a minority stake in the company. We therefore extend the model to take into account some special features that arise in the context of a bankruptcy procedure.

A. Bankruptcy

A bankruptcy procedure – or, even before bankruptcy, any restructuring in a situation of financial distress – has to dispose of the insolvent firm. Usually, creditors have control during the bankruptcy procedure but aim to transfer it in new hands. In other words, bankruptcy often leads to the sale of the company. This paper can therefore suggest a way to sell a bankrupt company that maximizes the creditors' proceeds from the sale.

Maximizing the creditors' proceeds from the sale of a bankrupt company is not the first quality of a bankruptcy procedure that comes to mind. Indeed, a bankruptcy procedure is usually considered efficient if it allocates the company assets in the hands of individuals that maximize the value of the company. This quality of a bankruptcy procedure is the *ex post* efficiency we discussed above. *Ex post* efficiency, however, does not take into account the effect that the disposal of the bankrupt company has on the incentives of the involved parties before the firm goes into bankruptcy, even before any clue of financial distress is at the horizon. A bankruptcy procedure that does a good job at promoting these incentives is *ex ante* efficient.

Two groups of stakeholders play a critical role in the life of a company. These are the entrepreneurs or managers of the company and its creditors. A bankruptcy procedure 'punishing' managers or entrepreneurs of the insolvent firm (for example, not giving them control even when it is *ex post* efficient to do so) may be seen as *ex ante* efficient. It provides entrepreneurs with the right incentives to manage the firm so as to avoid ending up in financial distress, for example, by not undertaking too many risks. The effects of different bankruptcy procedures on the managers' and entrepreneurs' incentives have been extensively studied in the literature (e.g., Aghion and Bolton 1992; Berkovitch et al. 1993; Bolton and Scharfstein 1996).

i. Toeholds

The main result of our analysis can shed light on some of the features of observed bankruptcy cases. Usually, an observed increase in the creditors' equity stake at the end of a bankruptcy restructuring is explained by the need to

increase monitoring by large shareholders (Gilson 1990), or more generally by the fact that an increase in the creditors' stake might affect the value of the company. This paper suggests that allowing creditors to retain minority stakes in the company is the best way for the creditors to sell the firm and recuperate as much as possible of their credits.

Our proposal of allowing creditors of a bankrupt company to retain a minority stake in the company may also reduce the magnitude of a well-known problem in the use of auctions in bankruptcy procedures. There is a large body of evidence suggesting that some of the potential acquirers of bankrupt companies are coalitions, that include one or more creditors. This is a special feature of bankruptcy procedures that has been documented, for example, by Eckbo and Thorburn (2009) for Sweden. In Sweden, a court-appointed trustee arranges an open auction. However, they argue that the bank who is the main creditor can influence the auction by financing a bidder in return for a strategy that maximizes the bank-bidder coalition's expected revenue. Under certain conditions, the bank-bidder coalition optimally bids more than the private valuation of the bank's coalition partner (overbidding). The intuition is the same as the one of the literature on toehold bidding in takeovers (Burkart 1995): the bidder is going to receive part of the payment and thus at least in part does not find costly to raise his bid. Hotchkiss and Mooradian (2003) also show that a coalition of creditors and management may overbid. In what follows, we consider a setup similar to theirs and show that reducing the control stake of the company sold also reduces the incentives to overbid.

We focus on the case of two potential buyers, and assume that one of the buyers is a coalition including some of the creditors. Following Hotchkiss and Mooradian (2003), we assume that the creditors in the coalition are entitled to a fraction γ of the revenues from the sale of the company.²²

Whether buyers overbid depends on whether the creditors' coalition is the efficient one (the one which maximizes the *ex post* share value V_i) or not. In our case $V_2 > V_1$, if the creditors' coalition is buyer 2, then they participate in the efficient coalitions. In such a case, creditors have no incentives to overbid, when using the mechanism we proposed in Section III above. In the case of no trade-off between public values and private benefits, $\underline{\alpha}V_2 + B_2 \geq \underline{\alpha}V_1 + B_1$, the creditors win by bidding the valuation of bidder 1 that clearly does not exceed their valuation $\underline{\alpha}V_2 + B_2$ of the control stake of the company. In the case of a trade-off between public values and private benefits, $\underline{\alpha}V_2 + B_2 < \underline{\alpha}V_1 + B_1$, the optimal mechanism prescribes the sale of a fraction μ of the company such that $\mu V_2 + B_2 = \mu V_1 + B_1$, thus the creditors win by bidding their valuation.²³

22 Hotchkiss and Mooradian (2003) also consider a setup, that takes into account the seniority structure of the credits. In such a case, creditors are repaid up to their debt face value (when they have priority) and do not obtain any additional fraction of the revenues. This setup does not add much to our main point and hence we abstract from seniority considerations.

23 In case such fraction μ does not exist, the entire firm is sold and thus we are back to the Hotchkiss and Mooradian (2003) framework.

Consider now the case where the creditors' coalition is the inefficient one (buyer 1). In the case of no trade-off, $\alpha V_2 + B_2 \geq \alpha V_1 + B_1$, for every $\alpha \geq \underline{\alpha}$ coalition 1 may have an incentive to overbid and pay $\alpha V_2 + B_2$. The payoff to coalition 1 is

$$\alpha V_1 + B_1 - (1 - \gamma)[\alpha V_2 + B_2] + \gamma(1 - \alpha)V_1 \quad (27)$$

The first two terms represent the payoff that the coalition will have from owning a fraction α of the firm and extracting the private benefits of control. The third term represents the amount that the buyer has to pay to the seller. Since the creditors in the coalition will appropriate a fraction γ of the revenues, the actual payment is only a fraction $1 - \gamma$ of the revenues. Finally, the last term is due to the fact that a fraction $(1 - \alpha)$ of the firm remains in the hands of the creditors. We assume that the creditors' coalition also obtains a fraction γ of these shares.

The incentive to overbid is given by the term:

$$(1 - \gamma)[\alpha V_2 + B_2] + \gamma(1 - \alpha)V_1$$

This term is clearly increasing in α , its derivative is $\gamma[V_2 - V_1]$. In other words, as α increases, the incentives to overbid increase. Thus, our mechanism that reduces α to the minimum $\underline{\alpha}$ also reduces the incentives to overbid.

The intuition is the following. On the one hand, the toehold effect is in place since the bidder effectively pays less than its full bid. By reducing the fraction of shares that are sold, the saving through the toehold is also proportionally reduced, and therefore, the advantage of the bidder with the toehold is reduced. On the other hand, more shares remain in the creditors hand, of which the winning coalition will appropriate of a fraction γ . This should therefore increase the incentives to overbid. However, since the shares remain in the hands of the inefficient buyer (who overpays an amount equal to the value, they would have had in the hands of the efficient buyer), the first effect dominates.

Consider now the case where there is a trade-off between public values and private benefits. The optimal mechanism prescribes the sale of a fraction μ of the company such that $\mu V_2 + B_2 = \mu V_1 + B_1$. In this case, the creditors coalition cannot overbid by much. In particular, the creditors may choose a fraction of the company such that no overbidding occurs.

ii. Privatization of bankruptcy

A natural question that comes to mind, when applying the optimal mechanism derived in Proposition 1 above to the sale of a bankrupt company is whether such a mechanism can be implemented in a decentralized way. The key to our optimal mechanism is clearly to leave the creditors the option to sell less than 100% of the shares of the bankrupt company. This objective can be practically implemented in a number of ways.

One way to proceed would be, for example, to transform the bankrupt firm in a all equity firm. Then allocate the shares of this new firm to the creditors

following whatever procedure is most suitable for the creditors.²⁴ Once this is done, the creditors are required to sell α of their share so as to transfer the control to the buyer with the highest valuation and retain the $(1 - \alpha)$ of their shares. The percentage α can be chosen so as to maximize the creditors proceed in the way described in Proposition 1 above.

Alternatively, the same procedure could be implemented by selling in a centralized manner α of the shares and distributing – according to whatever criterion is preferred by the creditors – both the monetary revenues from the sale and the residual percentage $(1 - \alpha)$ of shares to the creditors *ex post*. Either way, the final result would be identical.

One could argue that there is no need to centralize and discipline the way in which creditors sell their shares. In other words, we could simply transform the company in an all equity firm, allocate all shares of the new company to the creditors (following any chosen priority rule) and then let the creditors, now shareholders, decide what to do with the firm. This would be equivalent to privatizing the bankruptcy procedure: it is only necessary to define clearly the ownership rights of the creditors on the firm and then they optimally decide what to do with it.

Here below, we show that in the privatized procedure, there always exists an equilibrium that coincides with the one derived in Proposition 1, one in which the optimally chosen control stake of the equity, α , is allocated in the hands of the buyer who maximizes the firm's value. However, in the privatized procedure, there exist also other equilibria, which are both *ex post* and *ex ante* inefficient. Hence, disciplining the way the creditors proceed in allocating the bankrupt firm is a way to select the efficient equilibrium.

Assume that each creditor i is allocated s_i shares and that creditors have to decide whether to sell an amount \bar{s}_i of their shares, $\bar{s}_i \leq s_i$. To keep the treatment as simple as possible, we restrict attention to the perfect information environment in which there are only two potential buyers, 1 and 2, for the firm and there are no private benefits of control.²⁵ We also assume that the creditors only decision is whether to sell or not the amount \bar{s}_i of shares. In other words, provided that creditors are willing to tender their amount \bar{s}_i of shares, these shares are allocated to the buyer in the way suggested in Proposition 1 above.

Assume that the decision whether to tender an amount \bar{s}_i of shares is taken by each creditor simultaneously and independently. We denote p the share price paid by buyer 2 and take $V_1/S \leq p < V_2/S$ where $S = \sum_i s_i$. Clearly a creditor can always decide to sell the remaining shares in his hands ($s_i - \bar{s}_i$) immediately after the control of the company is transferred in the hands of buyer 2 at the share price (V_2/S).

24 In particular, the creditors might want to follow absolute priority rule using, for example, the procedure suggested in Bebchuk (1988) or might decide not to follow absolute priority rule. Notice that our main point is completely independent of the distribution of shares.

25 The discussion can be easily extended to the case in which there are private benefits of control.

The game we just described has a multiplicity of equilibria. In particular, in the case in which $s_i < \underline{\alpha}$ for any $i = 1, \dots, N$, there always exists an equilibrium in which each creditor tenders zero shares since he expects the other creditors to tender zero shares as well. In other words, $\bar{s}_i = 0$ for every $i = 1, \dots, N$, is always an equilibrium of this tendering game. This equilibrium is clearly *ex post* inefficient since the firm has no (or very low) value in the hands of the creditors while it has value V_2 in the hands of buyer 2. It is also *ex ante* inefficient since the creditors revenues are not maximized. The problem is the coordination failure among the creditors.²⁶

It should be noticed, however, that there also exists an equilibrium, which reproduces exactly the allocation of shares in Proposition 1 above as the outcome of our suggested procedure. Indeed, if creditor i believes that the other creditors will sell exactly the percentage of shares $(\underline{\alpha} - \pi)\%$, where $\pi \leq (s_i/S)$, then creditor i feels pivotal. It is therefore a best reply for creditor i to tender an amount of shares πS . The result is that the control is transferred to buyer 2, the firm value is V_2 and the total revenue obtained by the creditors is $[\underline{\alpha}pS + (1 - \underline{\alpha})V_2]$. The allocation implemented by this equilibrium is equivalent to the one derived in Proposition 1. Indeed, in the event that $p = (V_1/S)$, the creditors' revenue coincides with the one in equation (2) above when $B_1 = 0$.

Disciplining and centralizing the procedure the creditors are supposed to use solves the creditors' coordination problem. In other words, it isolates as the unique outcome the one which achieves *ex post* as well as *ex ante* efficiency. It is possible to reinterpret this discussion in favor of a bankruptcy procedure that disciplines the way creditors behave in the event of a corporate reorganization.

B. Carve-outs

In an equity carve-out, a firm offers to sell shares in a wholly owned subsidiary to the public. As such, a carve-out can be viewed as the sale of an asset. There are two issues that render them particularly interesting from a corporate control perspective. First, carve-outs are mainly conducted by large conglomerate firms, which are prime candidates to expect agency problems due to the separation of ownership and control, to be present. Second, a carve-out is an event where a firm's management raises funds at the expense of control rights in the sold subsidiary. Hence, a carve-out is always an event where a change in the governance structure occurs and a new market valuation can be observed. Allen and McConnell (1998) argue that although the parent company often still holds significant stakes in the subsidiary after the carve-out, management of the parent company has lost significant control rights.

Schipper and Smith (1986) show that the market has a positive reaction to carve-out announcements. This can be interpreted in different ways, but one of them is that the subsidiary's control is reallocated in the hands of the company that can increase the subsidiary's value because of better complementarities and

26 The logic is exactly the same of Grossman and Hart (1980) and Shleifer and Vishny (1986).

skills. Vijh (2002) scrutinizes the *Wall Street Journal* reported motives for carve-outs and finds that a majority of the reports mention lack of fit or focus and a desire to restructure operations by divesting subsidiary assets. This is thus consistent with the view that assets are reallocated in the hands, which can increase their value. Consistently, there are positive abnormal returns at the announcement of a carve-out. The case of equity carve-outs is particularly important for our analysis since in an equity carve-out, the parent company maintains an equity stake in the subsidiary. It is therefore evidence that companies selling subsidiaries prefer to sell less than the entire subsidiary, which is consistent with the point of this paper, and, in so doing, the seller maximizes his revenues. To be precise, in some carve-outs, the selling company is still retaining a control stake. However, even in those cases, the literature associates these carve-outs with a reduction in control by the selling company (these cases may thus be consistent with our analysis of the case where the size of the benefits of control depends on the size of the controlling stake).

Very interestingly, Klein et al. (1991) find that often carve-outs are associated with subsequent events, either a sell-off of the parent's remaining interest or a reacquisition of the subsidiary's outstanding shares. Moreover, it finds that the shorter is the period between the carve-out and the second event, the more likely it is that the second event will be a divestiture. This suggests that in these cases, the carve-out is the first stage of a divestiture, where first, a fraction is sold to a new buyer and then the rest of the shares are sold, as our result suggests. In some of these cases, the second event is a spin-off: thus first, the firm sells the control to a new buyer and afterwards, it distributes the remaining shares to the existing shareholders, who can choose to sell them or hold on to them. This is similar to what would happen in the case of bankruptcy under our proposal: creditors would divide among themselves the unsold shares and then would be free to sell them.

Consistent with our paper, Klein et al. (1991) also find that when the percentage of shares retained by the parent company is below 50% (i.e., the parent company is relinquishing control, as in our proposal) it is (statistically significantly) more likely that the second event is a divestiture. Taken together, these results suggest that a carve-out, especially one where control is sold, may often be the first stage of a planned two-step divestiture, as in the optimal mechanism of Proposition 1 above.

VI. CONCLUDING REMARKS

In this paper, we propose a way to sell a company that maximizes the proceeds from the sale. In the absence of private benefits from control, it is optimal to auction off the majority of the voting rights retaining as much as possible of the cash flow rights. This can be done by both selling a fraction of the shares or by changing the voting structure of the shares. When private benefits are present, it may not be optimal any more to completely separate voting and cash flow rights but it may still be optimal to retain part of the cash flow rights of the

company. Therefore, maximization of the sale proceeds may in general lead to a violation of the one-share-one vote principle. (Grossman and Hart 1988).

This way to sell a company implies an optimal choice of the minimum stake of the company α necessary to transfer control. In the absence of private benefits from control, it is clearly optimal to minimize such stake, for example, by auctioning off a minimal number of shares (possibly one share) with all the voting rights. However, the (public) value of a firm under the control of a given buyer (that is the expected cash flows when this buyer is in control) may depend on the fraction of cash flow rights this buyer has. In other words, if a buyer owns too little cash flow rights in that company, he may not invest any effort in it and not maximize its value. As a result, the choice of the number of shares with voting rights would not be so extreme (one share with all the voting rights would not be optimal). We do not model directly this issue since it is not crucial for our analysis. In principle, one may define α as the fraction of the cash flow rights, which solves this trade-off, and our analysis would then apply unchanged to this α .

The presence of private benefits from control may also provide an incentive not to sell the minimum number of shares. In fact, if the private benefits of control are larger, the larger is the control stake the buyer obtains (Burkart et al. 1998), the seller may want to increase the number of shares sold. Once again, our analysis could be extended to consider α as the share that maximizes the seller's surplus. Our result will apply unchanged once we redefine α in this way.

Leonardo Felli
London School of Economics
Houghton Street
London WC2A 2AE
UK
lfelli@econ.lse.ac.uk

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APPENDIX

A1. Derivation of the first- and second-order condition

The incentive compatibility constraint, equation (9), can be expressed as $V_i = \arg \max_{\tilde{V}_i} U_i(V_i, \tilde{V}_i)$. Assuming differentiability, by envelope theorem

$$\frac{dU_i}{dV_i}(V_i, V_i) = \int_{V_{-i}} (\alpha + \beta) p_i(V_i, V_{-i}) g_{-i}(V_{-i}) dV_{-i}. \quad (A1)$$

Reintegrating it, we get:

$$U_i(V_i, V_i) = \int_0^{V_i} \int_{V_{-i}} (\alpha + \beta) p_i(x, V_{-i}) g_{-i}(V_{-i}) dV_{-i} dx + U_i(0, 0). \quad (A2)$$

Comparing the expression for $U_i(V_i, V_i)$ in equation (A2) and its definition in equation (7), solving for t_i , we obtain:

$$\int_V t_i(V) g(V) dV = \int_V [\bar{B} + (\alpha + \beta) V_i] p_i(V) g(V) dV - U_i(0, 0) + \int_{V_{-i}} g_{-i}(V_{-i}) \int_0^{\tilde{V}_i} g_i(V_i) \int_0^{V_i} (\alpha + \beta) p_i(x, V_{-i}) dx dV_i dV_{-i} \quad (A3)$$

where $g_i(V_i) = f(V_i)$. Integrating by parts, the above expression can be transformed into:

$$\int_V t_i(V) g(V) dV = \int_V \left\{ \bar{B} + (\alpha + \beta) \left[V_i - \frac{1 - F_i(V_i)}{f_i(V_i)} \right] \right\} p_i(V) g(V) dV - U_i(0, 0). \quad (A4)$$

Substituting equation (A4) into equation (8) we obtain equation (13).

The second-order condition for the maximization is: $\left. \frac{\partial^2 U_i(V_i, \tilde{V}_i)}{\partial \tilde{V}_i^2} \right|_{\tilde{V}_i=V_i} \leq 0$.

Recall the first-order condition: $\left. \frac{\partial U_i(V_i, \tilde{V}_i)}{\partial \tilde{V}_i} \right|_{\tilde{V}_i=V_i} = 0$. Differentiating this first-order

condition with respect to \tilde{V}_i , we have

$$\left. \frac{\partial^2 U_i(V_i, \tilde{V}_i)}{\partial V_i \partial \tilde{V}_i} \right|_{\tilde{V}_i=V_i} + \left. \frac{\partial^2 U_i(V_i, \tilde{V}_i)}{\partial \tilde{V}_i^2} \right|_{\tilde{V}_i=V_i} = 0.$$

Therefore, the second-order condition is satisfied if $\left. \frac{\partial^2 U_i(V_i, \tilde{V}_i)}{\partial V_i \partial \tilde{V}_i} \right|_{\tilde{V}_i=V_i} \geq 0$ which can be rewritten as

$$\int_{V_{-i}} (\alpha + \beta) \frac{\partial p_i(V)}{\partial V_i} g_{-i}(V_{-i}) dV_{-i} \geq 0, \quad \forall i \in N, \forall V_i \in [0, \bar{V}] \quad (A5)$$

If $\underline{\alpha} + \beta > 0$, then the second-order condition are satisfied at $\underline{\alpha}$ and thus, this is the optimal solution. If instead $\underline{\alpha} + \beta < 0$, then the second-order conditions are satisfied if $\alpha = \beta$ or, if $\alpha = \underline{\alpha}$ and $\frac{\partial p_i(V)}{\partial V_i} < 0$.

A2. Derivation of the first- and second-order condition with trading

Proceeding as in the case above, by envelope theorem

$$\begin{aligned} \frac{dU_i}{dV_i}(V_i, V_i) = & \int_{V_{-i}} \alpha p_i(V_i, V_{-i}) g_{-i}(V_{-i}) dV_{-i} + \\ & + (1 - \psi) \int_{V_{-i}^+} \alpha p_i(V_i, V_{-i}) g_{-i}(V_{-i}) dV_{-i} \end{aligned} \quad (A6)$$

(the effects of a change of V_i on the extremes of integration compensate each other). Reintegrating it, we get

$$\begin{aligned} U_i(V_i, V_i) = & U_i(0, 0) + \int_0^{V_i} \left\{ \int_{V_{-i}} \alpha p_i(x, V_{-i}) g_{-i}(V_{-i}) dV_{-i} + \right. \\ & \left. + (1 - \psi) \int_{V_{-i}^+} \alpha p_i(x, V_{-i}) g_{-i}(V_{-i}) dV_{-i} \right\} dx. \end{aligned} \quad (A7)$$

We can set $U_i(0, 0) = 0$ using the individual rationality constraint. Then, comparing the expression for $U_i(V_i, V_i)$ in equation (A7) and its definition in equation (25), solving for t_i , we obtain

$$\begin{aligned} \int_V t_i(V) g(V) dV = & \int_0^{\bar{V}} \left\{ \int_{V_{-i}} \alpha V_i p_i(V_{-i}) g(V_{-i}) dV_{-i} + \right. \\ & \left. + \int_{V_{-i}^+} \alpha [(1 - \psi)V_i + \psi V_j^{max}] p_i(V_{-i}) g(V_{-i}) dV_{-i} \right\} f(V_i) dV_i + \\ & - \int_0^{\bar{V}} \left\{ \int_{V_{-i}} \int_0^{V_i} \alpha p_i(x, V_{-i}) g_{-i}(V_{-i}) dx dV_{-i} + \right. \\ & \left. - \int_{V_{-i}^+} \int_0^{V_i} \alpha (1 - \psi) p_i(x, V_{-i}) g_{-i}(V_{-i}) dV_{-i} \right\} f(V_i) dV_i. \end{aligned} \quad (A8)$$

Integrating by parts, the above expression can be transformed into:

$$\begin{aligned} \int_V t_i(V) g(V) dV = & \int_0^{\bar{V}} \left\{ \int_{V_{-i}} \alpha \left[V_i - \frac{1 - F(V_i)}{f(V_i)} \right] p_i(V) dG_{-i}(V_{-i}) + \right. \\ & \left. + \alpha \int_{V_{-i}^+} \left[(1 - \psi) \left[V_i - \frac{1 - F(V_i)}{f(V_i)} \right] + \psi V_j^{max} \right] p_i(V) dG_{-i}(V_{-i}) \right\} dF(V_i). \end{aligned} \quad (A9)$$

Substituting equation (A9) into equation (8), we obtain equation (26).