Course Outline

The Law and Economics of Contracts

- **Lecture 1**: Contracts and Enforcement: Complete Contracts. Coase Theorem and its failures.
- **Lecture 2**: Transaction Costs, Why parties go to Court?
- **Lecture 3**: The role of Courts: insurance, filling the gaps, disclosure.
- **Lecture 4**: Legal Systems: efficiency and tradeoffs.
- **Lecture 5**: Enforcement, Power, Crime and Punishment.
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- **Course Material:** available at: http://econ.lse.ac.uk/staff/lfelli/teaching
References: Contract Theory


References: Contract Law

The first natural question that needs to be answered is:

*What is a contract?*

**Definition**

A contract is the ruling of an economic transaction: the description of the performance that the contracting parties agree to complete at a (possibly future) date.
Example: a contract for the purchase of a specific item, say a meal. It specifies:

- the restaurant’s performance (number of courses, quality of food, cooking details, etc...),
- the customer’s performance (payment in full upon completion).

Contracts involve not only the contracting parties, but also outsiders (enforcing authority: the court).
We distinguish between *implicit* and *explicit* contracts.

A contract is *implicit* or *self-enforcing* whenever the environment in which the contracting parties operate corresponds to the extensive form of a game whose (unique) subgame perfect Nash equilibrium (PBE) exactly corresponds to the outcome the parties would like to implement.

If you believe in *SPE or PBE* then there is no need for explicit communication. The two rational individuals will behave in the way required.
If the outcome the parties would like to implement is *not* the subgame perfect Nash equilibrium of the environment in which they operate the parties might want to modify the environment.

This is accomplished through and *explicit contract*.

An explicit contract is a *commitment device* requiring:

- an *explicit agreement* between the parties,
- the intervention of a third party: *the court.*
The role of the court is to force the parties to behave in a way that differs from the one that would arise in the absence of any agreement.

An explicit contract therefore specifies a new extensive form corresponding to a new game for the parties.

The usual way for the court to guarantee that the parties operate in this new environment is by modifying the parties’ payoffs, when necessary.

By agreeing to bring in a court in the game the parties commit to play a game that differs from the initial one they were in.
To see how the presence of a court may work consider the following example: *(Kreps, 1984)*

A buyer $B$ and a seller $S$ wish to trade an indivisible item at date 1.

The buyer’s valuation: $v$,

The seller’s delivery cost: $c$.

Let $v > c$

In other words, trade is *socially efficient*. 
Let $p$ be a reasonable price level (we abstract for the moment from bargaining) such that:

$$v > p > c.$$ 

$B$’s and $S$’s situation may be described by the following normal form:

<table>
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<tr>
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<th>deliver</th>
<th>not deliver</th>
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</thead>
<tbody>
<tr>
<td>$pay \ p$</td>
<td>$v - p$, $p - c$</td>
<td>$-p$, $p$</td>
</tr>
<tr>
<td>$not \ pay \ p$</td>
<td>$v$, $-c$</td>
<td>$0$, $0$</td>
</tr>
</tbody>
</table>
- The *unique Nash equilibrium* (dominant solvable) is:

$$ (B \text{ does not pay}, S \text{ does not deliver}) $$

- This is clearly an *inefficient outcome: no trade*.

- The situation does not change if any of the following two extensive forms are played.
The **unique SPE** of the following game is:

\[ \{ B \text{ does not pay, } S \text{ does not deliver at both nodes} \}. \]
The unique SPE of the following game is:

\[ \{ S \text{ does not deliver, } B \text{ does not pay at both nodes, } \}\]
- **Solution**: to this inefficiency is an explicit contract enforced by a court.

- It specifies:
  - the payment $p$ that $B$ is supposed to make contingent on $S$ delivering the item,
  - the punishment $F_B > p$ (implicit in the legal system) imposed by the court on $B$ in the event that $S$ delivers and $B$ does not pay,
  - the punishment $F_S > c$ (implicit in the legal system) imposed by the court on $S$ in the event that $B$ pays but $S$ does not deliver.
In this case the normal form describing the contracting parties problem once the contract is in place is:

<table>
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<tr>
<th></th>
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<th>not deliver</th>
</tr>
</thead>
<tbody>
<tr>
<td>pay p</td>
<td>$v - p, p - c$</td>
<td>$F_S - p, p - F_S$</td>
</tr>
<tr>
<td>not pay p</td>
<td>$v - F_B, F_B - c$</td>
<td>0, 0</td>
</tr>
</tbody>
</table>

The unique Nash equilibrium is now:

$$(B \text{ pays } p, S \text{ delivers}).$$

Notice that the particular contract considered is budget balanced off-the-equilibrium-path: it is renegotiation proof. The latter property does not always hold.
Consider now an environment in which when the plaintiff goes to court detection is costly ($\kappa$) and is successful only with probability $\pi$.

The payoffs associated with (not pay $p$, deliver) are:

$$\nu - \pi (F_B + \kappa), \quad \pi F_B - (1 - \pi) \kappa - c$$

The payoffs associated with (pay $p$, not deliver) are:

$$\pi F_S - (1 - \pi) \kappa - p, \quad p - \pi (F_S + \kappa)$$
- Notice that as deterrence goes: the detection probability (policing, monitoring) $\pi$ and the size of the punishment, $F_B$ and $F_S$, are substitutes (Becker 1968).

- The game is solved assuming that court’s costs $\kappa$ are paid by the loosing party (British system).

- If court’s costs $\kappa$ are too high the game has multiple Nash equilibria: (pay $p$, deliver) and (not pay $p$, not deliver).
This example clearly shows the need for an enforcement mechanism.

This mechanism may be due to:

- the parties being involved in a repeated relationship *relationship/implicit contracting*, (multiplicity might be a problem).

- the presence of a legal system that through a court enforces the parties agreement *explicit contracting*.

Notice that according to this interpretation the court is essentially a commitment device available to the parties that can be used when the parties agree to call it in.
An alternative interpretation is that the court itself is one of the players of the game.

It should therefore be endowed with a payoff function and an action space and should be explicitly considered in the analysis of the contractual situation (we will come back to this).

It should be mentioned that using this line of argument one could obtain a rather extreme interpretation of a contract (a law) *(Mailath, Morris and Postlewaite 2000).*
The view is that enforcement/punishment is the only relevant activity.

A contract (a law) can at best be interpreted as cheap talk that allows the parties to coordinate on a particular equilibrium of the game.

No new equilibrium is introduced by the parties agreeing on a contract or by the parliament passing a law.
From now on we will assume that the two (or more) parties involved in the contractual relationship operate in a *market economy* with a *well functioning legal system*.

Whatever contract the parties agree to it will be enforced by the court.

The penalties for breaching the contract will be assumed to be *sufficiently severe* that no contracting party will ever consider the possibility of not honoring the contract.

We will abstract from explicitly specifying these penalties.
Coase Theorem:

- Once we have established what a contract is and how it works the next natural question is:

  - What could parties achieve in an economic environment in which they can costlessly negotiate a contractual agreement?

- The answer to this question is the celebrated Coase Theorem.
Theorem (Coase Theorem (Coase 1960))

In an economy where ownership rights are well defined and transacting is costless gains from trade will be exploited (a contract will be agreed upon) and efficiency achieved whatever the distribution of entitlements.

That is rational agents write contracts that are *individually rational* and *Pareto efficient*.

A contract is *individually rational* if each contracting party is not worse off by deciding to sign the contract rather than choosing not to sign it.
- This is the reflection of an other basic principle of a well functioning legal system known as: *freedom of contract*.

- This is equivalent to assume that the action space of the contracting parties always contains the option *not to sign the contract*.

- A contract is *Pareto efficient* if there does not exist another feasible contract that makes at least one of the contracting party strictly better off without making any other contracting party worse off.
Consider the following simple model of a production externality.

Consider two parties, labelled $A$ and $B$.

Party $A$ generates revenue $R_A(e_A)$ (strictly concave) by choosing the input $e_A$ at a linear cost $c e_A$ ($c > 0$).

$A$’s payoff function is then:

$$\Pi_A(e_A) = R_A(e_A) - c e_A$$
- Party $B$ generates revenue $R_B(e_B)$ (strictly concave) by choosing the input $e_B$ at the linear cost $c e_B$ ($c > 0$).

- Party $B$ also suffers from an externality $\gamma e_A$ ($x > 0$) imposed by $A$ on $B$.

- $B$’s payoff function is then:

$$\Pi_B(e_B) - \gamma e_A = R_B(e_B) - c e_B - \gamma e_A$$
Assume first that the parties choose the amounts of input $e_A$ and $e_B$ simultaneously and independently without any prior agreement.

- Party $A$’s problem:

$$\max_{e_A} \Pi_A(e_A)$$

- Party $B$’s problem:

$$\max_{e_B} \Pi_B(e_B) - \gamma e_A$$

- In equilibrium the inputs chosen $(\hat{e}_A, \hat{e}_B)$ are:

$$R'_A(\hat{e}_A) = c, \quad R'_B(\hat{e}_B) = c$$
Consider now the *social efficient* amounts of input $e_A^*$ and $e_B^*$.

These solve the problem:

$$\max_{e_A, e_B} \Pi_A(e_A) + \Pi_B(e_B) - \gamma e_A$$

In other words $(e_A^*, e_B^*)$ are such that:

$$R'_A(e_A^*) = c + \gamma$$

$$R'_B(e_B^*) = c$$
Comparing \((\hat{e}_A, \hat{e}_B)\) and \((e^*_A, e^*_B)\) we obtain using concavity of \(R_A(\cdot)\):

\[
e^*_B = \hat{e}_B, \quad e^*_A < \hat{e}_A
\]

In other words:

\[
\Pi_A(e^*_A) + \Pi_B(e^*_B) - \gamma e^*_A - \left[\Pi_A(\hat{e}_A) + \Pi_B(\hat{e}_B) - \gamma \hat{e}_A\right] = \\
= [\Pi_A(e^*_A) - \Pi_A(\hat{e}_A)] + \gamma (\hat{e}_A - e^*_A) > 0
\]

The joint surplus is reduced by the inefficiency generated by the externality.
Assume now that the two contracting parties have the opportunity to get together and agree on a contract before the amounts of input are chosen.

There exists strictly positive \textit{gains from trade}.

A reduction in the amount of input $e_A$ from $\hat{e}_A$ to $e_A^*$ will generate:

- a decrease in the net revenues from $A$’s technology:
  \[ \Pi_A(e_A^*) < \Pi_A(\hat{e}_A) \]

- reduction in the negative externality
  \[ \gamma e_A^* < \gamma \hat{e}_A \]
The former effect is more than compensated by the latter one. This may create room for negotiation.

Normalize for simplicity the total size of the surplus that is available to share between the two contracting parties to have size 1 (simple normalization).

To establish a well defined negotiation ownership rights need to be specified.

Entitlements/ownership rights define the outside option of each party to the contract.

In other words they define the payoff each party is entitled to without need for the other party to agree.
Denote $w_A$ and $w_B$ the entitlements of party $A$, respectively $B$ where:

$$w_A + w_B < 1.$$ 

We assume the following extensive form for the costless negotiation between the two parties:

- **Infinite horizon, alternating offers bargaining game with discounting and outside options.**
Denote:

- \( \delta \) the parties’ common discount factor,
- \( x \) the share of the pie to party \( A \),
- \( (1 - x) \) the share of the pie to party \( B \).
Odd periods:

**Stage I:** $A$ makes an offer $x_A$ to $B$,

**Stage II:** $B$ observes the offer and has three alternatives:

- he can accept the offer, then $x = x_A$ and the game terminates;
- he can reject the offer and take his outside option $w_B$ and the game terminates;
- he can reject the offer and do not take his outside option, then the game moves to Stage I of the following period.
Even periods:

Stage I: $B$ makes an offer $x_B$ to $A$,

Stage II: $A$ observes the offer and has three alternative choices:

- he can accept the offer, then $x = x_B$ and the game terminates;
- he can reject the offer and take his outside option $w_A$ and the game terminates;
- he can reject the offer and do not take his outside option, then the game moves to Stage I of the following period.
Payoffs:

- If parties agree on $x$ in period $n + 1$:
  \[ \pi_A(\sigma_A, \sigma_B) = \delta^n x, \quad \pi_B(\sigma_A, \sigma_B) = \delta^n (1 - x), \]

- If they do not agree and either party takes his outside option in period $n + 1$:
  \[ \pi_A(\sigma_A, \sigma_B) = \delta^n w_A, \quad \pi_B(\sigma_A, \sigma_B) = \delta^n w_B. \]
Theorem (Deal Me Out)

For any discount factor \( \delta < 1 \), and any pair \((w_A, w_B)\), \(w_A + w_B < 1\), the bargaining game has a unique subgame perfect equilibrium.

Agreement between the parties is immediate and the outside options are never exercised.
Proof: (sketch)

- Denote $x^H_i$, respectively $x^L_i$, $i \in \{A, B\}$, the highest, respectively the lowest, possible share that $A$ can receive in a subgame that starts with $i$ making the offer.

- We then have that:

\[
x^H_B \leq \max\{w_A, \delta x^H_A\}, \quad 1 - x^L_A \leq \max\{w_B, \delta \left(1 - x^L_B\right)\}
\]

- Moreover:

\[
x^L_B \geq \max\{w_A, \delta x^L_A\}, \quad 1 - x^H_A \leq \max\{w_B, \delta \left(1 - x^H_B\right)\}
\]
Solving these inequalities we obtain:

\[ x_A^H = x_A^L = x_A, \quad x_B^H = x_B^L = x_B \]

We also obtain that:

- If

\[ w_A \leq \frac{\delta}{1 + \delta}, \quad w_B \leq \frac{\delta}{1 + \delta} \]

then

\[ x_A = \frac{1}{1 + \delta}, \quad x_B = \frac{\delta}{1 + \delta} \]
If
\[ w_A \geq \frac{\delta}{1 + \delta}, \quad w_B \leq \delta(1 - w_A) \]
then
\[ x_A = 1 - \delta(1 - w_A), \quad x_B = w_A \]

If
\[ w_A \leq \delta(1 - w_B), \quad w_B \geq \frac{\delta}{1 + \delta} \]
then
\[ x_A = 1 - w_B, \quad x_B = \delta(1 - w_B) \]

If
\[ w_A \geq \delta(1 - w_B), \quad w_B \geq \delta(1 - w_A) \]
then
\[ x_A = 1 - w_B, \quad x_B = w_A \]
These offers characterize a pair of strategies \((\sigma_A, \sigma_B)\).

It is easy to show that these strategies constitute the unique subgame perfect equilibrium of the bargaining game.

Notice that an efficient agreement is reached independently of the size of the entitlements.
In particular if each party is entitled to the choice of his input, then:

\[ w_A = \frac{\Pi_A(\hat{e}_A)}{\Pi_A(e_A^*) + \Pi_B(e_B^*) - \gamma e_A^*} \]

\[ w_B = \frac{\Pi_B(\hat{e}_B) - \gamma \hat{e}_A}{\Pi_A(e_A^*) + \Pi_B(e_B^*) - \gamma e_A^*} \]

If instead party B is entitled to preclude party A from operating his technology, then:

\[ w_A = 0, \quad w_B = \frac{\Pi_B(\hat{e}_B)}{\Pi_A(e_A^*) + \Pi_B(e_B^*) - \gamma e_A^*} \]
In either case the result above implies that we would get the efficient outcome: \((e_A^*, e_B^*)\).

However, the share that accrue to each party depends on the entitlements \(w_A\) and \(w_B\).

The equilibrium contract specifies a transfer between the two parties and \(A\)’s choice of input \(e_A^*\).

Also the transfer depend on the entitlements \(w_A\) and \(w_B\).
From now on we are going to focus on models in which the Coase Theorem fails.

The classic cause for the failure of the Coase Theorem is the presence of asymmetric information between the parties.

This is a situation in which each party has private information on his own preferences (hidden information model).

Recall that a game of incomplete information (a player does not know the preferences of one opponent) can always be recast as a game of imperfect information (a player does not know the history of the game) (Harsanyi 1967).
We are going to consider first a very simple model of *bargaining under bilateral asymmetric information* (a specific extensive form) with no externalities.

We will show that in this situation *efficiency cannot be achieved*.

Recall that the Coase Theorem implies efficiency even in the presence of externalities therefore if inefficiency arises in the absence of externalities we can conclude that the *Theorem fails*. 
Notice however that this does not imply that we cannot find an extensive form that will achieve efficiency.

Fortunately an other fundamental principle of contract theory will help in this case: *Revelation Principle*.

Using the revelation principle we will be able to conclude that efficiency cannot be achieved whatever extensive form governs the bargaining between the two parties under bilateral asymmetric information.
Consider the following simple model of *bilateral trade* (double auction).

- Two players, a buyer and a seller: \( N = \{ b, s \} \).

- The seller names an *asking price*: \( p_s \).

- The buyer names an *offer price*: \( p_b \).
The action spaces:

\[ A_s = \{p_s \geq 0\}, \quad A_b = \{p_b \geq 0\}. \]

- The seller owns and attaches value \( v_s \) to an indivisible unit of a good.
- The buyer attaches value \( v_b \) to the unit of the good and is willing to pay up to \( v_b \) for it.
- The valuations for the unit of the good of the seller and the buyer are their private information of each player.
- Player \( i \in \{b, s\} \) believes that the valuation of the opponent \( v_{-i} \) takes values in the unit interval.
The type spaces:

\[ T_s = \{0 \leq v_s \leq 1\}, \quad T_b = \{0 \leq v_b \leq 1\} \]

Player \( i \in \{b, s\} \) also believes that the valuation of the opponent is uniformly distributed on \([0, 1]\):

\[ \mu_s = 1, \quad \mu_b = 1. \]

The extensive form of the game is such that:

- If \( p_b \geq p_s \) then they trade at the average price:

\[ p = \frac{(p_s + p_b)}{2}. \]

- If \( p_b < p_s \) then no trade occurs.
The payoffs to both the seller and the buyer are then:

\[
    u_s(p_s, p_b; v_s, v_b) = \begin{cases} 
        \frac{(p_s + p_b)}{2} & \text{if } p_b \geq p_s \\
        v_s & \text{if } p_b < p_s 
    \end{cases}
\]

and

\[
    u_b(p_s, p_b; v_s, v_b) = \begin{cases} 
        v_b - \frac{(p_s + p_b)}{2} & \text{if } p_b \geq p_s \\
        0 & \text{if } p_b < p_s 
    \end{cases}
\]

Players’ strategies: \( p_s(v_s) \) and \( p_b(v_b) \). We consider strictly monotonic and differentiable strategies.
Consider now the seller’s best reply.

This is defined by the following maximization problem:

$$\max_{p_s} E_{v_b} \{ u_s(p_s, p_b; v_s, v_b) \mid v_s, p_b(v_b) \}$$

Consider now the seller’s payoff, substituting $p_b(v_b)$ we have:

$$u_s = \begin{cases} 
\frac{(p_s + p_b(v_b))}{2} & \text{if } p_b(v_b) \geq p_s \\
v_s & \text{if } p_b(v_b) < p_s
\end{cases}$$
or

\[ u_s = \begin{cases} \frac{(p_s + p_b(v_b))}{2} & \text{if } v_b \geq p_b^{-1}(p_s) \\ v_s & \text{if } v_b < p_b^{-1}(p_s) \end{cases} \]

The seller’s maximization problem is then:

\[
\max_{p_s} \int_{v_b=0}^{p_b^{-1}(p_s)} v_s \, dv_b + \int_{v_b=p_b^{-1}(p_s)}^{1} \frac{(p_s + p_b(v_b))}{2} \, dv_b
\]
Recall that by Leibniz’s rule:

\[
\frac{\partial}{\partial y} \left( \int_{\alpha(y)}^{\beta(y)} G(x, y) dx \right) =
\]

\[
= G(\beta(y), y) \beta'(y) - G(\alpha(y), y)\alpha'(y) +
\]

\[
+ \int_{\alpha(y)}^{\beta(y)} \frac{\partial G(x, y)}{\partial y} dx
\]
Therefore the first order conditions are:

\[
\nu_s \frac{dp_b^{-1}(p_s)}{dp_s} - \frac{1}{2} \left[ p_s + p_b(p_b^{-1}(p_s)) \right] \frac{dp_b^{-1}(p_s)}{dp_s} + \\
+ \int_{p_b^{-1}(p_s)}^{1} \frac{1}{2} d\nu_b = 0
\]

or from \( p_s = p_b(p_b^{-1}(p_s)) \):

\[
(v_s - p_s) \frac{dp_b^{-1}(p_s)}{dp_s} + \frac{1}{2} \left[ \nu_b \right]_{p_b^{-1}(p_s)}^{1} = 0
\]

which gives:

\[
(v_s - p_s) \frac{dp_b^{-1}(p_s)}{dp_s} + \frac{1}{2} \left[ 1 - p_b^{-1}(p_s) \right] = 0
\]
The **buyer’s best reply** is instead defined by:

$$\max_{p_b} E_{v_s} \{ u_b(p_s, p_b; v_s, v_b) \mid v_b, p_s(v_s)\}$$

Consider now the buyer’s payoff obtained substituting $p_s(v_s)$:

$$u_b = \begin{cases} 
  v_b - \frac{(p_s(v_s) + p_b)}{2} & \text{if } v_s \leq p_s^{-1}(p_b) \\
  0 & \text{if } v_s > p_s^{-1}(p_b)
\end{cases}$$

we then get

$$\max_{p_b} \int_{v_s=0}^{p_s^{-1}(p_b)} \left[ v_b - \frac{(p_s(v_s) + p_b)}{2} \right] dv_s$$
Therefore the first order conditions are:

$$\left[ v_b - \frac{p_s(p_s^{-1}(p_b)) + p_b}{2} \right] \frac{dp_s^{-1}(p_b)}{dp_b} +$$

$$\frac{1}{2} \int_{v_s=0}^{p_s^{-1}(p_b)} d v_s = 0$$

or

$$v_b - p_b \frac{dp_s^{-1}(p_b)}{dp_b} - \frac{1}{2} v_s \bigg|_{0}^{p_s^{-1}(p_b)} = 0$$

which gives:

$$(v_b - p_b) \frac{dp_s^{-1}(p_b)}{dp_b} - \frac{1}{2} p_s^{-1}(p_b) = 0$$
To simplify notation we re-write $p_b^{-1}(\cdot) = q_b(\cdot)$ and $p_s^{-1}(\cdot) = q_s(\cdot)$.

The two differential equations that define the best reply of the seller and the buyer are then:

$$[q_s(p_s) - p_s] q'_b(p_s) - \frac{1}{2} [1 - q_b(p_s)] = 0$$

$$[q_b(p_b) - p_b] q'_s(p_b) - \frac{1}{2} q_s(p_b) = 0$$
Solving the second equation for \( q_b(p_b) \) and differentiating yields:

\[
q'_b(p_b) = \frac{1}{2} \left[ 3 - \frac{q_s(p_b)q'''(p_b)}{[q'_s(p_b)]^2} \right]
\]

Substituting this expression into the first differential equation we get:

\[
[q_s(p_s) - p_s] \left[ 3 - \frac{q_s(p_s)q'''(p_s)}{[q'_s(p_s)]^2} \right] - \left[ 1 - p_s - \frac{q_s(p_s)}{q'_s(p_s)} \right] = 0
\]
This is a second-order differential equation in $q_s(\cdot)$ that has a two parameter family of solutions.

The simplest family of solution takes the form:

$$q_s(p_s) = \alpha \ p_s + \beta$$

Then the values $\alpha = 3/2$ and $\beta = -3/8$ solve the second-order differential equation.

The definition of $q_s(\cdot)$ and $q_b(\cdot)$ imply that:

$$p_s = \frac{2}{3} \ v_s + \frac{1}{4}, \quad p_b = \frac{2}{3} \ v_b + \frac{1}{12}$$
- This is the *(unique) Bayesian Nash equilibrium* of this game.

- Notice now that it is **efficient to trade** whenever:

  $$v_b \geq v_s$$

- However in this double auction game **trade occurs** whenever:

  $$p_b \geq p_s$$

  or

  $$\frac{2}{3}v_b + \frac{1}{12} \geq \frac{2}{3}v_s + \frac{1}{4}$$
In other words, in equilibrium trade occurs whenever:

\[ v_b \geq v_s + \frac{1}{4} \]
The obvious question is now: *how can we make sure that there does not exists an alternative way for the parties to achieve efficiency?*

The tool that allows us to give an answer to this question is: *Revelation Principle*

This Revelation principle says that there is no loss in generality in restricting attention to direct revelation mechanisms that satisfy truth-telling constraints.

Recall:

- the indirect mechanism is the one in which parties agree to a trade set prices etc. . .
- the direct mechanism is the one in which parties report their private information to a mechanism designer who according to the reports enforces the mechanism.

Looking for the truth-telling equilibrium of the direct mechanism that maximizes the principal’s utility is the way to identify the best possible indirect mechanism from the principal’s view point.
Since every BNE of every indirect mechanism has an associated truth-telling BNE of a direct mechanism if we find the truth-telling BNE of the direct mechanism that maximizes the principal’s utility there cannot exist any BNE of the indirect mechanism that is better for the principal.

Notice that this way to proceed does not require us to specify the space of all possible indirect mechanisms.

It is critical that the principal can commit to the mechanism in advance: renegotiation may lead to a failure of the revelation mechanisms (Dewatripont, 1989).
In our setting there is no principal, but the two parties at an ex-ante stage — before they learn their private information — will commit to a mechanism via the contract.

They will choose their contract in a way that maximizes their ex-ante welfare.

Assume further that this is a pure bilateral contract transfers cannot involve a third party.

In jargon the contract has to be budget balancing.
A seller and a buyer trade a single unit of a good.

The seller’s cost of delivering is $c$ and it is the seller’s private information:

$$c \sim P_S(c), \quad c \in [c, \bar{c}]$$

The buyer’s valuation is $v$ and it is the buyer’s private information:

$$v \sim P_B(v), \quad v \in [v, \bar{v}]$$

A contract in this environment is a pair $(\phi, t)$ where

- $\phi$ is the probability of trade,
- $t$ is the transfer from the buyer to the seller.
By revelation principle restrict attention to truth-telling direct mechanisms.

The seller’s indirect utility is then:

\[ U_S(\hat{c}, v | c) = t(\hat{c}, v) - \phi(\hat{c}, v) c \]

The buyer’s indirect utility is instead:

\[ U_B(c, \hat{v} | v) = \phi(c, \hat{v}) v - t(c, \hat{v}) \]

Denote:

\[ U_S(\hat{c}) = E_v [t(\hat{c}, v) - \phi(\hat{c}, v) c] = t(\hat{c}) - \phi(\hat{c}) c \]

\[ U_B(\hat{v}) = E_c [\phi(c, \hat{v}) v - t(c, \hat{v})] = \phi(\hat{v}) v - t(\hat{v}) \]
Therefore every *truth-telling BNE direct mechanism* has to satisfy the following set of *incentive compatibility constraints (IC)*:

\[
U_S(c) \geq t(\hat{c}) - \phi(\hat{c}) c \quad \forall c, \hat{c} \in [c, \bar{c}]
\]

\[
U_B(v) \geq \phi(\hat{v}) v - t(\hat{v}) \quad \forall v, \hat{v} \in [v, \bar{v}]
\]

Since once again we insist on *freedom of contract* we also require the following *individual rationality constraints (IR)* to be satisfied:

\[
U_S(c) \geq 0, \quad \forall c, \hat{c} \in [c, \bar{c}]
\]

\[
U_B(v) \geq 0, \quad \forall v, \hat{v} \in [v, \bar{v}]
\]
Notice now that both parties’ preferences satisfy the \textit{Spence-Mirrlees} single crossing conditions:

\[
\frac{\partial}{\partial v} \left[ -\frac{\partial U_B/\partial t}{\partial U_B/\partial \phi} \right] > 0
\]

The following result helps us to write (IC) and (IR) constraints in a manageable form.
Theorem (Myerson and Satterthwaite 1983)

For any probability $\phi(c, v)$ there exists a transfer function $t(c, v)$ that satisfies (IR) and (IC) if and only if:

$$E_{c,v} [\phi(c, v) (J_B(v) - J_S(c))] \geq 0$$

where

$$J_B(v) = \left( v - \frac{1 - P_B(v)}{p_B(v)} \right), \quad J_S(c) = \left( c + \frac{P_S(c)}{p_S(c)} \right)$$

and

$$\frac{d\phi(c)}{dc} \leq 0, \quad \frac{d\phi(v)}{dv} \geq 0$$
Proof:

- From the (IC) constraints since for every $\hat{v} > v$ we must have:

$$U_B(v) \geq \phi(\hat{v}) \cdot v - t(\hat{v}) = U_B(\hat{v}) - (\hat{v} - v) \phi(\hat{v}),$$

$$U_B(\hat{v}) \geq \phi(v) \cdot \hat{v} - t(v) = U_B(v) - (v - \hat{v}) \phi(v),$$

- Summing the two inequalities we get:

$$(\hat{v} - v) \phi(\hat{v}) \geq (\hat{v} - v) \phi(v)$$

- Dividing by $(\hat{v} - v)$ and letting $\hat{v}$ tend to $v$ we obtain:

$$U'_B(v) = \phi(v)$$

- Since $\phi(v, c) \in [0, 1]$ we obtain for every $v \in [v, \overline{v}]$:

$$U_B(v) \geq U_B(\overline{v})$$
We therefore conclude that the only relevant (IR) constraint is:

\[ U_B(v) \geq 0 \]

Notice that since for every \( \hat{v} > v \) we have:

\[ (\hat{v} - v) \phi(\hat{v}) \geq (\hat{v} - v) \phi(v) \]

we also obtain that:

\[ \frac{d\phi(v)}{dv} \geq 0 \]

Symmetrically for the seller we can prove that (IC) constraint implies:

\[ \frac{d\phi(c)}{dc} \leq 0 \]

Consider now the differential equation obtained above:

\[ U'_B(v) = \phi(v) \]
Integrating it we obtain:

\[ U_B(v) = U_B(\bar{v}) + \int_{\bar{v}}^{v} \phi(v) dv \]

and symmetrically for the seller we obtain:

\[ U_S(c) = U_S(\bar{c}) + \int_{c}^{\bar{c}} \phi(\gamma) d\gamma \]

By budget balancing we now get:

\[ 0 = E_c [t(c)] - E_v [t(v)] = \]

\[ = \int_{\bar{c}}^{\bar{c}} \left( \phi(c) c + \int_{c}^{\bar{c}} \phi(\gamma) d\gamma \right) p_S(c) dc + U_S(\bar{c}) + \]

\[ + \int_{\bar{v}}^{\bar{v}} \left( \int_{\bar{v}}^{v} \phi(v) dv - v \phi(v) \right) p_B(v) dv + U_B(\bar{v}) \]
Integrating by parts we get:

\[ U_{S}(\overline{c}) + U_{B}(\overline{v}) = \]

\[ = - \int_{c}^{\overline{c}} \left( c + \frac{P_{S}(c)}{p_{S}(c)} \right) \phi(c) p_{S}(c) \, dc + \]

\[ + \int_{\overline{v}}^{\overline{c}} \left( v - \frac{1 - P_{B}(v)}{p_{B}(v)} \right) \phi(v) p_{B}(v) \, dv \]

or

\[ U_{S}(\overline{c}) + U_{B}(\overline{v}) = E_{c,v} [\phi(c, v) (J_{B}(v) - J_{S}(c))] \]

Since (IR) is such that \( U_{S}(\overline{c}) \geq 0 \) and \( U_{B}(\overline{v}) \geq 0 \) then:

\[ E_{c,v} [\phi(c, v) (J_{B}(v) - J_{S}(c))] \geq 0 \]
Sufficiency is a bit more complex to prove it requires us to solve the partial differential equation that is represented by the FOC of the (IC) constraints.

The *parties’ ex-ante problem* is now:

\[
\max_{\phi_i} \quad E_{c,v} [\phi(c, v) (v - c)]
\]

s.t. \[
E_{c,v} [\phi(c, v) (J_B(v) - J_S(c))] \geq 0
\]

\[
\frac{d\phi(c)}{dc} \leq 0, \quad \frac{d\phi(v)}{dv} \geq 0
\]
Ignoring monotonicity conditions and denoting $\mu$ the lagrange multiplier of the remaining constraint we get a lagrangian function that is linear in $\phi_i$:

$$
E_{c,v} \left[ \phi(c, v) \left\{ (v - c) - \frac{\mu}{1 - \mu} \left( \frac{1 - P_B(v)}{p_B(v)} - \frac{P_S(c)}{p_S(c)} \right) \right\} \right]
$$

The solution is to set $\phi = 1$ if and only if the term in brackets is strictly positive.

In other words trade occurs if and only if, for $\mu \geq 0$:

$$
v - \frac{\mu}{1 - \mu} \left( \frac{1 - P_B(v)}{p_B(v)} \right) \geq c + \frac{\mu}{1 - \mu} \left( \frac{P_S(c)}{p_S(c)} \right)
$$
This $\phi(c, v)$ is weakly monotonic in:

$$v - \frac{\mu}{1 - \mu} \left( \frac{1 - P_B(v)}{p_B(v)} \right)$$

and

$$c + \frac{\mu}{1 - \mu} \left( \frac{P_S(c)}{p_S(c)} \right)$$

Then MHRP implies that both monotonicity conditions are satisfied and hence local and global (IC) holds.

Clearly if $\mu > 0$ there will be inefficiencies in trade.
Theorem (Myerson and Satterthwaite 1983)

If $\bar{c} > \bar{v}$ and $\bar{v} > c$ then necessarily $\mu > 0$.

**Proof:** Immediate by substituting the efficient probabilities $\phi_i$ into the constraint of the parties problem and showing that is violated.

Clearly with bilateral asymmetric information the Coase Theorem fails in a very relevant sense: *efficiency is no longer guaranteed by the use of contracts.*