Trade and the Skill Premium Puzzle with Capital Market Imperfections

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Abstract

An interesting puzzle is that trade liberalization in the 1980s and 1990s has been associated with a sharp increase in the skill premium in both developed and developing countries. This is in contrast with neoclassical theory, according to which trade should increase the relative return of the relatively abundant factor. We develop a simple model of trade with capital market imperfections, and show that trade can increase the skill premium in both the North and the South, and both in the short run as well as in the long run. We show that trade with a skill-intensive economy has two effects: it reduces the skilled wage, and thus discourages non talented agents out of the skilled labor force; and it reduces the cost of subsistence, thus allowing the talented offspring of unskilled workers to go to school. This compositional effect has a positive effect on the observed skill premium, possibly strong enough to counterweight the decrease in the skilled wage.

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1 Introduction

One of the most important result in Heckscher-Ohlin models of international trade, the Stolper-Samuelson theorem, predicts that when a country opens up to international trade, the rewards to the factor of production in which it is relatively abundant should increase, relative to the reward of other factors. This prediction has been confirmed in a number of unskilled labor-abundant

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“early globalizers” (such as Italy, Singapore, South Korea and Taiwan) where trade has increased the unskilled wage relative to the skilled wage (thus decreasing the skill premium). However in the case of unskilled labor-abundant countries that have globalized in the 1980s and 1990s (such as most of Latin America, India and Hong Kong), trade seems to have induced an increase in the skill premium, rather than a reduction.¹

This fact, sometimes called the “skill premium puzzle” has attracted a fair bit of attention. In particular, the trade literature has sought to reconcile the Latin American experience with Heckscher-Ohlin (HO from now on) theory by arguing that trade liberalization affected disproportionately unskilled labor-intensive industries (Revenga, 1997), or that countries such as China, Indonesia and Pakistan made the world outside Latin America actually unskilled labor-intensive (Davis, 1996; Wood, 1999). In these contexts, the Stolper-Samuelson theorem would correctly predict an increase in the skill premium in Latin America. The problem with these interpretations is that they rely on inter-sectoral movements toward skill-intensive sectors to explain the increase in the skill premium, and they predict that skill intensity should have decreased across sectors in Latin America. Both of these predictions have not been confirmed in the data.² In response to these shortcomings, much of the literature has turned to non-trade explanations for the increase in the skill premium in Latin America, most notably skill biased technical change.³

In this paper, we show how to reconcile a standard HO model of trade liberalization between an unskilled labor-abundant South and a skill labor-abundant North with an increase of the skill premium in both North and South. We do so by drawing on the literature on trade liberalization and human capital accumulation in the presence of credit constraints (which we review in detail at the end of this section). This has convincingly argued that trade may increase human capital accumulation, by weakening the credit constraints of the poor and thus improving their access to

¹This has been documented my micro studies of at least 7 countries: Chile, Mexico, Colombia, Argentina, Brazil, India and Hong Kong. See the survey by Goldberg and Pavcnik (2007) for more details.

²For example, Wacziarg and Wallack (2004) find little evidence of labor re-allocation across sectors following to trade liberalization in a sample of 20 countries. For Mexico, Verhoogen (2008) finds evidence of labor re-location towards unskilled labor-intensive industries. See Goldberg and Pavcnik (2007, p. 59) for a list of empirical papers finding that skill intensity has increased across most industries in Latin America.

³There are two important exceptions to this. Feenstra and Hanson (1996) study the impact of trade liberalization when this is associated with significant outsourcing flows from North to South. They find that this specific type of liberalization may increase the skill premium in both countries. Verhoogen (2008) builds a “new” trade model where firms differ in productivity and quality of production, and shows that quality upgrading following to trade liberalization may result in a higher relative white-collar wage and higher sectoral wage inequality.
the education system. We show that when this is the case, trade may improve the allocation of
talent to the skilled labor force, both in the short and in the long run. This compositional effect
generates an upward pressure on the observed skilled wage, which can well be strong enough
to overturn the Stolper-Samuelson prediction of a lower skill premium in South following to
trade liberalization. While reconciling the Stolper-Samuelson theorem with the Latin American
experience, our model preserves the other main features of standard HO theory - namely, that
all industries in South become more skill intensive after trade liberalization, and that there is
inter-sectoral movements towards unskilled labor-intensive industries. Importantly, however, we
argue that the mechanism would survive if we allowed for labor market frictions (such as a high
cost of firing) to slow down the inter-sectoral reallocation of labor.

Our mechanism works as follows. Because of capital market imperfections, young agents
cannot borrow to pay for their subsistence while attending school. Thus, only those whose
parents have a high wage can possibly go to school. In an economy with little human capital,
the cost of subsistence is very high relative to the wage of unskilled workers. This creates one
equilibrium in which there are few skilled workers, the skilled wage is high, and all and only the
offspring of skilled workers go to school. With heterogeneous talent, this equilibrium is “bad” in
efficiency terms, in that many talented offspring of unskilled workers are prevented from going
to school while many offspring of skilled workers go to school despite being non talented. This
compares with a “good” equilibrium in which there are many skilled workers, the skilled wage is
low, and all and only the talented workers go to school independently on their social origins.

We consider an economy that is skill-scarce because it is stuck at the bad equilibrium, and
study its reaction to the liberalization of trade with an economy that, being at the good equilib-
rrium, is skill-abundant. By putting a downward pressure on the skilled wage, trade may induce
many non-talented skilled workers to drop out of the skilled labor force. At the same time, it
reduces the cost of subsistence for unskilled workers, thus making it easier for their offspring to
go to school. Because many of these previously-excluded agents are high talent, they may still
find it optimal to join the skilled labor force despite the trade-induced drop in the skilled wage.

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4Rigid labor market have been indicated as one of the main reasons why labor reallocation to comparative
advantage industries has been very slow in many countries, see for example Kambourov (2009).
These two effects may move the economy from its initial equilibrium to the good equilibrium, thus increasing the average quality of the skilled labor force. This creates an upward force on the average observed skill premium, that can more than compensate the negative effect of trade on the skilled wage. Thus, while the skill premium always increases in the skill-abundant economy, it may increase in the skill-scarce country as well, both in the short run and in the long run. Although trade disappears in this model in the long run, we argue that our results are robust to allowing for more “structural” sources of comparative advantage, such as cross-country differences in the quality of the education system. Our results suggests that the Stolper-Samuelson theorem needs to be modified in the context of imperfect credit markets, to account for the possibility of compositional changes in the skilled labor force.

The literature on trade with capital market imperfections is now quite large. An important part of it has focused on how comparative advantage and the pattern of trade are determined by cross-country heterogeneity in the efficiency of capital markets (see for example Kletzer and Bardhan, 1987; Wynne, 2005; and Manova, 2008). Although our result is compatible with the idea that comparative advantage in the export of skill-intensive products may be driven by different capital market development, the focus of our paper is different. More connected to our paper is the literature on trade liberalization and occupational choice, in the presence of credit market frictions. Starting with the seminal work by Cartiglia (1997), this literature has shown that, by lessening credit constraints, trade may induce an increase in human capital accumulation in both the skill intensive North and in the non skill-intensive South. This result is in contrast with the classical literature on this topic, and can help explain a rapid increase in capital accumulation in the early East Asian globalizers and, to a lesser extent, in Latin America. Among the various papers in this literature, the closest to our own is Ranjan (2003). Although our mechanism has much in common with the mechanism proposed in this paper, one key difference stands out: while Ranjan (2003) finds that trade has a standard Stolper-Samuelson effect on relative wages, we show that, if talent is positively correlated with productivity, trade

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6Eicher (1999), Ranjan (2001a), Das (2006), and Chesnokova and Krishna (2009) are some of the other important contributions in this literature.
may result in an increase in the skill premium in both North and South.\footnote{The paper is also related to the literature on trade, credit constraints and child labor, see in particular Ranjan (2001b).}

The paper is organized as follows. Section 2 presents our basic economic model. We make a series of modeling choices that keep the model extremely simple, even after building on it in subsequent sections. In Section 3 we endogeneize the schooling decisions of agents, and their participation in the skilled labor force. We then summarize the timing of our model 4 and solve for the autarchy equilibrium 5. Our main results are contained in Section 6, where we open up the economy to trade with a skill-intensive county and study the consequences on the skill premium. In Section 7 we conclude.

## 2 The Model

Consider the case of a “Home” country ($H$) where two tradable intermediate goods, $x$ and $y$, are produced using skilled and unskilled labor. Production of $x$ is relatively unskilled labor-intensive, while production of $y$ is relatively skilled labor-intensive. We capture this by the following functional forms:

$$
x = S^\alpha U^{1-\alpha}
$$

$$
y = S^{1-\alpha} U^\alpha
$$

where $\alpha < \frac{1}{2}$. The two intermediate goods are assembled into a non tradable final good $z$, also using a Cobb-Douglas technology:

$$
z = Ax^\frac{1}{2} y^{\frac{1}{2}}
$$

where $A$ is total factor productivity in the $z$ sector. There is also another non tradable final good $f$ in this economy, produced with constant return to scale and using unskilled labor only ($f = U$). To fix ideas, we may think of $f$ as products and services typical of the “traditional” economy (e.g. subsistence agriculture; constructions; retail trade; simple business services) and of $z$ as services of the “modern” economy (e.g. utilities; telecommunications; financial services;
health care). In this context, \( x \) could represent materials and basic manufactures, while \( y \) could represent more complex machines (such as computers).\(^8\)

The economy is populated by overlapping generations of agents, each containing a continuum of agents with mass \( n \). Agents live for two periods. We will call “generation \( t \)” the generation born in period \( t \). In her second period of life, each member of generation \( t \) gives birth to exactly one agent of generation \( t+1 \). This ensures that, in any given period, there is a mass \( 2n \) of agents.

Agents have identical preferences. In each period of life, agent \( i \) from generation \( t \) obtains utility from consuming the two final goods:\(^9\)

\[
 u_{i,s}^t = f_{i,s}^t + \phi \log z_{i,s}^t
\]

where \( s = t, t + 1 \) denotes whether the agent is in the first or second period of life, and \( \phi \) is a parameter. We assume that \( \phi \) is small enough, so that all agents can afford to spend \( \phi \) on good \( z \). This will imply that good \( f \) will always be produced, both in autarchy and in the free trade equilibrium.\(^10\)

In their second period of life, agents also care about the bequest they leave to their offspring. Their total inter-temporal utility is:

\[
 U_i^t = u_{i,t}^t + 2(b_i^t)^{\frac{1}{2}}(u_{i,t+1}^t)^{\frac{1}{2}}
\]

where \( b_i^t \) denotes the bequest (in terms of utility) left by the agent to her offspring. We assume that there is a “survival constraint”, in the sense that if an agent’s per-period consumption utility falls below a threshold \( \pi \) in any of the two periods, this agent dies of starvation. We capture

\(^8\)Notice that we don’t strictly need \( z \) to be non tradable; however this simplifies the description of the trade equilibrium.

\(^9\)Our results are robust to using other functional forms from the widely-used class \( u = f + v(z) \), where \( v(\cdot) \) is differentiable, increasing, and strictly concave. The choice \( v(\cdot) = \phi \log(\cdot) \) simplifies the calculations, however, by ensuring that expenditure on good \( z \) is fixed.

\(^10\)More details on this are provided in sections 3.2 and Appendix B.
this by constraining the agent’s optimization problem as follows:

\[
\max \ u_{i,t} + 2(b_i^t) \\
\text{s.t.} \ u_{i,t}, u_{i,t+1} > u.
\]

Notice that, in both periods, the marginal utility of income is constant and equal to 1 when the survival constraint does not bind.\(^\text{11}\) In this context, the role of the survival constraint is to introduce a decreasing marginal utility of income at a single point of the income distribution. As will be clear in section 3.2, this will allow us to model the impact of credit constraints in a very tractable way.

Finally, we assume that goods are perishable, and there are no financial markets. Thus, income cannot be transferred across periods.

### 2.1 Equilibrium with exogenous labor supply

Suppose that, in some period \(t\), the supply of skilled and unskilled labor is fixed, and that skilled workers are a share \(s\) of parents.\(^\text{12}\) Thus:

\[
s \equiv \frac{S}{n}.
\]

A competitive equilibrium of this economy consists of an unskilled salary \((w)\), a skilled salary \((v)\), two prices of the intermediate goods \((p_x \text{ and } p_y)\), and two prices of the final goods \((p_z \text{ and } p_f)\) such that these markets clear, given that all agents behave optimally. We normalize the unskilled wage to 1. Since good \(f\) is always produced in equilibrium, the zero-profit condition in the \(f\) industry implies that \(p_f = 1\). To simplify the notation, we rename the price of \(z\) to simply \(p\). Summing up, we have: \(w \equiv 1\) which implies \(p_f = 1\), and \(p_z \equiv p\).

\(^{11}\)A low \(\phi\) - the optimal expense on good \(z\) - ensure that the marginal utility of income is 1 in the first period. The equal power on \(b_i^t\) and \(u_{i,t+1}\) and the discount factor on second-period utility ensure that the marginal utility of income is constant and equal to 1 in the second period as well (see Appendix E for more details). Notice that to allow for more general functional forms would have greatly complicated the math without undermining the logic of the argument.

\(^{12}\)Only parents can work as skilled in this model, see section 3.
We can then denote a competitive equilibrium of this economy as a vector of prices \([v, p_x, p_y, p_x]\) such that all markets clear, given that all agents behave optimally. In what follows, we will denote by \([v_t, (p_x)_t, (p_y)_t, p_t]\) the competitive equilibrium in period \(t\). It turns out that the competitive equilibrium is straightforward to derive. Because \(p_z = 1\) and \(\phi\) is low enough, all agents allocate expenditure \(\phi\) to good \(z\). This implies that total revenues in the \(z\) sector will be \(2n\phi\), and total revenues in the \(x\) and \(y\) sector \(n\phi\) each. As skilled labor receives a share \(\alpha\) of revenues in the \(x\) sector and a share \(1 - \alpha\) of revenues in the \(y\) sector, the total reward to skilled labor is also \(n\phi\).

But equilibrium in the skilled labor market then requires that \(\frac{n\phi}{v_t} = s_t n\), or:

\[
v_t = \frac{\phi}{s_t}.
\]  

(1)

It is then easy to derive the equilibrium prices in all industries using the relevant zero-profit condition. We report here only the final expressions for \((p_x)_t\), \((p_y)_t\) and \(p_t\), presenting the derivations in Appendix A. To our purposes, these are best presented as functions of \(v\):

\[
(p_x)_t = \frac{v^\alpha}{\tilde{\alpha}}
\]

(2)

\[
(p_y)_t = \frac{v^{1-\alpha}}{\tilde{\alpha}}
\]

(3)

\[
p_t = \frac{2}{A} \frac{v^{\frac{1}{2}}}{\tilde{\alpha}}
\]

(4)

where \(\tilde{\alpha} \equiv \alpha^\alpha (1 - \alpha)^{1-\alpha}\). Equation (1) illustrates the simplicity and tractability of the economic model that we have chosen to use. In any period \(t\), the equilibrium skilled wage is only a function of the contemporary stock of skilled labor. Furthermore, it is a linear function of the inverse of the share of skilled labor in the population of parents. In the next sections, we will show how this simple economic framework may provide a very convenient building block of a dynamic trade model with endogenous schooling decisions and aggregate human capital accumulation.
3 Schooling

All agents are endowed with one unit of time in each period, and are born unskilled. In their first period of life, they can choose whether to use their time to work as unskilled, or to go to school. If they work, they can only work as unskilled in their second period. If they go to school, on the contrary, they become “educated” and acquire the option of working as skilled in the second period. Educated parents retain the option to work as unskilled in the second period. Thus, there may be three distinct groups of parents in any period $t$: non educated parents, educated parents that work as skilled, and educated parents that work as unskilled.

Agents differ in their level of talent, $\Theta$. For simplicity, talent can take only two values, $\Theta = 1$ and $\Theta = \theta > 1$. We will say that an agent is “non talented” when she has talent 1, while it is “talented” when she has talent $\theta$. We assume that talent is not inherited, but that is distributed randomly across the population with a probability of observing a talented agent equal to $\delta$. Thus, each generation has a mass $\delta n$ of talented agents and a mass $(1 - \delta)n$ of non talented agents, and the average talent in the population is:

$$\hat{\theta} \equiv \delta \theta + 1 - \delta.$$

While all agents are equally productive when they work as unskilled, there is a one-to-one mapping between an agent’s level of talent and its productivity as a skilled worker. In other words, a non talented worker can only contribute 1 efficiency unit of skilled labor, while a talented worker can contribute $\theta$ units. This maps into a higher salary awarded by a competitive skilled labor market to talented agents (who receive $\theta v$) than to non talented agents (who receive $v$). While in what follows we continue to refer to $v$ as to the skilled wage, it should be born in mind that what this symbol really indicates is the skilled wage of non-talented agents, or the skilled wage per efficiency unit.
3.1 Participation constraints

We assume that school is free of charge, as would be the case of a country where education is fully subsidized by the government.\textsuperscript{13} Despite being free, however, school comes at the cost of a lost unskilled wage in the agent’s first period of life. When choosing whether to go to school or not, agents compare this cost to the difference in salary that they can expect to receive working as skilled workers in their second period of life. Since agents with different levels of talent can expect to receive different skilled wages, they will also have different participation constraints.

Given that the marginal utility of income is constant and equal in the two periods, we can write generation $t$’s participation constraints as follows:

\begin{align*}
v_{t+1}^e \theta &> 2 \quad (5) \\
v_{t+1}^e &> 2 \quad (6)
\end{align*}

where $v_{t+1}^e$ is the expected equilibrium skilled wage at period $t + 1$. Notice that it always \textit{ex-post} optimal for an educated agent to work as skilled in period $t + 1$, provided that her expectations in period $t$ were correct. This is clear from the fact that the \textit{ex-post} participation constraints of the two types are $v_{t+1} \theta > 1$ and $v_{t+1}^e > 1$, which are always satisfied if (5) and (6) hold and $v_{t+1} = v_{t+1}^e$. Intuitively, the cost of education is sunk in period $t + 1$, while the agent’s productivity is unchanged. Rational initial decisions must then be optimal, unless external conditions have changed.

3.2 Credit constraints

Under the assumption that credit markets do not exist, we now move to consider how the schooling decisions of agents are driven not only by income maximization (as described in equations 5 and 6), but also by parental wealth. While school is free of charge, agents can only afford to go to school if their bequest is high enough to cover their subsistence expense while in school. If this is not the case, agents must join the unskilled labor force and pay for their own subsistence expense, or else die out of starvation. Thus, the distribution of bequests determines which young

\textsuperscript{13}This is only a simplifying assumption; see section 3.2 for further discussion of this point.
agents can afford to go to school, and which one cannot. We will refer to the latter group as the credit constrained young agents.\footnote{The literature introduces educational credit constraints in two ways. On one hand, some papers assume a constant marginal utility of income and a schooling fee that depends on the skilled wage (e.g. Cartiglia, 1997). On the other, a few other papers assume a decreasing marginal utility of income (e.g. Ranjan, 2003). Our paper belongs to the latter group, with the innovation that the marginal utility of income is assumed to be infinite at the subsistence level of utility, finite and constant above that level (more on this below). A constant marginal utility of income for all surviving agents (for all agents) offers obvious modeling advantages. We stress, however, that our results do not hinge on the specific way in which we introduce educational credit constraints.} This way of modeling educational credit constraints is in line with abundant empirical evidence suggesting that a major force keeping the poor out of school is the high opportunity cost from lost labor opportunities, given their proximity to the subsistence level of income.\footnote{For example, Cartwright (1999) argue that Colombian boys become increasingly likely to drop out of school as they grow older, as their opportunity cost from lost labor opportunities increases. He also find that a 1% increase in household expenditure map into a .11% (.19%) lower probability of work for a rural (urban) child. Cartwright and Patrinos (1999) find similar results for urban Bolivia. In both cases, there is also evidence that the existence of household assets decrease the probability of work, witnessing to the importance of credit constraints for schooling decisions.}

We now proceed to study the shape of the distribution of bequests, and its impacts on the schooling decisions of agents. In any period $t$, parents can fall into three income brackets, depending on whether they work as unskilled (income 1), skilled being non talented (income $v$), or skilled being talented (income $v\theta$). Because educated parents are free to choose whether to work in the skilled or unskilled labor force, income in the two latter brackets cannot fall below 1 in equilibrium. Thus, the minimum bequest that is transferred in period $t$ is $\frac{1}{2}$, which is what the offspring of unskilled workers receive. For any $\phi < \frac{1}{2}$, it is then indeed the case that all agents can afford to spend $\phi$ on good $f$.\footnote{This ensures that good $f$ is produced in autarchy, since total income must be higher than total revenues in the $z$ sector.}

These agents are the first to become credit constrained when the cost of achieving the subsistence level of utility increases. Specifically, for these agents not to be credit constrained in period $t$ the following condition must hold:

$$e(u_t, p_t) \leq \frac{1}{2}$$  \hspace{1cm} (7)

where $e(u_t, p_t)$ is the consumer’s expenditure function valued at the subsistence level of utility and at current prices (notice that we can omit the price of $z$ from the expression, as this is
always unity). When condition (7) does not hold, the offspring of unskilled workers must work as unskilled in the first period of her life, or else die from starvation.

The consumer’s expenditure function is straightforward to derive. Importantly, we can focus on that portion of the expenditure function that takes value around the threshold $\frac{1}{2}$: since $\phi < \frac{1}{2}$ by assumption, and $\phi$ is the optimal expenditure on good $z$, the relevant portion of the expenditure function is the one at which consumption of good $z$ is at its optimum ($\frac{\phi}{p_t}$). Since the utility from consuming this amount of $z$ is $\log\frac{\phi^2}{p_t}$, the relevant portion of the expenditure function is simply:

$$e(\bar{u}, p_t) = \phi + \bar{u} - \log\frac{\phi^2}{p_t}.$$  

That is, the cost of achieving utility $\bar{u}$ is given by the cost of the optimal consumption of $z$ ($\phi$), augmented by the cost of increasing utility from $\log\frac{\phi^2}{p_t}$ to $\bar{u}$ through consumption of $f$. We can now re-write the expenditure function in terms of $v_t$ by plugging in (4):

$$e[\bar{u}, p_t(v_t)] = F(\bar{u}, A) + \frac{1}{2} \log v_t$$  

where $F(\bar{u}, A) \equiv \phi + \bar{u} - \log\frac{\phi^2 A}{2}$. Not surprisingly, the cost of subsistence is always strictly increasing in $v$ - a measure of the scarcity of skilled labor in the economy, and thus the cost of producing $z$. Moreover, the cost of subsistence takes value in $(-\infty, \infty)$ for $v_t$ in $(0, \infty)$. Plugging (8) into (7), we obtain:

$$v_t \leq \exp\{1 - 2F(\bar{u}, A)\} \equiv v_{cc}. \quad (9)$$  

Inequality (9) defines a threshold for the current skilled wage that is critical to determine whether the offspring of unskilled agents are credit constrained or not. In particular, if $v_t \leq v_{cc}$ the offspring of unskilled agents are not credit constrained, while they are if $v_t > v_{cc}$. Intuitively, a increase in $v_t$ (that is, a scarcer skilled labor force) is associated with a high cost of production in the $z$ sector, a higher $p_t$, and a higher subsistence cost (recall that $e(\bar{u}, p)$ is increasing in $p$). For the offspring of unskilled workers, this increase in subsistence cost is not matched by an increase in bequest, which is only linked to the unskilled wage. Thus, a high enough $v_t$ implies that the offspring of unskilled agents must work as unskilled in the first period of their life to
meet their subsistence needs. Not surprisingly, the threshold \( v_{cc} \) is always increasing in \( A \), the total factor productivity in the \( z \) sector. Intuitively, a more productive economy makes the cost of subsistence smaller for any existing stock of skilled labor, thus reducing the probability that the offspring of unskilled workers are credit constrained. In fact, we can always set \( A \) large enough so that \( v_t \leq v_{cc} \), and credit constraints are not an issue for this group of agents.

We have thus determined that a high skilled wage may be bad news for the offspring of unskilled workers, in that it may force them to remain in the unskilled labor force as well. But how about the credit constraints of the offspring of skilled workers? As we have already noticed, these agents must be at least as wealthy as the offspring of unskilled workers in equilibrium. Thus, if the latter are not credit constrained in equilibrium (it is \( v_t \leq v_{cc} \)), nor can the offspring of skilled workers be. If the offspring of unskilled workers are credit constrained (it is \( v_t > v_{cc} \)), we show in Appendix C that a sufficient condition for the offspring of skilled workers not to be credit constrained is that \( v_{cc} > \frac{1}{\theta} \). Intuitively, a high enough \( v_t \) must be good news for the offspring of skilled workers: this is because the bequest that these agents receive is linear in \( v_t \), while the cost of subsistence is concave. It must then be the case that the former overtake the latter for \( v_t \) higher than a certain threshold. The condition \( v_{cc} > \frac{1}{\theta} \) makes sure that this threshold be lower than \( v_{cc} \), implying that the offspring of skilled workers will never be credit constrained for \( v_t > v_{cc} \). Since we will anyway want to restrict our attention to cases in which \( v_{cc} \) is high (and higher than \( \frac{1}{\theta} \), see Section 5), we can safely conclude that the offspring of skilled workers will never be credit constrained in equilibrium.

In the rest of the paper, we will study how a trade-induced decrease in \( v_t \) may lead to a relaxation in the credit constraints of the offspring of unskilled workers. In this context, the results derived above may be seen as a corollary of the classic 2x2x2 Stolper-Samuelson theorem. This states that a trade-induced rise in the relative price of a good raises the real return of the factor used intensively in the production of that good, and lowers the real return of the other factor, in terms of both goods in the economy. In our model, trade will increase the price of good \( x \), thus increasing the real return to unskilled labor in terms of both \( x \) and \( y \) (and thus \( z \)). This will then result into a lower cost of subsistence for the offspring of unskilled workers, relaxing their credit constraints.
Notice that the existence of a subsistence level of utility must put a constraint on the minimum endowment of productive factors in an economy. In this model with human capital accumulation, this requires to rule out that there are too few educated agents at any point in time. To see this, notice that if \( e(\pi, p) > 1 \) the unskilled workers are never able to reach the subsistence level of utility - not even if they leave no bequest to their offspring. Thus, all unskilled workers would pass away in this case, and the economy would collapse. To avoid this, we must impose the following:

\[
v < \arg \{ e(\pi, p) = 1 \} = v_{cc} \exp \{ 1 \} \equiv \overline{v}.
\]

Notice that it is always \( v_{cc} < \overline{v} \). Furthermore, being a linear function of \( v_{cc} \), \( \overline{v} \) depends on \( A \) just as \( v_{cc} \) does. In what follow, we will only consider the case in which the economy never collapses, or in which \( v_t < \overline{v} \) at all periods. This will only require to impose some restrictions on initial conditions, as \( v_t < \overline{v} \) will then emerge naturally from the agents' schooling decisions under the model's assumptions.

### 4 Timing and equilibrium concepts

The following events take place in each period \( t \):

- **t.1** Generation \( t \) is born from parents of generation \( t - 1 \);
- **t.2** Educated parents decide whether to work as skilled or unskilled. At the same time, generation \( t \) decide whether to join the unskilled labor force or go to school.
- **t.3** Production takes place; all markets clear.
- **t.4** Bequests are transferred, consumption takes place.
- **t.5** Parents pass away.
Just as in section 2, a competitive equilibrium at time $t$ is defined as a vector of prices $[v_t, (p_t)_x, (p_t)_y, p_t]$ such that all markets clear, given that all agents behave optimally and a positive amount of good $f$ is produced. It is important to notice that, with endogenous schooling and working decision, this definition includes two additional requirements of an optimal behavior of agents. First, the schooling decisions of young agents in $t - 1$ (and thus the supply of educated parents in $t$) must be optimal given the prices that realize in $t$. We consider this to be a feature of period $t$’s equilibrium (rather than period $t - 1$’s) because schooling decisions in $t - 1$ only affect prices in $t$.\footnote{This important feature of the model relies on the fact that good $f$ is always produced, and the relative wage does not depend on the amount of unskilled workers in the economy.} Second, the working decisions of educated parents at time $t$ (and thus the supply of skilled labor in $t$) must also be optimal given the prices in $t$. To distinguish it from a different equilibrium concept that we shall introduce shortly, we will refer to this equilibrium concept as to the \textit{short-run equilibrium} in period $t$.

While the short-run equilibrium in period $t$ is never affected by the short-run equilibrium in period $t + 1$, it may well be affected by the short-run equilibrium in period $t - 1$. To illustrate this, it is useful to consider a specific parametric specification where this “path dependency” does not take place, and then contrast it with a more general specification of the model. Suppose that $\pi \to -\infty$, so that $e(\pi, p_{t-1}) \to 0$. In this case, no agents is ever credit constrained, and schooling decisions at time $t - 1$ are only affected by expected prices at time $t$: in other words, the short-run equilibrium at time $t$ is not affected by any conditions prevailing at any previous time. Suppose now that $e(\pi, p_{t-1}) > 0$. In this case, the offspring of unskilled agents will be credit constrained for $v > v_{cc}$. Thus schooling decisions at time $t - 1$ (and the short run equilibrium at time $t$) will be affected by the level of prices at time $t - 1$ (or by the short-run equilibrium at time $t - 1$).

Whenever the model displays path dependency, it is interesting to define a concept of long-run equilibrium to clarify which different short-term equilibria may become persistent over time. We will then say that a “long-run equilibrium” is formed at time $t$ whenever the short-term equilibrium realizing at time $t$ is identical to the short-term equilibria realizing at times $t + s$, were $s = 1, \ldots, \infty$. That is, in a long run equilibrium all generations make the same schooling and occupational choices, and prices remain constant over time.
Having defined our equilibrium concept, we now move to describing the autarchy equilibrium of country $H$.

5 Autarchy equilibrium

We begin by making two assumptions on the distribution of talent and on the level of credit constraints in the economy:

Assumption 1 $\delta \in \left(\frac{\phi}{2\theta}, \frac{\phi}{2}\right)$

Assumption 2 $v_{cc} > 2$.

Assumption 1 requires that the number of talented agents in the population be “intermediate”, where the lower bound of the allowed range is decreasing in the ratio of the high level of talent to the low level of talent ($\theta$). As will be clear below, this assumption is merely useful to simplify the description of the equilibrium, given the discreteness of the distribution of talent. Assumption 2 requires that productivity in the economy be high enough, so that the threshold above which the offspring of unskilled agents are credit constrained be not too small. This assumption is necessary to make sure that there exists an equilibrium in which the talented agents are not credit constraints. This is the interesting case for us, as it is the one in which trade may lead to a better allocation of talent in $H$, thus affecting the skill premium in a non-standard way. Notice also that assumptions 1 and 2 together imply that $v_{cc} > \frac{1}{\theta}$, and we are then safe to assume that the offspring of skilled workers are never credit constrained in equilibrium (see section 3.2).

We are now ready to describe the long-run autarchy equilibrium of country $H$. For conciseness, we only report the equilibrium schooling decisions of agents and the equilibrium skilled wage:

Proposition 1 From any (feasible) initial stock of educated parents in $t = 0$, the economy converges to a unique long-run equilibrium no later than $t = 3$. Depending on the initial stock, this can be:
• A “good equilibrium” in which \( v = \frac{\phi}{\delta \theta} \), and all and only the talented agents go to school.

• One from a continuum of “bad equilibria” in which \( v \in (v_{cc}, \bar{v}] > \frac{\phi}{\delta \theta} \), and all and only the offspring of skilled workers go to school.

**Proof.** Denote by \( a_t \) and \( b_t \) the number of talented and non talented agents that go to school in generation \( t \) (as a share of the total number of agents in generation \( t \)). Furthermore, define \( e_t \equiv a_t + b_t \). We then focus on period \( t = 1 \) and show that, for any feasible initial conditions \( a_0, b_0 \), the economy converges to one of the two types of long-run equilibria no later than \( t = 3 \). Feasibility of initial conditions requires that \( \theta a_0 + b_0 > \frac{\phi}{\theta} \), so that \( v_1 \leq \bar{v} \) (since \( \bar{v} > 1 \) by assumption, all educated parents will join the skilled labor force before this high level of the skilled wage is reached). We can then distinguish two cases: \( v_1 \leq v_{cc} \) and \( v_1 \in (v_{cc}, \bar{v}] \). If \( v_1 \leq v_{cc} \), no agent is credit constrained, and schooling decisions in period 1 maximise lifetime income given an expected skilled salary in period 2 (\( v_2^e \)). With rational expectations and certainty, it is always optimal for an agent who has gone to school in \( t = 1 \) to work as skilled in \( t = 2 \). Thus, the supply of skilled labor in \( t = 2 \) depends only on \( v_2^e \):

\[
s^s_2(v_2^e) = \begin{cases} 
0 & \text{if } v_2^e < \frac{2}{\theta} \\
[0, \delta \theta] & \text{if } v_2^e = \frac{2}{\theta} \\
\delta \theta & \text{if } \frac{2}{\theta} < v_2^e < 2 \\
[\delta \theta, \hat{\theta}] & \text{if } v_2^e = 2 \\
\hat{\theta} & \text{if } 2 < v_2^e \leq \bar{v}
\end{cases}
\]  

(10)

Which is continuous and monotonically increasing in \( v_2^e \). In every period, the demand for skilled labor (expressed as a share of the population of parents) is continuous and monotonically decreasing in the current skilled salary, \( s^d(v) = \frac{2}{v} \) (for \( v \leq \bar{v} \)). Furthermore, it is \( s^d(0) > s^s_2(0) \) and \( s^d(\bar{v}) < s^s_2(\bar{v}) \). With rational expectations, we can substitute \( v_2 \) (the equilibrium skilled salary at \( t = 2 \)) for \( v_2^e \) in \( s^s_2(.) \), and solve for the unique \( v_2 \in (0, \bar{v}) \) and \( s_2 \) (the equilibrium stock of skilled labor at \( t = 2 \)) by equating demand and supply, \( s^s_2(v_2) = s^d(v_2) \). By Assumption 1, \( s^d(\frac{2}{\theta}) > \delta \theta \) and \( s^d(2) < \delta \theta \); this makes sure that \( s_2 = \delta \theta \) and \( v_2 = \frac{\phi}{\delta \theta} \). By Assumption 2, \( \frac{\phi}{\delta \theta} < v_{cc} \), making
sure that $s_3^e(\cdot) = s_2^e(\cdot)$. Since $s^d(\cdot)$ is the same at all times, the short-run equilibrium at $t = 3$ is identical to that at $t = 2$. Since this can be said for all $t = 3, 4, \ldots, \infty$, we have shown that a long-run equilibrium is achieved in $t = 2$, whereby $s_t = \delta \theta$ and $v_t = \frac{\phi}{\varepsilon_0}$ for all $t = 2, \ldots, \infty$.

If $v_1 \in (v_{cc}, v]$, all offspring of unskilled workers are credit constrained. This implies that the supply of skilled labor at $t = 2$ is:

$$s_2^s(v_2^e) = \begin{cases} 0 & \text{if } v_2^e < \frac{2}{\delta} \\ [0, e_0 \delta \theta] & \text{if } v_2^e = \frac{2}{\delta} \\ e_0 \delta \theta & \text{if } \frac{2}{\delta} < v_2^e < 2 \\ \left[e_0 \delta \theta, e_0 \hat{\theta}\right] & \text{if } v_2^e = 2 \\ e_0 \hat{\theta} & \text{if } 2 < v_2^e \leq v \\ \end{cases} \quad (11)$$

Which is again continuous and monotonically increasing in $v_2^e$, but lies always below (10) (except for the case in which $e_0 = 1$, where the two schedules are identical). Since $s^d(\cdot)$ is the same as before, it will then be $v_2 \geq \frac{\phi}{\varepsilon_0}$. Feasibility of initial conditions requires that $\hat{\theta} e_0 \geq \frac{\phi}{\varepsilon_0}$, so that $v_2 \leq v$ (notice that for at least some values of $a_0$ and $b_0$ this is a stricter requirement than $\theta a_0 + b_0 \geq \frac{\phi}{\varepsilon_0}$). We can then distinguish two cases, $v_2 \in \left(\frac{\phi}{\varepsilon_0}, v_{cc}\right]$ and $v_2 \in (v_{cc}, v]$. If $v_2 \in \left(\frac{\phi}{\varepsilon_0}, v_{cc}\right]$, the economy converges to the good equilibrium in period 3 (by the same logic used above). If $v_2 \in (v_{cc}, v]$ it must be that $s_3^e = s_2^e$, since $e_1 = e_0$ (recall that $v_{cc} > 2$ by assumption; then, all offspring of skilled workers must have gone to school in $t = 1$). Since $s^d$ is the same at all times, the short run equilibrium at $t = 3$ is identical to that at $t = 2$. Since this can be said for all $t = 3, 4, \ldots, \infty$, a long-run equilibrium is achieved in $t = 2$ in which $s_t = \hat{\theta} e_0$ and $v_t = \frac{\phi}{\varepsilon_0} \in (v_{cc}, v]$ for all $t = 2, \ldots, \infty$. ■

Proposition 1 argues that from any initial stock of educated parents (that cannot be too small, or the economy would immediately die out of starvation) the economy converges to a long-run equilibrium, and that there is a one-to-one mapping between the initial stock and the type of long-run equilibrium that is converged to. There are two, quite distinct types of long-run equilibria. One one hand, there is unique “good” long-run equilibrium in which all and only the talented agents go to school, and the skilled wage is relatively low. This is an equilibrium in
which credit constraints are not binding, and talent is efficiently allocated to the skilled labor force. On the other hand, there is a whole class of “bad” long-run equilibria in which all and only the offspring of skilled workers go to school, and the skilled wage is relatively high. In these equilibria credit constraints are binding for a fraction of the population, and the allocation of talent to the skilled labor force is inefficient. Intuitively, the offspring of unskilled workers cannot afford to go to school if \( v > v_{cc} \), while the offspring of skilled workers can. This implies that it will be \textit{only} the offspring as skilled workers to go to school for \( v > v_{cc} \). Furthermore, since \( v_{cc} > 2 \), it will be \textit{all} of them to go to school, creating an equilibrium in which the skilled labor force self-perpetuates itself over time.\(^{18}\) The only upper constraint on this class of equilibria is that \( v \) cannot be greater than \( \bar{\pi} \), or the economy would collapse.\(^{19}\)

Figure 1 illustrates convergence to the long-run equilibrium from any share of educated parents \( e_0 \). Because the working decisions of young agents do not affect the short-run equilibrium in period 1, we can treat \( v_1 \) as exogenous to the schooling decisions of generation 1. The left-hand panel represents the case in which \( v_1 \geq v_{cc} \). In this case, no one is credit constrained in period 1. Because of rational expectations and certainty, this implies that the skilled labor supply in \( t = 2 \) will include all of the talented agents of generation 1 if \( v_2 > \frac{2}{\delta} \), all of generation 1 if \( v_2 > 2 \). Under Assumption 1, demand and supply meet in the first vertical portion of the supply schedule. Thus, the short-run equilibrium in period 2 is such that all and only the talented agents go to school in period 1, and \( v_2 = \frac{\phi}{\delta \theta} \). Because \( \frac{\phi}{\delta \theta} < v_{cc} \) by Assumption 2, this is also a long-run equilibrium (the “good” equilibrium described in Proposition 1).

The right-hand panel represents the case in which \( v_1 > v_{cc} \). In this case, the offspring of skilled workers are the only ones who can go to school in period 1. The supply of skilled labor in period 2 is then everywhere lower than in the previous case, reflecting the fact that schools can only collect their intake from a portion \( e_0 < 1 \) of the population. There are then two distinct cases. If educated parents in period 1 are not too few, supply of skilled labor in period 2 is

\(^{18}\)Notice that we could relax Assumption 2 to \( v_{cc} > \frac{\phi}{\delta \theta} \). This would imply a slightly more complicated transitional dynamics for the case in which \( v_1 \in (v_{cc}, 2] \), but not change the result that the economy converges to one of the “bad” long run equilibria when \( v_1 > v_{cc} \).

\(^{19}\)Notice that the cost of subsistence ranges from \( \frac{1}{2} \) to 1 as \( v \) ranges from \( v_{cc} \) to \( \bar{\pi} \). For these high values of the cost of subsistence, unskilled parents bequeath less than a share \( \frac{1}{2} \) of their second period’s income to their offspring. This does matter for our result, however, as these offspring would have been credit constrained anyway for \( v > v_{cc} \).
Case 1: $v_1 \leq v_{cc}$

Case 2: $v_1 > v_{cc}$

Good eq.

Bad eq.

Figure 1: Long-run equilibria in autarchy

not too low ($s_2'$ schedule), and $v_2 \leq v_{cc}$. Because credit constraints are not binding in period 2, supply in period 3 ($s_3'$ schedule) is identical to supply in the right-hand panel, and the economy converges to the good long-run equilibrium in period 3. If instead there are very few educated parents and $v_2 > v_{cc}$ ($s_2'$ schedule), credit constraints are still binding in period 2, and the offspring of skilled workers are again the only ones that can go to school. Because $v_1 > v_{cc}$ and $v_2 > 2$, the number of skilled workers in $t = 2$ must be the same as in $t = 1$. It follows that the supply of skilled labor in $t = 3$ ($s_3'$ schedule) will be identical to supply in $t = 2$, and a “bad” long run equilibrium is reached.

Having found all the possible long-run equilibria at which $H$ can be in autarchy, we next move to consider how the long-run equilibrium of this country may be affected by opening up to trade with a foreign country.

6 Trade equilibrium

We now introduce a foreign country, $F$, and study the effect of trade on the skill premium. Since we are interested in explaining the effect of trade liberalization in relatively unskilled labor-abundant Latin America, we want $H$ to start out from a lower $s$ than $F$. The simplest way to obtain this in our framework is to assume that the two countries are identical, but $H$ is initially
at a bad long-run equilibrium while $F$ is at the good long-run equilibrium. In this environment, the only reason for trade between $H$ and $F$ is $H$ being stuck at an equilibrium where all and only the offspring of skilled workers can go to school, while in $F$ school is open to all talented agents independently on their social background. As a very first approximation, this resonates well with the very scarce social mobility of Latin America until the 1980s, at least relative to the United States or Europe.

While we assume that the main difference between $H$ and $F$ is the long-run equilibrium they start out at, we also want to capture the fact that $H$ may be more or less large relative to the world it opens to. To that purpose, we parameterize the size of $F$ by $\gamma > 1$: more precisely, the population of $F$ has mass $\gamma n$. This formulation will allow us to consider various scenarios for the impact of trade on prices in $H$. To the limit, for $\gamma \to \infty$ we will be considering the case in which $H$ is a small economy that opens to a world where prices are exogenously given. Apart from the difference in size, the two countries are identical in terms of preferences, technology and distribution of talent. In what follows, we denote by small letters with the superscript (*) the endogenous variables in $F$, by small letters without any super or sub-script the endogenous variables in $H$, and by capital letters the common value prices when free trade occurs between the two countries.

For simplicity, we chose to focus on the case in which trade equalizes the reward to factors in the two countries (factor price equalization, FPE from now on).\textsuperscript{20} We show in Appendix B that there exists an $\alpha \in [0, 1)$ such that FPE obtains iff $\alpha < \alpha$. We thus impose:

**Assumption 3** $\alpha < \alpha$, so that factor price equalization obtains.

In words, Assumption 3 requires that the two tradable sectors of the economy be different enough in their skilled labor-intensity. Intuitively, if production of $x$ was not enough unskilled labor-intensive, the economy would not be able to absorb $H$’s large unskilled labor force at the higher relative cost of unskilled labor dictated by international trade. The economy would then be forced to specialize in the production of $x$, and the relative reward to unskilled labor would have to remain lower than in $F$.

\textsuperscript{20}This is not necessary for our results: all we need is that trade decrease (increase) the relative skilled wage in $H$ ($F$), a result that holds independently on FPE.
6.1 Trade equilibrium with exogenous skilled labor

Before studying the effect of trade on schooling and working decisions, we investigate its effect in the case of fixed endowments. Suppose that the supply of skilled workers is exogenously given at $s_t$ and $s_t^*$ in the two countries. Then, we show in Appendix B that opening up trade in period $t$ results in the following equilibrium prices:

\begin{align}
V_t &= \frac{(1 + \gamma)\phi}{s_t + \gamma s_t^*} \\
(P_x)_t &= \frac{(V_t)^{\alpha}}{\tilde{\alpha}} \\
(P_y)_t &= \frac{(V_t)^{1-\alpha}}{\tilde{\alpha}} \\
P_t &= \frac{2}{A} \sqrt{V_t} \frac{\alpha}{\tilde{\alpha}}.
\end{align}

Comparing equations (12)-(15) to equations (1)-(4) reveals a very convenient fact: the common prices that realize in the two countries after trade is opened can be calculated as the equilibrium autarchy prices of a hypothetical country created by combining the two countries’ labor endowments. This result (known in trade theory as the “integrated trade equilibrium” result) only holds when FPE holds.

Looking at equation (12), we see that the skilled wage under free trade is always included between $\frac{\phi}{s_t}$ (the autarchy skilled wage in $H$) and $\frac{\phi}{s_t^*}$ (the autarchy skilled wage in $F$), is strictly decreasing in $\gamma$, and converges to $\frac{\phi}{s_t^*}$ as $\gamma$ goes to infinity. This implies that trade increases the relative price of $y$ versus $x$ \((\frac{(P_y)_t}{(P_x)_t} = V_t^{1-2\alpha})\) in $H$, decreases it in $F$. Not surprisingly, the pattern of trade will then be that $H$ (the unskilled labor-abundant country) export good $x$ (the unskilled labor-abundant good), in exchange for $y$. Notice that the absolute price of both $x$ and $y$ decreases in $H$. This result is due to the fact that we have fixed the level of the unskilled wage by adding a constant-return-to scale, non-tradable sector ($f$). It is in line with the Stolper-Samuelson prediction that the purchasing power of unskilled labor must increase in $H$, in terms of both tradable goods.\(^{21}\)

\[^{21}\text{Notice that the expansion of the } x \text{ industry in } H \text{ implies a re-allocation of labor from sector } y \text{ to sector } x \text{ in this country. If firing costs prevented this reallocation, } p_x \text{ and } p_y \text{ would still fall in response to a cheaper supply from } F. \text{ The zero-profit condition would then require wages to fall as well. However since } p_x \text{ must decrease less
Anticipating our definition of the skill premium as the ratio of observed skilled to unskilled wage (see Section 6.2 for more details), we notice that, in this model with exogenous endowments, the opening up to trade must be associated with an increase of the skill premium in $F$ and a decrease in $H$. This is because trade increases the skilled wage in $F$ and decreases in $H$ (for constant unskilled wage), while the average talent of the skilled labor force remains the same. Thus, the simplified model fully satisfies the Stolper-Samuelson predictions for the impact of trade on to skill premium. In the next section, we show that this does need to be the case after endogeneizing the schooling and working decisions of agents.

6.2 Trade equilibrium with endogenous skilled labor

Before investigating the impact of trade on schooling and working decisions, we must specify the timing of trade liberalization and its relations to agents’ expectations. A full-fledged analysis of trade liberalization in a model with rational expectations would require to specify to what extent trade liberalization at time $t$ is foreseen by agents in previous periods. This is important because, for example, the possibility of trade liberalization affects the expected skilled wage in $t$, thus influencing the schooling decisions at time $t-1$. While making the analysis more coherent, to account for the expectation of trade liberalization would greatly complicate the model, as it would link the autarchy equilibria of the two countries to each other even before trade is opened. At the same time, this extension would not undermine the logic of our argument unless trade liberalization is fully foreseen - a case that we consider unlikely. For these reasons, we choose to model trade liberalization as a fully unexpected event in period $t$. To facilitate the intuition, this simple model can be thought of as substantially equivalent to one in which trade is opened in period $t$, and the probability perceived beforehand that this would happen was very low.

To introduce trade in our framework, we enrich the timing at period $t$ (and at period $t$ only) by adding the following event:

\[ t.0 \text{ Trade is opened (to remain open forever after).} \]

than $p_y$ (as this is the comparative advantage industry for $F$), the unskilled wage must, on average, decrease less than the skilled wage. The basic results that trade decreases the credit constraints of the poor (see below) would then be preserved.
That is, trade is opened immediately before generation $t$ is born in $t$.1, and remains open forever after. At time $t$.2, educated parents decide whether to join the skilled labor force or not. As in autarchy, this choice and the prices that form in period $t$.3 must constitute a short-run equilibrium. Because conditions have unexpectedly changed, however, schooling decisions in period $t-1$ need not be optimal anymore. In particular, it may well be the case that some of the educated parents decide to stay out of the skilled labor force, as trade has depressed the skilled wage to a level well below what they expected when they decided to go to school. This misalignment between expected and real prices lasts for one period only. Because there is no uncertainty after period $t$ - trade remains forever open, and this is common knowledge - generation $t$’s schooling decisions must correctly reflect prices as they will form under free trade in period $t+1$. Notice that trade affects the schooling decisions of generation $t$ in two ways. It may affect their participation constraints, by changing the level of the skilled wage in $t+1$; and it may affect their credit constraints, by changing the level of the skilled wage in $t$.

Our first result is presented in the following proposition:

**Proposition 2** Suppose that country $H$ is at a bad long-run equilibrium while country $F$ is at the good long-run equilibrium. If trade between the two countries is opened in period $t$, and $F$ is large enough, both countries end up at the good long-run equilibrium in $t+1$.

**Proof.** Distinguish two cases. If $v_{t-1}^* \geq 1$, it is:

$$V_t = \frac{(1 + \gamma)\phi}{(e_{t-1})\bar{\theta} + \gamma\delta\theta}.$$  

Recall that $V_t(\gamma)$ is included between $\frac{\phi}{s_t}$ (the autarchy skilled wage in $H$) and $\frac{\phi}{s_t^*}$ (the autarchy skilled wage in $F$), is strictly decreasing in $\gamma$, and converges to $\frac{\phi}{s_t^*}$ as $\gamma$ goes to infinity. Because $\frac{\phi}{s_t^*} < v_{cc} < v_{t-1}$, there exists a $\tilde{\gamma} > 0$ such that $V_t < v_{cc}$ if $\gamma > \tilde{\gamma}$. Since no one is credit constrained for this level of the skilled wage, the supply of skilled labor in $t+1$ is equal to (10) in both countries. This implies that the joint supply (as a share of the total population of parents) is also equal to (10). Similarly, since demand for skilled labor (as a share of the total population
of parents) is $\phi_v$ in both countries, the joint demand is also equal to $\phi_v$. We can thus use a procedure identical to that in the proof to Proposition 1 to show that a long-run equilibrium is achieved in $t + 1$ whereby $s_{t+s} = s^{*}_{t+s} = \delta \theta$ and $V_{t+s} = \frac{\phi}{\delta \theta}$ for all $s = 1, \ldots, \infty$.

If $v^*_{t-1} < 1$, it is:

$$V_t = \begin{cases} 
\frac{(1+\gamma)\phi}{\varepsilon_{t-1}\theta+\gamma\delta \theta} > 1 & \text{if } \gamma < \bar{\gamma} \\
1 & \text{if } \gamma \in [\gamma, \bar{\gamma}] \\
\frac{(1+\gamma)\phi}{\varepsilon_{t-1}\delta \theta+\gamma\delta \theta} < 1 & \text{if } \gamma > \bar{\gamma}
\end{cases} \quad (16)$$

where $\gamma \equiv \arg\{\frac{(1+\gamma)\phi}{\varepsilon_{t-1}\theta+\gamma\delta \theta} = 1\}$ and $\bar{\gamma} = \arg\{\frac{(1+\gamma)\phi}{\varepsilon_{t-1}\delta \theta+\gamma\delta \theta} = 1\}$. Since $V_t$ has the same properties as before (except that it is now weakly decreasing in $\gamma$), there still exists a $\hat{\gamma} > 0$ such that $V_t < v_{cc}$ if $\gamma > \hat{\gamma}$, and the proof can proceed as before.

Proposition 2 suggests that if $H$ is at a bad long-run equilibrium and it opens up to a relatively skill-abundant country (or rest of the to world), trade may trigger a mechanism that shifts $H$ to the good long-run equilibrium within a generation. The intuition for this result is straightforward. Because $F$ is relatively skilled labor-intensive, trade reduces the skilled wage in $H$. Since $v^*_{t-1} < v_{cc}$, this reduction must be large enough to take the skilled wage below the threshold $v_{cc}$ when $F$ is large enough. But for $V_t < v_{cc}$ the offspring of unskilled workers are not credit constrained anymore, and all of generation $t$'s talented agents can choose to go to school in period $t$. Furthermore, while the (expected) skilled wage has fallen relative to its pre-trade level, it is still high enough to motivate all of these agents into school. Of course, this is only possible because credit constraints, and not participation constraints, prevented these agents from joining the skilled labor force in autarchy. On the contrary, the non-talented offspring of skilled workers - that would have gone to school had the (expected) skilled wage remained at its pre-trade level - are now better off opting out of the schooling system. It follows that all and only the talented agents go to school after trade is opened, and the economy moves to the good long-run equilibrium in period $t + 1$.\footnote{The spirit of the model would be preserved if fired costs prevented (or slowed down) the reallocation of labor from sector the $x$ to sector $y$, because the average unskilled wage would still increase relative to the average skilled wage (see also footnote 21).}

The result that trade may move the unskilled labor-intensive country from a low human
capital equilibrium to a high human capital equilibrium is very similar to the results found by Ranjan (2003). More in general, that trade may lead to an increase in school enrollment among the offspring of unskilled workers is a key result of the literature on trade with credit market frictions and trade, beginning with Cartiglia (1997). These results are consistent with a few recent empirical studies on the impact of trade liberalization on child labor. For example, Edmonds and Pavcnik (2005) find that a trade-induced increase in the price of rise in Vietnam was associated with a higher decrease in child labor in regions that were exposed to a higher price change. Similarly, Kis-Katos and Sparrow (2009) exploits variation in the degree of trade liberalization across Indonesian districts to argue that trade liberalization is associated with a decrease in child labor, and that this effect is strongest for children from low-skill backgrounds.

We now turn to the main focus of the paper, which is to consider the consequences of trade for the skill premium in both countries. To that purpose, we begin by defining the skill premium in $H$ in period $t$ as:

$$\pi_t \equiv (\hat{\theta}_S)_t v_t$$

where $(\hat{\theta}_S)_t$ denotes the average talent of members of $H$’s skilled labor force in period $t$ (the skill premium in $F$ is defined symmetrically as $\pi^* \equiv (\hat{\theta}_S^*)_t v_t^*$). Following the empirical literature, we define the skill premium as the ratio of the average wage of members of the skilled labor force by the average wage of the members of the unskilled labor force (which is always 1 in equilibrium), not controlling for the unobservable talent of workers. We begin by considering the impact of trade on the skill premium in the long-run (in period $t+1$ and after) and we will then comment on its impact in the short-run (in period $t$).

The long-run effect of trade on the skill premium are described in the following proposition:

**Proposition 3** Suppose that trade is opened between $H$ and $F$ in period $t$, and this moves $H$ to the good long-run equilibrium. Then, the skill premia from period $t+1$ onwards are:

$$\pi_{t+1} = \frac{\theta V_{t+1}}{\theta v_{t-1} \pi_{t-1}}$$

$$\pi^*_{t+1} = \pi^*_{t-1}.$$
Proof. From Proposition 1 we know that, if \( \gamma > \hat{\gamma} \), it is \( V_{t+s} = v^*_{t-1} \) and \( (\hat{\theta}^s)_{t+s} = (\hat{\theta}^s)_{t-1} = \theta \) for \( s = 1, ..., \infty \). Thus, it must be the case that \( \pi^*_{t+s} = \pi^*_{t-1} \) for \( s = 1, ..., \infty \). As for \( H \), we may write:

\[
\pi_{t+1} = \theta V_{t+1} = \frac{\theta V_{t+1}}{\theta v_{t-1}} \pi_{t-1} \quad (17)
\]

since \( \pi_{t-1} = \hat{\theta} v_{t-1} \).

Proposition 3 studies the long-run effect of trade on the skill premia for the case in which trade moves \( H \) to the good equilibrium (see Proposition 2). Not surprisingly, the long-run effect of trade on the skill premium in \( F \) is none \( (\pi^*_{t+1} = \pi^*_{t-1}) \). This follows from the fact that skilled wage goes back to its original value in this country, and agents’ schooling and working decisions remain unaffected throughout the adjustment process. Intuitively, we are working with a model where the only source of comparative advantage is the multiplicity of equilibria in the human capital accumulation process. As suggested by Proposition 2, trade may have the effect of lifting all trade participants to the same “good” equilibrium in the long run. When this is the case, any effect of trade on the country that was at that equilibrium before trade liberalization (and trade itself) must disappear in the long run.

More interestingly, proposition 3 suggests that the long-run skill premium in \( H \) is the product of its pre-trade liberalization level and two distinct terms. The first is the ratio of the level of the skilled wage post-trade liberalization to its level pre-trade liberalization \( (\frac{V_{t+1}}{v_{t-1}}) \). Because \( H \) is on the unskilled labor-abundant side of the trade relation, this ratio must be smaller than 1 in equilibrium. Thus, the impact of trade on the skill premium as described by the first term is just as in a standard Stolper-Samuelson world: trade decreases the skill premium in the unskilled labor-abundant country, because it decreases the skilled wage relative to the unskilled wage. The second term in equation (17), however \( (\frac{\theta}{\hat{\theta}}) \), introduces an important qualification to this conclusion. The term captures the extent of talent re-allocation following to trade liberalization, or alternatively the degree of talent misallocation in \( H \) before trade liberalization. Since \( \theta > \hat{\theta} \) - trade always improves the allocation of talent to the skilled labor force - this term must be greater than 1. Thus, the immediate gist of Proposition 3 is that the Stolper-Samuelson effect on the skill premium is always moderated by the reallocation of talent. In Appendix D we show
that it can actually be $\frac{\theta}{\theta} > \frac{V_{t+1}}{V_{t-1}}$ in our parameter space, or that the Stolper-Samuelson effect may be reversed when the initial degree of talent misallocation in $H$ is large enough.\footnote{In passing, we note that the effect of given reallocation of talent would be even stronger if productivity depended on talent in the unskilled labor force as well, as an increase in the average talent of the skilled labor force would then be accompanied by a decrease in the average talent in the unskilled labor force.}

We have thus found that in world with educational credit constraints, trade with a large skill-abundant world will have a less negative, and possibly a positive effect on the long-run skill-premium of a skill-scarce country. The intuition for this result is straightforward. By increasing the real purchasing power of unskilled workers in terms of the tradable goods in the economy, trade increases the capacity of the talented offspring of unskilled workers to pay for their subsistence expenses when going to school. At the same time, by lowering the skilled wage per efficiency unit, trade discourages the non talented offspring of skilled workers to go to school as they would have done in a pre-trade world. These two forces increase the average talent of those first in school after trade liberalization, leading to higher remunerations and possibly to a reversal of the Stolper-Samuelson theorem.

It is important to notice that the increase in the skill premium after trade liberalization is only possible if, for a given increase in the school enrollment of talented people, there is a sufficiently large decrease in the school enrollment of non talented people. These opposite movements (in and out of the schooling system) might explain why aggregate school enrollment increased relatively little after trade liberalization in Latin America, while the skill premium increased significantly. This compares with the case of South-East Asia, where trade liberalization was followed by a much more large increase in aggregate enrollment and by a decrease in the skill premium. This latter pattern is consistent with the prediction of the Stolper-Samuelson theorem and of our model for the case in which the movement out of the schooling system is very small relative to the movement into the schooling system.

We next consider the effect of trade liberalization on the skill premium in the short run. This is summarized in the following proposition:

**Proposition 4** Suppose that trade is opened between $H$ and $F$ in period $t$, and this moves $H$ to
the good long-run equilibrium. Then, the skill premia in period $t$ are:

$$\pi_t = \Phi \frac{V_t}{v_{t-1}^*} \pi_{t-1}$$

$$\pi_t^* = \frac{V_t}{v_{t-1}^*} \pi_{t-1}^*.$$ 

If $v_{t-1}^* \geq 1$, it is $\Phi = 1$. Otherwise, it is $\Phi \in [1, \frac{\theta}{\delta}]$, and $\Phi$ is monotonically increasing in the size of $F$.

**Proof.** Since $(\hat{\theta}_S)_{t-1} = (\hat{\theta}_S)_t = \theta$ we can write:

$$\pi_t^* = \theta V_t = \theta V_t \frac{\pi_{t-1}^*}{v_{t-1}^*} = \frac{V_t}{v_{t-1}^*} \pi_{t-1}^*.$$  

(18)

As for country $H$, we can always write:

$$\pi_t = (\hat{\theta}_S)_t V_t = (\hat{\theta}_S)_t \frac{V_t}{v_{t-1}^*} \pi_{t-1} = \Phi \frac{V_t}{v_{t-1}^*} \pi_{t-1}.$$ 

If $v_{t-1}^* \geq 1$, it is always $V_t > 1$ and $(\hat{\theta}_S)_t = \hat{\theta}$: it follows that $\Phi = 1$. If $v_{t-1}^* < 1$, it is $V_t > 1$, $(\hat{\theta}_S)_t = \hat{\theta}$ and $\Phi = 1$ for $\gamma < \underline{\gamma}$, while it is $V_t < 1$, $(\hat{\theta}_S)_t = \theta$ and $\Phi = \frac{\theta}{\delta}$ for $\gamma > \bar{\gamma}$ (see equation 16). For $\gamma \in [\underline{\gamma}, \bar{\gamma}]$, it must be $\frac{(1+\gamma)\delta}{\epsilon_t (\delta \theta + \psi)} = 1$, where $\psi$ is the number of non talented parents that join the skilled labor force (as a share of the total number of educated parents), and $(\hat{\theta}_S)_t = \frac{\delta \theta + \psi}{\delta + \psi}$. Since $\psi$ is strictly decreasing in $\gamma$ and must range between $1 - \delta$ and $0$ as $\gamma$ ranges between $\underline{\gamma}$ and $\bar{\gamma}$, it follows that $\Phi$ is strictly increasing in $\gamma$ for $\gamma \in [\underline{\gamma}, \bar{\gamma}]$, and ranges between $1$ and $\frac{\theta}{\delta}$ for $\gamma$ in this range. ■

Proposition 4 studies the short-run effect of trade on the skill premia, for the case in which trade moves $H$ to the good equilibrium. In other words, it studies the transitional dynamics of the skill premia, before these converge to the values described in Proposition 3. The effect of trade on the skill premium in $F$ is unambiguously positive in the short run (since it is always $\frac{V_t}{v_{t-1}^*} > 1$). This is because trade increases the value of skilled labor in this country, while the average composition of talent stays the same (as all educated parents are already part of
the skilled labor force). Thus, although in this model $F$’s skilled premium gets back to its original level in the long-run, the transitional dynamics contains the standard features of a trade liberalization for a skill-abundant country.

Just as in the long-run, our model may display non standard predictions for the impact of trade on $H$’s skill premium in the short run. Proposition 4 distinguish two cases. If $v^*_t-1 < 1$, the effect of trade on the skill premium is unambiguously negative in the short run (since it is always $\frac{V}{v^*_t-1} < 1$).\(^{24}\) Intuitively, the trade-induced decrease in the skilled wage cannot be too large in this case, and all educated parents must then remain in the skilled labor force. This implies that the average talent in the skilled labor force does not change, and the skill premium must then decrease. Thus, when $H$ opens up to a world that is not too skill abundant, the Stolper-Samuelson predictions are satisfied both in $F$ and in $H$, since our talent-reallocation channel is effectively shut down. If $v^*_t-1 > 1$, on the contrary, this channel can be fully at play. In particular, when $F$ is large enough, the extent of talent reallocation can be as high as in the long run ($\theta$). This is for a partially different reason, however. In the short run, the initial misallocation of talent can only be corrected through an exit of the non-talented educated parents from the skilled labor force. This re-allocation is only partial, as all talented, non educated parents do not have an option to join in. Because it increases the average talent of the skilled labor force, however, this partial re-allocation obtains a very similar effect as the full re-allocation in the long run. In fact, if $F$ is large enough both effects push up the skill premium by the same amount $\theta$. Because the decrease in the skilled wage is more modest in the short run ($\frac{V}{v^*_t-1} > \frac{V}{v^*_t-1}$), the skill premium will then increase in the short run whenever it does in the long run, and increase by more.\(^{25}\)

Thus, we have shown that trade-induced compositional change may result in an increase in the skill premium in the unskilled labor-abundant country, both in the short run and in the long run. In the short run, the downward pressure put by trade on the skilled wage may induce the least talented of the existing skilled workers to drop out of the skilled labor force, thus increasing its average quality. When it happens in the short run, this compositional change always extends

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\(^{24}\)Notice that Assumptions 1 and 2 are compatible with the case $\frac{\delta}{\theta} < 1$. For example, if $\delta = \frac{\phi}{2}$ this result holds for all $\theta > 2$.

\(^{25}\)This outflow of labor from the tradable sectors is consistent with evidence from Brazil suggesting that trade liberalization induced labor displacement from the formal sector to the informal and self-employment sector (see Menezes-Filho and Muenler, 2007).
to the long run, as only talented young agents find it optimal to go to school after trade has been opened. Even if it doesn’t happen in the short run, however, this compositional change may still realize in the long run. This is because non talented agents are more prone to joining the unskilled labor force when they are young and unskilled, rather than when they are old and already skilled.

In the short run, these results are associated with an increase in the skill premium in the skilled labor-abundant country as well. Thus, our results are consistent with the fact that trade between unskilled labor-intensive Latin America and various skilled labor-intensive parts of the world results in an increase in the skill premium in both places. Because it preserve the standard Heckscher-Ohlin structure in which the skilled wage per efficiency unit decreases in $H$, our model is also consistent with the finding that skill intensity in most Latin American industries increased after trade liberalization. Finally, our results are also compatible with the observation that the skill premium increased homogeneously across industries in skill-scarce countries, independently on the degree of trade liberalization to which each industry had been exposed (see Attanasio, Goldberg and Pavcnik, 2004, for the case of Colombia).

To better focus on our mechanism, we have set up a model where the only source of comparative advantage is the different initial equilibria that countries find themselves at. This simple model displays several unrealistic features, such as the fact that trade disappears in the long run and the skill premium in $F$ goes back to its pre-trade value. Also, as a consequence of the latter fact, the model can only generate a trade-induced increase in the skill premium in $H$ when $H$’s pre-trade skill premium is higher than $F$’s - a case that does not match the data well.\textsuperscript{26}

To bring more “structural” sources of comparative advantage to the model would take care of these problems. For example, we could allow for $F$’s skill-intensive sector to benefit from a better technological or institutional environment, or from a better educational system. Consider a straightforward example. Suppose that a better educational system maps into a larger proportion of agents that can reach high productivity in $F$ than in $H$ (thus, $\delta^* > \delta$). In this environment, a trade-induced shift of $H$ to the good equilibrium does not equalize the stock of human capital in

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\textsuperscript{26}To see this, notice that it must be $\pi_{t+1} = \pi_{t+1}$. If $\pi_{t+1} > \pi_{t-1}$ and $\pi_{t+1} = \pi_{t-1}$, it follows that $\pi_{t-1} < \pi_{t+1}$. This result is in contrast with evidence that the skill premium is generally higher in skill-scarce countries before trade liberalization.
the two countries. Positive trade will then be a feature of the long-run equilibrium, allowing for a common post-trade skill premium that is higher than $F$’s pre-trade skill premium. In this case the model may well predict an increase in the skill premium in both countries in the long run as well. Furthermore, even if the common post-trade skill premium is higher than $H$’s pre-trade skill premium, it is then possible that the latter is higher than $F$’s pre-trade skill premium.

7 Conclusion

In this paper we develop a model of trade liberalization and occupational choice, with capital market imperfections. When an economy is unskilled labor-intensive because of credit constraints that affect the schooling decisions of agents, trade liberalization may have a non-standard effect on the skill premium. This is for two reasons. First, credit constraints may have allowed a large number of non talented agents in the skilled labor force. Having been attracted to the skilled labor force by a high autarchic skilled wage, these agents may find it optimal to join the unskilled labor force when trade puts a downward pressure on the skilled wage. Second, credit constraints may have kept many talented agents out of the skilled labor force. To these agents, a trade-induced decrease in the cost of subsistence implies a lessening of credit constraints, and thus a better chance to move up the social scale. Both of these effects result in an increase in the average talent of the skilled labor force, which may lead to an increase in the skill premium despite the trade-induced decrease in the skilled wage.

Our results provide a possible explanation for the fact that trade liberalization in unskilled labor-intensive Latin America led to an increase in the skill premium in both Latin America and its skilled-intensive trade partners. One implication of this is that the increase in the skill premium in Latin America does not necessarily need to result in a massive increase in income inequality, as it may be (at least partly) due to a better allocation of talent and more intergenerational mobility.

While reconciling the predictions of the Stolper-Samuelson theorem with the Latin American experience, our mechanism is not incompatible with alternative explanations that have highlighted the role of skill-biased technical change or of quality upgrading. In fact, one interesting
extension of our model is to consider the interaction of talent re-allocation with these other trade-induced changes in the structure of production. Other needed extensions include studying our mechanism in the context of more structural sources of comparative advantage (such as differences in physical capital, quality of schooling, etc.), and letting the decisions of agents be affected by the wealth distribution.

Appendices

A

The zero profit conditions in the \( x, y \) and \( z \) sectors are:

\[
\begin{align*}
    p_x &= \left( \frac{1 - \alpha}{\alpha} v \right)^{\alpha} + v \left( \frac{\alpha}{1 - \alpha} \frac{1}{v} \right)^{1-\alpha} = \frac{v^{\alpha}}{\alpha} \\
    p_y &= \left( \frac{\alpha}{1 - \alpha} \right)^{1-\alpha} + v \left( \frac{1 - \alpha}{\alpha} \frac{1}{v} \right)^{\alpha} = \frac{v^{1-\alpha}}{\alpha} \\
    p_z &= p_x \frac{1}{A} \left( \frac{p_y}{p_x} \right)^{\frac{1}{2}} + p_y \frac{1}{A} \left( \frac{p_x}{p_y} \right)^{\frac{1}{2}} = 2 \frac{v^{\frac{1}{2}}}{A \alpha}.
\end{align*}
\]

Provided that all agents can afford to spend \( \phi \) on good \( z \), the autarchy equilibrium is defined by equilibrium in four markets:

\[
\begin{align*}
    p &= \frac{2 v^{\frac{1}{2}}}{\alpha A} \quad (19) \\
    p_x &= \frac{v^{\alpha}}{\alpha} \quad (20) \\
    p_y &= \frac{v^{1-\alpha}}{\alpha} \quad (21) \\
    \overline{S} &= \left( \frac{\alpha}{1 - \alpha} v \right)^{1-\alpha} \pi + \left( \frac{1 - \alpha}{\alpha} \frac{1}{v} \right)^{\alpha} \overline{y} \quad (22)
\end{align*}
\]

where we have replaced market clearing with the corresponding zero-profit condition in sectors displaying both constant returns to scale and a positive demand (in the \( f \) sector, also all individual
demands are positive by assumption), and we have dropped the equations for the $z$ and $U$ markets. Replacing $x = \frac{\phi}{p_x}$ and $y = \frac{\phi}{p_y}$ in (22) and using (20) and (21) yields $v = \frac{\phi_s}{s}$.

B

Provided that all agents can afford to spend $\phi$ on good $z$, the trade equilibrium is defined by the following 12 equations:

\begin{align*}
    p_f^* &= w^* \\
    p_z &= \frac{2v^{\frac{1}{2}}}{\alpha A} \\
    p_z^* &= \frac{2(w^*)^{\frac{1}{2}}(v^*)^{\frac{1}{2}}}{\alpha A} \\
    p_x &= \min \left[ \frac{v^\alpha}{\alpha}, \frac{(v^*)^\alpha(w^*)^{1-\alpha}}{\alpha} \right] \\
    p_y &= \min \left[ \frac{v^{1-\alpha}}{\alpha}, \frac{(v^*)^{1-\alpha}(w^*)^\alpha}{\alpha} \right] \\
    U &= \left( \frac{1-\alpha}{\alpha} v \right)^\alpha \bar{x} + \left( \frac{\alpha}{1-\alpha} v \right)^{1-\alpha} \bar{y} + \overline{\bar{U}} + v\overline{\bar{S}} - 2n\phi \\
    U^* &= \left( \frac{1-\alpha}{\alpha} v^* \right)^\alpha \bar{x}^* + \left( \frac{\alpha}{1-\alpha} v^* \right)^{1-\alpha} \bar{y}^* + \frac{w^*\overline{\bar{U}}^* + v^*\overline{\bar{S}}^* - 2n\phi}{p_f^*} \\
    S &= \left( \frac{\alpha}{1-\alpha} v \right)^{1-\alpha} \bar{x} + \left( \frac{1-\alpha}{\alpha} v \right)^\alpha \bar{y} \\
    S^* &= \left( \frac{\alpha}{1-\alpha} v^* \right)^{1-\alpha} \bar{x}^* + \left( \frac{1-\alpha}{\alpha} v^* \right)^\alpha \bar{y}^* \\
    \bar{x} + \bar{x}^* &= \frac{\phi(n + n^*)}{p_x} \\
    \bar{y} + \bar{y}^* &= \frac{\phi(n + n^*)}{p_y} \\
    0 &= p_x \left( \bar{x} - \frac{n\phi}{p_x} \right) + p_y \left( \bar{y} - \frac{n\phi}{p_y} \right)
\end{align*}

where equation (34) is the balanced trade condition for country $H$. Assume factor price equalization. Then $w^* = 1$ and $v^* = v = V$, implying that $p_f^* = 1$ and $p_z^* = p_z$. We can then drop 4
equations and simplify:

\[ p_x = \frac{2V^{\frac{1}{2}}}{\hat{\alpha}A} \]  

(35)

\[ p_x = \frac{V}{\bar{\alpha}} \]  

(36)

\[ p_y = \frac{V^{1-\alpha}}{\bar{\alpha}} \]  

(37)

\[ \bar{S} = \left( \frac{\alpha}{1-\alpha V} \right)^{1-\alpha} \bar{x} + \left( \frac{1-\alpha}{\alpha} \frac{1}{V} \right)^{\alpha} \bar{y} \]  

(38)

\[ \bar{S}^* = \left( \frac{\alpha}{1-\alpha V} \right)^{1-\alpha} \bar{x}^* + \left( \frac{1-\alpha}{\alpha} \frac{1}{V} \right)^{\alpha} \bar{y}^* \]  

(39)

\[ \bar{x} + \bar{x}^* = \frac{\phi(n+n^*)}{p_x} \]  

(40)

\[ \bar{y} + \bar{y}^* = \frac{\phi(n+n^*)}{p_y} \]  

(41)

\[ 0 = p_x \left( \bar{x} - \frac{n\phi}{p_x} \right) + p_y \left( \bar{y} - \frac{n\phi}{p_y} \right) . \]  

(42)

Summing (38) and (39) and plugging in (36), (37), (40) and (41) yields \( V = \frac{\phi(n+n^*)}{S+S^*} \). We now need to make sure that \( \bar{x}, \bar{x}^*, \bar{y} \) and \( \bar{y}^* \) are positive. Using (38) together with (42) yields:

\[ \bar{x} = \tilde{\alpha} v^{1-\alpha} S^{\gamma^*} \left( \frac{\xi^*}{1-2\alpha} + 1 - 2\alpha \right) \]  

(43)

\[ \bar{y} = \tilde{\alpha} v^{\alpha} S^{\gamma^*} \left( \frac{1-2\alpha\gamma}{1-2\alpha} \right) \]  

(44)

where \( \gamma \equiv \frac{s n + s^* n^*}{s(n+n^*)} \) and \( v \) is the pre-trade level of the skilled wage in \( H \). Using symmetry we then obtain:

\[ \bar{x}^* = \tilde{\alpha} (v^*)^{1-\alpha} S^{\gamma^*} \left( \frac{\xi^*-1}{\xi^*} + 1 - 2\alpha \right) \]  

(45)

\[ \bar{y}^* = \tilde{\alpha} (v^*)^{\alpha} S^{\gamma^*} \left( \frac{1-2\alpha\xi^*}{1-2\alpha} \right) \]  

(46)

where \( \xi \equiv \frac{s n + s^* n^*}{s^*(n+n^*)} \) and \( v^* \) is the pre-trade level of the skilled wage in \( F \). Several things are apparent. First, if \( s = s^* \), \( \xi = \xi^* = 1 \) for all \( n \) and \( n^* \), and countries always produce their autarchy level of \( x \) and \( y \) (and there is no trade). Second, if \( s^* > s \), \( \xi > 1 > \xi^* > \frac{1}{2} \) for all
n and \( n^* \) such that \( n^* \geq n \), and \( H \) produces more \( x \) (less \( y \)) than in autarchy, thus exporting (importing) this good. Finally, because \( \xi \geq 1 \) and \( \xi^* > \frac{1}{2} \), there exists \( \sigma \) such that, if \( \alpha > \sigma \), production of both intermediates is positive in both countries (notice that \( \widehat{\alpha} \to 1 \) as \( \alpha \to 0 \)). Notice also that, as \( \alpha \to 0 \), production of \( y \) in country \( j \) converges to \( S^j \).

We conclude by checking that local demand for unskilled labor in the \( x \) and \( y \) sectors does not exceed supply in \( H \) - the country where trade pushes up employment in the unskilled labor-intensive sector:

\[
U^x_x + U^y_y = 2\phi n - vS
\]

\[
= \phi n \left( 1 + \frac{n^*(s^* - s)}{sn + s^*n^*} \right).
\]

Although trade with a skill-intensive partner soaks up additional unskilled labor to \( H \)'s tradable sectors relative to autarchy (where \( U^x_x + U^y_y = \phi n \)), there will always be sufficient supply as long as \( \phi \) is small enough.

\section{C}

A sufficient condition for an offspring \( i \) of a skilled worker not to be credit constrained in period \( t \) is:

\[
F(\bar{u}, A) + \frac{1}{2} \log v_t \leq \frac{1}{2} \Theta^p_i v_t
\]

(47)

where \( \Theta^p_i \) is level of talent of the parent of agent \( i \). Because \( \Theta^p_i v_t > 1 \), condition (47) must hold at \( v_t = v_{cc} \). Next, we notice that the LHS of (47) is strictly concave in \( v_t \), while the RHS is linear. Since \( \frac{1}{2} v_t < \frac{1}{2} \Theta^p_i \) (recall that it must be \( \Theta^p_i = \theta \) if \( v_t \in (\frac{1}{\theta}, 1] \)), (47) must hold for any \( v_t > v_{cc} \).

\section{D}

The skill premium increases whenever:

\[
v_{t-1} < \frac{\theta}{\theta} V_{t+1} = \frac{\theta}{\theta} \frac{\phi}{\delta} = \frac{\phi}{(\theta - 1)\delta^2 + \delta}.
\]

(48)
Since the RHS of (48) is decreasing in $\delta$, we experiment by plugging in the lowest possible value for $\delta$ allowed by Assumption 1:

$$v_{t-1} < \frac{\phi}{(\theta - 1) \frac{\phi}{4\theta^2} + \frac{\phi}{2\theta}} = \frac{4\theta^2}{\theta(2 + \phi) - \phi}.$$  \hfill (49)

The RHS of (49) ranges between 2 and $\infty$ as $\theta$ ranges between 1 and $\infty$. Since $v_{t-1}$ may take value in $(2, \overline{v}]$, it may well be the case that condition (49) is satisfied.

\textbf{E}

Call $m_{t,i}^i$ the income of agent $i$ in generation $t$. Because it must be $m_{t,i}^i > \phi$ in equilibrium (see Section 3.2), the marginal utility of income in period $t$ is 1. Assume now that $\overline{\pi}$ is low enough, so that the survival constraint is not binding. Utility maximization then requires that $m_{t,t+1}^i$ be split equally between $b_t^i$ and $u_{t,t+1}^i$. Since $\frac{m_{t,t+1}^i}{2} > \phi$ in equilibrium (see Section 3.2), it must be at a maximum:

$$\left(\frac{b_t^i}{2}\right)^{\frac{1}{2}} \left(\frac{u_{t,t+1}^i}{2}\right)^{\frac{1}{2}} = 2 \left(\frac{m_{t,t+1}^i}{2} - \phi + \log \phi \frac{\phi^2}{p_t+t+1}\right)^{\frac{1}{2}} \left(\frac{1}{2} m_{t,t+1}^i - \phi + \log \phi \frac{\phi^2}{p_t+t+1}\right)^{\frac{1}{2}}.$$  \hfill (50)

From (50), it is clear that $\frac{\partial U_i}{\partial m_{t,t+1}^i} = 1$.

\textbf{References}


