Industrialization and Compensation for Displaced Farmers *

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1 Introduction

A major problem of contemporary development policy concerns compensation paid to those whose traditional livelihoods are uprooted by modern industrial projects.

This involves both equity and efficiency considerations. In the absence of a welfare state those who are rendered unemployed by economic change are left at the mercy of market forces. The political and social fallout of this is a major cost of undertaking industrialization, a point that is often ignored in standard economic discussions about the costs and benefits of industrialization. Also, in the absence of a well defined compensation policy, those who fear displacement due to the process of industrial development, will tend to under-invest in the assets (e.g., land) which will affect the productivity of these assets in their existing use, as well as the willingness of the owners to convert them to alternative uses. In a world without frictions, complete contracts will take care of investment incentives, make sure that losers are compensated adequately by the gainers, and industrialization will only occur when the net social benefits exceed the net social costs. Our point of departure is to model explicitly frictions that typically characterize

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agricultural production, including problems of incentives and commitment, as well as departures from the assumption of transferable utility.

These problems have surfaced quite prominently in China and India recently, as these countries have embarked on rapid industrialization programs in the past two decades. As described in more detail in Section 2 below, the transition to industrialization in these countries have been marked by conversion of agricultural land into land earmarked for industrial projects and urban real estate development. The process has been facilitated by local or regional governments anxious to raise the rate of growth in their jurisdictions, which generate large spillover effects and/or raise government revenues. At the same time, the livelihoods of farmers cultivating these lands and workers employed by these farmers get uprooted. The compensations paid to those displaced has been criticized as being inadequate. The process of determining and implementing these compensations have been described as arbitrary, ad hoc and lacking transparency. There have also been complaints of the lack of any rights or participation of those displaced in the process of transition.

These problems of compensation have created widespread social and political tensions. For instance, Cao, Feng and Tao (2008) report that in the first nine months of 2006, China reported a total of 17,900 cases of “massive rural incidents”, in which a total of 385,000 farmers protested against the government. They go on to state that

“there are currently over 40 million dispossessed farmers due to urban expansion and transportation networking and 70% of the complaints lodged from farmers in the past 5 years are related to rural land requisition in urbanization.” (ibid, pp. 21-22)

Likewise in the eastern state of West Bengal in India, farmers were displaced by a motor car project started in 2007 for which land had been compulsorily acquired by the state government. A significant proportion of these protested that the compensation paid to them was inadequate. These protests were orchestrated by the principal opposition party to the party controlling the state government. The resulting tension and confrontations eventually led to the industrial group in question moving its factory to a different state in India in 2008. Despite agreement between most parties that the land ought to be converted to industrial use, the problem of inadequacy of compensation caused the process of conversion to be reversed.

These events raise important questions regarding economic principles that should guide the design and implementation of compensation for agents displaced by industrial development projects. A newspaper article (Banerjee
et al (2007)) on the transition and compensation issues in West Bengal by a group of nine economists (including the two of us) stated:

“With growing population pressure on the land and stagnant yields in agriculture, there is no alternative to industrialization. The key question then is how to carry out while making sure that the rural population does not remain disaffected and gets its fair share of the benefits of industrialization. In the long run, a part of the answer has to be more skill formation for the time being however, the focus has to be on actual compensation policy.”

The specific issues were described as follows:

“First, how should the compensation formulae be designed? ... Second, who should have claims to compensation? For example, how should de facto owners without the right legal titles to the land be treated? What about unregistered sharecroppers and agricultural workers who stand to lose their access to tenancy or work? Third, how should such compensations be administered?”

The second issue raised above involves a fundamental question concerning property rights. According to most legal frameworks, property owners do not require the permission of their current tenants or workers in order to sell the property. Nor are they required to compensate them in the event that the tenant gets evicted or the workers lose their jobs. Ownership rights include both freedom to decide how the property is to be used as well as over the sale of the property. Yet the preceding events in China and India raise the question whether tenants or workers employed by a landlord should be legally entitled to some compensation if the owner were to sell the property. And if so, what principles should guide the design of such compensation.

Analogous issues arise in executive compensation or severance payments to workers that lose their jobs when their employer sells the firm to a new owner. For instance, top executives frequently have compensation packages with ‘golden parachute’ provisions which go into effect when the firm is sold to a new owner. Workers participate in stock purchase plans which cushion losses of employment associated with sales of the firm to new owners, and ensure they obtain a proportion of gains in stock price appreciation resulting from the sale.

There is a substantial theoretical literature on the optimal allocation and regulation of use rights inherent in ownership of property. For instance,
the literature on incomplete contracts and the nature of the firm following Grossman and Hart (1986) or Hart and Moore (1990) examines the implications of different assignment of use rights over property used jointly for productive purposes by a set of agents. There is a literature in development economics on the incentive effects of sharecropping tenancy and its regulation (Singh (1989), Mookherjee (1997) or Banerjee, Gertler and Ghatak (2002)). But there is no comparable analysis that we are aware of concerning analogous issues in the allocation of *exchange rights* which is also an important component of ownership rights.

The purpose of this paper is to initiate such a theoretical analysis of compensation arrangements for incentives of concerned parties to invest in productivity-enhancing investments or actions. Like most existing literature, we focus on implications for efficiency, as evaluated by a utilitarian social welfare function which neglects issues of distributive equity. In particular, we examine whether there is an efficiency argument for restricting the rights of owners over the sale of assets in the sense of mandating compensation of displaced tenants. If so, inclusion of considerations of distributive justice would further strengthen the argument.

We consider a simple stylized example of a single plot of agricultural land owned by a landlord (or local government which is the *de facto* owner) which is leased to a tenant farmer who cultivates the land. A third party industrial developer (I) may arrive with some probability $\theta$ with a high valuation $V$ on the land relative to the agricultural incomes currently generated. The landlord and the industrial developer negotiate a price for sale of the land to the latter. To keep matters simple, we assume that the incomes or valuations of the property are common knowledge among the landlord, industrialist and tenant, though not necessarily by outsiders. The question we analyze concerns regulations of the compensation paid to the tenant in the event of a sale. These can be based either on the price at which it is sold, or on an assessment (by the regulator) of the agricultural income lost by the tenant. We consider this in two different settings: an *incomplete contract* (IC) setting extending the Grossman-Hart-Moore (GHM) analysis, as well as a *complete contract* setting extending standard models of sharecropping contracts based on moral hazard and limited liability (MHLL). In the former, limitations on the ability of the landlord to commit to a long-term contract for the tenant generates inefficient investment incentives. In the latter, there are no commitment problems. Instead, limited liability constraints combined with low wealth of tenants limit the ability of landlords to extract benefits of productivity improvements, which distort investment incentives. Nevertheless, the analysis of the two models turn out to yield
qualitatively similar results.

As in either of these traditional approaches, we allow for \textit{ex ante} investments by the landlord and tenant which may be affected by the nature of property rights. Indeed, in the absence of these, the only allocative role of property rights concerns their implications for decisions for whether or not the property will be sold or not. If the decision rests with the landlord, standard economic analysis yields a straightforward answer to the question of compensation. Optimal resource allocation necessitates paying compensation to the tenant so that the landlord correctly internalizes the cost imposed on the latter as a result of the property sale. This will be traded off against the various benefits that will accrue to the landlord or industrialist. If the rental market for property operates without distortion, the current rent captures the value to the tenant of leasing the asset. Since the landlord earns this rent which will be foregone upon selling the property, vesting the sole decision right over the sale to the landlord results in an efficient outcome. The argument is further strengthened if the landlord makes \textit{ex ante} investments in the construction and upkeep of the property. Retaining full rights over sale will generate the correct (i.e., first-best) incentives to the landlord for making such investments.

The argument for vesting full rights of sale in the owner is less clear when the tenant also makes specific investments in the asset. While this is less relevant in the context of urban real estate or industrial machinery, it is important in the context of agricultural land. Empirical evidence in the context of Indian agriculture for the relative importance of tenant’s incentives has been provided by Shaban (1987), Banerjee, Gertler and Ghatak (2002) and Bardhan and Mookherjee (2007). The latter two papers study the effect of a sharecropping regulation program in West Bengal which would be expected to lower the landlord’s incentive and raise the tenant’s incentives to raise agricultural productivity. They both find a net increase in agricultural productivity as a result of the reform, indicating that the enhancement of tenant’s incentives outweighed the reduction in the landlord’s incentive.

In such contexts, efficiency requires providing both landlord and tenant with appropriate investment incentives. Owing to the well-known problem of free-riding or ‘moral hazard in teams’, it is not possible to provide both with first-best incentives. Each will tend to under-invest as they will not receive the full return on their respective investments. Even if the landlord has all the bargaining power over the terms of the tenancy contract, it will typically be in his interest to leave some surplus to the tenant to induce the latter to make some investments. But giving the tenant higher investment incentives also requires leaving the tenant with a larger fraction of the overall
economic rent from the property. So the landlord will tend to induce too little investment from the tenant in a *laissez faire* situation from a social standpoint. This is precisely the argument for regulating sharecropping contracts (e.g., with rent control or restrictions on eviction) as stressed in the literature on use-rights (Banerjee, Gertler and Ghatak (2002)).

Analogously, vesting sole decision rights with the landlord concerning sale of the asset will generate socially excessive incentives to sell to third parties when the opportunity arises. This is because the landlord will neglect the effect of the sale on the loss of surplus by the tenant. Moreover, offering compensation to the tenant which is linked to the agricultural value of the land will augment his *ex ante* investment incentives, though at the cost of lowering the landlord’s investment incentives. It is not immediately obvious what the net effect will be for overall efficiency, taking into account all three channels of impact (*ex ante* investment of the two parties respectively, and the decision concerning conversion). This is precisely what we study in this paper.

For the reasons explained above, we focus our analysis on the case where the productivity of the tenant’s investments exceed those of the landlord’s. In addition, to keep the analysis tractable we assume that land productivity is linear in the investments made by the landlord and tenant, and that their respective costs are quadratic in the investments made. Finally, we focus on the case where the industrial value $V$ of the land is large relative to agricultural productivity so that the *ex post* optimal allocation requires the land to be converted to industrial use when the opportunity arises.

Our analysis verifies the intuition that there is an efficiency rationale for mandating compensation to tenants when the investments of the latter are important relative to the landlord. In that case tenants do and should obtain a substantial surplus. Mandating compensation implies tenants earn a higher share of returns to their investment, which raises the tenant’s investment and lowers the landlord’s investment. Since the tenant under-invests this generates an improvement. What is less obvious is the welfare effect of the landlord’s investment reduction. For a large class of relevant cases we find that the landlord has a tendency to over-invest. This is for a strategic reason: raising the agricultural value of the land raises the sale price of the land to the industrialist (by increasing the opportunity cost to the landlord of selling the land). Hence the reduction in the landlord’s investment also results in a welfare improvement. Finally, mandating compensation reduces the *ex post* incentive of the landlord to convert the land to industrial use. Since the landlord does not adequately internalize the effect of the conversion on the surplus of the tenant, this channel also generates a welfare...
improvement. Hence on all three counts, net economic efficiency rises for a large set of parameter values.

The argument is easier in the case of incomplete contracts where the landlord is unable to make any credible ex ante promises of compensation to the tenant. In this case the compensation mandate serves as a commitment device, which may result in a Pareto improvement, i.e., benefit the landlord as well as the tenant. The argument for a mandate is more complicated in the complete contract setting where the landlord can make such commitments. We show that compensation mandates are needed from a welfare standpoint in that setting as well, essentially because the landlord does not adequately internalize the incentive rents of the tenant, or the effects of the sale decision on these rents. In such a case, the landlord and industrialist are worse off as a result of the mandate. But these are outweighed by the gains of the tenant.

These results are shown for the case where the compensation is based on the sale price. In a later section we consider where it is based instead on agricultural yields or incomes foregone. In that context we provide conditions for the optimal compensation to exceed more than 100% of the income lost by the tenant. It is also worth reiterating that these arguments for mandating compensation rest on economic efficiency rather than distributive or political economy considerations.

The paper is organized as follows. Section 2 provides a more detailed description of the problems faced in China concerning compensation of displaced farmers. Section 3 lays out the model in detail. Section 4 first considers the case where no opportunities for land conversion arise, and shows how the standard analysis of sharecropping contracts with moral hazard and limited liability extends to the context of a two-sided investment problem. Section 5 then examines the case when $\theta > 0$ where the opportunity for industrial conversion arises, and compensations are based on sale price. Section 6 turns to the case where they are based on agricultural yields. Finally Section 7 concludes with a summary of the policy implications and other applications, followed by qualifications and extensions needed to be explored in future work.

2 Issues in Chinese Land Policy

The fast pace of urban growth and industrialization in China since the late 1970s have been aided and accompanied by far-reaching changes in policies allowing markets for land-use rights to develop. Initially adopted in special
economic zones to facilitate the process of direct foreign investment in the 1980s, the most far-reaching of these new policies were adopted in the late 1980s and early 1990s. Land-use rights were allowed to be sold by municipal governments for fixed periods to buyers via auctions, tender or negotiation, and for these use-rights to be traded among private buyers. Ding (2003) reports that between 1986 and 1995 at least 1.9 million hectares of farmland were converted; 31 big cities expanded their areas by more than 50%. Urban areas expanded at the rate of 7.5% per annum between 1985 and 1994.

Ding (2003) describes how these had positive impacts on urban development, transportation, and municipal government finances. At the same time social conflicts resulted from inadequate compensation of peasants from whom land was acquired compulsorily by municipal governments. Ding (2007) points out that the land management law is silent on the issue of labor resettlement. Moreover, absence of guidelines for just compensation often results in wide ranges of compensation that seem arbitrary and ad hoc. With compensations based on agricultural production valued at state-controlled prices which understate their true value, the compensations turned out to be too low relative to the real losses experienced. Chan (2003) points out failure to include costs of resettlement, to base compensations on market value of the land, and absence of compensation for temporary losses resulted in inadequacy of compensation. Both Ding and Chan point out that tenants had no legal right to claim compensation, and the former calls for a ‘radical and fundamental changes in property rights’.

Su (2005) describes problems of lack of transparency of the land conversion process, and refers to it as a ‘land grab’ in which the taking of the land and the terms of compensation were unilaterally imposed on farmers. Local governments were always on the lookout for financial support, and given the nature of fiscal and monetary policies in the country, land conversion represented one of the few effective ways for them to raise money. Land use rights were cheap for them to obtain, and were marketable at huge profits. This author estimates that in a typical land conversion, the local government obtains 20-30%, the developer gets 40-50%, the village organization gets 30% and the peasant gets only 5-10% of the gains. Peasants who lost their land were in a worse position than before, with the only source of relief being handouts provided by the government. The expropriation of their lands have led to a decline in legitimacy of the governments involved.

These views are echoed by Cao, Feng and Tao (2008), who declare that “unfair compensation for land requisition during urban expansion has become the most visible and contentious rural issue over the past decade in China” (ibid, pp.21-22) They point out that the majority of land is leased
out by local governments by negotiation and usually at a very low price. They can take advantage of their monopolistic positions in local urban land supply and can extract extra-budgetary revenues as much as possible. They make “every effort to take land from farmers by evading central regulations on arable land use” (ibid, p.26) And “the current land requisition system by depriving farmers of power in negotiating land prices and compensation packages, is to blame for both the under-compensation in land acquisition, and the excessive expansion in urban and industrial land” (ibid, p.28) Accordingly they propose marketization of the land requisition system by allowing farmers to directly negotiate their compensation packages with prospective land users, while local governments can earn revenues by imposing property and value added taxes on such land sales.

Note that the land rights are ultimately owned by the local government, though subject to regulations of the central government. Given the above accounts, it seems appropriate to view the local governments as the effective landlord, and farmers as tenants to whom cultivation rights have been leased. The local governments are more fundamentally motivated by revenue considerations rather than the welfare of the peasants. Accordingly we shall assume that their main objective is to maximize their revenues net of costs of infrastructural development (i.e., costs of specific investments). The question to be posed concerns appropriate regulations to be implemented by the central government, which is motivated to maximize national welfare, inclusive of the interests of industrial developers, local governments and peasants in equal measure.

3 Model

There is an asset or plot of land whose productivity depends on two inputs: $x$ and $y$. The land is owned by a landlord $L$ who provides input $x$ at a personal cost of $c_L(x)$, and is operated by a tenant $T$ who provides input $y$ at a personal cost of $c_T(x)$. These inputs are noncontractible. In the incomplete contract (IC) model, they are mutually observed prior to contract renegotiation. In the MHLL model these are not observed except by the concerned investor. Our version of the IC model will pertain to an urban real estate setting, where the asset generates a deterministic return of $f(x, y)$. And the MHLL model will pertain to the agricultural setting, where output is stochastic: it is 1 with probability $f(x, y)$ and 0 otherwise.

For most part, we shall consider the example of a linear, separable pro-
duction function \( f(x, y) = \alpha x + \beta y \), and quadratic costs \( c_L(x) = \frac{x^2}{2} \), \( c_T(y) = \frac{y^2}{2} \). Utility is transferable, so the payoff of each agent equals share of the output less costs incurred. The tenant is wealth constrained: he has zero wealth and zero autarkic payoff. The landlord is not wealth constrained.

The sequence of moves is as follows. We first consider the setting where there is no prospect of sale of the asset to a third party.

At \( t = 0 \), an \textit{ex ante} contract is signed. At this stage, there is a single \( L \) and many potential tenants, all identical to \( T \). Hence at this stage \( L \) will have all the bargaining power. An \textit{ex ante} contract in the real estate setting involves \( \bar{q}, \bar{r} \), where \( \bar{q} \) is the probability that \( T \) will be allowed to stay \textit{ex post} in the property upon payment of fixed rent \( \bar{r} \). In the agricultural setting it involves a share \( s \) of the output that accrues to \( T \), the remaining accruing to \( L \).

After the contract is signed, at \( t = 1 \) both agents select their respective investments.

Then at \( t = 2 \) the \textit{ex ante} contract can be renegotiated (in the IC model). The investment of each agent is characterized by specificity parameter \( \delta \in [0, 1] \). The renegotiated contract is formed on the basis of Nash bargaining between \( L \) and \( T \), with a status quo formed on the basis of the \textit{ex ante} contract. In this status quo, with probability \( 1 - \bar{q} \) the landlord can evict the tenant and replace him with another tenant. The latter will be able to generate an output of \( f(x, \delta y) \). Since \( L \) owns the asset, he will then be able to charge the new tenant a rent equal to \( f(x, \delta y) \).

In the MHLL model there is no scope for such renegotiation. The \textit{ex ante} contract is implemented.

We now explain how this setting is modified by the possibility of sale of the asset to a third party \( I \). At \( t = 2 \), with probability \( \theta \), \( I \) arrives and wants to purchase the asset for industrial development which yields a return of \( V \) to \( I \). In that case \( T \) is evicted and there is no agricultural production possible. We shall assume \( V \) is deterministic and common knowledge amongst \( L \) and \( I \), so they will bargain under complete information. In the event of a sale at price \( P \), the landlord can offer a compensation to \( T \) equal to \( \sigma.P + T \). (In the last section we shall consider a setting where compensation is on the basis of agricultural production or income foregone, rather than a part of the sale price received by \( L \)). The parameters \( \sigma, T \) are part of the \textit{ex ante} contract, apart from the share \( s \) of the crop in the event that no \( I \) arrives. Or this may be subject to a legal mandate. We shall be considering contexts where \( V \) is ‘large’ relative to agricultural productivity, so that efficiency will dictate that the land be converted to industrial use. The only relevant consideration
is how this affects incentives for agricultural development.

Note that this version of the IC model allows $L$ to make commitments via the \textit{ex ante} contract, which forms the basis of subsequent contract renegotiation. This is similar to the models by Edlin and Reichelstein (1996) and Che and Hausch (1999) which extended the GHM framework to allow for some \textit{ex ante} contracting. However, there may be restrictions on what courts will \textit{ex post} enforce. If courts are biased in favor of $L$ — as is perhaps appropriate in the Chinese context where the courts are controlled by local governments and the peasants lack legal rights to the land or to compensation — the only feasible value of $\bar{q}$ equals zero. Then that is exactly the GHM model: $L$ cannot commit to not evicting $T$ \textit{ex post}. This damages $T$'s investment incentives below the first-best level, while generating first-best incentives to $L$. If $T$'s incentives involve no specificity at all ($\delta = 0$), $T$ has zero incentives. This is exactly the same as in a sharecropping contract with a share $s = 0$ for the tenant.

On the other hand, if courts are biased in favor of $T$ — owing to the sway of populist sentiment or considerations of protecting the poor — the only feasible value of $\bar{q}$ is 1. This may be more appropriate in the case of West Bengal where the state government has been controlled by a Left-front coalition over the past three decades, which implemented a regulation to protect tenurial rights of sharecroppers. In that case $T$ is the effective owner, and thus ends up with first-best incentives, while $L$ ends up with zero incentives. This corresponds to the case of sharecropping where $s = 1$.

More generally, if $\bar{q}$ is a number between 0 and 1, it allows $L$ to make intermediate degrees of commitment. These are similar to providing $T$ with an intermediate share of the returns. Indeed, it can be checked that (provided $\delta = 1$) it is exactly equivalent to the sharecropping model with a share $s = \bar{q}$. \textit{In terms of its implications for investment and efficiency, the IC model with zero asset specificity is equivalent to the MHLL model.} We may as well thus consider the MHLL model with this interpretation in mind.

Another reason for considering the MHLL model is that the incentive effects there are not driven by a lack of commitment. The fact that with zero commitment $L$ cannot provide $T$ with any incentives provides a transparent argument in favor of mandating sales compensation, which substitutes for $L$’s lack of commitment power. $L$ cannot commit to providing any sales compensation as well for the same reason as he cannot commit to letting the tenant stay and enjoy the fruits of his investment. A legal mandate for sale compensation then gives some stake to the tenant, promoting his investment incentive. If the tenant’s incentive problem is strong relative to the landlord, this will not only promote productive efficiency but may also

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benefit the landlord. It amounts to a commitment device.

In the MHLL model, in contrast, there is no scope for lack of commitment. If providing sales compensation to the tenant is in the landlord’s interest, the latter will do so through an \textit{ex ante} contract. This is commonly observed in the form of employee compensation plans in firms which offer severance payments in the form of company stock to workers if the firm happens to be sold to a third party. So possible welfare benefits for mandated sales compensation are not due to any commitment role they provide. And as we shall see, \( L \) has two instruments of providing incentives: through the share of the crop offered to \( T \), and the offer of sales compensation to \( T \) in the event of a sale to a third party. One needs to understand the interplay between these two instruments of incentivization, and between the corresponding legal regulations of these.

4 Sharecropping with Two-Sided Investment and No Sale Possible

In this section we explain how the standard MHLL model of sharecropping extends to the context of a two-sided investment. The standard model has been developed by most authors for the case where only the tenant has to be provided incentives. Our context necessitates an incentive problem for the landlord as well. We abstract from the issue of land conversion in this section, by assuming \( \theta = 0 \). This will set the stage for the rest of the paper which will subsequently extend the analysis of this section to the case where \( \theta \) is positive, and then focus on the land conversion issue.

In the absence of any incentive problems, the first-best allocation will solve

\[
\max_{\{x,y\}} S(x, y) = \alpha x + \beta y - \frac{x^2}{2} - \frac{y^2}{2}
\]

which yields the first-order conditions

\[
x = \alpha \\
y = \beta.
\]

Since \( f(x, y) = \alpha x + \beta y \) is a probability, for an interior solution, we assume

\[
\alpha^2 + \beta^2 < 1.
\]

The first-best social surplus is

\[
S^{**} = \frac{1}{2} (\alpha^2 + \beta^2)
\]
Suppose now that both $x$ and $y$ are both subject to moral hazard. In addition $T$ has zero wealth and is liquidity-constrained: his income in all states of the world cannot be negative. A contract is a pair $(s, w)$ where $s$ is $T$’s share of incremental output, and $w$ is a flat wage component. Since output takes two values and is verifiable (as opposed to the inputs, which are unobservable and hence unverifiable), this is the most general form of a contract. $L$’s payoff is 

\[ (1 - s) f(x, y) - w - \frac{1}{2} x^2 \]

and $T$’s payoff is 

\[ sf(x, y) + w - \frac{1}{2} y^2. \]

Since both $x$ and $y$ are unobservable, the incentive-compatibility constraints (ICs) are 

\[ x = \arg\max \left\{ (1 - s) f(x, y) - w - \frac{1}{2} x^2 \right\} = (1 - s) \alpha \]

\[ y = \arg\max \left\{ sf(x, y) + w - \frac{1}{2} y^2 \right\} = s \beta. \]

The optimal contract maximizes the payoff of either $L$ or $T$ subject to the two incentive-compatibility constraints (ICs), the participation constraint (PC) of the other party, and the limited liability constraint (LLC) that requires $w \geq 0$. Let $u$ denote the reservation payoff of $T$. In the current setting it can be treated as exogenous. We treat it as a parameter to illustrate how the selected sharecropping contract depends on $u$. The allocation of use-rights between $L$ and $T$ will affect $u$, and thus affect incentives, as shown in Mookherjee (1997). Specifically, if $T$ owns the rights then $u$, the autarkic option for $T$ is raised: consequently the effect of shifting use-rights from $L$ to $T$ correspond to an increase in $u$. In subsequent sections of the paper, however, we shall abstract from the allocation of use-rights between $L$ and $T$, and assume that they rest with $L$, so that $u$ will be set equal to zero.

We now proceed to characterize the second-best contract chosen by $L$. 

\[
\max_{\{s,w\}} (1 - s) f(x, y) - w - \frac{1}{2} x^2 
\]

subject to 

\[ x = (1 - s) \alpha \]
\[ \begin{align*}
\frac{y}{2} &= s\beta \\
\frac{sf(x, y)}{2} + w - \frac{y^2}{2} &\geq u \\
w &\geq 0.
\end{align*} \]

Substituting the values of \( x \) and \( y \) from the ICs throughout, we can rewrite this in a more compact form:

\[ \max_{\{s, w\}} \frac{1}{2}(1 - s)^2 \alpha^2 + s(1 - s) \beta^2 - w \]

subject to:

\[ \frac{1}{2} s^2 \beta^2 + s(1 - s) \alpha^2 + w \geq u \\
w \geq 0. \]

The nature of the second-best contract for various possible values of \( u \) is described below.

**Proposition 1** Consider the MHLL model with \( \theta = 0 \), and \( u \) the autarkic payoff of \( T \). The nature of the sharecropping contract chosen by \( L \) is as follows. There exists \( \bar{u} > u > 0 \) such that \( s = \frac{\beta^2}{\beta^2 + \alpha^2} \) for \( u \geq \bar{u} \) and \( w > 0 \), \( s = \hat{s}(u) \) where \( \hat{s}(\cdot) \) is strictly increasing and \( w = 0 \) for \( u \in [\bar{u}, \bar{u}] \); and \( s = \hat{s} \) and \( w = 0 \) for \( u \in [0, \bar{u}] \).

See Figures 1 and 2 for a depiction of the optimal share as a function of \( u \) and the corresponding utility possibility frontier. The chosen share is initially rising in \( u \); over this part of the parameter space the LL constraint is binding. The landlord responds to a rise in \( u \) by giving the tenant a larger share of the output. This raises the tenant’s incentives and lowers the landlord’s incentives. The net effect is a rise in social surplus, as the landlord tends to award the tenant ‘too low’ a share from a social standpoint. Once \( u \) is sufficiently high, the tenant is given a fixed and high share of \( s^* \equiv \frac{\beta^2}{\beta^2 + \alpha^2} \), which maximizes social surplus. Over this range the LL constraint is not binding; there is a fixed rent component which is varied in response to changes in \( u \) while the share is held fixed. This share depends on the relative significance of the tenant’s and landlord’s respective incentive problems (i.e., their relative productivity, since their investment costs are similar).

Proposition 1 implies reallocations of use rights in favor of the tenant would raise social surplus by raising \( u \), if the latter were low enough to start with. Regulations of the sharecropping contract (e.g., which mandate...
a minimum share $\tilde{s}$ accruing to the tenant) will raise social surplus if $s < \tilde{s} < s^\ast$. However, if $s > s^\ast$ then social surplus may decline as a result of the regulation: here the reduction in the landlord’s net return induces a reduction in the landlord’s investment which outweighs the increase in the tenant’s investment. Indeed, it is possible even that the regulation if implemented may lower the tenant’s return also — in which case the implementation of the regulation is in neither party’s interest. This may explain why a significant fraction of tenants in West Bengal chose not to register their tenancy contracts following the implementation of Operation Barga.

5 Sales-Price-Based Compensation

Now we turn to the primary case of interest: where $\theta > 0$. We keep with the MHLL model for the time being, and in due course explain how related results arise for the IC model.

In this section we consider the case where only the sale price $P$ for the land (when it is sold) is observable publicly, so compensation can be conditioned on $P$. We assume the compensation is linear in $P$. This is a useful simplifying assumption which may be restrictive in the current context, though not in an extended setting.\footnote{For instance, $L$ could offer instead an option contract where the compensation is a fixed fraction of the excess of the sale price over some target level. We are grateful to Rocco Machiavello for this observation. Such a contract will enable $L$ to generate a comparable incentive effect for $T$, without having to give up the same amount of surplus from the sale. However, the choice of the target price will depend on $V$. If there is a continuous distribution over $V$, such an option contract will not provide comparable incentives when the realized value of $V$ is low. In order to extract a similar incentive effect for all values of $V$, the target price will have to be progressively lowered, which converges to a linear contract if it is lowered sufficiently. However, detailed analysis of the case of a continuous distribution over $V$ is needed in future extensions of the current analysis.} Effectively, $L$ offers (or may be required to offer) some “shares in the assets of the enterprise” to $T$, which the latter can encash in the event of the “enterprise” being sold to a third party. And in the event of the “enterprise” not being sold, $L$ continues to offer a sharecropping contract to $T$.

5.1 Where $L$ Decides Compensation Policy

We first consider the case where $L$ commits to a compensation policy as part of the ex ante contract with $T$. Then the overall policy consists of these two parts: a sharecropping contract in the event of no sale, and a
share in the sale proceeds in the event of a sale. The former is represented by a fixed share \(s\) of the agricultural output, and a fixed rent of \(w\). The latter is represented by a payment of a sales compensation of \(\sigma P + T\) if the sale occurs. Using \(t\) to denote \(-w\), the policy is represented by four parameters \(s, t, \sigma, T\).

Since the tenant has zero wealth, the limited liability constraint imposes \(t, T \geq 0\). We also restrict \(s, \sigma\) to lie in \([0, 1]\), though these will not turn out to be binding. Since \(T\) has zero autarkic utility it is clear that \(t = T = 0\): paying any fixed amount to \(T\) has no incentive effects. So neither \(L\) nor a utilitarian social planner will want to make positive lump sum payments to \(T\).

For now we will assume laissez faire and see what commitments \(L\) would like to make in his own self-interest. Later we shall examine the effect of regulations concerning compensation policy.

When \(I\) arrives, a sale takes place if there exists sale price \(P\) such that \(V \geq P \geq \frac{\pi + T}{1 - s}\), where \(\pi \equiv (1 - s)(\alpha x + \beta y) - t\).

With the sale price \(P\) determined via Nash bargaining between \(L\) and \(I\), the ex ante payoffs of \(L\) and \(T\) are respectively:

\[
\Pi_L = [(1 - \theta)(1 - s) + \theta \frac{1 - \sigma}{2 - \sigma}(1 - s)][\alpha x + \beta y] + \theta \frac{1 - \sigma}{2 - \sigma} V - \frac{x^2}{2} \tag{1}
\]

\[
\Pi_T = [s(1 - \theta) + \theta \frac{\sigma}{2 - \sigma}(1 - s)][\alpha x + \beta y] + \theta \frac{\sigma}{2 - \sigma} V - \frac{y^2}{2} \tag{2}
\]

It is evident from (2) that a higher \(s\) increases \(T\)'s incentives only if \(1 - \theta - \theta \frac{\sigma}{2 - \sigma} > 0\), or \(\theta < 1 - \frac{\sigma}{2}\). If this condition is not satisfied, i.e., \(\sigma \geq 2(1 - \theta)\) then it is optimal for \(L\) to set \(s = 0\). A higher value of \(s\) reduces \(\pi\), thus reduces the sale price, which makes \(L\) worse off. The reduction in the sale price causes \(T\)'s incentive to go down as he receives a large portion of the sale price. In this case \(T\) can be incentivized solely through a positive \(\sigma\) — the “dot-com” employee policy where they are paid zero wages and incentivized via company stock which will yield returns to the employees when the company is bought by someone else.

**Lemma 2** It is never optimal for \(L\) to select \(\sigma \geq 2(1 - \theta)\).

Hence \(L\) will not want to raise \(\sigma\) so high that a higher crop share actually disincentivizes the tenant. It follows that over the relevant range \(\sigma < 2(1 - \theta)\), then, a higher share will increase the tenant’s effort incentive.
Using the change of variable $\mu = \frac{1}{2 - \sigma}$, we now restrict attention to the range $\mu \in [\frac{1}{2}, \min\{\frac{1}{2\theta}, 1\})$ and $s \in [0, 1]$. The landlord then selects $\mu$ and $s$ in these ranges to maximize

$$\Pi_L = \frac{1}{2} \alpha^2 (1-s)^2 (1-\mu\theta)^2 + \beta^2 (1-s)(1-\mu\theta)[s(1-\theta) + (2\mu - 1)\theta(1-s)] + \theta(1-\mu)V$$

subject to the constraint that the sale goes through, i.e.,

$$V \geq (1-s)\frac{\mu}{1-\mu} [\alpha^2 (1-s)(1-\mu\theta)] + \beta^2 \{s(1-\theta) + (2\mu - 1)\theta(1-s)]\}$$

The following Lemma is a useful step in our subsequent analysis. In what follows we restrict ourselves to the case where $\beta \geq \frac{\alpha^2}{2}$.

**Lemma 3** Assume T’s incentive problem is important relative to L’s in the sense that $\beta \geq \frac{\alpha^2}{2}$. Then $\frac{\partial^2 \Pi_L}{\partial \mu^2} < 0$ over the range $\mu \in [\frac{1}{2}, \min\{\frac{1}{2\theta}, 1\})$ and $s \in [0, 1]$. Moreover, $\mu^* > \frac{1}{2}$ implies $\frac{\partial^2 \Pi_L}{\partial \mu \partial s} < 0$ for all $\mu \leq \mu^*, s \leq s^*$ in this range.

The first part of this says that the landlord’s payoff is concave in $\mu$, so we can use first order conditions to characterize the optimal $\mu^*$. The second part shows that over the relevant range incentivizing the tenant via a higher crop share $s$ and a higher sales compensation $\mu$ are “strategic substitutes”. This property does not hold globally, e.g., when $s$ is high enough.\footnote{The two instruments are strategic substitutes from the standpoint of determining $y^*$. But they also have a direct effect on $\Pi_L$, and for a fixed $y$ they are ‘strategic complements’ in their effect on the landlord’s profits. So there is a conflict between the effect on the tenants incentives, and that on the landlord profits. The lemma shows that the former effect prevails over the range of shares less than the chosen levels.}

This latter property provides a necessary condition for positive sale compensation, i.e., for $\mu^* > \frac{1}{2}$.

**Proposition 4** Assume T’s incentive problem is important relative to L’s in the sense that $\beta \geq \frac{\alpha^2}{2}$. Then it is optimal for L to set $\sigma^* = 0$ whenever

$$V \geq (2\beta^2 - \alpha^2)(1-\theta^2).$$

Under this condition, $s^* > 0$ if and only if

$$\beta^2 > \alpha^2 \frac{1-\theta}{1-\theta}$$

The following Lemma is a useful step in our subsequent analysis. In what follows we restrict ourselves to the case where $\beta \geq \frac{\alpha^2}{2}$.\footnote{The two instruments are strategic substitutes from the standpoint of determining $y^*$. But they also have a direct effect on $\Pi_L$, and for a fixed $y$ they are ‘strategic complements’ in their effect on the landlord’s profits. So there is a conflict between the effect on the tenants incentives, and that on the landlord profits. The lemma shows that the former effect prevails over the range of shares less than the chosen levels.}
Under conditions (5) and (6), then, the optimal policy entails no sales compensation \( \sigma^* = 0 \) and

\[
s^* = \frac{\beta^2(1 - \theta) - \alpha^2(1 - \frac{\theta}{2})}{2\beta^2(1 - \theta) - \alpha^2(1 - \frac{\theta}{2})}.
\]  

(7)

Hence for large enough \( V \), the landlord will offer no sales compensation at all. The cost of giving up a part of the sale price is too high in that case, relative to the benefits from motivating the tenant to invest more in the land. Given zero sales compensation, it pays to offer the tenant a positive share only if the latter’s incentive problem is strong enough relative to the landlord’s, as expressed by condition (6).

Using the preceding result, the following partial characterization can be given of the landlord’s optimal policy for different parameter ranges.

**Proposition 5**  
(a) If \( \beta = \alpha \) then \( \sigma^* = 0 = s^* \).

(b) If

\[
\frac{\alpha^2(1 - \theta)}{1 - \theta} > \beta^2 > \alpha^2
\]

then \( \sigma^* = 0 = s^* \) if (5) is satisfied. If (5) does not hold, and

\[
\alpha^2(1 - \frac{\theta}{2}) < V < (2\beta^2 - \alpha^2)(1 - \frac{\theta}{2})
\]

then \( \sigma^* > 0, s^* = 0 \).

(c) If

\[
\beta^2 > \frac{\alpha^2(1 - \theta)}{1 - \theta}
\]

and

\[
V \geq (2\beta^2 - \alpha^2)(1 - \frac{\theta}{2})
\]

then \( \sigma^* = 0, s^* = \frac{1 - \gamma}{2 - \gamma}, \) where \( \gamma \equiv \frac{\alpha^2 - \theta}{\beta^2(1 - \theta)} \in (0, 1) \).

(d) If (10) holds and

\[
\frac{\beta^2}{(2 - \gamma)^2}[(1 - \theta)\gamma + \frac{1 - \gamma}{2 - \gamma}] < V < \frac{\beta^2}{(2 - \gamma)^2}
\]

then \( \sigma^* > 0 \).
Case (a) and the first part of (b) give conditions for \( L \) to provide \( T \) with no incentives at all. The second part of (b) gives conditions for the ‘dot-com’ strategy to be optimal. Case (c) provides conditions for the opposite: a positive share and no sales compensation (this repeats the result of the previous Proposition, and is included here only for completeness). Part (d) provides a general condition for positive sales compensation to be provided. We have not yet identified a case where \( L \) uses both instruments of incentivizing \( T \) simultaneously.

Figure 3 illustrates the chosen policies.

5.2 The Welfare Benefits of Regulating Sales Compensation

The utilitarian expression for welfare is simply

\[
W = (1 - \theta)[\alpha x + \beta y] + \theta V - \frac{x^2}{2} - \frac{y^2}{2}
\]  

(13)

where incentives are a function of \( s \) and \( \sigma \) as follows:

\[
y^* = [s(1 - \theta) + \theta \frac{\sigma}{2 - \sigma}(1 - s)]\beta, x^* = (1 - s)(1 - \theta + \theta \frac{1 - \sigma}{2 - \sigma})\alpha
\]  

(14)

and the parameters are such that a sale takes place whenever \( \tilde{V} = V \).

The first-best efforts are

\[
y^{FB} = \beta(1 - \theta), x^{FB} = \alpha(1 - \theta)
\]  

We continue to focus on the situation where \( \beta > \frac{\alpha}{2} \), so the results of the preceding section apply. Let us consider the case where \( V \) is large (in the sense that (5) is satisfied), so \( L \) offers no sales compensation. We shall consider the implications of a legal rule which requires a small positive level of sales compensation. We first consider the context where \( T \)’s incentive problem is not strong enough relative to \( L \) so that the optimal policy for \( L \) is to choose \( s^* = 0 \) as well.

**Proposition 6** Suppose (8) holds, so \( L \) selects \( s^* = 0 \). Then \( T \) under-invests and \( L \) over-invests relative to the first-best. Mandating a small rise in \( \sigma \) from the level \( (\sigma^*) \) currently chosen by \( L \) will result in a reduction in the extent of underinvestment for \( T \) and overinvestment for \( L \), so welfare will rise.
Note here that the prospect of a sale to I motivates L to over-invest in this case: the motive is to manipulate the sale price upward by investing more than the first-best level. This raises the agricultural value of the land, raising the opportunity cost to L of selling to I. A legal mandate to provide tenants with sales compensation reduces the incentive to L to over-invest, while providing T with an incentive to select positive effort. On both counts welfare rises.

Consider next the case where V is high enough that L does not offer any sales compensation to T in the absence of a legal mandate, but T’s incentive problem is strong enough that it pays L to offer T a positive share of the crop.

**Proposition 7** Suppose case (c) of Proposition 5 applies, so in the absence of a legal mandate L chooses $s^* > 0 = \sigma^*$. Suppose also that

$$\beta^2 \theta^2 > \beta^2 (1 - \theta) - \alpha^2 (1 - \theta^2).$$

(15)

Then T underinvests and L over-invests relative to the first-best. A legal mandate to raise \( \sigma \) from 0 will improve welfare if it is accompanied by a rule that L cannot react by reducing the tenant’s share $s$. If it is not accompanied by such a rule, the mandate to raise $\sigma$ above 0 will reduce welfare (by inducing L to reduce $s$ below $s^*$ which more than outweighs the benefits of a rise in $\sigma$.)

Care must therefore be taken to avoid L’s reacting to a mandate of positive sales compensation by reducing the tenant’s cropshare. Otherwise, welfare will fall!

It may be noted that in the case of a one-sided investment problem ($\alpha = 0 < \beta$) where $V$ is ‘large’, case (c) of Proposition 5 does apply. In this case we do not need condition (15) as there is no need to worry about effects on L’s investment. Moreover, it is easy to check in this case that T will always under-invest under laissez-faire. Hence the conclusion of Proposition 7 always applies to the context of one-sided investment problems. With $V$ large, L will not voluntarily offer any sales compensation to T. But a legal mandate to offer such compensation will reduce the under-investment, provided of course L cannot reduce $s$ in response.

5.3 Interactions between Sharecropping and Sales Compensation Regulations

Land reforms of the West Bengal Operation Barga variety happen to regulate $s$ rather than $\sigma$. So the question that may arise concerns how the welfare
benefit of legally mandated compensation rules vary with the mandated $s$. If tenants are already protected via legally mandated floors on $s$, does that reduce the argument for mandated sales compensation?

This question bears also on the case where $L$ faces problems in committing to not evict tenants, owing to $L$-biases of courts that enforce tenancy contracts. In our IC model, it is easily checked that the case of $L$-biased courts is a special case of the current model with $s$ fixed at 0: $L$ cannot commit to letting $T$ stay and enjoy the fruits of his investments. The case of $T$-biased courts corresponds to $s$ fixed at 1: here courts will not allow tenants to be evicted. How does the argument for sales compensation very between these two cases?

Intuitively, one expects the argument for mandated sales compensation to become weaker as the mandated $s$ rises. In other words, the two instruments of policy — mandated $s$ and mandated $\sigma$ — are also strategic substitutes from an efficiency standpoint. We show a variety of senses in which this is indeed true.

Consider first the interpretation of varying enforcement constraints as corresponding to the mandated $s$ changing from 0 to 1. Note that if $s$ is fixed at 0, the benefits of a sales compensation mandate are exactly as described in Proposition 6. On the other hand, if $s = 1$ then $T$ invests the first-best level, while $L$ does not invest at all. Moreover, it is easily checked that $\frac{\partial \pi_L}{\partial \mu} = -V$ in this case, so $L$ will select $\sigma^* = 0$. But with $s = 1$, $y^* = y^{FB}$, $x^* = 0$ and a sales compensation mandate will have no effect at all.

Notice also that if there is a mandated floor to the share $\underline{s}$ which is small, the optimal response of $L$ will continue to induce under-investment for $T$ and over-investment for $L$. The extent of these distortions will be smaller, the larger $\underline{s}$ is. Hence the welfare benefits of a given increase in $\mu$ will then be smaller. Moreover, since $y^* = \underline{s}(1-\theta) + \theta(2\mu-1)(1-\underline{s})\beta$, $x^* = (1-\underline{s})(1-\mu\theta)$, a rise in $\underline{s}$ means a given rise in $\mu$ translates into smaller investment effects for both parties, and the distortions are reduced at a slower pace. So over this range the welfare benefits of a given rise in $\mu$ are indeed smaller, as the mandated share for the tenant rises.

6 Output-Based Compensation

An alternative basis for compensation of the tenant in case of the land being sold to $I$, is to base it on what the tenant lost, which is $sf(x, y)$. This is an alternative basis for compensation: the income or agricultural
output lost, rather than the sale price received by $L$. We now investigate the efficiency implications of the tenant being compensated a fixed proportion $c$ of the income loss $sf(x, y)$. We do not study the nature of output-based compensation that $L$ may wish herself to offer ex ante. Instead we take the mandated compensation proportion $c$ as determined by the regulation and examine the welfare implications of varying $c$. A later version of this paper will also contract the welfare optimal choice of $c$ with what $L$ may choose to offer voluntarily.

In the event of a sale the landlord receives $P - csf$ and his opportunity cost in terms of his share of agricultural output is $(1 - s)f$. Therefore, his net payoff is $P - csf - (1 - s)f = P - f\{1 - s(1 - c)\}$. Assuming Nash bargaining, $P$ is given by

$$P = \frac{V + f\{1 - s(1 - c)\}}{2}.$$ 

The condition for trade to go through is, $V > P$ which is equivalent to $V > f\{1 - s(1 - c)\}$. It is evident from this that the land conversion decision is ex post efficient if and only if $c = 1$, i.e., the tenant must be compensated in full. If $c$ is less (resp. greater) than one, the tenant is under-compensated (resp. over-compensated) and this gives rise to a socially excessive (resp. insufficient) rate of conversion.

Nevertheless we are interested in efficiency implications, after accounting for effects on ex ante investments as well.

**Proposition 8** Suppose $\beta \geq \alpha$, $\theta < \frac{2}{3}$, and tenant compensation is constrained to be a constant fraction $c$ of agricultural income foregone by the tenant in the case of sale of the land. If the investment problem is one-sided ($\alpha = 0$) or if $\frac{2(3\beta^2 - \alpha^2)}{3\beta^2 - 2\alpha^2} < \theta$ holds, welfare is strictly increasing in $c$ for all $c \leq 1$. Hence under these circumstances the optimal regulation involves over-compensation.

If the investment problem is only on the tenant’s side, a rise in the compensation proportion will reduce the extent of under-investment. It will also reduce ex post allocative distortions resulting from a socially excessive rate of conversion of the land to industrial purposes. On both counts, therefore, there is an improvement in efficiency. The argument is more complicated when there are incentive problems on both sides, since a rise in $c$ will lower the landlord’s ex ante investment incentives. The conditions provided ensure that the landlord over-invests, in which case the reduction in the landlord’s
investment constitutes a third source of efficiency improvement. The proposition therefore provides sufficient conditions for under-compensation to be suboptimal. We suspect that the same result holds for a wider set of cases as well.

7 Concluding Comments

In this paper we have provided an analysis of compensation policy for farmers displaced by the process of industrialization. In a world without frictions, complete contracts will take care of investment incentives and industrialization will only occur when the net social benefits exceed the net social costs. In the presence of contracting frictions (moral hazard and/or commitment problems) and absence of transferable utility (due to limited wealth and credit market frictions) this is unlikely to hold. We have shown that land compensation policies can typically be designed that will be beneficial from the point of view of both efficiency and equity. Socially optimal policies will typically not be voluntarily adopted by individual economic agents, thereby highlighting the need for a clear policy framework for regulation of exchange rights and compensation of displaced agents.

Our analysis was based on a very simple model, which abstracted from a number of significant issues. Even within the context of this model, some ranges of cases remain to be explored more thoroughly — e.g., where the possible industrial values $V$ form a continuous distribution, or where there is asymmetric information concerning these valuations.

More significant perhaps is the need to extend the analysis to the case of many farmers. It is quite often the case that an industrial project necessitates a large land area, where many farmers are involved. So it is important to consider an extension to the case where there are many farmers, and one industrialist. In the case where compensation is based on the sale price, it is necessary to allow each farmer to negotiate separately with the industrialist, in order to provide appropriate incentives to individual farmers. But such a framework of decentralized bargaining will create the problem of *hold-out*: each farmer will hold-out for a higher price as they will neglect the consequences on other farmers if the overall deal does not go through. Then there will be a tendency to under-industrialize. To get around this problem, it is important for local governments to use rights of eminent domain, and structure compensation arrangements for displaced farmers. This is exactly how the process operates in China and India.

In the design of the compensation, it is important to base compensation
to each farmer based on the evaluation of agricultural productivity on their respective plots. Basing compensation on average yields in the area will remove the incentive advantages of compensation, apart from creating inequitable treatment of heterogenous plots and farmers. This complicates the administration of the process of compensation considerably. Given farmer heterogeneity, some of which is unobserved, also generates problems of asymmetric information wherein some farmers will claim that they are damaged more than the estimated yields suggest. So the kind of tensions witnessed recently are inescapable.

However, our analysis suggests that compensation arrangements are necessary that provide sufficient compensation for most of the displaced farmers. On average at least the compensation has to be sufficient. Perhaps some kind of majority or supermajority approval should be necessary from those displaced. This is not only the result of the logic of economic efficiency but also distributive justice and political sustainability of the industrialization process. In addition the process of compensation needs to be administered in a transparent manner. To gain credibility, it is necessary to delegate tasks of valuation and compensation determination to a regulatory authority that operates at arms length from the local government. If a significant fraction of the affected farmers feel that the compensation is insufficient, they should have the right to appeal to independent judicial authorities. A more detailed analysis of these issues will have to await further research.

Finally, it would be interesting to obtain empirical estimates of the extent of agricultural income loss experienced by displaced farmers in recent Chinese and Indian experiences, and assess the adequacy of the compensation that has been provided. This will help assess the extent of undercompensation that has taken place in practice.

References


APPENDIX: PROOFS

Proof of Proposition 1:
If the LLC is not binding, the PC must be binding, because otherwise $w$ can be reduced and $T$ would be better off. First, let us consider this case. Substituting $w$ from the binding PC the problem reduces to

$$\max_s \pi(s) = \frac{1}{2} (1 - s)^2 \alpha^2 + s(1 - s) (\alpha^2 + \beta^2) + \frac{1}{2} s^2 \beta^2 - u.$$  

This yields the following first-order condition

$$-(1 - s)\alpha^2 + s\beta^2 + (1 - 2s) (\alpha^2 + \beta^2) = 0.$$  

It is a globally concave problem, as $\pi''(s) = -(\alpha^2 + \beta^2) < 0$. Therefore, the interior solution is

$$s^* = \frac{\beta^2}{\alpha^2 + \beta^2}.$$  

Plugging into the objective function,

$$\pi^* = \frac{1}{2} (\alpha^2 + \beta^2) - \frac{\alpha^2 \beta^2}{2(\alpha^2 + \beta^2)} - u.$$  

The social surplus gross of the reservation payoff of $T$ is

$$S^* = \frac{1}{2} (\alpha^2 + \beta^2) - \frac{\alpha^2 \beta^2}{2(\alpha^2 + \beta^2)}.$$  

Now we turn to the case where the limited liability constraint is binding. In this case $w = 0$ and the optimal contracting problem reduces to

$$\max_{\{s\}} \pi(s) = \frac{1}{2} (1 - s)^2 \alpha^2 + s(1 - s) \beta^2$$  

subject to:

$$\frac{1}{2} s^2 \beta^2 + s(1 - s) \alpha^2 \geq u.$$  

Ignoring the PC, if $L$ maximized his payoff with respect to $s$, the first-order condition would be:

$$-(1 - s)\alpha^2 + (1 - 2s)\beta^2 \leq 0.$$  

$$s = \min \left\{ 0, \frac{\beta^2 - \alpha^2}{2\beta^2 - \alpha^2} \right\}$$
An interior solution exists if $\beta^2 > \alpha^2$. In this case, the second-order conditions are satisfied, as $\pi''(s) = \alpha^2 - 2\beta^2$. Also, in this case, $T$'s expected payoff is

$$
\frac{1}{2} \left( \frac{\beta^2 - \alpha^2}{2\beta^2 - \alpha^2} \right)^2 \beta^2 + \frac{\beta^2 - \alpha^2}{2\beta^2 - \alpha^2} \frac{\beta^2}{2\beta^2 - \alpha^2} \alpha^2
$$

$$
= \beta^2 \frac{\beta^2 - \alpha^2}{(2\beta^2 - \alpha^2)^2} \left( \alpha^2 + \frac{\beta^2 - \alpha^2}{2} \right)
$$

$$
= \frac{1}{2} \beta^2 \frac{\beta^2 - \alpha^2}{(2\beta^2 - \alpha^2)^2} = u.
$$

For $0 \leq u \leq u_0$ the optimal contract is as characterized above. The expected payoff of $L$ is

$$
\frac{1}{2} \left( \frac{\beta^2 - \alpha^2}{2\beta^2 - \alpha^2} \right)^2 \alpha^2 + \frac{\beta^2 - \alpha^2}{2\beta^2 - \alpha^2} \frac{\beta^2}{2\beta^2 - \alpha^2} \alpha^2
$$

$$
= \frac{\beta^4}{(2\beta^2 - \alpha^2)^2} \left( \beta^2 - \alpha^2 + \frac{\alpha^2}{2} \right)
$$

$$
= \frac{1}{2} \frac{\beta^4}{(2\beta^2 - \alpha^2)^2}.
$$

If $\beta^2 \leq \alpha^2$ then we have a corner solution: $s = 0$. In this case, $T$’s expected payoff is 0 which clearly violates the PC as long as $u > 0$.

Even when $\beta^2 > \alpha^2$, for $u > u_0$ the PC will be violated. In this case, $s$ will be the solution to the binding PC

$$
\frac{1}{2} s^2 \beta^2 + s(1 - s) \alpha^2 = u.
$$

Let us denote the left hand side by $u(s)$. Clearly, $u(0) = 0$. It is readily verified that

$$
u'(s) = s \beta^2 + (1 - 2s) \alpha^2 = \alpha^2 + (\beta^2 - 2\alpha^2) s
$$

$$
u''(s) = \beta^2 - 2\alpha^2.
$$

Let us denote the value of $s$ that satisfies $u(s) = u$ and yields the highest expected payoff to $L$ (in case several values of $s$ satisfy $u(s) = u$) as $\hat{s}(u)$.

Recall that $s^*$ is chosen for any value of $u$ when $T$ is not liquidity-constrained (or the LLC is not binding). Suppose in the present case that
$u$ is such that $T$ has to be given $s^*$ when there is a binding LLC, and a binding PC. Formally, let $\bar{u}$ be such that:

$$\frac{1}{2} (s^*)^2 \beta^2 + s^*(1-s^*)\alpha^2 = \bar{u}.$$  

We claim that if $u$ is higher than $\bar{u}$ (say, $\bar{u} + \varepsilon$ where $\varepsilon > 0$), $T$ should still be given $s^*$ since $L$ is not liquidity constrained and can pay $T$ through the flat wage $w$. Suppose not and so there is a $s' > s^*$ such that $T$ gets $u + \varepsilon$ and $L$ gets $\pi'$. Now suppose $T$ is given $s^*$ and a flat transfer $w = \varepsilon$ such that his expected payoff is $u$. This is feasible, as $L$ is not liquidity constrained. Also, then $L$ must then be getting $\pi$ but by the definition of $\pi$ it must be higher than $\pi'$.

There are several cases two consider.

If $\beta^2 > 2\alpha^2$ then $u(s)$ is increasing and convex, and for non-negative values of $s$, reaches a minimum at $s = 0$. In this case, for a given value of $u$ there is a unique value of $s$ that satisfies $u(s) = u$. This is given by

$$\hat{s}(u) = \frac{\sqrt{\alpha^4 + 2u(\beta^2 - 2\alpha^2)} - \alpha^2}{(\beta^2 - 2\alpha^2)}.$$  

It is clear upon inspection that $\hat{s}(u)$ is increasing and strictly concave in $u$.

In contrast, if $\beta^2 < 2\alpha^2$ then $u(s)$ is globally concave, strictly increasing at $s = 0$, reaching a maximum at $s = \frac{\alpha^2}{2\alpha^2 - \beta^2}$ and then decreasing. As a result, there are two non-negative values of $\hat{s}(u)$ that satisfies $u(s) = u$:

$$\frac{\alpha^2 \pm \sqrt{\alpha^4 - 2u(2\alpha^2 - \beta^2)}}{2\alpha^2 - \beta^2}$$

for $0 < u < \frac{\alpha^4}{2(2\alpha^2 - \beta^2)}$ (the upper bound is derived by setting $\sqrt{\alpha^4 - 2u(2\alpha^2 - \beta^2)} = 0$). We show that only the lower root is relevant. The lowest value the higher root can take is when $\sqrt{\alpha^4 - 2u(2\alpha^2 - \beta^2)} = 0$ and $s = \frac{\alpha^2}{2\alpha^2 - \beta^2}$. However, it is easy to verify that

$$\frac{\alpha^2}{2\alpha^2 - \beta^2} > \frac{\beta^2}{\beta^2 + \alpha^2} = s^*$$

as this is equivalent to $\alpha^4 + \beta^4 - \alpha^2 \beta^2 = (\alpha^2 - \beta^2)^2 + \alpha^2 \beta^2 > 0$. Therefore, the higher root will never be chosen and in this case:

$$\hat{s}(u) = \frac{\alpha^2 - \sqrt{\alpha^4 - 2u(2\alpha^2 - \beta^2)}}{2\alpha^2 - \beta^2}.$$
It is clear upon inspection that \( \hat{s}(u) \) is increasing and convex in \( u \).

Finally, if \( \beta^2 = 2\alpha^2 \) then the PC can be written as:

\[
\alpha^2 \{ s^2 + s(1 - s) \} = u
\]

which yields

\[
s = \frac{u}{\alpha^2}.
\]

(This can also be derived from \( \hat{s}(u) \) by applying L’Hôpital’s rule. In particular, let \( 2\alpha^2 - \beta^2 = z \). Then \( \hat{s}(u) = \frac{\alpha^2 - \sqrt{\alpha^4 - 2uz}}{z} \). Both the numerator and denominator go to zero as \( z \) goes to 0 and so by applying standard methods, \( \lim_{z \to 0} \hat{s}(u) = \frac{u}{\alpha^2} \).)

Therefore, we have

\[
\hat{s}(u) = \frac{\sqrt{\alpha^4 + 2u(\beta^2 - 2\alpha^2)} - \alpha^2}{(\beta^2 - 2\alpha^2)} \text{ for } \beta^2 > 2\alpha^2
\]

\[
= \frac{u}{\alpha^2} \text{ for } \beta^2 = 2\alpha^2
\]

\[
= \frac{\alpha^2 - \sqrt{\alpha^4 - 2u(2\alpha^2 - \beta^2)}}{(2\alpha^2 - \beta^2)} \text{ for } \beta^2 < 2\alpha^2.
\]

This completes the proof of Proposition 1.

Proof of Lemma 2: Use the following change of variable \( \mu = \frac{\sigma}{\tau - \theta} \), which is a monotone increasing function of \( \sigma \). \( \mu \) lies in the range \([\frac{1}{2}, 1]\), with \( \frac{1}{2} \) corresponding to \( \sigma = 0 \) and \( \mu = 1 \) if \( \sigma = 1 \). Then \( \mu = \frac{1}{2\theta} \) corresponds to \( \sigma = 2(1 - \theta) \), and we have to show that it is not optimal for \( L \) to raise \( \mu \) above \( \frac{1}{2\theta} \). As explained above in this case its optimal to set \( s = 0 \), and so

\[
\Pi_L = \frac{1}{2} \alpha^2 (1 - \theta \mu)^2 + \beta^2 (1 - \theta \mu) \theta \mu (2 - \frac{1}{\mu}) + \theta (1 - \mu) V
\]

(16)

Now it can be checked directly that \( \Pi_L \) is strictly decreasing in \( \mu \) over the entire range \([\frac{1}{2\theta}, 1]\).

Proof of Lemma 3: From (3),

\[
\frac{1}{\theta} \frac{\partial \Pi_L}{\theta \mu} = (\beta^2 - \alpha^2)(1-s)^2(1-\theta \mu) - V + (1-s)\beta^2[(1-s)(1-\theta \mu) - s(1-\theta) - \theta(1-s)(2\mu - 1)]
\]

(17)
which implies that
\[ \frac{1}{\theta} \frac{\partial^2 \Pi_L}{\partial \mu^2} = -\theta(4\beta^2 - \alpha^2)(1-s)^2 < 0 \] (18)

if \( \beta \geq \frac{\alpha}{2} \). Next, note that
\[ \frac{1}{\theta} \frac{\partial^2 \Pi_L}{\partial \mu \partial s} = -2(\beta^2 - \alpha^2)(1-s)^2(1-\theta \mu) - \beta^2[3(1-s)(1-2\mu \theta) + \theta - s]. \] (19)

Now \( \mu^* > \frac{1}{2} \) implies
\[ \frac{1}{\theta} \frac{\partial \Pi_L(\mu^*, s^*)}{\partial \mu} = -\alpha^2(1-s)^2(1-\theta \mu^*) - V + \beta^2\{2(1-s^*)^2(1-2\mu^* \theta) + (\theta - s^*)(1-s^*)\} \geq 0 \] (20)

which in turn implies
\[ 2(1-s)(1-2\mu \theta) > s - \theta \] (21)
at \((\mu^*, s^*)\). Since the L.H.S. of (21) is decreasing in \( s \) and \( \mu \), while the R.H.S. is increasing in \( s \), it follows that inequality (21) holds for all \( \mu \leq \mu^*, s \leq s^* \). It follows that \( 3(1-s)(1-2\mu \theta) + \theta - s > 0 \) for all \( \mu \leq \mu^*, s \leq s^* \). The result then follows from (19).

Proof of Proposition 4: If \( \mu^* > \frac{1}{2} \) we have \( \frac{1}{\theta} \frac{\partial \Pi_L(\mu^*, s^*)}{\partial \mu} \geq 0 \). Since \( \frac{\partial^2 \Pi_L}{\partial \mu \partial s} < 0 \) for all \( \mu \leq \mu^*, s \leq s^* \), it follows that \( \frac{1}{\theta} \frac{\partial \Pi_L(\mu^*, 0)}{\partial \mu} \geq 0 \). Since \( \Pi_L \) is strictly concave in \( \mu \), we have \( \frac{1}{\theta} \frac{\partial \Pi_L(\frac{1}{2}, 0)}{\partial \mu} > 0 \). From (17) we obtain \( (2\beta^2 - \alpha^2)(1-\frac{\theta}{2}) > V \), contradicting (5).

Given (5), we must have \( \mu^* = \frac{1}{2} \). Now
\[ \frac{\partial \Pi_L(\frac{1}{2}, s)}{\partial s} = -\alpha^2(1-s)(1-\frac{\theta}{2})^2 + \beta^2(1-\frac{\theta}{2})(1-\theta)(1-2s). \] (22)

Hence \( s^* > 0 \) implies \( 1 - \frac{\alpha}{\beta(1-s)} \geq \gamma \), where \( \gamma \equiv \frac{\alpha^2(1-\frac{\theta}{2})}{\beta(1-\theta)} \). Hence \( \gamma < 1 \), which is equivalent to (6). This establishes necessity of (6) for \( s^* > 0 \). For sufficiency note that from (22), \( \frac{\partial \Pi_L(\frac{1}{2}, 0)}{\partial s} = (1-\frac{\theta}{2})[\alpha^2(1-\theta) + \beta^2(1-\theta)] \) which is positive if (6) holds.

Proof of Proposition 5: (a) Recall (4), the condition that the land sale occurs. Note that the RHS of this condition has a derivative of \( \beta^2 s(1-\theta) > 0 \) with respect to \( \mu \). Hence the RHS is at least as large as \( (1-s)[\alpha^2(1-s)(1-
\[ \frac{\theta}{2} + \beta^2 s(1 - \theta) \], its value evaluated at \( \mu = \frac{1}{2} \). If \( s = 0 \), it is necessary that \( V \geq \beta^2 (1 - \frac{\theta}{2}) \) (using the hypothesis that \( \alpha = \beta \)).

We claim that if \( \alpha = \beta \) then \( s^* = 0 \). Otherwise \( \frac{\partial \Pi_L(\mu^*, s^*)}{\partial s} \geq 0 \). Given the restricted strategic substitute property in Lemma 3, it follows that \( \frac{\partial \Pi_L(\mu^*, s^*)}{\partial s} \geq 0 \), so that by the reasoning in Proposition 5, \( \gamma \) must be less than 1, which is impossible if \( \alpha = \beta \).

Now given \( s^* = 0 \), the reasoning in the first part of Proposition 5 implies that \( \mu^* > \frac{1}{2} \) only if \( V < (2\beta^2 - \alpha^2)(1 - \frac{\theta}{2}) = \beta^2 (1 - \frac{\theta}{2}) \), since \( \beta = \alpha \) by hypothesis. This contradicts the condition in the first para above for the sale to occur. So we must have \( \mu^* = \frac{1}{2} \).

(b) The first part follows from Proposition 5. The second part follows from analogous reasoning: \( s^* \) must equal zero by virtue of the first inequality in (8) and the restricted strategic substitute property. Given \( s^* = 0 \), the second inequality in (9) implies that the landlord’s profit is strictly increasing in \( \mu \) at \( \mu = \frac{1}{2} \). And the first inequality in (9) implies that the condition (4) for the sale to occur holds with a strict inequality. Hence it is feasible and desirable for \( \mu \) to be raised above \( \frac{1}{2} \), implying \( \mu^* > \frac{1}{2} \).

(c) This is merely a restatement of Proposition 5.

(d) Suppose \( \mu^* = \frac{1}{2} \). Since \( \gamma < 1 \), we have by the reasoning in Proposition 5: \( s^* = \frac{1 - \gamma}{2 - \gamma} \in (0, 1) \). Hence \( \frac{\partial \Pi_L(\mu^*, s^*)}{\partial \mu} = \frac{\beta^2}{(2 - \gamma)^2} - V > 0 \) by virtue of the second inequality in (12). The first inequality implies that the condition (4) for the sale to occur holds with a strict inequality at \( \mu = \frac{1}{2}, s = s^* \). Hence it is feasible and desirable for \( \mu \) to be raised above \( \frac{1}{2} \), a contradiction.

**Proof of Proposition 6:** Suppose first that (5) holds, so \( L \) selects \( \sigma^* = 0 \). Under the policy \( s^* = 0 = \sigma^* \) we have from (14): \( y^* = 0 < y^{FB} \) while \( x^* = (1 - \frac{\theta}{2})\alpha > x^{FB} \). A small rise in \( \sigma \) from 0 will induce \( L \) to keep \( s^* = 0 \), since condition (8) implies that \( \frac{\partial \Pi_L(\mu, s)}{\partial s} \) is negative at \( \mu = \frac{1}{2}, s = 0 \), so this must continue to hold for \( \mu \) slightly larger than \( \frac{1}{2} \). This will cause \( y^* \) to rise slightly and \( x^* \) to fall slightly, both of which contribute to a welfare improvement.

In the case where (5) does not hold, \( L \) selects \( \sigma^* > 0 \). Indeed, the optimal policy involves

\[
\mu^* = \frac{(2 + \theta)\beta^2 - \alpha^2 - V}{(4\beta^2 - \alpha^2)\theta}
\]

(23)

Now \( y^* = \theta(2\mu^* - 1)\beta \) which is less than \( y^{FB} = (1 - \theta)\beta \) since \( \mu^* < \frac{1}{2\theta} \) by Lemma 2. And \( x^* = (1 - \mu^*\theta)\alpha \) which exceeds \( x^{FB} = (1 - \theta)\alpha \). Hence we
continue to get underinvestment for $T$ and overinvestment for $L$ in this case.

**Proof of Proposition 7:** With $s = s^*$ as given by case (c) of Proposition 5, and $\sigma = 0$, it follows from (14) that $y^* = s^*(1 - \theta)\beta < (1 - \theta)\beta = y^{FB}$. And

$$x^* = (1 - s^*)(1 - \frac{\theta}{2})\alpha$$

which is greater than $(1 - \theta)\alpha = x^{FB}$ if $(1 - s^*)(1 - \frac{\theta}{2}) > (1 - \theta)$, which reduces to condition (15). In this case, a small rise in $\mu$ with $s$ fixed at $s^*$ will reduce $L$’s over-investment as well as $T$’s underinvestment. However, given a rise in $\sigma$, $L$ will want to reduce $s$ below $s^*$ owing to the strategic substitute property. Taking $\mu$ as fixed, we can work out $L$’s optimal response in terms of choice of $s$: we can denote this as $s^*(\mu)$. It is readily checked that when case (c) of Proposition 5 applies, $\Pi_L$ is concave in $s$, and so $s^*(\mu)$ is characterized by the first-order condition:

$$s^*(\mu) = \frac{\beta^2[1 - \theta - 2\theta(2\mu - 1)] - \alpha^2[1 - \theta + \theta(1 - \mu)]}{\beta^2[2(1 - \theta) - 2\theta(2\mu - 1)] - \alpha^2[1 - \theta + \theta(1 - \mu)]}$$

Direct and tedious computation shows that a rise in $\mu$ causes $T$’s effort to fall, as $s^*(\mu)(1 - \theta) + \theta(2\mu - 1)(1 - s^*(\mu))$ is decreasing in $\mu$, while it causes $L$’s effort to rise ($(1 - s^*(\mu))(1 - \theta + \theta(1 - \mu))$ is rising in $\mu$).

**Proof of Proposition 8:**

$$\Pi_L = (1 - \theta)(1 - s)f(x, y) + \theta \left[ \frac{V}{2} + \left\{ \frac{1 - s(1 - c)}{2} - cs \right\} f(x, y) \right] - \frac{x^2}{2}.$$ 

implying that his own effort response will be

$$x^* = \left\{ (1 - \theta)(1 - s) + \frac{\theta}{2}(1 - s(1 + c)) \right\} \alpha.$$ 

The payoff of $T$ is

$$\{(1 - \theta)s + \theta cs \} f(x, y) - \frac{y^2}{2}$$

which means that his choice of $y$ is

$$y^* = \{1 - \theta(1 - c)\} s\beta.$$ 

Substituting $x^*$ an $y^*$ in $L$’s objective function we get:
\[ \Pi_L = \frac{\alpha^2}{2} \left[ (1 - \theta)(1 - s) + \frac{\theta}{2} \{1 - s(1 + c)\} \right]^2 + \frac{\beta^2}{2} \left[ (1 - \theta)(1 - s) + \frac{\theta}{2} \{1 - s(1 + c)\} \right] [1 - \theta(1 - c)] s + \frac{\theta V}{2} \]

if \( V > f \{1 - s(1 - c)\} \) and \( \frac{\alpha^2}{2} (1 - s)^2 + \beta^2 s(1 - s) \) otherwise.

The first-order condition is
\[
\frac{\partial \Pi_L}{\partial s} = -\alpha^2 \left[ (1 - \theta)(1 - s) + \frac{\theta}{2} \{1 - s(1 + c)\} \right] \{1 - \theta\} + \frac{\theta}{2} (1 + c) \}
- \beta^2 \left[1 - \theta(1 - c)\right] \{1 - \theta\} + \frac{\theta}{2} (1 + c) \} s
+ \beta^2 \left[1 - \theta(1 - c)\right] \left[ (1 - \theta)(1 - s) + \frac{\theta}{2} \{1 - s(1 + c)\} \right]
= 0
\]

which yields
\[ s^* = \frac{b - a}{2b - a} \left(1 - \frac{\theta}{2}\right) \]

where
\[ a \equiv \alpha^2 \{1 - \frac{\theta}{2}(1 - c)\} \]
\[ b \equiv \beta^2 \left[1 - \theta(1 - c)\right]. \]

The second-order condition is
\[
\frac{\partial^2 \Pi_L}{\partial s^2} = \{1 - \theta\} + \frac{\theta}{2} (1 + c)\}\{1 - \theta\} + \frac{\theta}{2} (1 + c) \}
- 2\beta^2 \{1 - \theta(1 - c)\}. \]

It can be easily verified that so long as \( \beta \geq \alpha \), the second-order condition holds for all \( c \geq 0 \) if \( \theta < \frac{2}{3} \), and for \( c \geq 1 \), the second-order condition holds for all \( \theta \).

The next question we explore is how do the levels of \( x \) and \( y \) compare with the first-best. Under the first-best \( x^{**} = (1 - \theta) \alpha \) and \( y^{**} = (1 - \theta) \beta \). Using the expressions for (25 and (26), and simplifying, we get
\[
x^{**} - x^* = \alpha \left[ \frac{\theta}{2} c + s(1 - \theta) - \frac{\theta}{2} \right]
\]
\[
y^{**} - y^* = \beta \left[ (1 - \theta)(1 - s) - s \theta c \right]. \]
Observe that if \( x^{**} - x^* < 0 \) for \( c = 1 \) then it continues to be negative for \( 0 \leq c < 1 \). Analogously, if \( y^{**} - y^* > 0 \) for \( c = 1 \) then it continues to be negative for \( 0 \leq c < 1 \). It is straightforward to check that for \( c = 1 \), the condition for \( x^{**} - x^* < 0 \) is \( s < \frac{\theta}{2} \) or

\[
\theta > \frac{2(\beta^2 - \alpha^2)}{3\beta^2 - 2\alpha^2}.
\]

Similarly, for \( c = 1 \), the condition for \( y^{**} - y^* > 0 \) is \( s < (1 - \theta) \) or

\[
\theta < \frac{2\beta^2}{3\beta^2 - \alpha^2}.
\]

As we assume \( \beta \geq \alpha \) it is straightforward to check that \( \frac{2\beta^2}{3\beta^2 - \alpha^2} > \frac{2(\beta^2 - \alpha^2)}{3\beta^2 - 2\alpha^2} \). It is also straightforward to check that \( \frac{2\beta^2}{3\beta^2 - \alpha^2} > \frac{2}{3} \) and so \( \theta < \frac{2\beta^2}{3\beta^2 - \alpha^2} \) is consistent with our assumptions about the second-order conditions. Therefore, there are two relevant cases.

First, if \( \frac{2(\beta^2 - \alpha^2)}{3\beta^2 - 2\alpha^2} < \theta < \frac{2}{3} \) then we have overinvestment of \( x \) and underinvestment of \( y \) for \( c = 1 \). From the expressions for \( x^{**} - x^* \) and \( y^{**} - y^* \) these results continue to hold for \( c \leq 1 \).

Second, if \( \theta < \frac{2(\beta^2 - \alpha^2)}{3\beta^2 - 2\alpha^2} \) then there is underinvestment in both \( x \) and \( y \) for \( c = 1 \).

It is straightforward to check that

\[
\frac{\partial x^*}{\partial c} = -\frac{\theta}{2} s \alpha - \left\{ (1 - \theta) + \frac{\theta}{2} (1 + c) \right\} \alpha \frac{\partial s}{\partial c},
\]

\[
\frac{\partial y^*}{\partial c} = \theta s \beta + (1 - \theta + c\theta) \beta \frac{\partial s}{\partial c}.
\]

Now

\[
\frac{\partial s}{\partial c} = \left( 1 - \theta \right) \frac{\alpha^2 \beta^2 \theta}{2 (2b - a)^2} > 0.
\]

As \( \frac{\partial s}{\partial c} > 0 \) we therefore know that \( \frac{\partial x^*}{\partial c} < 0 \) and \( \frac{\partial y^*}{\partial c} > 0 \).

Now social surplus is

\[
W = \theta V + \alpha x + \beta y - \frac{x^2}{2} - \frac{y^2}{2},
\]

\[
\frac{\partial W}{\partial c} = (x^{**} - x^*) \frac{\partial x^*}{\partial c} + (y^{**} - y^*) \frac{\partial y^*}{\partial c}.
\]
Therefore, for $\frac{2(\beta^2 - \alpha^2)}{3(\beta^2 - 2\alpha^2)} < \theta < \frac{2}{3}$ we have $\frac{\partial W}{\partial c} > 0$ for $c \leq 1$.

If we have a one-sided investment problem then $x^{**} = x^* = 0$ and in that case $\theta < \frac{2}{3}$ is a sufficient condition for $\frac{\partial W}{\partial c} > 0$ for $c \leq 1$. This completes the proof of Proposition 8. \qed
Figure 1
Figure 2

First-best Pareto Frontier
\[ s^* = 0 \quad s^* > 0 \]

\[ \sigma^* = 0 \]

\[ \sigma^* > 0 \]

conversion occurs

no conversion

\[ \alpha^2(1-\theta) \]

\[ \beta^2 \]