Ec428, Topic 3: Coordination Failure and Sorting

Introduction

- Standard economic models feature a unique stable equilbrium

- It also have some efficiency properties: Pareto-efficient allocation (first welfare theorem)

- Reason for unique equilibrium: negative feedback mechanism

  - Example 1: Supply-demand model (dim. MP, MU) (see Fig 1)

  - Example 2: Solow model (dim. MP)

- What happens if we allow positive feedback mechanisms?

- The more you do something, or others do something, the more attractive it becomes.

- Multiple stable equilbria can result

- Downside: Lose predictive power.

- Upsides
  - More realistic (creates a role for history)
  - More optimistic (underdevelopment can be viewed as a bad equilbrium & not because of intrinsically bad parameters)
  - Greater role for policy: one shot policies can have permanent effects. Can remove them once new equilbrium is reached.

Increasing Returns (based on Ray, Chapter 5)

- This is an example of multiple equilibria due to increasing returns

- Two firms, incumbent (I) &entrant (E)

- Average costs are decreasing in output (i=E,I):

$$
\begin{aligned}
TC_i &= F + c_i q \\
AC_i &= \frac{F}{q} + c_i.
\end{aligned}
$$

- The incumbent (e.g., a firm in a developed country or a multinational) will have cost advantage which will make entry hard for the entrant (e.g., developing country firm)

- This is true even if the entrant has a better technology, say, $c_E < c_I$. See Figure 2.

- If $F = 0$ then by standard Bertrand competition argument, you get the most efficient firm getting all the market

- If initially price p incumbent's cost a, entrant's b

- To stop making losses entrant must produce at least $Q*$

- Why could this lead to multiple equilbria?

- Positive feedback mechanism. Let us posit a behavioral rule that says how much you supply next period is an increasing function of your margin of profit in the current period. As Figure 3 shows, this results in multiple equilbria. Not true with decreasing returns.

- Increasing returns not sufficient for multiple equilibria. Two implicit assumptions
  - Customers switch slowly, not instantaneously
  - Credit markets are imperfect & the firm is not very rich

Complementarities

- Now we look at multiple equilbria due to strategic complementarities: how many others are doing something affects my returns from doing it positively

Example 1: Technological Complementarities

- Why don't developing countries adopt efficient technologies?

- Returns from adoption of technology may depend on how many others are adopting it
  - Obvious example of network externalities: fax machines, email
  - Less obvious: repair facilties or trained workforce are not going to develop unless a critical threshold of people adopt some technology

Example 2: Demand Complementarities

- Why don't developing countries industrialize?

- Rosenstein-Rodan's parable of a shoe factory.
  - Poor economy - agriculture + cottage industry
  - A shoe factory can make profits only if sales exceed some minimum level due to set up costs
  - In the investment stage, generate demand for inputs & consumption goods for workers but only a small part of this will be for shoes
  - Since the cottage industries have limited capacity & face decreasing returns, inflation will result.
  - Shoe factory will close down

- If a lot of different factories were set up simulteaneously, they could have generated demand & supply for each other.
- Critical assumption: closed economy.

Model of Technological Complementarities

- Continuum of agents in [0,1]

- Each decides whether to invest or not (say acquire a skill or buy a machine)

- Let π be the fraction of the population that has invested.

- An individual takes this as given when making his decision.

- However, your returns from investing is positively affected by how many others have also invested

$$\begin{aligned} y_s &= H(1+\pi) - c \\ y_u &= L(1+\pi) \end{aligned}$$

- Assumption H>L. Let $H - L \equiv \triangle$

- Note that the model indicates that there are positive externalities (my payoff goes up if you invest) AND complementarities (my marginal return from investing, $y_s - y_u$, goes up if you invest):

$$y_s - y_u \equiv MR(\pi) = \triangle(1 + \pi) - c$$

- Three cases to consider (Figures 4-6):
  - $\triangle - c > 0$: Unique equilbrium, everyone invests
  - $2\triangle - c < 0$: Unique equilbrium, no one invests
  - $2\triangle - c \geq 0 \geq \triangle - c$: Multiple Equilbria. Three equilbria, $\pi^* = 1, \pi^* = \frac{c}{\triangle} - 1$ & $\pi^* = 0$. The interior one unstable.

- Which equilbrium would you prefer? Per capita income

$$y = \pi\{H(1+\pi) - c\} + (1-\pi)\{L(1+\pi)\}$$

is increasing in π as H>L.

- So the $\pi = 1$ equilbrium is the best.

- What are the conditions needed for multiple equilbria?

  – Externalities necessary but not sufficient. Consider a slightly different model:

$$\begin{aligned} y_s &= H + \pi - c \\ y_u &= L + \pi \end{aligned}$$

  – Here the choice does not depend on π,unique eqm

  - Need complementarities. Even with this, need further parameter restrictions (only case 3). Not only MR(π) is increasing in π (for which we need $\triangle > 0$) but fast enough (MR(1) $> 0 >$MR(0))

- General case: if payoff is $f(x_1, x_2)$

$$\frac{\partial^2 f}{\partial x_1 \partial x_2} > 0$$

- See appendix for supplementary material on it (not required reading)

History vs. Expectations

- How about expectations? Suppose everyone, in a wild burst of optimism, thinks $\pi = 1$ tomorrow. Then history does not matter. Expectations will be self-fulfilling.
  - If you introduce costs of adjustment then again history matters.
    - Returns take time to adjust
    - Each player will think, let others go first, I will go next
    - But then no one invests.
- One shot policy enough: if you announce subsidizing skill acquisition, then in equilbrium you can withdraw subsidies.

Group Inequality (Bowles, Loury, and Sethi, 2008)

- Conditions under which inequality among groups can persist in the long run despite equality of economic opportunity

- There are spillovers in human capital accumulation: your costs are lower, the more educated people you interact with

- You interact more with people of your social group (race, ethnicity, class)

- Therefore, it is possible that a person with the same talent but who is in a social group where not many people are educated, will not invest compared to another person who is in a social group where many people are educated

- Their general model shows the importance of three factors: the extent of segregation, the strength of interpersonal spillovers, and how responsive are wages to the skill composition of the population

- We will focus on a simpler case (based on section 5 of the paper) where the wage differential is constant.

- Population normalized to 1

- Everyone lives for two periods: in the first period get education (or not) and in the second period work

- skilled wage w_s and unskilled wage w_u

- Let the gap be δ

- Let s_t be the fraction of skilled workers at time t

- Therefore, $1 - s_t$ is the fraction of unskilled workers

- Let there be two groups 1 and 2 (racial or ethnic or economic) with fraction β and $1 - \beta$ and

$$s_t = \beta s_t^1 + (1 - \beta)\, s_t^2.$$

- The costs of skill acquisition for a person depends on how many of your social affiliates are skilled

- Let η be the fraction of people from your own group that you interact with, and $1 - \eta$ from the other group

- η is the measure of integration (lower it is, more integrated)

- $\eta = 1$ means completely segregated and $\eta = 0$ means completely integrated

- σ_t^i is the mean level of human capital among a person's social affiliates:

$$\sigma_t^i = \eta s_t^i + (1 - \eta)s_t.$$

- Let $c(a, \sigma)$ denote the cost of acquiring education where a is ability

- Decreasing in both arguments

- Assume δ is constant

- Assume that everyone has the same ability so that the cost function can be written as $c(\sigma)$

- By assumption $c(1) < c(0)$

- Three cases:

- $\delta > c(0)$
- $c(1) < \delta < c(0)$
- $\delta < c(1)$

- The first and the third cases are easy: in the first, everyone invests and in the third no one does

- Very similar to our model of technology adoption

- The second case is the interesting one

- We can apply our previous reasoning to see that both $(s^1, s^2) = (0, 0)$ and $(s^1, s^2) = (1, 1)$ are steady states for all η.

- (When no subscripts are used, it means steady state values)

- Suppose $\eta = 1$ (total segregation)

- Then the skill distribution $(s^1, s^2) = (0, 1)$ is a stable steady state

- Now consider the case of complete integration: $\eta = 0$

- Then $\sigma_t^1 = \sigma_t^2 = (1 - \eta)s_t$

- Let $\hat{\beta}$ be such that

$$c(1 - \hat{\beta}) = \delta.$$

- This exists as c is decreasing and continuous in σ (for example, $c = a - b\sigma$)

- If $\beta \leq \hat{\beta}$ then if group 2 is fully skilled ($s^2 = 1$) group 1 too will have incentives to invest

- But then $s^1 = 1$ even if you start with $s_0^1 = 0$

- If $\beta > \hat{\beta}$ this is not the case (not enough high skilled guys to hang out with)

- Therefore, low values of β and low values of η are conducive to catch up by the backward group

- Otherwise, you get segregation.

Predictive Content of Multiple Equilbria Models

- Some authors have thought hard about the predictive content of multiple equilibria models.

- Is it true that they suggest anything can happen?

- Trouble: see only one equilbrium even though potentially there could be multiple equilbria

- Any cross-sectional comparison contaminated by omitted variable problem

- Need a temporary and big shock

- Temporary, because you want to see if the shock goes away then if the economy reverts to the old equilbrium

- Big shock, since equilibria are locally robust

- A recent study by Donald Davis and David Weinstein ("A Search for Multiple Equilibria in Urban Industrial Structure", N.B.E.R. Working Paper 10252, January 2004)

  - Bombing of Japanese cities are industries in World War 2 provides a good test of multiple equilibria theory.

  - One implication of this theory is, a big shock can throw the system from one stable equilibrium to the other.

  - They show that in the aftermath of these immense shocks, a city not only typically recovered its population and its share of aggregate manufacturing, they also built the same industries they had before.

- This seems more consistent with "locational fundamentals" theory rather than increasing returns.
- As they themselves acknowledge, while thought provoking, this does not settle the issue.
- After all, even if buildings were destroyed, the land and ownership claims to it remained the same after the bombing.
- A labour force specialized to particular industries may have largely survived (even in Hiroshima 80% of the population survived).
- Infrastructure also remained largely unaffected.
- Therefore the pattern of economic activity prior to the bombing might have acted as a focal point for reconstruction.

- More promising approach: micro-level technology adoption decisions

- Take similar villages and then give them for free varying amounts of some technology that is likely to be subject to complementarities (e.g., mobile phones)

- Make available this technology for purchase at some resonable cost to others who did not get them for free

- See if adoption is higher in villages where the initial number of free mobiles crossed some threshold

Global Games Approach to Selection of Equilbria (optional material)

- Let there be two technologies, traditional and modern.

- There is a continuum of investors, and let π be the fraction of investors who invest in the modern technology.

- The returns from the traditional technology is $\gamma > 0$.

- The return from the modern technology to an investor when a fraction π of all investors are investing in the modern technology is

$$\alpha + \theta + \beta\pi$$

where $\alpha > 0, \beta > 0$ and $\theta \geq 0$.

- From our previous analysis, we can immediately conclude that no investors invest if

$$\alpha + \theta + \beta * 1 < \gamma$$

or

$$\theta < \gamma - \alpha - \beta \equiv \underline{\theta}$$

- All investors invest if

$$\alpha + \theta + \beta * 0 > \gamma$$

or

$$\theta > \gamma - \alpha \equiv \overline{\theta}.$$

- However multiple equilbria exist for $\theta \in [\underline{\theta}, \overline{\theta}]$

- In particular, for this range of θ

  – Everyone investing is a stable equilbrium as

$$\alpha + \theta + \beta \geq \gamma$$

– Similarly, no one investing is an equilbrium too as

$$\alpha + \theta \leq \gamma.$$

– There exists an unstable equilbrium at

$$\alpha + \theta + \beta\pi^* = \gamma$$

or,

$$\pi^* = \frac{\gamma - \alpha - \theta}{\beta}.$$

- As we noted earlier, since the "high" equilbrium dominates the "bad" equilbrium

- It is reasonable to expect that the economy will coordinate to the high equilbrium.

- The global games approach (Carlsson & Van Damme, Econometrica 1993 & Morris & Shin, American Economic Review 1998)* shows that changing the informational environment of the above game slightly can potentially get rid of multiple equilbria, and allow us to select a unique equilbrium.

- The parameter θ captures the state of the economy that affects all investors. There is
  - some uncertainty over its realization. In particular, it is common knowledge that it is distributed uniformly over the interval $[0, 1]$
  - investors receive a noisy signal regarding its realized value. In paricular, investor i receives a signal

$$x_i = \theta + e_i$$

*See “Rethinking Multiple Equilbria in Macroeconomic Modelling" by Morris and Shin (NBER Macroeconomics Annual 2000, 139-161. M.I.T. Press) for a simpler exposition (see also comments on this article by Atkeson published in the same volume).

- θ is the true realized value of θ and e_i is an error term that distributed uniformly over the support $[-\rho, \rho]$.
- Recall that the probability density function of a uniformly distributed random variable z with support $[-\rho, \rho]$ is $\int_{-\rho}^{\rho} k dz = 1$ or $k = \frac{1}{2\rho}$.

- We can prove that there is exists a critical value of the signal $x^* = \gamma - \alpha - \frac{1}{2}\beta \in (\underline{\theta}, \overline{\theta})$ such that each player i
  - invests if $x_i > x^*$
  - does not invest if $x_i < x^*$
  - indifferent if $x_i = x^*$

- This is a very striking result because it says that there is a unique equilbrium for this game.

- We will prove that this is in fact an equilbrium. (The proof of uniqueness will be sketched but not discussed in detail)

- Consider an agent who receives the signal $x_i = x^*$.

- He knows the true value of θ lies between $[x^* - \rho, x^* + \rho]$ (assuming $x^* - \rho > 0$ and $x^* + \rho < 1$)

- He also knows that others are following this strategy.

- Therefore, the crucial question is what fraction of the population has received a signal $x_i \leq x^*$?

- For any given θ the support of x is $[\theta - \rho, \theta + \rho]$ and the relevant density is $k = \frac{1}{2\rho}$.

- Therefore, this probability is

$$F(x^* \mid \theta) = \frac{1}{2\rho}\int_{\theta-\rho}^{x^*} dx = \frac{1}{2\rho}[x^* - (\theta - \rho)].$$

- However, as θ lies between $[x^* - \rho, x^* + \rho]$ we need to "add up" (i.e., integrate) these probabilities for all possible realizations of θ conditional on the observed signal to a player being x^*. (see Figure 11)

- This is obtained by

$$\begin{aligned}
& \int_{x^*-\rho}^{x^*+\rho}\left(\frac{1}{2\rho}[x^* - (\theta-\rho)]\right)\frac{1}{2\rho}d\theta \\
= \; & \frac{1}{2\rho}\left\{\frac{1}{2\rho}(x^*+\rho)\,[\theta]_{x^*-\rho}^{x^*+\rho} - \frac{1}{2\rho}\left[\frac{1}{2}\theta^2\right]_{x^*-\rho}^{x^*+\rho}\right\} \\
= \; & \frac{1}{2\rho}\left[(x^*+\rho) - \frac{1}{4\rho}\left\{(x^*+\rho)^2 - (x^*-\rho)^2\right\}\right] \\
= \; & \frac{1}{2\rho}\left[(x^*+\rho) - \frac{1}{4\rho}2x^*2\rho\right] \\
= \; & \frac{1}{2}.
\end{aligned}$$

- Since the agent who receives the signal x^* is indifferent between investing and not, we must have:

$$x^* + \alpha + \frac{1}{2}\beta = \gamma.$$

- This gives us

$$x^* = \gamma - \alpha - \frac{1}{2}\beta.$$

- Notice that if the actual value of $\theta < x^* - \rho$ no one will invest and similarly if $\theta > x^* - \rho$ everyone will invest.

- What about $\theta \in (x^* - \rho, x^*)$? Here a majority (a fraction $> \frac{1}{2}$) will receive a signal $x < x^*$ and will not invest. Those who will receive a signal $x > x^*$ will invest but ex post will regret it as by construction $\pi < \frac{1}{2}$ and so

$$x^* + \alpha + \pi\beta < \gamma.$$

- An analogous argument holds for $\theta \in (x^*, x^* + \rho)$: some individuals (a minority) will not invest and ex post regret it.

- Is the unique equilbrium efficient?

- No, because individuals do not internalize the effect of their decisions on others.

- If there was a social planner, he would check if $\theta + \alpha + \beta > \gamma$ or $\theta > \gamma - \alpha - \beta = \underline{\theta}$ and if so, will ask everyone to invest.

- However, in the outcome of the above game, people invest if $x \geq \gamma - \alpha - \frac{1}{2}\beta > \underline{\theta}$.

- As $E(x) = \theta$, there is inefficiency in the form of underinvestment.

- Intuition:
  - In multiple equilibria arguments, agents are homogeneous
  - Therefore, either everyone prefers doing something or not
  - If you add heterogeneity in terms of productivity or costs, then some people will adopt a technology anyway, and some never will
  - Given this the "middle" types will lean one way or the other

Sorting & Segregation

- Suppose your productivity depends positively on the productivity of your co-workers.

- What kind of a production function will generate this? One where skills of various workers are complements.

- Suppose output is produced by two tasks (theory, econometrics).

  – The skill of a worker in task 1 is denoted by q_i

  – The skill of a worker in task 2 is denoted by q_j

- The production function is:

$$y = f(q_i, q_j).$$

- The marginal product of a worker of skill q_i in task 1 (equal to the wage in a competitive market)

$$w_i = \frac{\partial f(q_i, q_j)}{\partial q_i}$$

- This is increasing in the type of his co-worker if

$$\frac{\partial w_i}{\partial q_j} = \frac{\partial^2 f(q_i, q_j)}{\partial q_i \partial q_j} > 0$$

- That is, the skills are complements.

- Similarly the wage of a worker of skill q_j in task 2 is

$$w_j = \frac{\partial f(q_i, q_j)}{\partial q_j}$$

  and

$$\frac{\partial w_j}{\partial q_i} = \frac{\partial^2 f(q_i, q_j)}{\partial q_i \partial q_j} > 0.$$

- Suppose there are two skills levels in both tasks, i.e., $q_i \in \{H, L\}$ and $q_j \in \{H, L\}$ with $H > L > 0$.

- We want to look at stable matchings of workers.

- These have the property that it is not possible for an individual worker to rematch and be better off.

- We allow unrestricted side payments: e.g., a worker can offer a higher wage to attract a potential partner, than what he is currently getting.

- Then we have the following important result that is widely used in a various contexts:

Result 1: The unique stable match involves positive assortative matching, i.e., workers of type H are matched with workers of type H, and workers of type L are matched with workers of tyep L.

- Suppose there are 4 workers, two of each type.

- Under the proposed match total output is

$$f(H, H) + f(L, L).$$

- If workers are matched non-assortatively, total output is

$$f(H, L) + f(L, H).$$

- The condition for the former to exceed the latter can be written as:

$$f(H, H) - f(H, L) > f(L, H) - f(L, L).$$

- But from the assumption of complementarity

$$\frac{\partial f(H, x)}{\partial x} > \frac{\partial f(L, x)}{\partial x}$$

- So switching from a L-type partner to a H-type partner must be more profitable for a H-type worker than a L-type worker.

- But that means a low type worker currently matched with another low type worker can never profitably bid away a high type worker who is currently working with another high type worker.■

Corollary: In a competitive market if the initial match is non-assortative, then assortative matching makes high types workers strictly better off, and low type workers strictly worse off.

- Directly follows from the fact that the wage rate is equal to the marginal product of a type of a worker, and the marginal product is increasing in the type of the co-worker.

- herefore, if you remove labour regulation and allow free "hiring and firing", efficiency will go up, but so will inequality.

- Other Applications:

  - Marriage Market due to Gary Becker

  - School choice (the quality of your education depends on the quality of your peers) - more generally, public goods

  - Brain drain (high skilled workers from less developed countries move to developed countries)

  - Industrial organization (the quality of your product depends on the quality of your suppliers)

Kremer's O-Ring Model

- Many production processes involve a sequence of tasks such that mistakes in any one of them can dramatically reduce the product's value (e.g., the o-rings in the Challenger space shuttle)

- Kremer (1993) proposes a production function that involves n tasks, all of which must be successfully completed for the product to have full value

- Each task requires a single worker.

- There are two outcomes of each task, success or failure and a worker's skill or quality at a task $q \in [0, 1]$ is the probability of success.

- The probability of failure of workers are independent.

- Capital k enters in conventional Cobb Douglas form.

- B is output per worker if all tasks are successfully carried out.

- Then expected output is :

$$E(y) = nB\left(\prod_{i=1}^{n} q_i\right)k^{\alpha}.$$

- Workers supply labor inelastically and there is no cost of effort.

- There is a perfectly elastic supply of credit at the world interest rate r.

- Firms and workers are all risk neutral.

- Question: what is odd about this production function?

- Answer: it seems to have increasing returns to scale: if you increase K as well as the qualities of all the workers by a multiple $\lambda > 1$, output will go up by $\lambda^{n+\alpha} > \lambda$.

- Is this consistent with perfect competition?

- Take the following more familiar looking production function:

$$f(K, L) = K^{\alpha} L^{\beta},\ \alpha + \beta > 1$$

- Clearly, if the cost of the factors are rK and wL then the answer is no, since the firm would want to hire infinite amounts of both inputs.

- However, notice if the cost of the factors are $rK^{2\alpha}$ and $wL^{2\beta}$ and using the notation $K^{2\alpha} = K'$ and $L^{2\beta} = L'$ we get:

$$\pi(K, L) = \left(K'\right)^{\frac{1}{2}} \left(L'\right)^{\frac{1}{2}} - rK' - wL'.$$

- This is the profit function of a competitive firm under CRS!

- Therefore, so long as the costs of the inputs are allowed to be non-linear, having a production function that is subject to increasing returns to scale is perfectly compatible with perfect competition.

- This is what Kremer does.

- A competitive equilibrium is defined as a an assignment of workers to firms, a set of wage rates that vary by quality, $w(q)$, a rental rate r such that firms maximize profits and the market clears for capital k and for workers of all skill levels.

- Firms facing a wage schedule of $w(q)$, a rental rate r chooses the skill level of workers for each task $(q_1, q_2, .., q_n)$ and the level of capital solves

$$\max_{k, q_1, q_2, .., q_n} k^{\alpha} \left(\prod_{i=1}^{n} q_i \right) nB - \sum_{i=1}^{n} w(q_i) - rk$$

$$nB \left(\prod_{j \neq i}^{n} q_j \right) k^{\alpha} = \frac{dw(q_i)}{dq_i}$$

- Notice that

$$\frac{d^2 y}{dq_i d \left(\prod_{j \neq i}^{n} q_j \right)} = nbk^{\alpha} > 0.$$

- This property implies that the search for equilibria can be restricted to those allocations of workers to firms such that all workers employed by any single firm have the same q, that is those displaying positive assortative matching as in Result 1.

- Generalizing, since $\frac{d^2 y}{dq_i d\left(\prod_{j \neq i}^{n} q_j\right)} = nbk^{\alpha} > 0$, the condition for positive assortative matching holds.

- It follows that in a zero profit equilibrium firms will be indifferent to the skill level of their workers so long as they are homogenous.

- Given that there is assortative matching, $q_i = q_j$ for all j in a given firm and so the first-order condition for q can be written as

$$\frac{dw}{dq} = nBq^{n-1}k^{\alpha}.$$

- The first-order condition on capital is

$$\alpha k^{\alpha-1} q^n n B = \bar{r}$$

or,

$$k = \left(\frac{\alpha q^n n B}{\bar{r}} \right)^{\frac{1}{1-\alpha}} .$$

- Notice that the payment to capital is

$$\bar{r} k = \alpha y = \bar{r}$$

- Substituting in

$$\frac{dw}{dq} = n q^{n-1} B \left(\frac{\alpha q^n n B}{\bar{r}} \right)^{\frac{\alpha}{1-\alpha}}$$

- Integrating we get

$$w(q) = (1-\alpha) \left(q^n B \right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha n}{\bar{r}} \right)^{\frac{\alpha}{1-\alpha}} + c$$

- Equivalently

$$w(q) = (1-\alpha)q^n Bk^\alpha + c$$

- c is the constant of integration which is the wage of the worker of skill zero.

- Multiplying the wage schedule by the number of workers we get the total wage bill to be

$$(1-\alpha)y + nc.$$

- Since payment to capital is αy, and firms earn zero profits, $c = 0$. It follows that expected wages and output are

$$\begin{aligned} w(q) &= Bq^n \\ E(y) &= nBq^n \end{aligned}$$

- This model has important implications for development

- Wage and productivity differentials among rich and poor countries are enormous.
  * If we interpret countries as firms in this model, then this follows directly.
  * It also follows if instead we assume that countries differ in terms of the distribution of skills, or there are some frictions in the matching process (such as search costs).
- Capital does not flow from rich to poor countries.
  * Capital is complementary with skills in this production function
- Income distribution is more skewed than skill distribution.
  * For a skill gap of $q_1 - q_0 > 0$, the income gap is $(q_1)^n - (q_0)^n$.
  * Since q^n is a convex function for $n > 1$, if $q_3 - q_2 = q_1 - q_0$ where $q_3 > q_1$ then by convexity $(q_3)^n - (q_3)^n > (q_1)^n - (q_0)^n$.

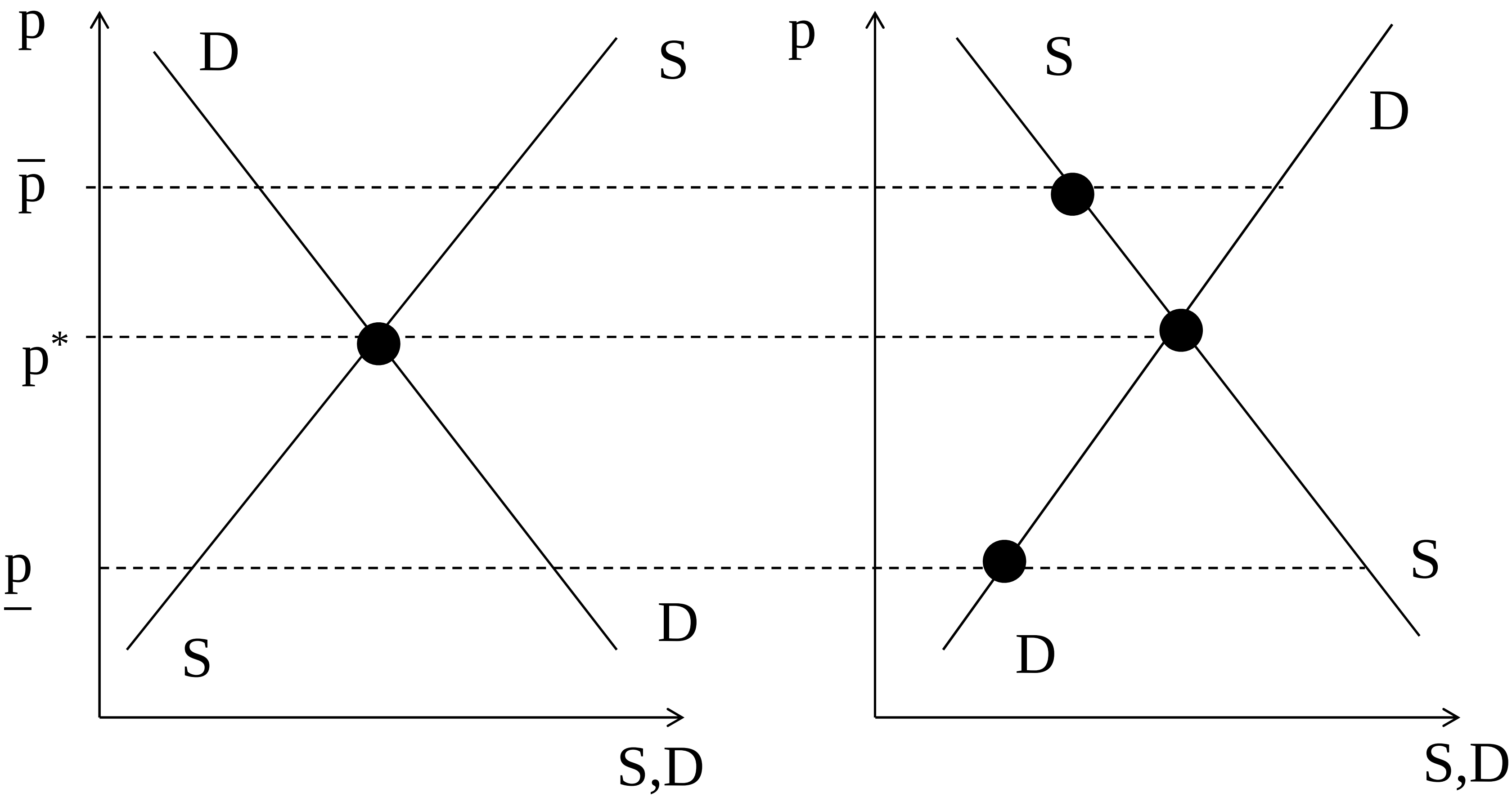
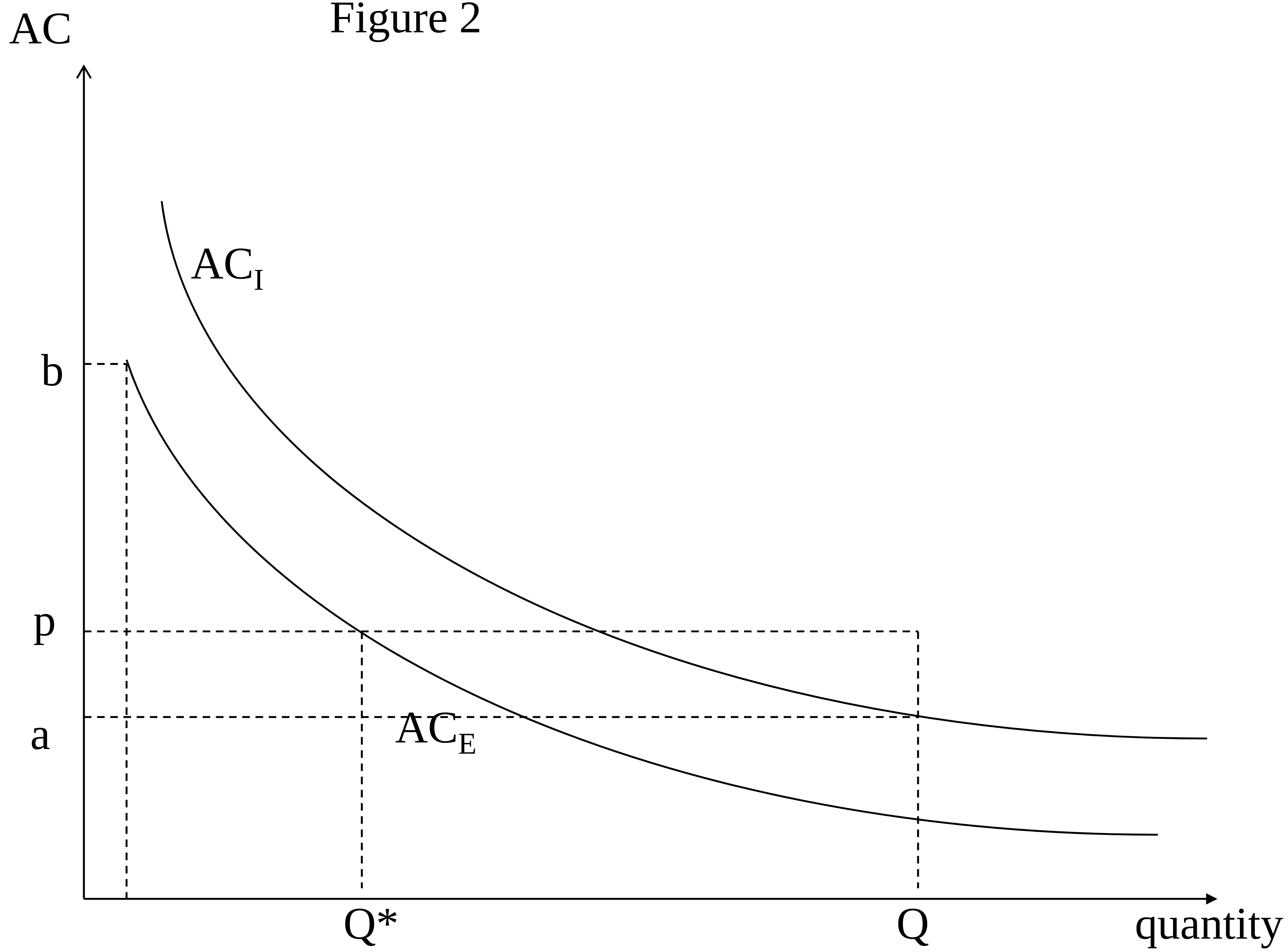


Figure 1: Examples of unique & multiple equilibria

Figure 2



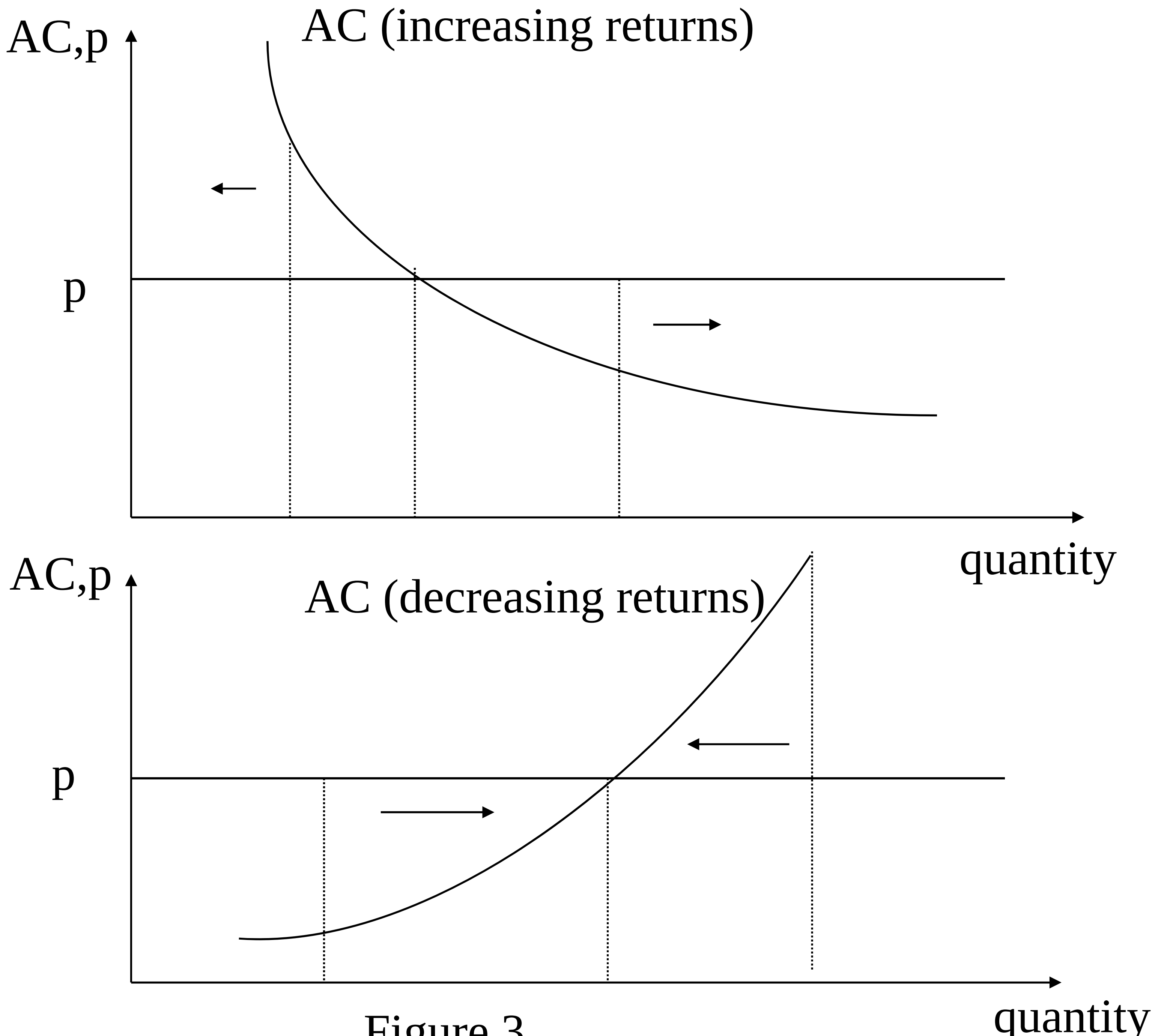

AC,p
AC (increasing returns)
p
quantity
AC,p
AC (decreasing returns)
p
quantity

Figure 3

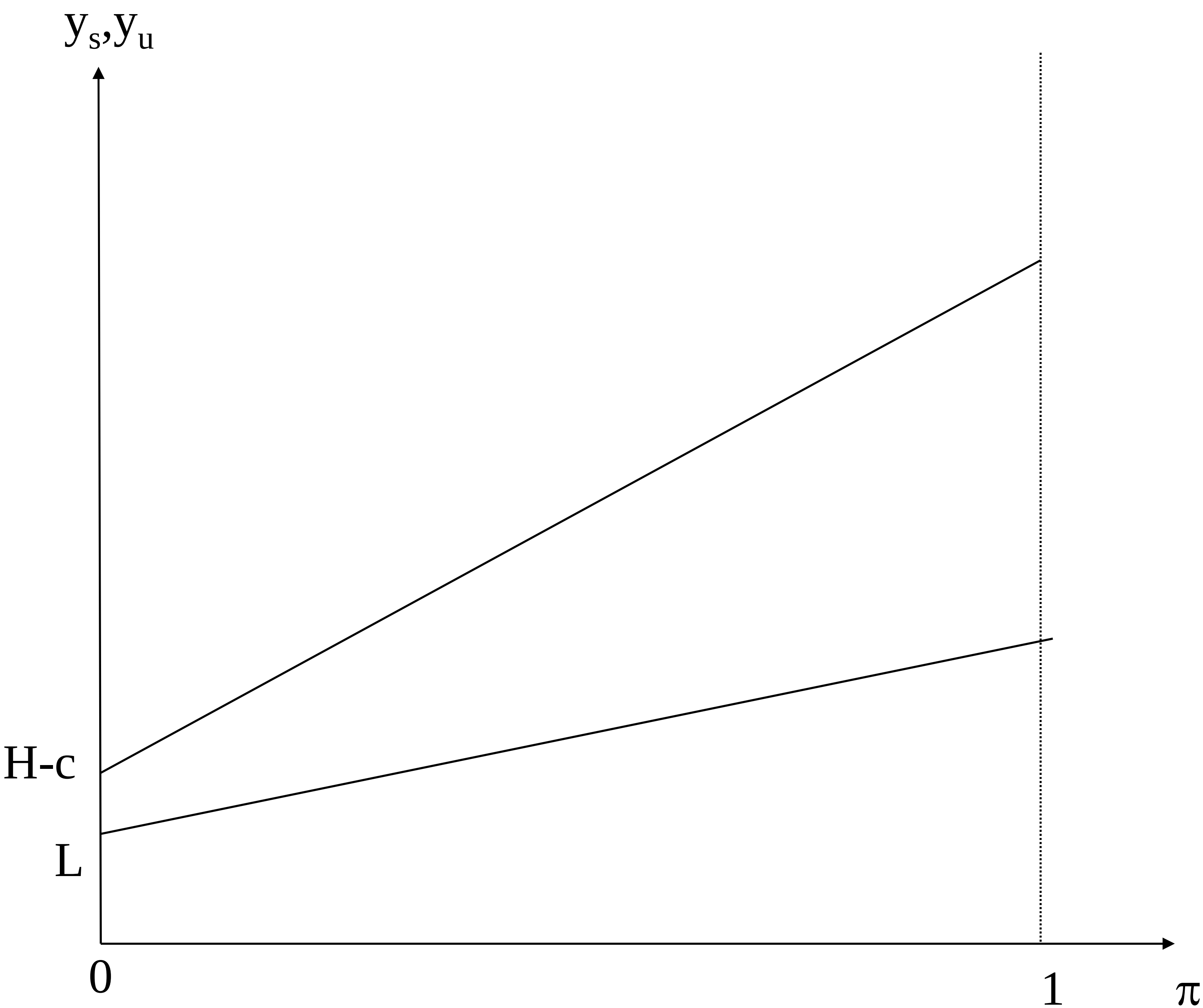


Figure 4: Case 1, Unique Equilibrium., π=1

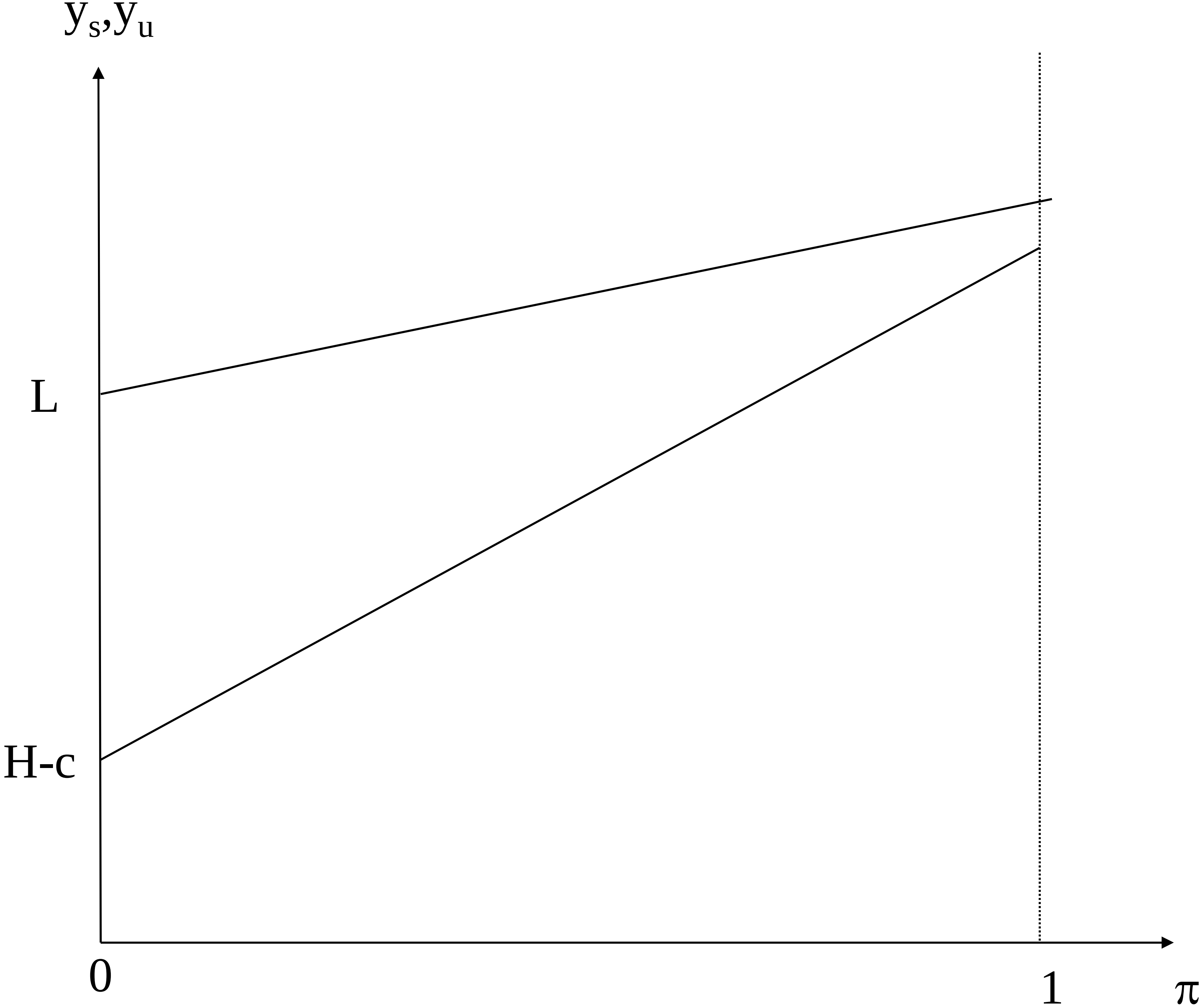


Figure 5: Case 2, Unique Equilibrium., π=0

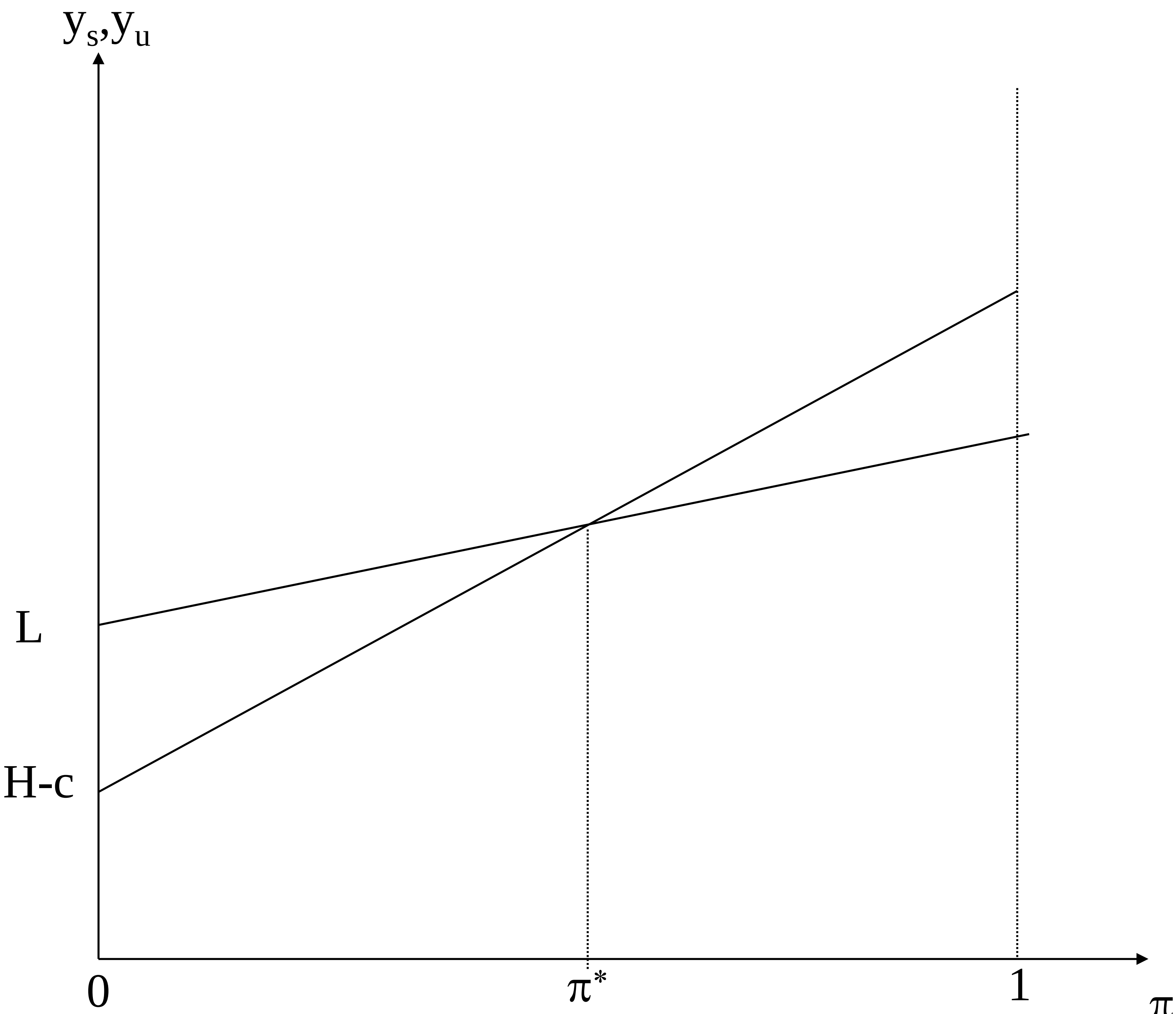


Figure 6: Case 3, Multiple Equilibria