Lectures in Growth and Development

(M. Ghatak, LSE, 2009-10)

Topic 2: Market Failure and Poverty Traps

These notes are not guaranteed to be error free. If you spot one, please let me know.

Also material beginning with * means optional material.

Conceptual Framework

- How does a person or economy grow richer?
  - You have some resources (skills, capital, land) which can be converted into output or income.
  - If you consume all your income in the current period, then clearly you cannot grow - at best you will be able to replicate what you did last period (i.e., provided the resources do not depreciate).
  - Savings, and investment are therefore key to growth.
- What are the limits to growth, if any? Can a person or an economy become infinitely rich?

- Typically, there is diminishing returns due to some fixed factor which slows down the growth rate (e.g. supervision time)

- People and economies reach their "steady states" where there is no growth barring shocks to technology or preferences.

- The notion of “convergence” in economic growth models.

- True both in time series and cross section.

- You grow faster when you are smaller but as you approach steady state, the growth rate slows down.

- If country A has more capital than country B, then it will grow slower. As the poor grow faster, they "catch up".

- Any persistent differences across countries must be pinned down differences in innate abilities of the people, its natural resources, attitudes regarding thrift, enterprise.

- One must have permanent policy measures in place (e.g., tax incentives to encourage savings) to do anything about it.

- Economists have proposed many theories of persistence of poverty.

- A lot of these are variations on the theme of a vicious cycle.

- The poor are malnourished, which makes them less productive, earn less, and this keeps them malnourished (Dasgupta and Ray, 1986; Banerjee and Mullainathan, 2008)

- The poor save less because they discount the future more heavily and that is why they stay poor (Moav, 2002)

- The poor have little to offer in the way of collateral, and as a result lenders are wary of lending to them, as a result they cannot expand their small businesses or acquire skills or afford education for their children. (Banerjee and Newman, 1993, Galor and Zeira, 1993, Ghatak and Jiang, 2002)

The Solow Model

- Robinson Crusoe (representative agent) economy
- Production function as a function of capital k_t (Fig. 1):

$$y_t = Ak_t^{\alpha}.$$

- Saves a constant fraction of his net investment so that capital next period is:

$$k_{t+1} = sy_t + (1 - \delta)k_t$$

or, $\triangle k_t = sy_t - \delta k_t$ where

$$\triangle k_t = k_{t+1} - k_t$$

and δ is the rate of depreciation.

- This defines a first-order non-linear difference equation in k (Fig. 2):

$$k_{t+1} = sAk_t^{\alpha} + (1 - \delta)k_t.$$

- The formula for the steady state capital stock is :

$$k_{t+1} = k_t$$

or

$$\triangle k_t = sy_t - \delta k_t = 0$$

or

$$sy^* = \delta k^*$$

or

$$sA(k^*)^{\alpha} = \delta k^*.$$

- Solve for steady state level of capital stock and output

$$k^* = \left(\frac{sA}{\delta}\right)^{\frac{1}{1-\alpha}}$$
$$y^* = A^{\frac{1}{1-\alpha}}\left(\frac{s}{\delta}\right)^{\frac{\alpha}{1-\alpha}}.$$

- Who will be richer in steady state? The model gives a simple answer : whoever has a higher value of s and A or a lower value of δ.

- The growth rate of the capital stock behaves in the following way:

$$\frac{\triangle k}{k} = sAk^{\alpha-1} - \delta$$

- Since $\alpha < 1$ the growth rate is declining in the level of the capital stock in the transition phase.

- Since

$$\log y = \alpha \log k + \log A$$

or,

$$\frac{\triangle y}{y} = \alpha \frac{\triangle k}{k}.$$

the same is true of per capita income. The poor grow faster.

- Example: Linear Model

Let

$$y = a + bk.$$

Then

$$\begin{aligned} k_{t+1} &= sy + (1-\delta)k_t \\ &= \{sb + (1-\delta)\}k_t + sa = \alpha k_t + \beta. \end{aligned}$$

This is a linear first-order difference equation

Solving it, we get:

$$k_t = \left(k_0 - \frac{\beta}{1-\alpha}\right)\alpha^t + \frac{\beta}{1-\alpha}.$$

Converges to steady state as $t \longrightarrow \infty$.

Growth rate:

$$\left(k_{t+1} - k_t\right)/k_t = \frac{\beta}{k_t} - (1 - \alpha).$$

Decreasing, and as $k_t \longrightarrow \frac{\beta}{1-\alpha}$, goes to 0.

*Digression: Solow vs. Neo-classical growth Models

- In the Solow model, the saving rate is exogenously given
- You save so that you can increase consumption in the future
- This choice is explicitly modeled in the neo-classical growth framework:

$$\max_{\{c_t, k_t\}} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

subject to $c_t + k_{t+1} \leq f(k_t)$

- β is the discount factor, assumed to lie in $(0, 1)$

- k_0 is given

- The intertemporal budget constraint says total output can be either be consumed or saved as next period's capital stock

- Recursive re-formulation* - define value function $V(.)$

$$V(k_t) = \max \{u(f(k_t) - k_{t+1}) + \beta V(k_{t+1})\}.$$

*See Ch 12 of R.K. Sundaram, *A First Course in Optimization Theory*, Cambridge U. Press 1996, for a discussion of this approach.

- This is like an indirect utility function which takes as given the existing capital stock k_t as a parameter

- Uses the insight that the problem looks exactly the same tomorrow as it does today except for tomorrow we start with k_{t+1}

- This is true as this is an infinite horizon model

- Let

$$g(k_t) = \arg\max_{k_{t+1}} \{u(f(k_t) - k_{t+1}) + \beta V(k_{t+1})\}$$

- This is called a policy function which tells us what is the optimal capital stock tomorrow, given k_t and given the consumer's preferences

- The first-order condition with respect to k_{t+1} is

$$u'(f(k_t) - k_{t+1}) = \beta V'(k_{t+1}).$$

- But differentiating the value function with respect to k_t

$$V'(k_t) = u'(f(k_t) - k_{t+1})f'(k_t)$$

- Therefore,

$$V'(k_{t+1}) = u'(f(k_{t+1}) - k_{t+2})f'(k_{t+1}).$$

- Substituting in the first-order condition

$$u'(f(k_t) - k_{t+1}) = \beta u'(f(k_{t+1}) - k_{t+2})f'(k_{t+1}).$$

- This is known as the Euler equation

- Nice interpretation: if you reduce consumption today by ε then you lose $u'(f(k_t) - k_{t+1})$, but this produces $f'(k_{t+1})$ next period, and boosts next period consumption by $u'(f(k_{t+1}) - k_{t+2})$

- You weight this gain in tomorrow's consumption by β

- Along the optimal path these must be equal

- In steady state $k_{t+1} = k_t = k_{t+2}$

- Therefore, the first-order condition gives us

$$1 = \beta f'(k).$$

- We can solve this out explicitly if we are given $f(.)$ (e.g., k^α)

- Very similar to steady condition is Solow with s being an increasing function of β (more patient people have a higher saving rate).

Role of Capital Markets in the Solow Model

- Now suppose that there are many Robinson Crusoes in an economy who differ in their initial capital endowment
- Without capital markets, each individual is on autarchy mode
- The poor will grow faster, and eventually there is convergence
- However:
  - Now the distribution of initial capital among people affects output in the short run

  - In an unequal economy, output will be lower compared to an more equal economy *with the same total capital stock*

  - Redistribution can raise output

  - In the long run these effects go away

- Presence of capital markets will speed up convergence

- Also, it will take away any effect of redistribution on output

- Once again, all we would need to keep track of is total capital, not its distribution

- What about the role of international capital markets in an open economy?

- Let $A = 1$. Recall that $k^* = \left(\frac{s}{\delta}\right)^{\frac{1}{1-\alpha}}$.

- What is the interest rate?

- The return from k_t is $f(k_t) + (1 - \delta)k_t$

- This is an accounting convention by which you treat net output and leftover capital stock all as return to capital

- Therefore, the interest rate should be the marginal product of capital: $f\prime(k_t) + (1 - \delta)$

- At k^* this is $r^* = \alpha (k^*)^{\alpha-1} + (1-\delta)$

- For simplicity set $\delta = 1$ (full depreciation)

- Since there is full depreciation,
$$r^* = \alpha (k^*)^{\alpha-1} = \frac{\alpha}{s}.$$

- Assume that the economy is small relative to the rest of the world, & that it can borrow freely at the world rate of interest r^*.

- As $k_0 < k^*$, capital will immediately flow in as marginal return is higher.

- The GDP of this economy will be immediately y^* but GNP will be less as loans have to be repaid.

- The equation of motion of the *domestically owned* capital stock is now:

$$\begin{aligned} k_{t+1} &= s\left\{y^* - r^*(k^* - k_t)\right\} \\ &= s\left[(s)^{\frac{\alpha}{1-\alpha}} - \frac{\alpha}{s}\{(s)^{\frac{1}{1-\alpha}} - k_t\}\right] \\ &= (s)^{\frac{1}{1-\alpha}}(1-\alpha) + \alpha k_t. \end{aligned}$$

- Does it converge? Yes, as $\alpha < 1$.

- Does it converge to k^*? Yes, if you set $k_{t+1} = k_t = k$ in the above equation, you get the solution $k = (s)^{\frac{1}{1-\alpha}}$.

- Compare this with autarchic equation of motion

$$k_{t+1} = sk_t^{\alpha}.$$

- Linearize around k^*.

- Slope? $s\alpha k_t^{\alpha-1}$, evaluated at k^* this is α

- This is the same as the slope of the new (linear) equation of motion

- Also, both lines pass through k^*

- As a result, they must be identical.

- Therefore, what the option of borrowing does is to allow countries to converge faster.

- Given this, the argument directly follows from Figure 3.

- There, the dashed line is the new equation of motion (with borrowing)

- It is tangent to the autarchic equation of motion at $k = k^*$ and has slope α

- Its intercept is equal to $(s)^{\frac{1}{1-\alpha}}(1-\alpha)$

- Notice that at $k = k^*$ the slope of the production function is $\alpha (k^*)^{\alpha - 1} = \frac{\alpha}{s}$ which is steeper than the slope of the equation of motion
- This is the point which on which there was confusion in the lecture.
- Intuition: you can borrow at a *low* rate and fast track your accumulation
- Earlier you were facing the constraint of a concave production function
- Had to "climb up" a fair bit
- Now you are facing a budget line that strictly expands your opportunity set

- This result (convergence in GNP as well as GDP) is specific to the Solow model and does not necessarily generalize with endogenous savings.

The Main Lessons

Lesson 1: Convergence. Being poor is no handicap in the long run. History does not matter.

Lesson 2: No long-run growth without technological progress. In the above model long run growth rate is 0. But A could be growing due to technological progress, & that would determine long-run rate of growth.

Lesson 3: (for closed economy version) With capital markets, all we need to keep track of is total capital stock of the economy as far as income is concerned. Without capital markets, how capital is distributed will affect output in the short run, but not in the long run.

- What happens if we introduce non-convexities in the production technology?

- Consider the Solow Model with Set-up costs or indivisibilities

- $y = Ak^{\alpha}$ for $k \geq \underline{k}$, $= 0$ otherwise. (See Figure 4)

- In this case, there will be multiple steady states & history would determine where you end up.

- The transition equation is

  – the horizontal line $O\underline{k}$ that coincides with the horizontal axis for $k < \underline{k}$

- the usual transition equation $k_{t+1} = sk_t^{\alpha} + (1-\delta)k_t$ for $k \geq \underline{k}$

- Implication: Poverty Trap: In this case if you are poor to start with then you end up more poor, whereas the rich gets richer.

- Exactly the opposite of convergence.

- With capital markets, however, there is convergence

- A capital market can help a poor economy escape a poverty trap in case 3. Just borrow $\underline{k}$ & repay in a few periods. Pareto improving.

- General Point: Markets can overcome technological indivisibilities & non-convexities.

- However, markets may not work perfectly in the presence of institutional failure

- Without market supporting institutions such as the legal system, there will be market failure

- That will have an impact on development.

- Even in the Solow model this will slow down convergence.
- With non-convexities the problems will be accentuated
- Can lead to poverty traps: anti-convergence.
- Have already noted this for Solow model with set up costs and missing capital markets
- Develop this point more generally by explicitly studying the evolution of the wealth distribution
- Two way interaction between wealth inequality and development

Occupational Choice Model with Borrowing Constraints: A Benchmark Model

- Banerjee-Newman, 1993; Galor-Zeira, 1993
- We do the version in Ghatak and Jiang (2002)
- Infinitely lived families, each generation lives for one period
- Population is size is normalized to 1, no population growth.
- Start of with wealth endowment of a_{it} & labor endow. of 1.

- Split end of period income into consumption $c_{it} = (1 - s)y_{it}$ & bequest $b_{it} = sy_{it}$ for next generation, which then becomes their initial endow. a_{it+1}.

- Save £1, get £$(1 + r)$ in the next period.

- Suppose there are three occupations:

- Subsistence: requires no investment, only labor, produces $\underline{w}$

- Worker: work for someone else at market wage w

- Entrepreneur produces q units of output using
  - Capital $I > 0$ (for training or buying a machine)
  - Two units of labor (his own labour and one hired labourer).
- The entrepreneurial technology is more efficient: $q - rI > 2\underline{w}$.
- A key assumption here is the presence of a technological indivisibility

Occupational Choice

- (a) Subsistence : The agent earns some income by using her labor endowment to produce $\underline{w}$ with the subsistence technology. She puts her inherited wealth in the bank, which yields $ra_{i,t}$. Therefore her income is

$$y_{i,t}^{S} = \underline{w} + ra_{i,t}.$$

- (b) Worker : The agent works for an entrepreneur for wage income w_t (which is determined endogenously). She puts her inherited wealth in the bank, which yields $ra_{i,t}$. Therefore her income is

$$y_{i,t}^{W} = w_t + ra_{i,t}.$$

- (c) Entrepreneur : The agent invests an amount I to start a firm and hires 1 worker to produce an output q with certainty. Her job is to monitor the worker. The agent's income as an entrepreneur is the output of the project less wage and capital costs:

$$y_{i,t}^{E} = q - w_t + r(a_{i,t} - I).$$

Credit Markets

- Enforcement Problem. A borrower may default on her loan (namely, $r(I - a)$), but the cost of this action is that she gets caught with some probability π & then has to pay a fixed non-monetary cost of F due to imprisonment or social sanctions.

- Thus only those individuals get loans whose wealth satisfies the incentive compatibility constraint (ICC):

$$\begin{aligned} (q - w_t) - r(I - a_{i,t}) &\geq (q - w_t) - \pi F \\ \text{or, } a_{i,t} &\geq I - \frac{\pi F}{r}. \end{aligned} \tag{1}$$

Set $\pi = 0$ for notational simplicity, so no borrowing possible.

Labor Market & Static Equilibrium

- The wage rate at which entrepreneurs are indifferent between working as wage laborers or hiring workers is given by

$$\overline{w} = \frac{q - rI}{2}.$$

- Labor supply:

$$\begin{aligned} 0 \text{ if } w_t &< \underline{w} \\ [0, G_t(I)] \text{ if } w_t &= \underline{w} \\ G_t(I) \text{ if } w_t &\in (\underline{w}, \overline{w}) \\ [G_t(I), 1] \text{ if } w_t &= \overline{w} \\ 1 \text{ if } w_t &> \overline{w}. \end{aligned}$$

- Labor demand:

$$\begin{aligned} 0 \text{ if } w_t &> \overline{w} \\ [0, 1 - G_t(I)] \text{ if } w_t &= \overline{w} \\ 1 - G_t(I) \text{ if } w_t &< \overline{w}. \end{aligned}$$

- They look odd, but they have the standard slopes (driven entirely by extensive margin, no intensive margin effect)

- Generically, two types of equilibria, high wage & low wage, depends on wealth distribution (Figures 5,6)

Bequests and Dynamics of Wealth Distribution

- With the knowledge of an individual's occupational choice and that the wage rate can take only two values ($\underline{w}$ and $\overline{w}$), we can write down the difference equations describing the evolution of a dynasty i's wealth as:

$$\begin{aligned} a_{i,t+1}(a_{i,t} \mid w_t = \underline{w}) &= s[ra_{i,t} + \underline{w}] && \text{if } a_{i,t} < I \\ &= s[r(a_{i,t} - I) + \ q - \underline{w}] && \text{if } a_{i,t} \geq I \\ a_{i,t+1}(a_{i,t} \mid w_t = \overline{w}) &= s[ra_{i,t} + \overline{w}] && \forall a_{i,t}. \end{aligned}$$

- Assume $sr < 1$ to make sure these are stable.

Long Run Behavior of Economy

- Let $a^J(w)$ be the stationary point of the difference equation describing the wealth transition of a dynasty engaged in occupation J (where $J = S, W, E$ denotes the three occupations : subsistence, worker and entrepreneur) when the wage rate is w.

- Then we have

$$\begin{aligned} a^S(w) &= \frac{s\underline{w}}{1-sr} \text{ for all } w. \\ a^W(\underline{w}) &= \frac{s\underline{w}}{1-sr} \\ a^E(\underline{w}) &= \frac{s(q - rI - \underline{w})}{1-sr} \\ a^W(\overline{w}) = a^E(\overline{w}) &= \frac{s(q-rI)}{2(1-sr)}. \end{aligned}$$

- By Assumption 1, $a^E(\underline{w}) > a^E(\overline{w}) = a^W(\overline{w}) > a^W(\underline{w})$.

- Its best being an entrepreneur when the wage is low, and its worst being a worker in this case

- If the wage is high, its the same whether you are entrepreneur or worker, but this must lie between the above two thresholds

- Given that the difference equations are stable, we should be able to predict the long run wealth distribution and the long run equilibrium wage rate

- Bad news: the transition equations depend on the wage rate, & they in turn depend on the wealth distribution (non-linear)

- Good news: we can show that the wage rate can change at most once.

- If w is constant then we have a simple linear system

- Intuition: there is no relative downward mobility.

- If dynasty i is richer than j at time t, the same holds at time $t+1$

- So can focus just on the median dynasty.

- But if this dynasty is poor at time t & becomes rich at $t+1$, the high wage will be reached, & it will stay rich in the future as well

- Analogously, if this dynasty is rich at time t & becomes poor at $t+1$, the low wage will be reached, & it will stay poor in the future as well.

- This means

Proposition 1: Given any initial wealth distribution, there exists a unique stationary wealth distribution to which it converges.

- Once the initial distribution is given, there is only one wage that can prevail in the long run, & so we can work out the long run distribution

- This is good news, but it does not say that there is a unique stationary wealth distribution for any given wealth distribution

- What it says is that there cannot be any cycles - once you have a initial distibution, there is a unique stationary state the system will head toward

- But there can be several stationary states overall

- For the same parameters regarding technology $(q, \underline{w}, I)$, preferences (s) & markets (π, F) what wage (high/low) will result & so which long run distribution you converge to depends on the initial distribution.

- This is characterized in

Proposition 2 : The initial distribution of wealth matters in determining the stationary distribution of wealth and the long run equilibrium wage rate if and only if

$$s(\ q - \underline{w}) \geq I \geq \frac{s\underline{w}}{1 - sr}.$$

Otherwise the economy converges to a high wage equilibrium (if $I < \frac{s\underline{w}}{1-sr}$) or a subsistence equilibrium (if $I > s(\ q - \underline{w})$) irrespective of initial conditions.

- Where do we get these inequalities from?

- $s(\ q - \underline{w}) \geq I$ is equivalent to $a^E(\underline{w}) = \frac{s(q - rI - \underline{w})}{1 - sr} \geq I$.

- $I \geq \frac{s\underline{w}}{1 - sr}$ is equivalent to $a^W(\underline{w}) \leq I$.

- In Figure 7 (a) we depict a situation where long run equilibrium has low wages, high inequality and low levels of per capita income

- In Figure 7 (b) we depict a situation where long run equilibrium has high wages, zero inequality and high levels of per capita income

- What is interesting is, if the condition mentioned in Proposition 2 holds, then for the same set of parameters, both types of long run equilbria are possible

- If you started unequal (in particular, $G(I) > \frac{1}{2}$), you end up as in Figure 7(a)

- If you started equal (in particular, $G(I) \leq \frac{1}{2}$), you end up as in Figure 7(b)

- Big implication: History matters i.e. convergence may not occur

- If you start too unequal, the wage will be low, upward mobility will be low & so converge to a low wage equilibrium with a small class of rich people & a large class of poor people.

- We don't need endogenous wages for this story: add a skilled wage w_s and an unskilled wage w_u and suppose I is the cost of getting skill

- Then can tell a similar story

- Here endogenous wages are accentuating the problem

- What parameters make "poverty traps" more likely?
  - Obvious ones: high I, low q, low s.
  - Less obvious: high $\underline{w}$. While it makes upward mobility for the very poor easier, it makes capital accumulation for the rich harder.
  - Also, if you increase F or π then capital market will improve.

- Now we formally prove why inequality can hurt development if capital markets are imperfect:

Proposition 3 : For parameter values for which initial conditions matter, the greater is the fraction of the population who are initially poor, the lower is steady state income.

- Total income of the economy:

$$Y = G(I)w + \{1 - G(I)\}\{\ q - w - Ir\}$$

- Decreasing in the number of non-entrepreneurs (workers + subsistence earners)

- Several implications are worth noting:
  - Credit market imperfections have real costs - long run per capita GNP is lower.
  - One shot policies can permanently raise the total income of the economy.
  - Lower per capita income is also associated with greater inequality so that redistribution can improve efficiency.
- Banerjee and Duflo (2003): negative relationship between growth and inequality lagged by one period
- Greater inequality - less investment - lower growth (consistent with above model)

- Policy Implications
  - Use of lotteries. ROSCAs an example.
  - Redistribution: can't be any redistribution, only those that aim to increase the number of entrepreneurs.
  - Credit subsidies
  - Improving institutions so that credit market works better - better courts, better titles (the de Soto effect)

*Wealth Shocks and Steady State Mobility

- So far, we did not allow any randomness

- The very lucky or talented poor will escape poverty traps & the unlucky rich will become poor.

- Ghatak-Jiang 2002 (section 3) considers this

- Suppose every individual's saving rate is subject to an idiosyncratic i.i.d. shock.

- In every period, each individual's saving rate could be high ($\overline{s}$) with probability p or low ($\underline{s}$) with probability $1 - p$.

- Assume

$$\overline{s} > \frac{I}{\underline{w} + rI} \text{ and } \underline{s} = 0 \qquad (\textit{Assumption 3})$$

- Notice that $\frac{I}{\underline{w}+rI} < \overline{s}$ implies there exists an integer m such that it takes at most m consecutive periods of good luck for a dynasty - even if it started with no initial wealth and even if wage rates remained low - to become rich.

- Formally,

$$m = \min\{n \in N : \overline{s}[\sum_{i=0}^{n-1} (r\overline{s})^i \underline{w}] \geq I\}.$$

- Let us first define m', similar to m, as the number of periods needed for a zero-wealth dynasty to become rich under *high* wages.

- Naturally, $m' \leq m$.

- We show
  - *The wage dynamics can be either stationary (with high or low wages), or display cycles.*

– *In addition to the above assumption, if* $\overline{s} \leq \frac{1}{r}$*, then the stationary wage dynamics can only be either always high wage or always low wage.*

- Then we are able to show

Proposition 4 : *The initial distribution of wealth matters in determining the stationary distribution of wealth and the long run equilibrium wage rate in the stochastic model if and only if*

$$p^{m'} \geq \frac{1}{2} > p^m.$$

Otherwise the economy converges to a high wage equilibrium (if $p^{m'} \geq p^m > \frac{1}{2}$*) or a low wage equilibrium (if* $\frac{1}{2} > p^{m'} \geq p^m$*) irrespective of initial conditions.*

- The wealth distribution is *stationary*: people move around, interchange positions, but at the aggregate level distribution looks the same.

- There can be multiple stationary wealth distributions with mobility : one corresponding to the low wage & the other the high wage.

- This was the key contribution of Banerjee and Newman 1993.

- Highlights the distinction between individual vs. aggregate poverty traps

- Under individual poverty traps, the poor never escape poverty and aggregate poverty traps are a corollary of this

- However, aggregate poverty traps can exist without individual poverty traps

- In a low wage equilibrium, a poor individual can and will become rich at some point

- However, on aggregate there will be a fraction $\geq \frac{1}{2}$ that will be poor

Extension: Poverty Traps without Credit Market Imperfections

- This is based on Banerjee-Mullainathan (2008)

- There are two goods: food (f) and comfort good (c) (such as, electricity supply or good baby sitter at home)

- The main idea is, the poor have to spend too much time worrying about domestic problems that the rich have a way of “buying out”

- As a result the rich can pay more attention at work, get a higher income, and this in turn keeps them rich

- It is a variation of the poverty trap argument but it does not require the assumption of capital market imperfections and/or non-convexities in production

- Another way of interpreting this is, the poor have an higher opportunity cost of time spent at work (but not because they are lazy)

- Let y be income and from the budget constraint (assuming both c and f have a price of 1), $f = y - c$.

- Let h be the level of human capital with y being increasing (and linear) in h

- Utility function

$$u(c, f, \theta) = (y - c)^{\alpha} - p(a - \delta\theta)(b - c)$$

- With p there is a problem at home, which creates a loss of amount $b - c$

- $\theta \in [0, 1]$ is the amount of time spent attending problems at home

- This reduces the loss by a the probability $a - \delta\theta$ where $1 \geq a > \delta$

- This is at the expense of time spent at work and so $y = h(1 - \theta)$

- What they are assuming is that there is an endowment of 1 unit of attention, and this is spent in proportions θ and $1 - \theta$ at home and work

- The first order condition with respect to c is

$$\alpha(y - c)^{\alpha - 1}(-1) + p(a - \delta\theta) = 0.$$

- Notice that if we differentiate again with respect to c then we get a negative sign, so the function is strictly concave and a global maximum exists

- Solving for c we get

$$c = y - \left[\frac{\alpha}{p(a - \delta\theta)}\right]^{\frac{1}{1-\alpha}}.$$

- Substituting this into the utility function we get the indirect utility function as a function of θ :

$$u(\theta) = \left[\frac{\alpha}{p(a - \delta\theta)}\right]^{\frac{\alpha}{1-\alpha}} - p(a - \delta\theta)\left[b - y + \left\{\frac{\alpha}{p(a - \delta\theta)}\right\}^{\frac{1}{1-\alpha}}\right].$$

- Since $y = h(1-\theta)$, substituting we get

$$u(\theta) = \left[\frac{\alpha}{p(a-\delta\theta)}\right]^{\frac{\alpha}{1-\alpha}} - p(a-\delta\theta)\left[b - h(1-\theta) + \left\{\frac{\alpha}{p(a-\delta\theta)}\right\}^{\frac{1}{1-\alpha}}\right].$$

- Collecting terms and simplifying we get

$$u(\theta) = (1-\alpha)\left(\frac{\alpha}{p}\right)^{\frac{\alpha}{1-\alpha}}(a-\delta\theta)^{-\frac{\alpha}{1-\alpha}} - pb(a-\delta\theta) + p(a-\delta\theta)h(1-\theta).$$

- Differentiating twice with respect to θ it can be verified that $u(\theta)$ is strictly convex in θ

- Therefore the optimal solution is either $\theta = 1$ or 0 : there is no interior solution

- Now $u(1) = (1-\alpha)\left(\frac{\alpha}{p}\right)^{\frac{\alpha}{1-\alpha}}(a-\delta)^{-\frac{\alpha}{1-\alpha}} - pb(a-\delta)$ and $u(0) = (1-\alpha)\left(\frac{\alpha}{p}\right)^{\frac{\alpha}{1-\alpha}} a^{-\frac{\alpha}{1-\alpha}} - pba + pah$

- Notice that if h is high, then $u(0) > u(1)$.

- Therefore, those who are rich (high h) will choose $\theta = 0$

- Intuitively, the rich put all attention at work but is not costly, as having a higher y means they can have a higher c and this reduces the loss from having problems at home

- The poor folks put all attention at home, but this means their income is lower, and so the loss from having problems at home is higher which justifies $\theta = 1$

- Now if you allowed intergenerational transfers as in the previous model, it is fairly straightforward to show that you will get a dynamic poverty trap (i.e., kids of poor folks get less human capital and therefore they too stay poor)

Other Extensions

- The interest rate is fixed in Banerjee-Newman, Galor-Zeira, Ghatak-Jiang
  - Piketty (1997), Aghion and Bolton (1997) tell related stories but there is no wage employment in their model.
  - The wealth distribution affects the interest rate which is endogenous.
  - As the rich goes richer the extra savings can reduce interest rate & help upward mobility (AB call this the trickle down effect).
  - In AB there is convergence to a unique wealth distribution, in Piketty there exists multiple steady state wealth distributions

- Heterogeneous ability and selection ("Entrepreneurial Talent, Occupational Choice, and Trickle Up Policies" by M. Ghatak, M. Morelli and T. Sjostrom, Forthcoming *Journal of Economic Theory*)

- Market frictions prevent entrepreneurs from undertaking potentially profitable investments - a major cause of underdevelopment

- These frictions act as entry barriers

- Obvious policy implication would be reduce these barriers

- But this ignores the role of heterogeneity in entrepreneurial quality and selection issues

- Consequence of adverse selection on entrepreneurship:
  - If good screening mechanisms are absent "bad" types cannot be sorted from "good" types
  - Good types do not receive full marginal returns from their efforts/skills/talents - loss of efficiency
- Credit Market: terms faced by good borrowers are worse: either don't get a loan unless have some minimum collateral, or get a loan but interest rate high because bad borrowers are present. Leads to lower investment.
- Severity of adverse selection depends on outside options (e.g., wages) if these are attractive bad types will have less temptations to enter the market (high types have a comparative advantage in entrepreneurship)

- We endogenize the outside option through occupational choice.

- Two way feedback:

  – When outside option improves efficiency improves due to better pool (e.g., lower interest rate or lower wealth threshold for a loan which means more investment)

  – This *pool quality effect* will have feedbacks in other markets via complementarities & may support the high outside option (e.g., more investment means higher labour demand & higher wages).

- Didn't study dynamic incentives
  - If savings are endogenous then capital market imperfections can actually promote savings & upward mobility.
  - The American Dream effect of Ghatak, Morelli and Sjostrom (Review of Economic Studies, 2001)

- Many investment levels and pecuniary non-convexities
  - Suppose there are many investment levels and the economy needs all these activities
  - Long run inequality may be inevitable given imperfect credit markets even if
    * there are no technological non-convexities
    * people are fully forward looking in their saving decisions

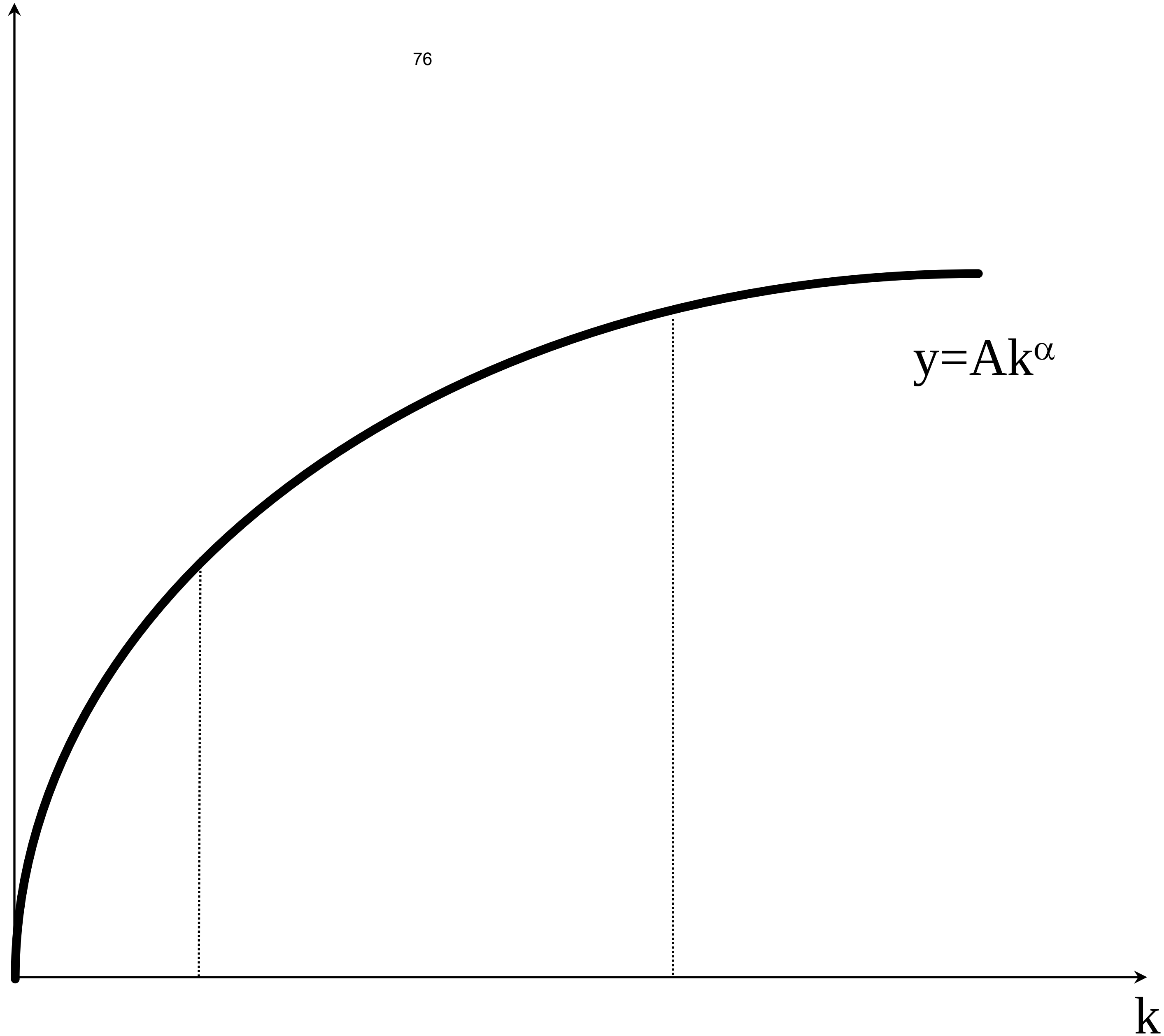

Figure 1: Production Function

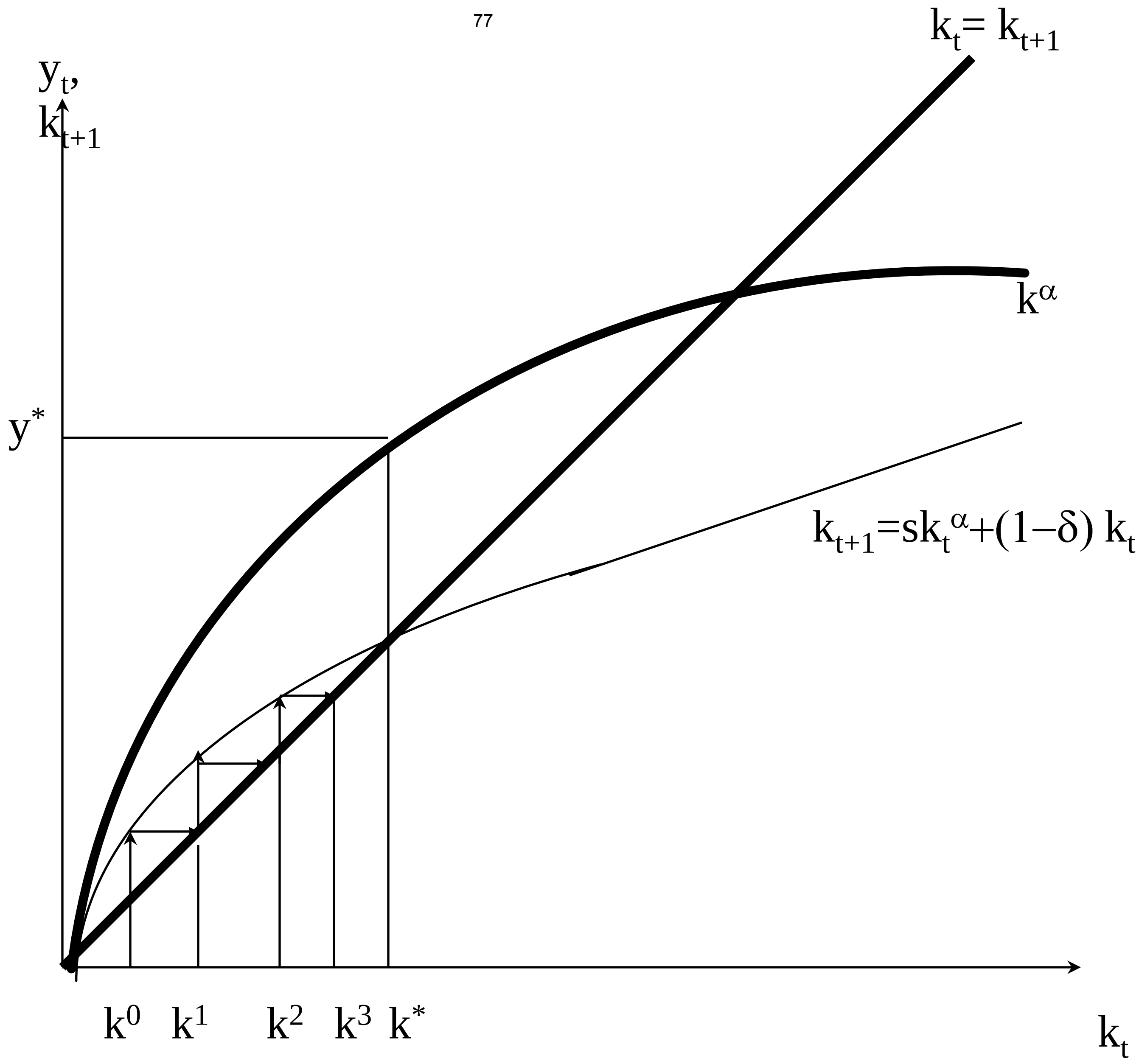

Figure 2: Convergence in Solow Model

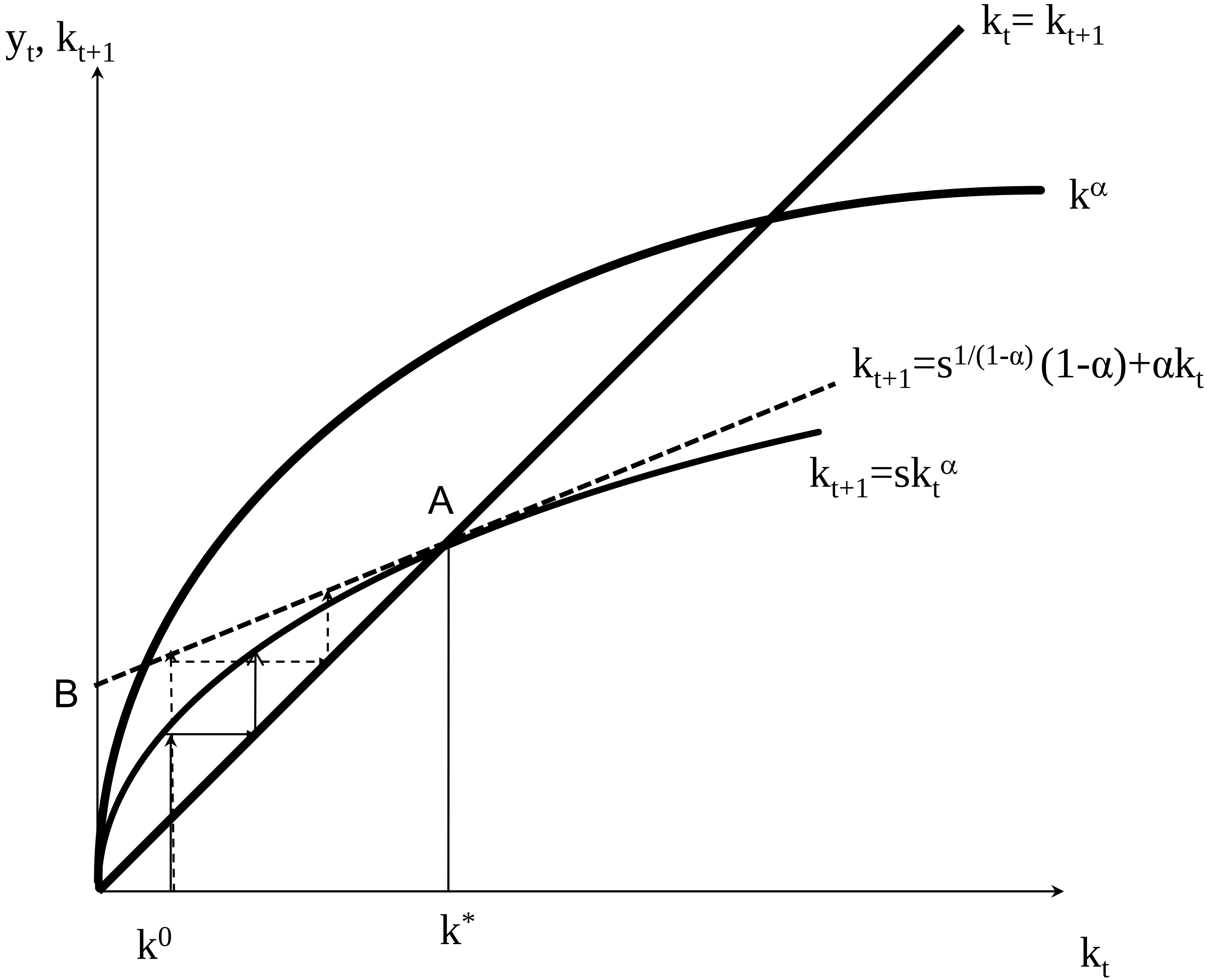

Figure 3: Convergence with Capital Market

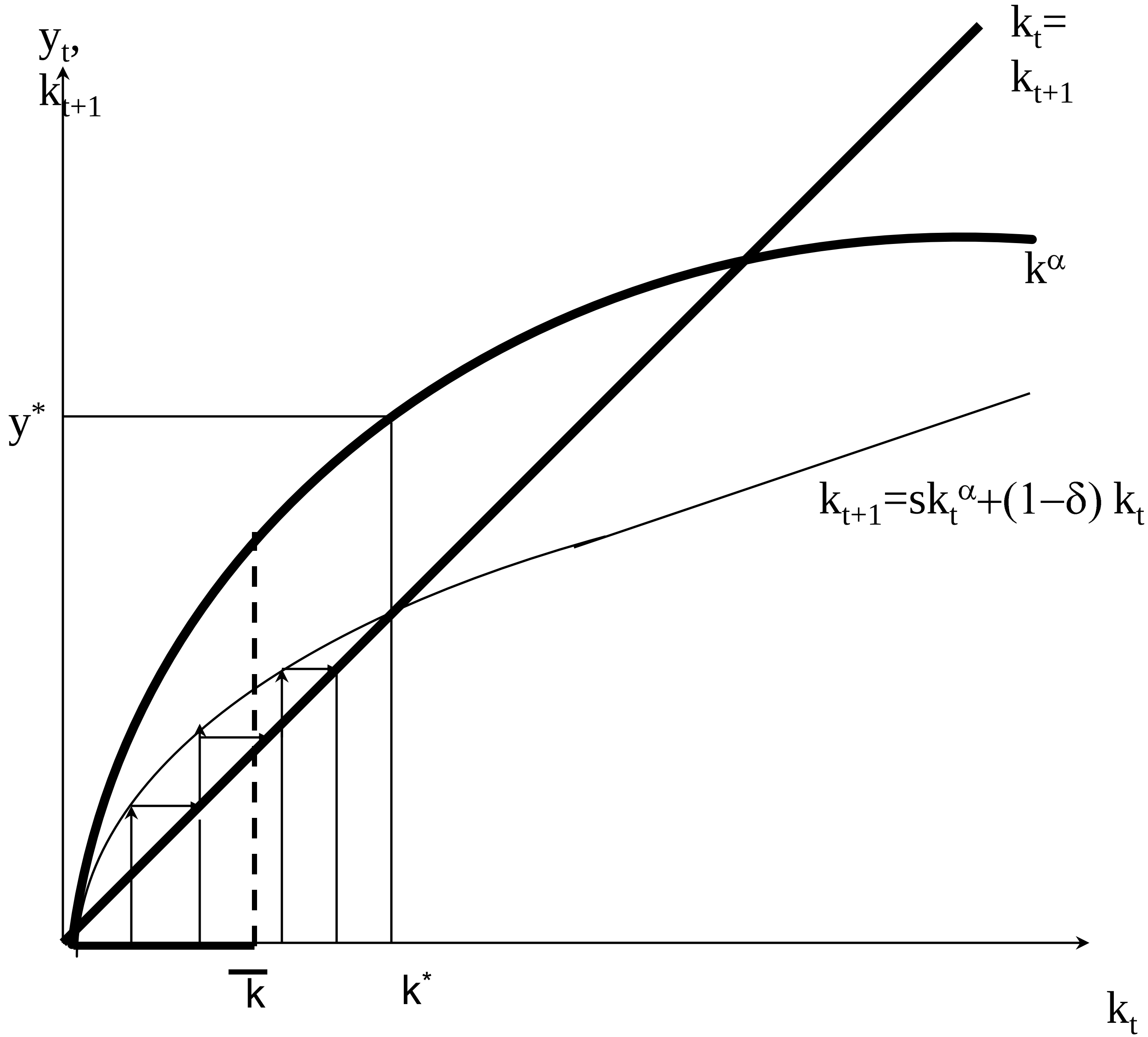

Figure 4: Set up Costs in Solow Model

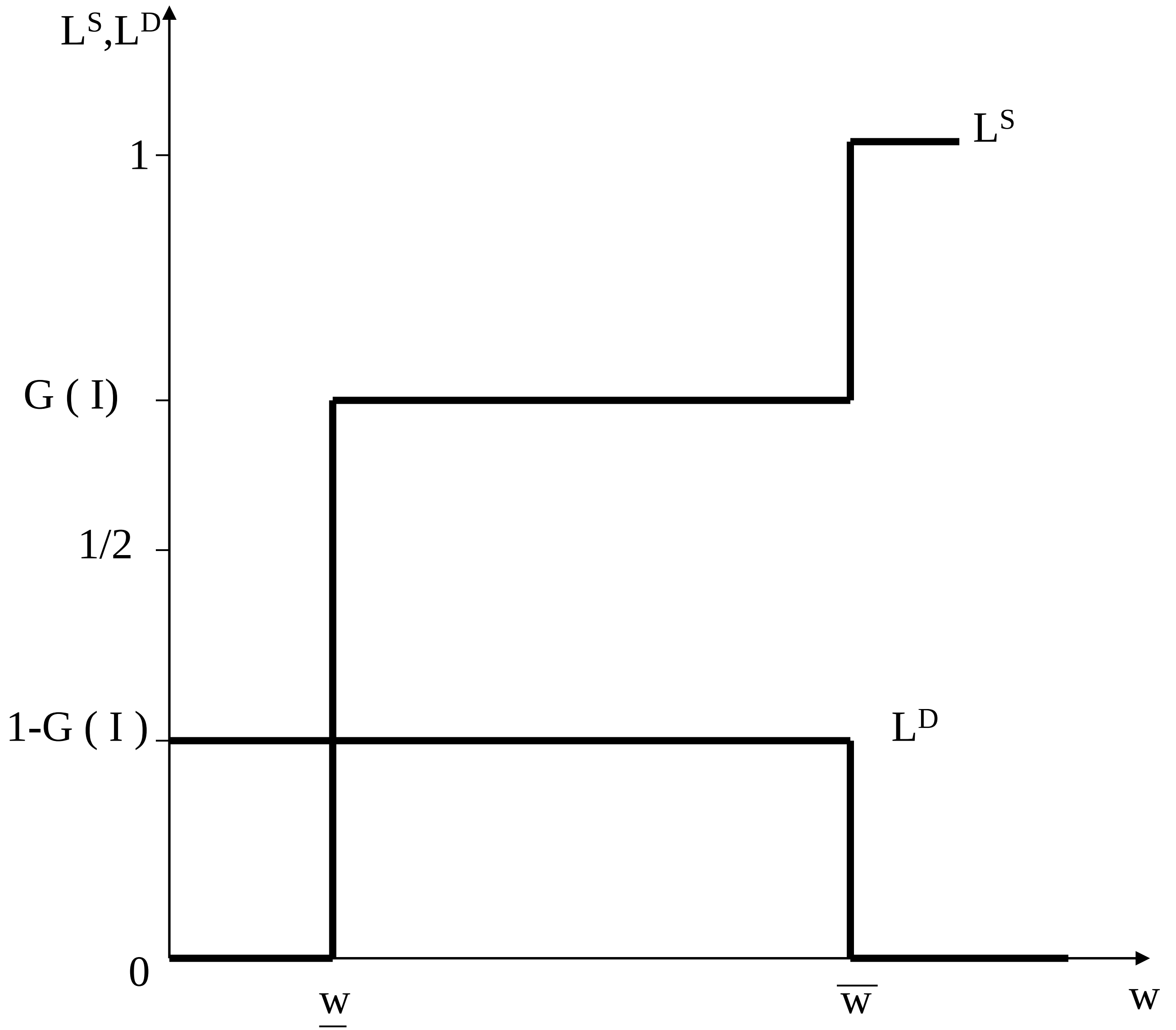

Fig 5: Low wage equilibrium

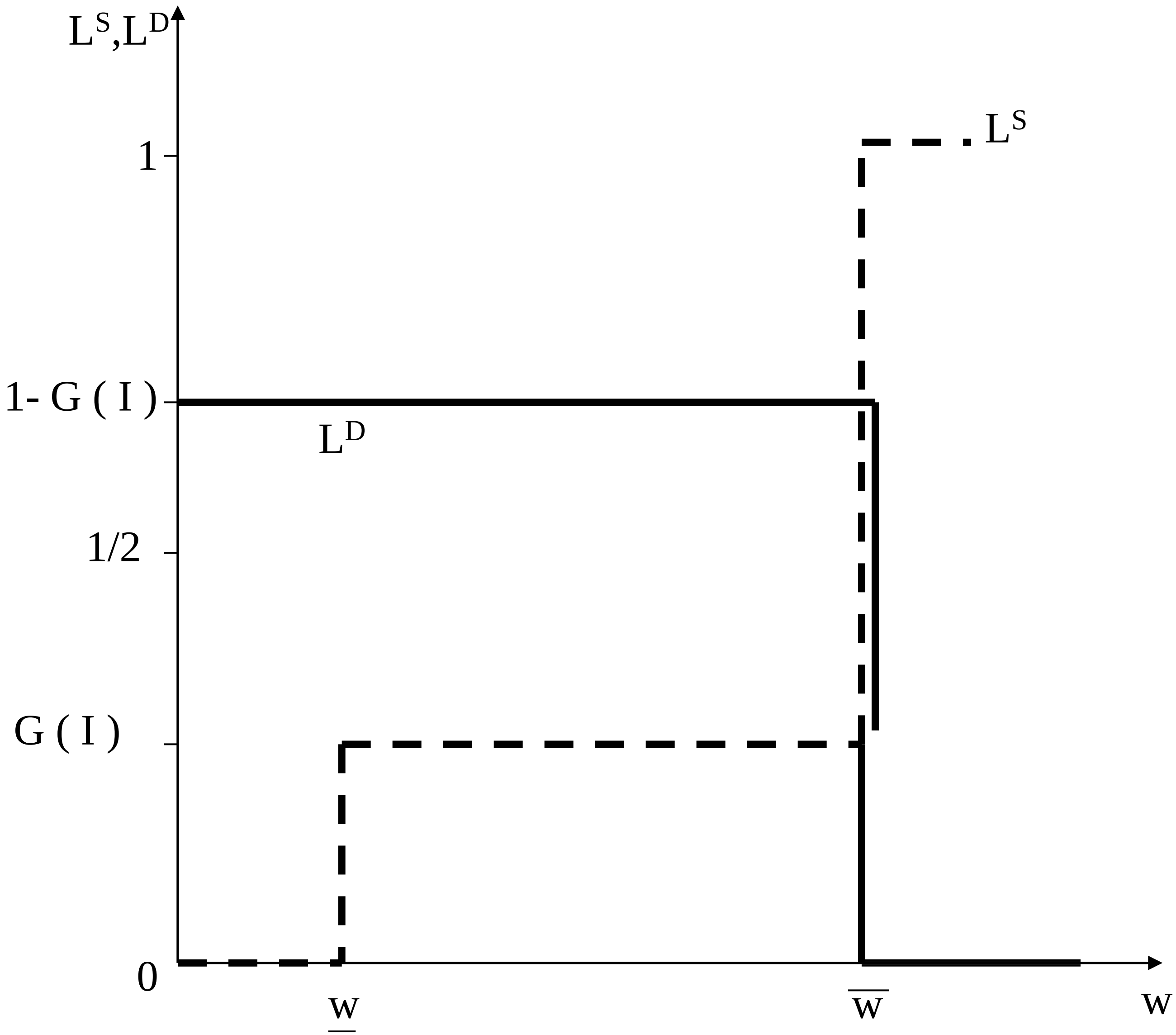

Fig 6: High wage equilibrium

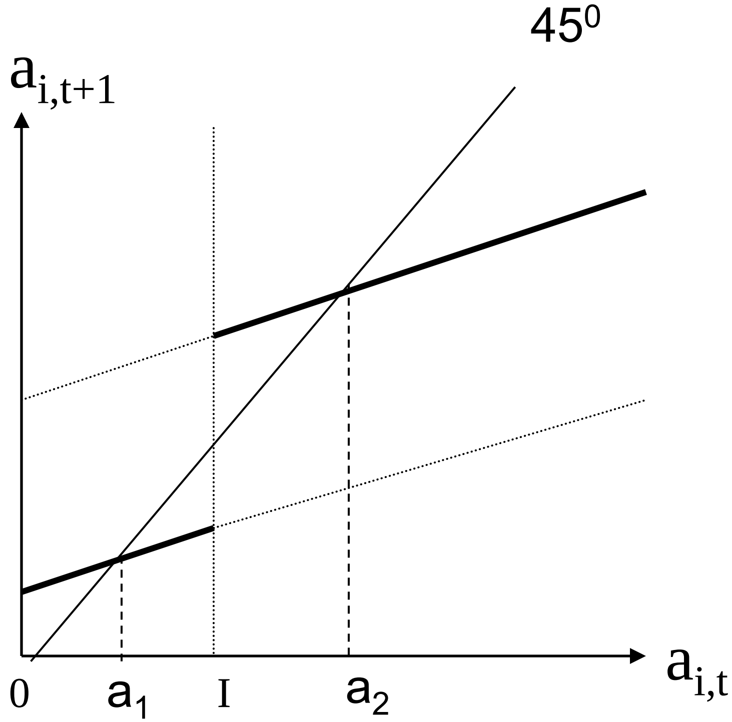

Fig. 7(a)

($w_t=\underline{w}$)

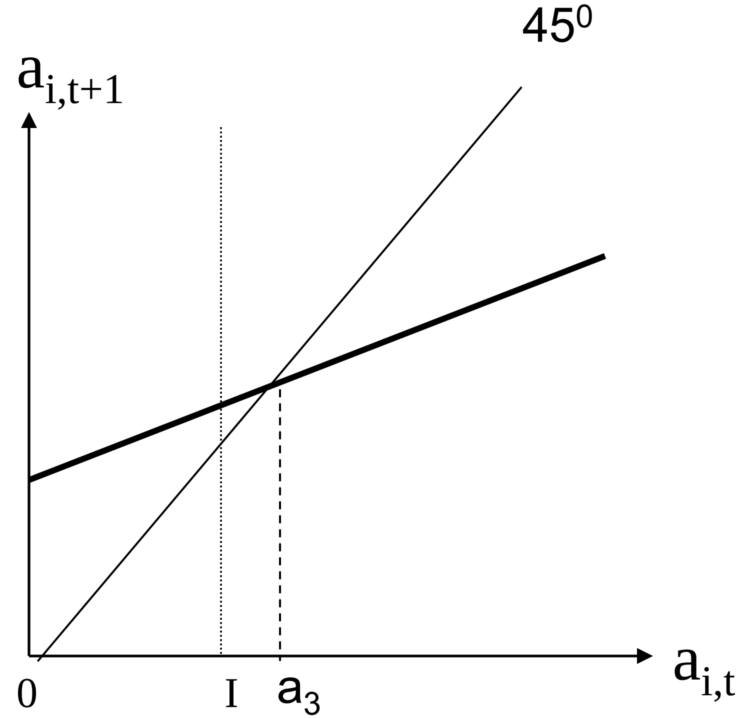

Fig. 7(b)

($w_t=\overline{w}$)