EC403
Michaelmas Test

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You have one hour. Explain all your answers. Answer question 1 and one other.

1. True, False, Explain.

   (a) When an intercept is not included in the X matrix, the usual regression R^2 can depend on the units of measurement of the included X variables.

   (b) When there are homogeneous linear restrictions $R\beta = 0$, the restricted least squares estimator (imposing these restrictions) is best linear unbiased.

   (c) A sufficient condition for the n by n matrix M to be a projection onto the linear space $\mathcal{L} = \{Mx : x \in \mathbb{R}^n\}$ (i.e. $(y - My)'z = 0$ for all $y \in \mathbb{R}^n$ and $z \in \mathcal{L}$) is that: M is idempotent.

   (d) The significance level of a test should be greater than or equal to the power of a test.

   (e) The necessary and sufficient condition for the central limit theorem to hold for a normalized sum, $T_n = \sum_{i=1}^{n} X/\sqrt{n}$, of independent and identically distributed random variables X_i is that $E(X_i^2) < \infty$.

   (f) Suppose that $T\widehat{\theta} \xrightarrow{d} \chi^2(1)$, then $T \ln \widehat{\theta} \xrightarrow{d} \ln \chi^2(1)$.

   (g) In a linear regression: omitted variable bias is large when the correlation between the left out variable and the included variable is large.

   (h) Generalized method of moments is better than maximum likelihood estimation because it does not require the specification of the data distribution.

   (i) If a moving average process is invertible, then it is stationary.

   (j) If a time series is non-stationary, then it can be made stationary by taking first differences.

2. Consider the multiple regression equation:

$$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{3i} + \varepsilon_i, \quad i = 1, \ldots, 1000.$$

   Show how you could test the following hypotheses. Take a significance level of 1% and give the critical region (this involves the test statistic and the critical value from the appropriate approximate distribution). You can assume that the usual regression assumptions hold, except that the errors are heteroskedastic, i.e. $\text{var}(\varepsilon_i) = \sigma_i^2$, where σ_i^2 are unknown.

(a) $\mathbf{H}_0 : 2\beta_2 - \beta_1 = 1$. $\mathbf{H}_\text{A} : 2\beta_2 - \beta_1 < 1$.

(b) $\mathbf{H}_0 : \beta_0 = \beta_1 = \beta_2 = \beta_3$ vs. $\mathbf{H}_\text{A}$: the general alternative.

(c) Now suppose that σ_i^2 are known exactly. How would you change your answers to parts (a) and (b)?

3. Consider the multiple regression equation:

$$y = X_1\beta_1 + X_2\beta_2 + X_3\beta_3 + \varepsilon,$$

where X_j is an $n \times k_j$ matrix, $j = 1, 2, 3$. The usual regression assumptions including normality may be assumed. Suppose that researcher A regresses y on X_1 and X_2 to obtain $\widehat{\beta}_{A1}$ and $\widehat{\beta}_{A2}$, and researcher B regresses y on X_1 and X_3 to obtain $\widehat{\beta}_{B1}$ and $\widehat{\beta}_{B3}$. Let $\widehat{\beta} = (\widehat{\beta}'_{A1}, \widehat{\beta}'_{A2}, \widehat{\beta}'_{B1}\widehat{\beta}'_{B3})'$.

(a) Derive the mean and variance covariance matrix of $\widehat{\beta}$

(b) Is it true to say that if researcher A finds a significant coefficient on X_1 then researcher B will also tend to find a significant effect for this variable? Discuss.

(c) Define a Hausman test of the null hypothesis that $E\widehat{\beta}_{A1} = E\widehat{\beta}_{B1}$, that is, define the test statistic in terms of observable quantities and give its limiting distribution under the null hypothesis. Discuss also under what circumstances the null hypothesis could be true.

4. Suppose that

$$y_t = \beta x^e_{t,t+1} + \varepsilon_t,$$

where ε_t are i.i.d. $N(0, \sigma^2)$ and $x^e_{t,t+1} = E[x_{t+1}|x_t, x_{t-1}, ..]$. In other words, $x^e_{t,t+1}$ is the rational expectation of future x given the history. Suppose also that

$$x_t = u_t - \theta u_{t-1},$$

where u_t are i.i.d. $N(0, s^2)$ and independent of ε_t, and $|\theta| < 1$. You may find it helpful below to use the lag operator L, where $Lx_t = x_{t-1}$.

(a) Derive the autocovariance function of x_t

(b) Show that $x_{t+1} = (-\theta)\sum_{j=0}^{\infty} \theta^j x_{t-j} + u_{t+1}$, and use this formula to obtain $x^e_{T+1,T+2}$. Now provide a forecast of y_{T+1} given only the history $\{y_t, x_t\}_{t=-\infty}^{T}$.

(c) Show that $x^e_{t,t+1} = \theta(x^e_{t-1,t} - x_t)$, and interpret this formula in the case that x_t is positively autocorrelated.

(d) Show that $y_t = \theta y_{t-1} - \beta\theta x_{t-1} + \varepsilon_t - \theta\varepsilon_{t-1}$, and explain how you would estimate β given knowledge of θ and a sample $\{x_t, y_t\}_{t=1}^{T}$.