Economics 403
Michaelmas Test

Oliver Linton

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Answer question one and one other. Explain all your answers.

1. Comment briefly on each of the following statements, indicating whether or not you agree. No formal proofs are required, but be sure to spell out your reasons and to explain technical terms.

   (a) One can have exact collinearity even if the variables are not highly correlated. Just consider the three-variable case where one variable is the sum of the other two.

   (b) Suppose a random sample is drawn from two populations each with mean μ, but the even-numbered observations have a lower population variance than the odd-numbered ones. In this case the average of just the even-numbered observations can be a better estimate of μ than the average of all the observations.

   (c) Suppose that $T\widehat{\theta} \xrightarrow{d} \chi^2(1)$, then $T \ln \widehat{\theta} \xrightarrow{d} \ln \chi^2(1)$.

   (d) When selecting regressors for a linear model, it is better to include too many variables than too few. Redundant variables may increase variance, but they do not produce bias. At least this is true for the fixed design regression model.

   (e) The significance level should be greater than or equal to the power of a test.

2. Consider the linear model

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + u. \tag{1}$$

   If we set $z = y - x_2$, $w_1 = x_1 - x_2$, and $w_2 = x_2$, we can rewrite (1) as

$$z = \alpha_0 + \alpha_1 w_1 + \alpha_2 w_2 + v. \tag{2}$$

   (a) Express α_0, α_1, α_2, and v in terms of β_0, β_1, β_2 and u.

(b) Compare the estimates $\hat{\beta}_0$, $\hat{\beta}_1$, $\hat{\beta}_2$, their estimated variance, the residuals $\hat{u}$, and R^2 obtained by applying OLS to (1), with the estimates $\hat{\alpha}_0$, $\hat{\alpha}_1$, $\hat{\alpha}_2$, their estimated variances, $\hat{v}$ and R^2 obtained by applying OLS to (2). Illustrate your conclusions by a graph.

(c) Generalize your conclusions to the comparison of the following regressions in the case of estimation of a production function:

i. Regress $\log Q$ on 1, $\log K$ and $\log L$.
ii. Regress $\log(\frac{Q}{L})$ on 1, $\log K$ and $\log L$.
iii. Regress $\log(\frac{Q}{L})$ on 1, $\log(\frac{K}{L})$
iv. Regress $\log(\frac{Q}{L})$ on 1, $\log(\frac{K}{L})$, and $\log(L)$.

Stating any additional assumptions you need, compare the sum of squared residuals and the R^2 of these three equations. Explain which regression equation is to be preferred.

(d) Now say how you might test the null hypohesis of constant returns to scale using only a standard t-test.

3. Consider the nonlinear regression model

$$y_i = \exp(\alpha + \beta x_i) + u_i \qquad i = 1, \ldots, n \tag{1}$$

where the u_i are iid $N(0, 1)$ random variables.

(a) Find the first order conditions for the maximum likelihood estimates of α and β.

(b) Find an expression for a derivative based algorithm for calculating the maximum likelihood estimates of α and β.

(c) Derive the asymptotic properties of the MLE $\widehat{\theta} = (\widehat{\alpha}, \widehat{\beta})$. In particular, give a brief discussion about consistency, and provide the asymptotic covariance matrix.

(d) Derive the LM (score) test statistic for the hypothesis $\beta = 0$. Discuss the advantages of this LM test over the Wald and LR tests of the hypothesis.