

# Economics 403

## Michaelmas Test

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**Answer question one and one other. Explain all your answers.**

1. Explain whether you agree or disagree with the following. Be sure to define basic terms and give reasons for your answers. No formal proofs are required, but be sure to spell out your reasons and to explain technical terms.
  - (a) In the regression model  $y = \alpha + \beta x + u$  where  $u$  is normal with mean zero and scalar covariance matrix, the best unbiased estimate of  $\beta^2$  is the square of the OLS estimate  $b$ .
  - (b) When the errors in a linear model are autocorrelated, the standard tests based on ordinary least squares always reject too often.
  - (c) Let  $\bar{X}$  be the sample mean of  $n$  independent normal random variables each with mean one and variance one. When  $n$  is large,  $\bar{X}$  and  $\frac{1}{\bar{X}}$  have approximately the same distribution.
  - (d) The Wald test is always more powerful than the LM test.
  - (e) Suppose a random sample is drawn from two populations each with mean  $\mu$ , but the even-numbered observations have a lower population variance than the odd-numbered ones. In this case the average of just the even-numbered observations can be a better estimate of  $\mu$  than the average of all the observations.

2. Consider the errors in variables model

$$y_i^* = \beta x_i^*$$

where  $y_i^*, x_i^*$  are subject to error

$$y_i = y_i^* + \varepsilon_i, \quad x_i = x_i^* + u_i.$$

Assume that  $(x_i^*, u_i, \varepsilon_i)$  are i.i.d. random variables mutually independent with  $E(\varepsilon_i) = E(u_i) = 0$ . We observe a sample  $\{y_1, x_1, \dots, y_n, x_n\}$ .

- (a) Show that the OLS estimator of  $\beta$  is biased

(b) Under what conditions is the estimator

$$\tilde{\beta} = \frac{\sum_{i=1}^n y_i}{\sum_{i=1}^n x_i}$$

consistent? Derive its asymptotic distribution in this case.

(c) Now consider the orthogonal regression estimator that minimizes the orthogonal distance between the points  $(x_i, y_i)$  and the line  $y_i = \beta x_i$

i. Explain why this is equivalent to minimizing the criterion

$$Q_n(\beta) = \frac{1}{1 + \beta^2} \sum_{i=1}^n (y_i - \beta x_i)^2$$

with respect to  $\beta$ .

ii. When will this estimator be consistent?

3. The following test of structural change was proposed by Chow. Suppose we have  $n_1 + n_2$  observations on a random outcome variable and on  $K$  condition variables. The expected outcome is a linear function of the conditions, but the coefficients might differ for the two groups of observations. In matrix notation, we have

$$\begin{aligned} y_1 &= X_1 \beta_1 + u_1 \\ y_2 &= X_2 \beta_2 + u_2 \end{aligned}$$

where (for  $i = 1, 2$ )  $y_i$  is an  $n_i$ -dimensional random outcome vector,  $x_i$  is an  $n_i \times K$  matrix of nonrandom condition variables,  $u_i$  is an  $n_i$ -dimensional error vector, and  $\beta_i$  is a  $K$ -dimensional unknown parameter vector. Define

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \quad u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad Z = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \quad W = \begin{bmatrix} 0 \\ X_2 \end{bmatrix} \quad X = \begin{bmatrix} X_1 & 0 \\ 0 & X_2 \end{bmatrix}$$

Suppose the elements of  $u$  are iid  $N(0, \sigma^2)$  random variables. Assuming that  $X$  has rank  $2K$ , define  $Q_i$  to be the sum of squared residuals from regressing  $y_i$  on  $X_i$ . Let  $Q$  be the sum of squared residuals from regressing  $y$  on  $X$  and let  $Q^*$  be the sum of squared residuals from regressing  $y$  on  $Z$ .

- Show that  $Q = Q_1 + Q_2$ . Using the variables  $Q_1$ ,  $Q_2$ , and  $Q^*$ , construct a test of the hypothesis that  $\beta_1 = \beta_2$ .
- Show that your test is equivalent to testing that  $W$  has zero coefficient in the regression of  $y$  on  $(Z, W)$ .
- Suppose  $X_2$  has rank  $n_2 < K$ . Show that your test in (a) is still valid if the numerator degrees of freedom is now  $n_2$  and the denominator degrees of freedom is  $n_1 - K$ . (Explain how you would calculate the residual vectors.)
- How would you modify your test in (b) if  $\text{rank}(X_2) < K$ ?
- How would you modify your tests in (a) and (b) if the error variances in the two groups of observations were unequal?