# Methods of Economic Investigation II (EC403)
# Problem Set #1

October 1, 2001

**Instructions**: Prepare for the week of 14th October.

1. If $X$ is a non zero matrix of order $n$ by $K$ show that $X'X$ is (a) symmetric and (b) positive semi-definite. Under what condition on $X$ is $X'X$ positive definite?

2. If $A$ and $B$ are square, non-singular matrices of the same order show that $(A')^{-1} = (A^{-1})'$ and $(AB)^{-1} = B^{-1}A^{-1}$.

3. Use the definition of non-singularity, namely that a square matrix is non-singular if and only if there exists no $x$ except $x = 0$ such that $Ax = 0$, to show that

   (a) If $A$ is positive definite, then $A$ is non-singular

   (b) If $X$ is an $n \times k$ matrix, $n > k$, of full column rank $k$, then the $k \times k$ matrix $X'X$ is positive definite and non-singular

4. If $x$ is an $n$ vector distributed as $N(\mu, \Sigma)$. What is the distribution of $a'x + b$, where $a, b$ are vectors of constants?

5. Show that for a positive definite symmetric matrix $A$, there exists a square non-singular matrix $P$ such that $A = PP'$. Use this result to show that if $x \sim N(\mu, \Sigma)$, where $\Sigma_{n \times n}$ is off full rank, the distribution of the scalar random variable $z = (x - \mu)'\Sigma^{-1}(x - \mu)$ is $\chi^2$ with $n$ degrees of freedom.

6. Let $x = (x_1, \dots, x_k)'$ and consider the function $y = f(x)$, where $y$ is a scalar. We define the Hessian of $f$ to be the $k \times k$ matrix with typical element

$$\frac{\partial^2 f}{\partial x_i \partial x_j}.$$

   Find the Hessian of the function $y = x'Ax$, where $A$ is a real square matrix.

7. If $X$ and $Y$ are independently distributed random variables, show that $E(X|Y) = E(X)$. Is the reverse true?

8. If $E(X|Y) = E(X)$, show that $cov(X, Y) = 0$. Is the reverse true?

9. Suppose that
$$y_i = \beta x_i + \varepsilon_i, \quad i = 1, \ldots, n,$$
where $\varepsilon_i$ are i.i.d. with mean zero and variance $\sigma^2$, while $x_i$ are fixed regressors such that $\sum_{i=1}^n x_i^2 > 0$. Consider the three estimators $\widehat{\beta} = \sum_{i=1}^n x_i y_i / \sum_{i=1}^n x_i^2$, $\widetilde{\beta} = \overline{y}/\overline{x}$, and $\overline{\beta} = \sum_{i=1}^n y_i/x_i$. Derive the mean and variance of all three estimators and say which estimator you prefer.

10. Consider the following regression
$$y = \alpha x + \beta z + u,$$
and let $\widehat{\alpha}, \widehat{\beta}$ be the coefficients from the regression of $y$ on $x$ and $z$. Now consider the following iterated least squares procedure. First regress $y$ on $x$, call the coefficient $\widetilde{\alpha}_1$. Then regress the residuals from this regression on $z$, call the coefficient $\widetilde{\beta}_1$. Now continue by regressing the residuals from the last regression on $x$, to get $\widetilde{\alpha}_2$ etc. Using the data in findat.wf1 let $y = sp500tr$, $x = 1$, and $z = tbill$, compute the bivariate regression. Comment on the regression coefficents. Now compute the iterated least squares procedure. What happens to the coefficients of the iterated least squares estimator as the number of iterations increases?