Methods of Economic Investigation II (EC403) Problem Set #2

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1. Suppose that I have two unbiased estimates $\widehat{\theta}_1, \widehat{\theta}_2$ of a parameter θ such that $\mathrm{var}(\widehat{\theta}_1) = \sigma_1^2, \mathrm{var}(\widehat{\theta}_2) = \sigma_2^2$, and $\mathrm{cov}(\widehat{\theta}_1, \widehat{\theta}_2) = \sigma_{12}$. Now consider the class of estimators
$$\widehat{\theta}(\omega_1, \omega_2) = \omega_1 \widehat{\theta}_1 + \omega_2 \widehat{\theta}_2.$$
   (a) What restriction on ω_1, ω_2 is necessary to make $\widehat{\theta}(\omega_1, \omega_2)$ unbiased.
   (b) What is the optimal value for ω_1, ω_2, where optimality is in the sense of Best Unbiased estimators.

2. Consider the multiple regression equation:
$$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{3i} + \varepsilon_i, i = 1, 2, \ldots, 25.$$
   Show how you could test the following hypotheses using only linear regressions. Using a significance level of 5% give the exact critical region involving the test statistic and the critical value from the appropriate distribution. You can assume that all of the usual regression assumptions hold.
   (a) $\mathbf{H}_0$: $\beta_0 = \beta_1 = \beta_2 = \beta_3 = 0$ vs. $\mathbf{H}_\mathrm{A}$: not $\mathbf{H}_0$.
   (b) $\mathbf{H}_0$: $\beta_2 = \beta_1$ vs. $\mathbf{H}_\mathrm{A}$: $\beta_2 > \beta_1$.
   (c) $\mathbf{H}_0$: $\frac{\beta_0}{\beta_1} - \frac{\beta_2}{\beta_1} = 1$ vs. $\mathbf{H}_\mathrm{A}$: not $\mathbf{H}_0$.

3. Consider the following three definitions of R^2
$$\begin{aligned} R_1^2 &= 1 - \frac{RSS}{TSS} = 1 - \frac{\sum_{i=1}^n (y_i - \widehat{y}_i)^2}{\sum_{i=1}^n (y_i - \overline{y})^2} \\ R_2^2 &= \frac{[\sum_{i=1}^n (y_i - \overline{y})(\widehat{y}_i - \overline{\widehat{y}})]^2}{\sum_{i=1}^n (y_i - \overline{y})^2 \sum_{i=1}^n (\widehat{y}_i - \overline{\widehat{y}})^2} \\ R_3^2 &= \frac{ESS}{TSS} = \frac{\sum_{i=1}^n (\widehat{y}_i - \overline{y})^2}{\sum_{i=1}^n (y_i - \overline{y})^2}. \end{aligned}$$
   Construct an example where $R_1^2 < 0$ and $R_3^2 > 1$.

4. Computer software gives you $t_1, t_2, \ldots, t_K$ and F, R^2. What can we expect to see in terms of the relationship between these quantities? The following question explores this issue. We consider the special case that $K = 3$. We also suppose that $u_i \sim N(0,1)$ and that $X'X/n = I_3$. The first regressor is just a column of ones. We have three hypotheses of interest.

$$\begin{aligned} H_0^a &: \beta_2 = 0 \text{ and } \beta_3 = 0, \quad H_A^a : \text{not} \\ H_0^b &: \beta_2 = 0 \ , \quad H_A^b \text{ not} \\ H_0^c &: \beta_3 = 0 \ , \quad H_A^c \text{ not} \end{aligned}$$

First of all, construct the acceptance regions for the usual tests of the three hypotheses. Is it possible to: (a) Accept H_0^b and H_0^c but reject H_0^a?(b) Accept H_0^a but reject both H_0^b and H_0^c?

5. True, False, Explain.

   (a) A one-sided 95% confidence interval strictly contains a two-sided 95% confidence interval.

   (b) The R^2 from a multiple regression always lies between 0 and 1.

   (c) In a regression of weight on an intercept and height, measuring the variables in metric units (meters and kilograms) or in US units (feet and pounds) will not affect any of the parameter estimates.

   (d) A significant F statistic but insignificant individual t-statistics is evidence of multicollinearity.