Methods of Economic Investigation II (EC403)
Problem Set #4

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1. Construct an example of a random sequence X_n such that $X_n \xrightarrow{p} 0$ but X_n does not converge in mean square. Suppose that $\sqrt{T}(\widehat{\theta} - 2\pi) \xrightarrow{d} N(0,1)$, then show that $T(1 - \cos\widehat{\theta}) \xrightarrow{d} \chi^2(1)$.

2. Consider the multiple regression equation:

$$y = X_1\beta_1 + X_2\beta_2 + \varepsilon,$$

where X_j is an $n \times k_j$ matrix, $j = 1, 2$. The usual regression assumptions including normality may be assumed.

   (a) Show how to construct a Hausman test for exclusion of the variables in X_2. Specifically, explain how to construct the test statistic and give the distribution from which critical values can be obtained. Compare this test with the usual regression F-test with special regard to the case where $X_1'X_2 = 0$.

   (b) Now suppose that $k_j = 1$, $j = 1, 2$, and consider the modified t-statistic

$$t^* = \frac{\widehat{\beta}_2}{s^*\sqrt{(X_2'M_1X_2)^{-1}}},$$

where $\widehat{\beta}_2$ is the OLS estimate of β_2 from the full regression, i.e. $\widehat{\beta}_2 = (X_2'M_1X_2)^{-1}X_2'M_1y$, but $s^{*2} = \varepsilon^{*\prime}\varepsilon^*/(n-1)$, where $\varepsilon^* = M_1y$ are the residuals from the restricted least squares estimation. Here, $M_1 = I - X_1(X_1'X_1)^{-1}X_1$ is the usual projection matrix. Explain why the exact distribution of t^* under the null hypothesis $\beta_2 = 0$ is not the $t(n-1)$ distribution. Show, however, that t^* is asymptotically standard normal. Compare t^* with the usual t-statistic in terms of their rejection rates when the normal approximation is used in both cases.

3. Suppose that

$y_i = \sin(\theta_0 x_i) + u_i, \quad i = 1, 2, \ldots, n,$

where u_i are i.i.d. mean zero and variance σ^2. Unfortunately, the scalar stochastic regressors x_i are correlated with u_i. However, there exist a $p \times 1$ vector of instrumental variables z_i that satisfy $E\left[z_i \left\{y_i - (\sin(\theta_0 x_i)\right\}\right] = 0$ (at only the true θ_0).

(a) Explain how to estimate θ_0 by the Generalized Method of Moments. In your answers include some discussion of the computational problems that may arise and also about the choice of weighting matrix.

(b) Derive the asymptotic distribution of the estimator (you can assume that the procedure is consistent). You do not have to be too rigorous; just try to get to the right answer.