Methods of Economic Investigation II (EC403)
Problem Set #5

Instructions: Prepare for the first week of next term.

1. Suppose that

$$y_t = \beta t + u_t; \qquad t = 1, 2, \ldots, T;$$
$$u_t = \varepsilon_t - 0.5\varepsilon_{t-1}; \qquad t = 1, 2, \ldots, T;$$

where ε_t are i.i.d, with $E(\varepsilon_t) = 0$ and $var(\varepsilon_t) = 1$:

(a) Calculate the autocorrelation function of u_t:

(b) Is u_t a stationary process? Is y_t a stationary process?

(c) Show how to construct an approximate 95% confidence intervals for the OLS estimator of β; $\hat{\beta} = \left\{\sum_{t=1}^{T} t^2\right\}^{-1} \sum_{t=1}^{T} t y_t$:

2. Suppose that

$$y_t = \gamma y_{t-1} + \beta x^e_{t;t+1} + \varepsilon_t;$$

where ε_t are iid $N(0; \sigma^2)$ and $x^e_{t;t+1} = E[x_{t+1} | x_t; x_{t-1}; \ldots]$: In other words, $x^e_{t;t+1}$ is the rational expectation of future x given the history. Suppose also that

$$x_t = \alpha + \rho x_{t-1} + u_t;$$

where u_t are iid $N(0; s^2)$ and independent of ε_t; and $|\rho| < 1$: You may find it helpful below to use the lag operator L; where $Lx_t = x_{t-1}$:

(a) Derive the autocovariance function of x_t

(b) Provide a point and interval forecast for x_{t+r}; for any $r > 0$; assuming that all parameters are known.

(c) Provide a point and interval forecast for y_{t+1} assuming that all parameters are known.

(d) Explain how you would estimate the unknown parameters $(\alpha; \rho; \gamma; \beta; \sigma^2; s^2)$ from a sample $\{x_t; y_t\}_{t=1}^{T}$: