Advance Econometric Theory (EC481)
Problem Set #2

1. Suppose that you would like to know the dimensions of a table, the width (W), and the length (L), but that you only can get observations on the area $A = WL$. In fact, you have two separate research assistants who measure each quantity many times, but unfortunately they multiply their measurements together before passing them onto you. Specifically, we have

$$\begin{aligned} L_i &= L_0 + \varepsilon_i \\ W_i &= W_0 + \eta_i, \quad i = 1, \ldots, n, \end{aligned}$$

where (ε_i, η_i) are independent and identically distributed with mean zero, and are mutually independent.

   (a) Suppose initially that ε_i, η_i are also normally distributed with the same unknown variance σ^2. Define a method of moments estimator of the unknown parameters W_0 and L_0 and prove that such a procedure will be consistent as $n \to \infty$.

   (b) Now suppose that ε_i, η_i are only symmetrically distributed and have potentially different variances σ_ε^2 and σ_η^2.

2. Suppose that we have a linear regression

$$y_i = \beta x_i + \varepsilon_i, \quad i = 1, \ldots, n,$$

where $x_i = i$. Suppose that ε_i is i.i.d. and known to satisfy $E(\varepsilon_i|x_i) = 0$ and $E(\varepsilon_i^3|x_i) = 0$. Consider estimators of β that solve the first order condition

$$\sum_{i=1}^{n} (y_i - \beta x_i)^3 h(x_i) = 0$$

for functions h. Discuss the conditions under which the resulting estimator will be consistent.

3. Suppose that

$$\Pr[y_t = 1] = \Phi(\beta_0 t),$$

where Φ is the standard normal c.d.f. Explain how you would compute an estimator of β_0 using simulation methods. Now show that this estimator is inconsistent unless $\beta_0 = 0$.

4. Suppose that we have a regression model

$$y_i = m(x_i, \theta) + \varepsilon_i, \quad i = 1, \ldots, n.$$

Suppose that ε_i is i.i.d. and known to satisfy $E(\varepsilon_i^l | x_i) = 0$, $l = 1, 3.$ Suppose also that we must compute by simulation from the relation

$$m(x_i, \theta) = \int M^2(x_i, \theta, \eta) dP(\eta) - \left(\int M(x_i, \theta, \eta) dP(\eta) \right)^2,$$

where M is some known function and P is a known probability distribution. Show how to get an unbiased simulator for $m(x_i, \theta)$, denoted $\widehat{m}(x_i, \theta)$. Now consider the estimator that solves the first order condition

$$\sum_{i=1}^{n} (y_i - \widehat{m}(x_i, \theta))^3 h(x_i) = 0$$

for some given function h with the same dimensions as θ. Explain why this estimator will be inconsistent. Then show, in general terms, how to modify the first order condition to ensure consistency.