Structural Transformation, Marketization and Female Employment

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Abstract

The rise in female participation to the labor market and the expansion of the services sector are two of the most remarkable stylized facts of the post-war period. We propose a model with three sectors: goods, services and home production, in which women have a comparative advantage in the production of services, both in the market and at home. Productivity growth is faster in market sectors than at home, and, within the market, it is faster in goods. Goods and services are poor substitutes in consumption, giving rise to structural transformation. On the other hand, market services are good substitutes to home production, driving marketization. Realistic differences in productivity growth across sectors can predict an important share of the rise in women’s market hours and the rise of services in the economy.

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1 Introduction

One of the most remarkable changes in labor markets since World War II is the increase in female participation to the labor market. To quote a simple fact, in the US the employment rate of prime age women has more than doubled from about 35% in 1945 to 77% at the end of the century, and similar trends can be detected in the majority of OECD countries. These developments have generated a large literature on the causes, characteristics and consequences of the increase in female involvement in the labor market. The main explanations put forward include a number of “supply-side” stories, and namely the introduction of oral contraceptives (Goldin and Katz, 2002), cheaper home appliances (Greenwood et al., 2005), changes in society’s views about the role of women’s work (Fernandez et al., 2004), and the increase in the return to experience for women (Olivetti, 2006).

Despite the substantial rise in female employment, the overall employment rate is fairly stable, as well as the average number of working hours per person, revealing that the sustained increase in female hours has been accompanied by a decrease in male hours (see Panel A in Figure 1). Interestingly, that the entire rise in female hours took place in the services sector, while the entire fall in male hours took sector producing “goods”, and namely the primary sector, manufacturing, construction and utilities (Panel C and D). This pattern is closely linked the process of structural transformation, namely, the reallocation of labor from goods to service industries (see Panel B). The rise in female hours in the service sector is accompanied by the observed decline in female in home production, declining from 38 to 28 weekly hours during 1968-2009. (Ramey and Francis 2009, Aguiar and Hurst 2007). This two observations are often linked up as the process of marketization of home production.

This paper’s objective is to investigate the role of structural transformation and marketization for our understanding of the rise in female hours of market work. The interaction between structural transformation, marketization and female employment has been largely overlooked in the literature. However there are at least two reasons why structural transformation and marketization can contribute to the rise in female hours of market work. First, service jobs have traditionally been perceived as more appropriate for women since the early 20th century, as they involved safer, cleaner working conditions and shorter working hours than jobs in factories (Goldin, 2006). While these factors may have become less relevant later in the century, other factors have been suggested to imply that women may still retain a comparative advantage in the service sector, like the relatively more intensive use of communication skills than in manufacturing, and the less intensive use of heavy manual skills (Galor and Weil 1996, Rendall 2010). Such comparative advantage is reflected in the initial allocation of women’s hours of market work. In 1968, 73% of market hours for a typical working woman in the US were supplied to the services sector, whereas the corresponding figure for men was only 50%. As the process of structural transformation implies an ex-
pansion of the sector in which women may have a comparative advantage, this could have important consequences for the evolution of women’s hours of market work.

The second reason is related to the allocation of women’s total working time to the market and the household. In 1965, women spent on average 60% of their total working time in home production, corresponding to roughly 38 hours per week, while men spent only about 11 hours in home production. Household work typically included child care, cleaning, food preparation, and in general activities that can be potentially marketized in the service sector. If the expansion of the service sector makes it cheaper to outsource these activities, one should expect a reallocation of women’s working time from the household to the market. This is indeed confirmed by the data from time use surveys.

Motivated by these stylized facts, we propose a dynamic model in which both the rise in female employment and service employment can be driven by uneven productivity growth across sectors. The model has three sectors (goods, services and home production) and two genders. Goods and services are imperfect substitutes in the consumer’s utility function, as they are inherently different commodities, e.g. cars and childcare in nurseries. Services and home production, however, are close substitutes, as they can encompass very similar commodities such as childcare in nurseries and childcare at home. Labor inputs of the two genders are imperfect substitutes in all the three sectors, and specifically females have a comparative advantage in both services and home production. The main driving force behind the dynamics of this model is uneven productivity growth across sectors, which is highest in the goods sector, followed by the services and finally home production. As goods and services are poor substitutes, faster productivity growth in the goods sector reallocates from the goods sector to services, resulting in structural transformation. At the same time, as services are good substitutes to home production, slower productivity growth in the home sector reallocates hours of work from the home to services, resulting in marketization.

Conditional on the initial hours allocations of the two genders, the combination of structural transformation and marketization may in turn lead to a rise in female market hours and a fall in male hours. Specifically, in the late 1960s, men were heavily employed in the goods sector and thus mostly suffered from the generalized decline in this sector. Women were instead mostly working in services or home production, and thus their employment rates were boosted by both the rise in services and the marketization of home production. As a consequence of such demand forces attracting women into the market, our model also predicts a rise in the gender wage ratio.

To quantitatively assess the relevance of the mechanisms described, we calibrate our model to match the initial 1968 time allocation by gender and sector, and then predict the final 2009 time allocation implied by uneven productivity growth. As another key labor market development during this time span was the rise in the relative human capital endowment of women relative to men, we also calibrate the efficiency level of female relative to male
labor by matching it to data on human capital levels. In our baseline calibration the model can account for 88 percent of the increase in female market hours, 75 percent of the increase in the service share, and 77 percent of marketization of female home hours. However, it can only account for 7 percent of the increase in gender wage ratio, as the direct positive effect of the demand forces on female wage is dampened in our general equilibrium framework by the generalized supply-side increase in female hours of work.

While there are extensive literatures that have independently studied the rise in female labor market participation and the rise of services, respectively, work on the interplay between the two phenomena is scant. Early work by Reid (1934), Fuchs (1968) and Lebergott (1993) has suggested that the two mechanisms could be related, without however proposing a unified theoretical framework. Our work is related to Galor and Weil (1996) and Rendall (2010), who illustrate that the rise in female employment may result as a consequence of brain-biased technological progress in a one-sector model in which females have a comparative advantage in the provision of “brain” (rather than “brawn”) inputs. In a similar vein, we assume women have a comparative advantage in service production in a model with two market sectors and home, in which the rise in female employment and service employment are outcomes of uneven sector-specific productivity growth.

One key implication of faster productivity growth in the market than the home sector is the marketization of home services, which contributes to both the rise of female employment and the share of services. This phenomenon is also featured in Akbulut (2011) and Rendall (2011). There are a number of differences between their and our model, but one key difference is that we let the gender wage ratio respond endogenously to sector-specific productivity growth, while the gender wage ratio is exogenous in Akbulut (2011) and Rendall (2011), as it is also done in most of the macro literature on the rise in female participation (Jones et al., 2003, Greenwood et al., 2004, and Heathcote et al. 2011).

The mechanisms driving the rise in service employment in our model were first studied by Ngai and Pissarides (2008) and Rogerson (2008), who focus on the dynamics of aggregate market hours and are thus agnostic about diverging trends by gender. Our paper introduces a gender dimension into their framework to explain gender specific trends in hours of work. Finally, motivated by Rogerson (2005), who observes a clear cross-country correlation between female and service employment, a number of papers have taken these ideas to an international perspective (Rogerson 2007, 2008, Rendall 2011, Olivetti and Petrongolo 2012) and relate lower female employment rates in Europe to the smaller weight of services in Europe relative to the US.

The paper is organized as follows. The next section describes our data set and documents the increase in female participation to the labor market, relative women’s wages, and the share of services in the US economy during 1968-2009. Section 3 proposes a model of a three-sector economy and shows predictions for the gender intensity in each sector and the
gender wage ratio. Section 4 provides a calibration of the main parameters in the model, and Section 5 shows the quantitative predictions of uneven productivity growth for the main variables of interest. Section 6 concludes.

2 Data and stylized facts

In order to construct evidence on the evolution of women’s position in the labor market and on the process of structural transformation we use micro data from the March Current Population Surveys (CPS) for survey years 1968 to 2009. This is the data source that offers the longest span on both the employment rates of various demographic groups and the industry structure. We complement the CPS with Time Use Data in order to obtain information on time use off the market and specifically on hours of home production.

Our working sample obtained from the CPS includes individuals ages 18-65 (both inclusive), who are not in full-time education, retired, or military. Our key labor input variable is represented by working hours. Annual hours are obtained from information on usual weekly hours and the number of weeks worked in the year prior to the survey year. Until 1975, weeks worked in the previous year are only reported in intervals (0, 1-13, 14-26, 26-39, 40-47, 48-49, 50-52), and to recode weeks worked during 1968-1975, we use within interval means obtained from later surveys. Similarly, usual weekly hours in the previous year are not available for 1968-1975, and thus we use hours worked during the survey week to measure usual weekly hours in the previous year. For individuals who did not work during the survey week we imputed usual weekly hours using the average of current hours for individuals of the same sex in the same year. Both adjustment methods have been previously applied to the March CPS by Katz and Murphy (1992). Our wage concept is represented by hourly earnings, obtained as wage and salary income in the previous year, divided by annual hours.

Figure 1 shows a number of interesting stylized facts about working hours over our sample period. Panel A plots usual weekly hours by gender, and shows a steady increase in female hours from about 16 weekly hours in 1968 to 25 hours in 2000, followed by a slight decline, while male hours were gradually declining throughout the sample period, from about 40 hours in 1968 to 30 hours in 2009. These diverging trend imply a near doubling of the gender hours ratio (female/male) from about 0.43 in 1968 to 0.84 in 2009, and a relatively stable aggregate market hours during this period.

To provide very simple evidence of the structural transformation process we classify hours of work into two broad sectors, which we define as goods and services. The goods sector includes the primary sector; manufacturing; construction and utilities. The service sector includes all the rest and namely transportation; post and telecommunications; wholesale trade; retail trade; finance, insurance and real estate; professional, business, repair and personal services; entertainment; health; education; welfare and non-profit organizations;
public administration. Panel B in Figure 1 plots the proportion of hours in services overall and by gender, and shows an increase of nearly 20 percentage points in the share of working time spent by both males and females in the service sector. For women such share was always substantially higher than for men, and rose from 74% to 91%, while for men it rose from 50% to 68%. Panel C further shows that all of the increase in female hours took place in the service sector, while Panel D shows that all of the fall in male hours took place in the goods sector. In other words, while women were moving (in net terms) from nonemployment into the service sector, with a corresponding fall in home production, men were moving from the goods sector to nonemployment, with a slight increase in their home production hours.

Figure 2 (Panel A) shows the evolution of the female to male wage ratio in the aggregate economy, obtained as the exponential of the mean log wage gap, unadjusted for characteristics. Women’s hourly wages remained relatively stable at or below 65% of male wages until about 1980, and then started rising up to about 80% in 2009. The combined increase in female hours and wages raised the female wage bill from 30% to two thirds of the male wage bill. When using hourly wages adjusted for human capital (age and age squared, race and four education groups), the rise in the gender wage ratio is only slightly attenuated, from 64% in 1968 to 77% in 2009 (see Panel B). Note finally that both the gender wage ratio and its trends are quite similar across broad sectors, and we use this piece of evidence to motivate free labor mobility and wage equalization across sectors in the model of the next section.

One very simple way to summarize the relationship between female employment and structural transformation consists in showing how much of the rise in female hours took place through the expansion of the service sector, rather than within each sector. We thus decompose the change in the female hours share between 1968 and 2009 into a term reflecting the change in the share of services, and a term reflecting changes in gender intensities within sectors. Having denoted by $L_m$ and $L_f$ the hours worked by men and women, respectively, and by $L$ their sum, the change in the female hours share between time 0 and time $t$ can be expressed as

$$\frac{L_{ft}}{L_t} - \frac{L_{f0}}{L_0} = \sum_i \alpha_{fi} \left( \frac{L_{it}}{L_t} - \frac{L_{i0}}{L_0} \right) + \sum_i \alpha_i \left( \frac{L_{fit}}{L_{it}} - \frac{L_{fi0}}{L_{i0}} \right),$$

where $i$ indexes sectors, $L_{fit}$ denotes female hours in sector $i$ at time $t$, $L_{it} = L_{mit} + L_{fit}$ denotes the sectoral hours, and finally $\alpha_{fi} = \left( \frac{L_{fit}}{L_{it}} + \frac{L_{fi0}}{L_{i0}} \right)/2$ and $\alpha_i = \left( \frac{L_{it}}{L_t} + \frac{L_{i0}}{L_0} \right)/2$ are decomposition weights. The first term in equation (1) represents the change in the female hours share that is attributable to structural transformation, while the second term reflects changes in the female intensity within sector. The $\alpha_{fi}$ and $\alpha_i$ terms serve as weights on the between- and within industry components, respectively, obtained as averages over the sample period. The results of this decomposition are reported in Table 1, both for the
Table 1: The decomposition of the share of female market hours into within- and between-sector components

<table>
<thead>
<tr>
<th>Time Period</th>
<th>Female Hours Change (x100)</th>
<th>% Between</th>
</tr>
</thead>
<tbody>
<tr>
<td>1968-2009</td>
<td>42.2 - 27.5 = 14.7</td>
<td>46.8</td>
</tr>
<tr>
<td>1968-1980</td>
<td>34.3 - 27.5 = 6.8</td>
<td>29.8</td>
</tr>
<tr>
<td>1980-1990</td>
<td>38.8 - 34.3 = 4.4</td>
<td>38.2</td>
</tr>
<tr>
<td>1990-2000</td>
<td>40.2 - 38.8 = 1.4</td>
<td>64.7</td>
</tr>
<tr>
<td>2000-2009</td>
<td>42.2 - 40.2 = 2.0</td>
<td>104.7</td>
</tr>
</tbody>
</table>

whole sample period and for each decade separately. The female hours share increased from 27.5% in 1968 to 42.2% in 2009, and nearly one half of this increase took place between industry, i.e. through the expansion of services. Looking across decades, one can notice a marked deceleration in the rise of the female hours share and an important increase in its between industry component.\(^1\) This decomposition illustrate the importance of the sectoral dimension in understanding the rise in female hours relative to male hours.

### 3 The model

To understand the facts presented, this Section proposes a model in which uneven productivity growth across sectors generates a rise in female hours via the rise of the service sector. The main outcome of the model is the time allocation of each gender across sectors.

We consider an economy that consists of a unit measure of representative male agents and a unit measure of representative female agents. Males and females are endowed with \(h_m(t)\) and \(h_f(t)\) units of human capital, respectively. Each male has one unit of time endowment, which can be allocated into three sectors at any time \(t\): goods, market services, and home sector. Each female has time endowment \(H(t)\).\(^2\)

As we abstract from capital in production, the model is essentially static. Its simple dynamics follows directly from the evolution of the sector-specific productivity growth, the gender-specific time endowment and the gender-specific human capital. We therefore omit time subscripts for simplicity.

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\(^1\)The between-industry component was actually explaining slightly more than 100% of the change in the female hours share since 2000, implying that the female intensity was slightly falling within sector during 2000-2009.

\(^2\)We focus on time allocation across types of work (market or home), and abstract from the allocation of the residual into leisure, sleeping and personal care. The value of \(H\) will be calibrated using the CPS population sample (see Section 4).
3.1 Firms

Production in sectors 1 and 2 only involves effective labor \( L_i \), where \( i = 1, 2 \) indexes sectors, according to \( y_i = A_i L_i \), where \( y_i \) denotes sector output and \( A_i \) denotes labor productivity. Effective labor is defined as a CES aggregator of male and female labor inputs:

\[
L_i \equiv \left( \xi_i \left( \frac{h_f L_f i}{h_f L_f i} \right)^{\frac{\eta-1}{\eta}} + (1 - \xi_i) \left( \frac{h_m L_m i}{h_m L_m i} \right)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}},
\]  

(2)

where \( \xi_i \) captures the comparative advantage of the female labor input in sector \( i \) and \( \eta > 1 \) indicates that female and male labor inputs are highly substitutable.

Profit maximization implies

\[
w_f = p_i A_i \xi_i h_f \left( \frac{L_i}{h_f L_f i} \right)^{1/\eta}; \quad w_m = p_i A_i (1 - \xi_i) h_m \left( \frac{L_i}{h_m L_m i} \right)^{1/\eta},
\]  

(3)

and relative labor demand is given by:

\[
\frac{L_f i}{L_m i} = \left( \frac{\xi_i w_m}{1 - \xi_i w_f} \right)^{\eta} \left( \frac{h_f h_m}{h_m h_f} \right)^{\eta-1},
\]  

(4)

which is a function of the endogenous wage ratio. As it will be shown later that the endogenous variables we are interested in will all depends on the wage ratio instead of the gender-specific wage. So for convenience, we let \( x \equiv w_f / w_m \) be the wage ratio which is the key endogenous variable in our model. Combining (2) and (4) gives an expression for (the inverse of) the female intensity in each sector:

\[
\frac{L_i}{h_f L_f i} = g_i (x) \equiv \xi_i^{\eta} \left[ 1 + \left( \frac{1 - \xi_i}{\xi_i} \right)^{\eta} \left( \frac{w_f h_m}{w_m h_f} \right)^{\eta-1} \right]^{-\frac{\eta}{\eta-1}} \quad i = 1, 2.
\]  

(5)

This is a useful expression that derive female intensity in each sector as a function of the endogenous wage ratio \( x \). The function \( g_i (x) \) will be shown to determine the female labour supply and eventually be the key for the implicit function of the the endogenous wage ratio.

3.2 Household

Each household consists of a male and a female who maximize a joint utility function, defined over two types of consumption goods \( c_g \) and \( c_s \):

\[
u (c_g, c_s) = \left( \omega c_g^{\frac{\varepsilon-1}{\varepsilon}} + (1 - \omega) c_s^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}},
\]  

(6)

where \( c_g \) can be purchased directly from the market sector 1 and \( c_s \) can be either purchased from the market sector 2 or produced at home (sector 3), and is defined as

\[
c_s = \left( \psi c_2^{\frac{\sigma-1}{\sigma}} + (1 - \psi) c_3^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}.
\]  

(7)
We assume $\varepsilon < 1 < \sigma$, implying that goods and services are poor substitutes while home services and market services are goods substitutes in consumption.

Home production has a similar structure as market production defined in equation (2):

$$c_3 = A_3 L_3; \quad L_3 \equiv \left( \xi_3 (h_{f3} L_{f3})^{\frac{\sigma}{\eta}} + (1 - \xi_3) (h_{m3} L_{m3})^{\frac{\sigma}{\eta}} \right)^{\frac{\eta}{\sigma}}. \quad (8)$$

The main difference between market production (2) and home production (8) is the home-specific return to human capital. This can be motivated e.g. by different usefulness of communication skills and the ability to operate capital in the home and market sectors.

The household budget constraint is given by

$$p_1 c_1 + p_2 c_2 = w_m (1 - L_{m3}) + w_f (H - L_{f3}), \quad (9)$$

where $H$ is the relative time endowment of females.

The first order conditions associated to the household maximization problem are:

$$(c_1) : \frac{\partial u}{\partial c_1} = \lambda p_1$$

$$(c_2) : \frac{\partial u}{\partial c_i} = \lambda p_2$$

$$(L_{f3}) : \frac{\partial u}{\partial c_s} A_2 \frac{\partial L_2}{\partial L_{f3}} = \lambda w_j, \quad j = m, f.$$  

They imply that the gender ratio in sector 3, $L_{f3}/L_{m3}$ and $L_3/(h_{f3} L_{f3}) = g_3(x)$ are in the same form as (4) and (5) with the home-specific return to human capital $h_{m3}/h_{f3}$ replacing $h_m/h_f$. We next define the implicit price of home production similarly as for market production in (3):

$$p_3 \equiv \frac{w_f}{A_3 \frac{\partial L_3}{\partial L_{f3}}} = \frac{w_f}{A_3 \xi_3 h_{f3} [g_3(x)]^{1/\eta}}. \quad (10)$$

Equations (3) and (10) together express the relative prices as functions of the endogenous wage ratio $x$.

We next derive demand for all goods and show that the relative demand across any two goods is a function of the corresponding relative price, thus a function of the endogenous wage ratio. By doing so we derive an implicit equation for the endogenous wage ratio using the labour market clearing condition.

The first-order conditions $(c_1)$ and $(c_2)$ imply the following demand for goods relative to total services:

$$\frac{c_1}{c_2} = \left( \frac{\omega \psi}{p_2} \frac{c_2}{1 - \omega p_1} \right)^{\frac{\varepsilon}{\sigma}} \left( \frac{c_s}{c_2} \right)^{\frac{(\sigma - \varepsilon)}{\sigma}}, \quad (11)$$

while conditions for $(c_2)$ and $(L_{f3})$ imply the following demand for market services relative to home services

$$\frac{c_2}{c_3} = \left( \frac{p_3}{p_2} \right)^{\sigma} \left( \frac{\psi}{1 - \psi} \right)^{\sigma}. \quad (12)$$
Finally, using (7) we can express \( \frac{c_1}{c_2} \) as
\[
\frac{c_1}{c_2} = \left( \frac{\omega \psi - p_2}{1 - \omega p_1} \right)^\varepsilon \psi^{\frac{\sigma - \varepsilon}{\sigma}} \left[ 1 + \left( \frac{1 - \psi}{\psi} \right) \left( \frac{c_3}{c_2} \right)^{\frac{\sigma - 1}{\sigma}} \right].
\] (13)

Equations (12) and (13) imply that relative demand are functions of relative prices, thus they are function of the wage ratio.

The household problem and the final consumption allocation allow us to obtain female hours of home production \( (L_{f3}) \) and, as a residual, female labor supply to the market \( (H - L_{f3}) \). In order to derive \( L_{f3} \), we rewrite the budget constraint (9) as:
\[
w_m + w_f H = \sum_{i=1}^{3} p_i c_i = p_3 c_3 \sum_{i=1}^{3} E_{i3},
\] (14)

where \( E_{ij} \equiv (p_i c_i)/(p_j c_j) \) is relative expenditure on goods \( i \) and \( j \). Given both relative demand and relative prices are function of the wage ratio, the relative expenditure \( E_{ij} \) is also a function of the wage ratio. Rearranging (14) to obtain
\[
L_{f3} = \left( H + \frac{w_m}{w_f} \right) \left( \sum_{i=1}^{3} \frac{E_{i3}}{I_3} \right)^{-1},
\] (15)

where \( I_3 \equiv (w_3 L_{f3}) / (p_3 c_3) \) is female’s income share in home production. Using the equilibrium wage (3) and the production function, we can solve for \( I_3 \):
\[
I_3 \equiv \frac{w_f L_{f3}}{p_i c_i} = \xi_3 [g_3 (x)]^{1/\eta - 1}.
\] (16)

Thus it follows from (15) that \( L_{f3} \) is a function of the wage ratio \( x \) as both \( E_{i3} \) and \( I_3 \) are function of \( x \). The female labor supply is obtained \( H - L_{f3} \) which of course is also a function of \( x \).

### 3.3 Market Equilibrium

We can now solve for the equilibrium gender wage gap \( w_f/w_m \), and the remaining labor allocation \( L_{ji} \) for \( j = m, f \) and \( i = 1, 2 \), using (3), labor market clearing
\[
L_{f1} + L_{f2} = H - L_{f3}; \quad L_{m1} + L_{m2} = 1 - L_{m3},
\] (17)
and goods market clearing
\[
c_i = y_i = A_i L_i; \quad i = 1, 2.
\] (18)
3.3.1 Relative Prices and Expenditure

Using (3) and (10), we can derive the relative prices as

\[ \frac{p_i}{p_j} = \frac{A_j \xi_j h_{fj}}{A_i \xi_i h_{fi}} \left( \frac{g_j(x)}{g_i(x)} \right)^{1/\eta}; \quad i, j = 1, 2, 3, \]  

(19)

where \( h_{f1} = h_{f2} = h_f \). Substituting (5) into (19) gives the following result:

**Lemma 1** Given \( \eta > 1 \), if \( \xi_i > \xi_j \), \( \frac{p_i}{p_j} \) is increasing in \( \frac{w_f}{w_m} \).

This result is intuitive: it means that, if sector \( i \) is more female intensive than sector \( j \) (because females have a comparative advantage in it), then an increase in female relative wages will increase the relative price of its output.

Relative expenditures \( E_{12} \) and \( E_{23} \) can be determined using the household consumption allocation (13) and (12). In particular, the choice between goods and market service implies

\[ E_{12} = \frac{p_1 c_1}{p_2 c_2} = \left( \frac{p_2}{p_1} \right)^{\varepsilon-1} \left( \frac{\omega \psi}{1-\omega} \right)^\varepsilon \psi^{\frac{\sigma-\varepsilon}{\sigma-1}} \left[ 1 + \left( \frac{1-\psi}{\psi} \right)^\sigma \left( \frac{p_2}{p_3} \right)^{\sigma-1} \right]^{\frac{\sigma-\varepsilon}{\sigma-1}}, \]  

(20)

which captures the extent of structural transformation between goods and services, and namely a declining \( E_{12} \) is consistent with a rising service economy.

The choice between home and market services implies

\[ E_{23} = \frac{p_2 c_2}{p_3 c_3} = \left( \frac{p_3}{p_2} \right)^{\sigma-1} \left( \frac{\psi}{1-\psi} \right)^\sigma, \]  

(21)

which captures the extent of marketization of services, and namely a higher \( E_{23} \) means that a larger share of services is marketized.

In the calibration section, we will calibrate \( \xi_i \) to match the female intensity in the three sectors using data on time allocation for 1968, the start of our sample period. As one would have expected, we find \( \xi_1 < \xi_2 < \xi_3 \). In this case the interpretation of Lemma 1 is that rising relative female wages imply rising \( p_2/p_1 \) and \( p_3/p_2 \). In other words, home service is becoming more expensive than market service, which in turns is becoming more expensive than goods. Given \( \sigma > 1 > \varepsilon \), this result in turn shows up as rising \( E_{23} \) and falling \( E_{12} \). In other words, rising female relative wages contribute to both the processes of marketization and structural transformation. Therefore, the gender dimension can potentially strengthen the prediction on the rise of service economy.

3.3.2 Time Allocation and Gender Wage Gap

We finally turn to time allocation to solve for the equilibrium gender wage gap. As the gender ratio of labor inputs in all sectors is given in (4) for \( i = 1, 2, 3 \), we only need to
solve for the female time allocation. Using market clearing conditions (17) and (18), and the production functions yields

\[
\frac{L_{fi}}{L_{fj}} = \frac{(L_j/L_{fj}) A_i c_i}{(L_i/L_{fi}) A_i c_j}, \quad i, j = 1, 2, 3. \tag{22}
\]

Using (19), this condition can be rewritten as

\[
\frac{L_{fi}}{L_{fj}} = E_{ij} \xi_i \left( \frac{g_i (x)}{g_j (x)} \right)^{1/\eta-1}, \quad i, j = 1, 2, 3. \tag{23}
\]

Combining (23) and the labor market clearing condition for females, we can solve for the demand for female home production time as

\[
L_{f3} = H \left[ 1 + E_{13} \xi_1 \left( \frac{g_1 (x)}{g_3 (x)} \right)^{1/\eta-1} + E_{23} \xi_2 \left( \frac{g_2 (x)}{g_3 (x)} \right)^{1/\eta-1} \right]^{-1}. \tag{24}
\]

We can now solve for the equilibrium gender wage gap by equating demand (24) and supply (15), i.e.

\[
\frac{\sum_i E_{ij} T_i}{H + \frac{1}{x}} = \frac{1 + E_{13} \xi_1 \left[ g_1 (x) \right]^{1/\eta-1} + E_{23} \xi_2 \left[ g_2 (x) \right]^{1/\eta-1}}{H}. \tag{25}
\]

Thus the equilibrium gender wage ratio \( x \) satisfies

\[
\frac{1 + E_{13} + E_{23}}{H + \frac{1}{x}} \left( 1 + E_{13} \xi_1 \left[ g_1 (x) \right]^{1/\eta-1} + E_{23} \xi_2 \left[ g_2 (x) \right]^{1/\eta-1} \right) = 0, \tag{26}
\]

which is an implicit equation that will solve for equilibrium wage ratio \( x \). The \( g_i (x) = L_i/(h_{fi} L_{fi}) \) term (defined in (5)) captures the equilibrium female intensity in sector \( i \), and the \( E_{ij} \) terms (defined in (21)-(20)) capture equilibrium expenditure relative to the value of home production.

**Lemma 2** Female labour supply and the gender wage gap depend on the extent of both structural transformation and marketization.

This can be seen directly from equations (24) and (26). Changes that are gender-neutral such as shocks to sector-specific productivity \( A_i \) can trigger marketization and structural transformation, which in turn affect female market hours \( H - L_{f3} \) and the gender wage gap \( w_f/w_m \).

Finally, to derive time allocation explicitly, we turn to marketization and structural transformation in terms of hours as opposed to expenditure. Marketization of hours can be expressed as

\[
\frac{L_{f2} + L_{m2}}{L_{f3} + L_{m3}} = \frac{L_{f2} \left( 1 + \frac{L_{m2}}{L_{f2}} \right)}{L_{f3} \left( 1 + \frac{L_{m3}}{L_{f3}} \right)},
\]
Using (23), this amounts to

\[
\frac{L_f^2 + L_m^2}{L_f^3 + L_m^3} = \frac{\xi_2}{\xi_3} \left( \frac{g_2(x)}{g_3(x)} \right)^{1/\eta-1} \left( \frac{1 + \frac{L_m^2}{L_f^2}}{1 + \frac{L_m^3}{L_f^3}} \right). \tag{27}
\]

Similarly for structural transformation:

\[
\frac{L_f^1 + L_m^1}{L_f^2 + L_m^2} = \frac{\xi_1}{\xi_2} \left( \frac{g_1(x)}{g_2(x)} \right)^{1/\eta-1} \left( \frac{1 + \frac{L_m^1}{L_f^1}}{1 + \frac{L_m^2}{L_f^2}} \right). \tag{28}
\]

**Lemma 3** When \(\xi_i\) and are identical across sectors, \(L_{fi}/L_{mi}\) are identical across sectors, and marketization and structural transformation in goods and hours are independent of gender-specific parameters \((H, \eta, \xi_i, h_{fi}/h_{mi})\).

The proof of Lemma 3 follows straight from equation (4) for \(\frac{L_{fi}}{L_{mi}}\), equation (5), and equations (21)-(20). Lemma 3 shows the importance of the presence of gender-specific comparative advantage. It states that if there is no difference in comparative advantage across gender, i.e. \(\xi_i = \xi\) for all sector \(i\), then gender-specific parameters plays no role in understanding marketization and structural transformation.\(^3\)

## 4 Calibration

Our model abstracts from the human capital accumulation decision, and in the quantitative exercise we simply match the growth in \(h_m\) and \(h_f\) to the gender-specific evolution of human capital observed in the data. Specifically, we estimate wage equations on the CPS for 1968-2009, including a female dummy, age and its square, a race dummy, education dummies and year dummies. The schooling categories are: high school dropout \((\text{drop})\), high school completed \((\text{hs})\), some college \((\text{sc})\), and college completed \((\text{cc})\). We use the coefficients on the education dummies to construct the human capital index for men in each year as

\[
h_{mt} = (x_{\text{drop},mt} + \exp(\beta_{\text{hs}})x_{\text{hs},mt} + \exp(\beta_{\text{sc}})x_{\text{sc},mt} + \exp(\beta_{\text{cc}})x_{\text{cc},mt}),
\]

where the \(x's\) are shares of the male population within each schooling category, and the \(\beta's\) are the associated coefficients from the wage regression (dropouts being the excluded category). An analogue expression gives \(h_{ft}\).

Using these indices, we find that the gender ratio in human capital has risen from 0.98 to 1.02 during 1968-2009, and we use these numbers to calibrate growth in \(h_f/h_m\). We finally assume \(h_{f3} = h_{m3}\) for all periods, implying that the usefulness of human capital for females and males is identical in the home sector.

\(^3\)Note that this is true even if one gender has absolute advantage in production of all goods, e.g. if \(\xi < 1/2\) i.e. male has absolute advantage in production of all goods.
We take the change in the relative female time endowment ($H_t$) as given. Changes in $H_t$ can be driven by changes in the gender mix in the population, or changes in gender time allocation to work (market and home) versus other activities (leisure, sleep and personal care). Our model is silent about both forces, and our strategy is to pick the growth in $H_t$ that matches changes in both. During the period 1968-2009, the gender ratio in the population according in the CPS decreases from 1.15 to 1.086, while the gender ratio in total hours of work rises from 1.078 to 1.12 (using information from both CPS and the American Time Use Survey). Together they imply a growth factor equal to 0.9995, thus we set $H$ constant in the calibration.

Taking the evolution of the human capital accumulation as given, the objective of the quantitative exercise is to investigate the importance of structural transformation and marketization in accounting for the rise in female labour supply and rise in the gender wage ratio. Key parameters in the calibration are the various elasticity of substitution parameters and the relative sectoral productivity growth rates.

We take $\sigma = 2.5, \varepsilon = 0.3$ from the literature on structural transformation and home production (see Ngai and Pissarides, 2008). As for the elasticity of substitution between male and female labor inputs, $\eta$, will be estimated on the CPS using data on hours and wages by gender aggregated at the state level. Our regression equation is

$$\ln \frac{w_{mst}}{w_{fst}} = \beta_0 + \beta_1 \ln \frac{L_{mst}}{L_{fst}} + \beta_2 \frac{h_{mst}}{h_{fst}} + \beta_s + \beta_t + \varepsilon_{st},$$

where $w_{mst}$ and $w_{fst}$ are average wages by gender in state $s$ and year $t$, $L_{mst}$ and $L_{fst}$ are the corresponding aggregate hours, $h_{mst}$ and $h_{fst}$ are controls for human capital, and $\beta_s$ and $\beta_t$ represent state and time fixed-effects, respectively. Human capital indicators are gender-specific vectors of proportions of college graduates, high-school graduates with some college and high school graduates, and $\beta_2$ denotes the vector of associated parameters. As labor supply is potentially endogenous with respect to wages, we instrument the hours ratio $L_{mst}/L_{fst}$ by its lagged value. Equation (29) is estimated for 1977-2009, as information on state of residence is only available in a consistent form from 1977 onwards. When we do not control for human capital ($\beta_2 = 0$), the estimate obtained for $\beta_1$ is -0.092, corresponding to an elasticity of substitution of 10.8. Controlling for human capital somewhat lowers the estimated elasticity of substitution to 8.4. In the simulations that follow we adopt a benchmark value for $\eta$ of 10, and perform some robustness checks using values around this benchmark.

The main driver for model dynamics is the difference in sectoral productivity growth $\gamma_i \equiv \dot{A}_i / A_i$. Note that $\gamma_i$ does not coincide with actual labor productivity growth, as labor productivity is based on total hours of work by both genders combined, while $A_i$ is productivity of effective labor, as defined in equation (2). In the Appendix, we explain the
mapping of actual labor productivity growth into $\gamma_i$ using the data on gender intensity from the CPS. The implied growth rates are $\gamma_1 = 2.3\%$ and $\gamma_2 = 1.4\%$.

Our quantitative exercise is simple. We match the initial gender wage gap and initial time allocation of genders across market and home activities. We then feed in the relative productivity growth for each sector and the relative human capital growth to predict the evolution of the gender wage gap and female labor supply until the end of the sample period. The exact calibration procedure is described in the Appendix. In brief, we choose $H$ to match the initial gender wage gap. We then set $\xi_i$ to match the initial within-sector gender hours ratio using (4), given data on relative human capital $h_f/h_m$, the gender wage gap $w_m/w_f$ and the within-sector gender hours ratio $L_{fi}/L_{mi}$. Finally, we choose the initial relative productivity $A_{ij} = A_i/A_j$ to match the allocation of hours across sectors. We show in the Appendix that the dynamics of the model depends on two growth rates, $\gamma_{12}$ and $\gamma_{23} = \gamma_{23} + \gamma_{h_f}$, where $\gamma_{h_f}$ is the growth in female human capital, which is equal to 0.5% over the sample period. Given values for $\gamma_1$ and $\gamma_2$, we obtain $\gamma_{12} = 0.9\%$. To obtain $\gamma_{23}$ we need a measure of $\gamma_{23}$, i.e. the difference in productivity growth between the service and the home sector. As the latter is not available, for our benchmark case we set $\gamma_{23} = 0.4\%$, which is equal to TFP growth in the service sector. This implicitly assumes that the home sector has zero TFP growth. We also consider the case in which $\gamma_2 - \gamma_3 = 0$, i.e. the only difference between the service and the home sector is due to the usefulness of human capital. We will conduct sensitivity analysis with regard to both $\gamma_{12}$ and $\gamma_{23}$. Baseline parameters are summarized in Table 2.

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>$\varepsilon$</th>
<th>$\eta$</th>
<th>$\gamma_{12}$</th>
<th>$\gamma_{23} = \gamma_{23} + \gamma_{h_f}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>0.3</td>
<td>10</td>
<td>0.9%</td>
<td>0.9%</td>
</tr>
</tbody>
</table>

5 Quantitative Results

5.1 Baseline Results

Table 3 reports our quantitative results. Using the baseline parameters (first row), our model replicates very well female market hours as a proportion of total work. In the data, an average female allocates 30% of her total working hours to market work in 1968, and this proportion rises to 45% by 2009. The model prediction for 2009 is 43%, implying that the model explains about 88% of the increase in female’s market hours. The model is also doing a very good job at predicting the rise in services. In the data, the share of service hours
increases from 0.57 to 0.77, while the model predicts an increase up to 0.72, i.e. it accounts for 75% of the observed increase. This is an improvement over Ngai and Pissarides (2008), who explains about 70% of the rise in services in a model without a gender dimension.\footnote{Another closely related paper is Rogerson (2008), who chooses model parameters to match the evolution of the US service employment share exactly, and then uses these parameters to predict European outcomes.}

Our model also gives a reasonable account for the change in the gender ratio within each sector. In the data, the ratio of female to male hours increases from 0.23 to 0.26 in the goods sector, and from 0.63 to 1.11 in the service sector, while it declines from 3.98 to 1.88 in the home sector. The model predicts this ratio to be 0.29 in 2009 in the goods sector, 0.79 in the service sector and 3.52 in the home sector. Thus the model over-predicts the increase in the gender ratio in the good sector, explains 34% of the increase in the service sector and 22% of the decline in the home sector. The main value-added of our framework to the literature on structural transformation consists in providing separate predictions for the dynamics of male and female hours, and for the change in the gender ratio within each sector, and these results show that this framework is performing reasonably well in all these aspects.

The model is, however, not doing a good job at explaining the rise in female relative wages. In the data, the wage ratio adjusted for human capital increases from 0.64 to 0.77, while the model predicts an increase to 0.65, corresponding to only 6.5% of the observed increase. In other words, due to the general equilibrium effect from the female labour supply decision the calibrated demand shock for female labor inputs is not strong enough to push the gender wage ratio close to observed levels.

Our model is among the very few which allow both the gender wage ratio and female market hours to be endogenously determined by the changing structure of labor demand. To interpret our results it is helpful to compare our calibration to that of Heathcote et al. (2011), who aim to explain the rise in gender wage ratio and female market hours in a one sector model in which female and male labor inputs perfect substitutes ($\eta \to \infty$). This assumption delivers a one-to-one mapping between the $\xi/(1 - \xi)$ ratio and the equilibrium wage ratio $w_f/w_m$:

$$\frac{w_f}{w_m} = \frac{\xi}{1 - \xi} \frac{h_f}{h_m},$$

obtained by setting $\eta \to \infty$ in equation (4) for a one-sector model. Heathcote et al. (2011) assume that $\xi/(1 - \xi)$ grows exogenously to match the observed rise in the wage ratio, and interpret the growth in $\xi$ as gender-specific technical change. In terms of our multi-sector model, their $\xi$ can be interpreted as a weighted average of $\xi_1$ and $\xi_2$ where the weight is the employment share of each sector. We do not assume exogenous changes in $\xi$ but instead calibrate the initial $\xi_i$’s to match the initial within-sector gender hours ratio. These satisfy $\xi_1 < \xi_2 < \xi_3$, reflecting the comparative advantage of females in services and home production.\footnote{In actual numbers $\xi_1 = 0.36$, $\xi_2 = 0.38$ and $\xi_3 = 0.42$.} We then predict the aggregate $\xi$ to be rising as labor reallocates from the goods sector.
to service sector (given $\xi_1 < \xi_2$). Quantitatively, though, this proposed mechanism is not strong enough to produce a substantial rise in $\xi$, explaining why we fall short of explaining the rise in the gender wage ratio. If we were to introduce some exogenous growth in $\xi_1$ and $\xi_2$, say 0.1% per year\textsuperscript{6}, the gender wage ratio would increase to 0.68, which amounts to about 30% of the observed increase. This would also improve model predictions for the gender hours ratio in both the service and home sectors, and for the fall in male market hours. However, in this case the model would overpredict female market hours and the gender ratio in the goods sector.

### 5.2 Mechanisms at Work

There are two major forces at work for the increase in female market hours and the change in the gender wage ratio. First the rise in female human capital under the assumption that human capital is useful in market production. Second, the process of structural transformation and marketization under the assumption that women have a comparative advantage in producing services. These mechanisms are related to the forces at work in Heathcote et al (2011), who find that skill-biased and gender-biased demand shifts are important for understanding the rise in female market hours. We do not model skill-biased demand shifts, and consider as exogenous the rise in relative female human capital. We, however, model gender-biased demand shifts as an outcome of structural transformation and marketization, insofar women have a comparative advantage in services.

To understand the contribution of these two forces, we conduct two counter-factual experiments. In the first, we shut down the human capital channel by setting $h_{fi} = h_{mi} = 1$ in all sectors $i$, and leaving $\gamma_{12} = 0.9\%$ and $\gamma_{23} = \gamma_{23} = 0.4\%$. In the second, we shut down the structural transformation channel by setting $\gamma_{12} = \gamma_{23} = 0$, but leaving $\gamma_{hf}$ unchanged at 0.5%. The results are shown in rows 2 and 3, respectively, of Table 2. Both experiments suggest that the increase in relative female human capital contributes to about two-thirds of the increase in female market hours and almost the entire rise in the wage ratio.

So far we have assumed that human capital is only useful for market production. We now relax this assumption and allow human capital to be equally important at home. In other words, we set $h_{fi} = h_f$ and $h_{mi} = h_m$ in all sectors, and $\gamma_{23} = \gamma_{23}$. Rows 4 and 5 now show a different picture about the relative power of structural transformation vs. human capital. In particular, the whole increase in female market hours is due to structural transformation, whereas the rise in the wage ratio continues to rely on the improvement in relative women’s human capital. But the overall effect on female hours is much weaker than in the baseline case, as the model now only predicts 20% of the rise in female market hours. Therefore, the

\textsuperscript{6}Note that if we follow Heathcote et al. (2011) by choosing the ratio of $\xi/(1 - \xi)$ to match the average annual growth rate of the wage ratio, which is 0.4% in the data, the implied growth rate for $\xi$ would be about 0.3%.
assumption that human capital is only useful in market production is crucial both in terms of the overall predictive power of the model and for the decomposition of the rise in female market hours into the two driving forces.

Finally, we examine the role of alternative values for productivity growth $\gamma_{12}$ and $\gamma_{23}$. We first assess the role of faster productivity growth in the goods sector by setting it equal to productivity growth in the service sector, $\gamma_{12} = 0$, but keeping productivity in both market sectors higher than in the home sector, i.e. $\gamma_{23} = 0.9\%$ as in the baseline case. Row 6 in Table 2 shows that the predictions on the wage ratio and female hours are not affected at all but as one would expect the model fails to capture the rise in the service share, now explaining less than half of the rise. We next assess the role of higher productivity growth in market sectors, by setting $\gamma_{23} = 0$, while keeping $\gamma_{12} = 0.9$. Row 7 in Table 2 shows that the predicted rise in female market hours is substantially reduced from 75% to 50% data, though the model predicts a slightly higher increase in the wage ratio, from 6.5% to 10%. This counterfactual exercise shows that while uneven productivity growth is an important driving force of female employment, it is mostly the growth differential between home and market sectors that matters, rather than the differential between the goods and service sectors. Indeed higher productivity growth in the goods sector than the service sector has limited predictive power for a similar reason why our model fails to explain the bulk of the rise in the gender wage ratio: both mechanisms hinge on the difference between $\xi_1$ and $\xi_2$ but their calibrated values do not differ enough to predict large gender changes. In the extreme case, if $\xi_1$ and $\xi_2$ were equal, differences in productivity growth between goods and service sectors would not contribute at all to both wage ratios and female employment.

6 Conclusions

The rise in female participation to the labor market and the rise of the service employment are two of the most remarkable stylized facts of the post-war period. We construct a three-sector model, in which women have a comparative advantage in the production of services, both in the market and in the household. Taking into account the uneven productivity growth across the three sectors and the rise in relative human capital of women relative to men, our model can account for a substantial fraction of the rise in female market hours, the increase in service share and the marketization of female home hours. In decomposing the mechanism at work, we find that, quantitatively, the important forces behind this result is the higher productivity growth in the market sectors relative to home sector and the relative increase in female human capital.

In addition to the observation of Rogerson (2005) that there is a strong cross-country correlation between female and service employment, using Multinational Time Use Study Fang and McDaniel (2012) show that cross-country differences in time allocation are driven
primarily by differences in female time allocation. By jointly modelling gender specific time allocation and the aggregate dynamics that drives time allocation, the model proposed in this paper could be extended to understand the observed cross-country time allocation pattern.

7 Appendix

7.1 Calibration

Here we give details on (1) how $H$ is chosen to match the initial gender wage gap and (2) how productivity growth differences $\gamma_{ij}$ and female human capital growth $\gamma_{hf}$ are used in the calibration.

To determine $H$, by definition we have

$$H = \sum_j L_{fj} = \sum_j \frac{L_{fj}}{L_{fj3}} L_{fj3} \implies L_{fj3} = \frac{H}{1 + \frac{L_{fj2}}{L_{fj3}} + \frac{L_{fj1}}{L_{fj3}}}.$$  

Substituting $L_{fj3}$ using household supply in (15) gives

$$\frac{H}{1 + \frac{L_{fj2}}{L_{fj3}} + \frac{L_{fj1}}{L_{fj3}}} = \frac{H + \frac{w_m}{w_f}}{(1 + E_{23} + E_{13})} I_3,$$

where

$$E_{ij} \equiv \frac{p_i c_i}{p_j c_j}, \quad I_j \equiv \frac{w_f L_{fj}}{p_j c_j} = \frac{L_{fj}}{L_{fj} + \frac{w_m}{w_f} L_{mj}}.$$

Equation (30) can be simplified as

$$H = \frac{w_m}{w_f} \left[ \frac{(1 + E_{23} + E_{13})}{(1 + \frac{L_{fj2}}{L_{fj3}} + \frac{L_{fj1}}{L_{fj3}}) I_3} - 1 \right]^{-1}.$$

Note given data on initial time allocation $(L_{fj}, L_{mj})_{j=1,2,3}$ and gender wage gap $(\frac{w_m}{w_f})$, we can compute $I_j$ and

$$E_{ij} = \frac{L_{fi}}{L_{fj} w_f L_{fj}} \frac{p_i c_i}{p_j c_j} \frac{w_f L_{fj}}{w_f L_{fj}} = \frac{L_{fi}}{L_{fj}} \frac{I_j}{I_i}.$$

Thus $H$ (1) is obtained given initial time allocation and initial gender wage gap.

Finally, we choose the initial relative productivity $A_{ij} \equiv \frac{L_{fi}}{L_{fj}}$ to match initial marketization and structural transformation. Using computed $\xi_i$ from equilibrium equation (4) and the data, we derive

$$\frac{L_i}{h_{fi} L_{fi}} = \left( \xi_i + (1 - \xi_i) \left( \frac{h_{mi} L_{mi}}{h_{fi} L_{fi}} \right)^{\frac{n_i - 1}{n_i}} \right)^{\frac{n_i}{n_i - 1}},$$

(31)
which give values for
\[
\frac{A_i h_i p_i}{A_j h_j p_j} = \frac{\xi_j (g_j(x))^{1/\eta_j}}{\xi_i (g_i(x))^{1/\eta_i}}.
\] (32)

From marketization (21), we can write
\[
E_{23} = \left(\frac{p_2}{p_3}\right)^{1-\sigma} \left(\frac{\psi}{1-\psi}\right)^\sigma = \left(\frac{A_2 h_{f2}}{A_3 h_{f3}}\right)^{1-\sigma} \left(\frac{A_3 h_{f3}}{A_2 h_{f2}}\right)^{1-\sigma} \left(\frac{\psi}{1-\psi}\right)^\sigma,
\]
so together with (32), we can compute an effective relative productivity
\[
\hat{A}_{23} \equiv \left(\frac{A_2 h_{f2}}{A_3 h_{f3}}\right) \left(\frac{1-\psi}{\psi}\right)^{\sigma-1} = \left(\frac{A_2 h_{f2}}{A_3 h_{f3}}\right) \left(\frac{1}{E_{23}}\right)^{1/\sigma}.
\] (33)

Similarly from structure transformation (20), we can write
\[
E_{12} = \left(\frac{\omega \psi}{1-\omega}\right)^{\varepsilon} \left(\frac{p_2}{p_1}\right)^{1-\varepsilon} M
\]
where
\[
M \equiv \left[1 + \left(\frac{1-\psi}{\psi}\right)^{\sigma-1} \left(\frac{p_2}{p_3}\right)^{\sigma-1}\right]^\frac{\varepsilon-\sigma}{\varepsilon-1} = \left[1 + \frac{1}{E_{23}}\right]^\frac{\varepsilon-\sigma}{\varepsilon-1},
\]
so together with (32), we can compute
\[
\hat{A}_{12} \equiv \left(\frac{A_1 h_{f1}}{A_2 h_{f2}}\right) \left(\frac{1-\omega}{\omega}\right)^{\frac{\nu}{\sigma-\varepsilon}} \left(\frac{p_2}{p_1}\right)^{\frac{\varepsilon-\sigma-\varepsilon}{\varepsilon-1}} = \frac{A_1 h_{f1} p_1}{A_2 h_{f2} p_2} \left(\frac{M}{E_{12}}\right)^{\frac{1}{\varepsilon}}.
\] (34)

Let \(\hat{\gamma}_{ij} \equiv \frac{\hat{A}_{ij}}{A_{ij}}\) be the growth rate of \(\hat{A}_{ij}\) and \(\gamma_{ij} \equiv \gamma_i - \gamma_j\), by definition we have
\[
\hat{\gamma}_{12} = \gamma_{12}; \quad \hat{\gamma}_{23} = \gamma_{23} + \gamma_{h_f},
\] (35)
where \(\gamma_{h_f}\) is the growth in female human capital.

We next describe how we obtain \(\gamma_1\) and \(\gamma_2\). Using KLEMS 2008 March release, we compute the real labour productivity growth in service and non-service sector, \(\hat{B}_i\), where
\[
Y_i = B_i (L_{fi} + L_{mi});
\] (36)
where \(Y_i\) is the real value-added of sector \(i\) and \(L_{gi}\) is the hours of work by gender \(g\). Using KLEMS data, \(B_i\) is simply real value-added per hour. We find \(\gamma_{B1} = 2.55\%\) and \(\gamma_{B2} = 1.48\%\), so the difference across the two sectors is 1.07\%. To link \(\gamma_{B_i}\) to \(\gamma_i\) in the model, rewrite (36) as
\[
Y_i = L_i B_i \left(\frac{L_{fi} + L_{mi}}{L_i}\right);
\]
where $L_i$ is the CES form of male and female labour hours as in the model (2), so we have

$$A_i = B_i \left( \frac{L_{fi} + L_{mi}}{L_i} \right)$$

We can rewrite it as

$$A_i = \frac{B_i}{h_f} \left[ \frac{L_{fi} + L_{mi}}{L_{fi}} \right] \left( \frac{h_{fi}L_{fi}}{L_i} \right),$$

where we can obtain the first two terms directly from the data and the last term from the model, where $\frac{L_i}{h_{fi}L_{fi}}$ is derived in before in (31).

To summarize, the growth rate we are interested is

$$\frac{\dot{A}_i}{A_i} = \dot{B}_i \frac{h_f}{B_i} - \frac{\dot{h}_f}{h_f} - \frac{\dot{I}_{fi}}{I_{fi}} - \frac{\dot{X}_i}{X_i},$$

(37)

where $I_{fi} \equiv \frac{L_{fi}}{L_{fi} + L_{mi}}$ is the intensity of female hours in sector $i$ and $X_i \equiv \frac{L_i}{h_{fi}L_{fi}}$. To compute growth rate of $X_i$, using (31)

$$\dot{X}_i = \left[ X_i^{\frac{\eta}{1 - \eta}} \right] \frac{1}{1 - \xi_i} \left( \frac{h_{mi}L_{mi}}{h_fL_{fi}} \right)^{\frac{1}{1 - \eta}} \hat{R}_i,$$

where $R_i = \frac{h_{mi}L_{mi}}{h_fL_{fi}}$ is the ratio of efficiency hour across gender, so

$$\frac{\dot{X}_i}{X_i} = X_i^{\frac{\eta}{1 - \eta}} \frac{1 - \xi_i}{1 - \xi_i} \left( \frac{h_{mi}L_{mi}}{h_fL_{fi}} \right)^{\frac{1}{1 - \eta}} \frac{\hat{R}_i}{R_i}$$

$$= \frac{1 - \xi_i}{\xi_i + (1 - \xi_i)} \left( \frac{h_{mi}L_{mi}}{h_fL_{fi}} \right)^{\frac{1}{1 - \eta}} \left( \frac{h_{m}/h_f + L_{mi}/L_{fi}}{h_{m}/h_f + L_{mi}/L_{fi}} \right)$$

So substituting into (37), the final formula is

$$\frac{\dot{A}_i}{A_i} = \dot{B}_i \frac{h_f}{B_i} - \dot{h}_f \frac{1}{h_f} - \frac{\dot{I}_{fi}}{I_{fi}} + \frac{(1 - \xi_i) \left( \frac{h_{fi}L_{fi}}{h_{m}L_{mi}} \right)^{\frac{1}{1 - \eta}} \left( \frac{h_{f}/h_{m} + L_{fi}/L_{mi}}{h_{m}/h_{f} + L_{mi}/L_{fi}} \right)}{\xi_i + (1 - \xi_i) \left( \frac{h_{fi}L_{fi}}{h_{m}L_{mi}} \right)^{\frac{1}{1 - \eta}}} \frac{\dot{R}_i}{R_i},$$

or as

$$\frac{\dot{A}_i}{A_i} = \dot{B}_i \frac{h_f}{B_i} - \left( \frac{\xi_i}{\xi_i + (1 - \xi_i)} \left( \frac{h_{fi}L_{fi}}{h_{m}L_{mi}} \right)^{\frac{1}{1 - \eta}} h_f + \frac{(1 - \xi_i) \left( \frac{h_{fi}L_{fi}}{h_{m}L_{mi}} \right)^{\frac{1}{1 - \eta}} h_m}{\left( \frac{h_{m}/h_{f} + L_{mi}/L_{fi}}{h_{m}/h_{f} + L_{mi}/L_{fi}} \right)} \right)$$

$$- \frac{\dot{I}_{fi}}{I_{fi}} + \frac{(1 - \xi_i) \left( \frac{h_{fi}L_{fi}}{h_{m}L_{mi}} \right)^{\frac{1}{1 - \eta}} L_{fi}/L_{mi}}{\xi_i + (1 - \xi_i) \left( \frac{h_{fi}L_{fi}}{h_{m}L_{mi}} \right)^{\frac{1}{1 - \eta}} L_{fi}/L_{mi}}.$$
Together with the average $\frac{\beta_i}{B_i}$ from 1970-2005 from KLEMS, we can calibrate $A_i$ using CPS data 1970-2005 related to gender, i.e. the growth rate of human capital for each gender $(h_t, h_m)$, and hours ratio across gender $\frac{L_{mi}/L_{ti}}{L_{mi}/L_{ti}}$ given computed $\xi_i$ from (4) using initial observation in the data. We obtain $\gamma_{A1} = 2.31\%$ and $\gamma_{A2} = 1.41\%$, so the difference across the two sectors is 0.89\%.

Given $\gamma_{12}$ and $\gamma_{23}$, we have the time series for $\hat{A}_{ij}$, together with $H$ and $\frac{h_t}{h_m} (t)$, we are now ready to derive the path of gender wage gap $\left( w_{m} / w_{f} (t) \right)$ from (25), female labour supply using (15) and share of service sector using (28).

References


Figure 1

Panel A:
Weekly Hours of Work per Person

Panel B:
Percentage Hours in Services

Panel C:
Female Weekly Hours of Work

Panel D:
Male Weekly Hours of Work

Source: Current Population Survey
Figure 2
Trends in gender wage ratios by sector

Panel A
Unadjusted Wage Ratios
(Female relative to Male)

Panel B
Wage Ratios Adjusted for Human Capital
(Female relative to Male)

Source: Current Population Survey
### Table 3

Data and model predictions

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