

# Accounting for Research and Productivity Growth Across Industries

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## Abstract

What factors underlie industry differences in research intensity and productivity growth? We develop a multi-sector endogenous growth model allowing for industry-specific parameters in the production functions for output and knowledge, and in consumer preferences. We find that long run industry differences in both productivity growth and R&D intensity mainly reflect differences in "technological opportunities", interpreted as the parameters of knowledge production. These include the capital intensity of R&D, knowledge spillovers, and diminishing returns to R&D. To investigate the quantitative importance of these factors, we calibrate the model using US industry data. We find that diminishing returns to research activity is the dominant factor.

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# 1 Introduction

Total factor productivity (TFP) growth rates differ widely across industries, and these differences appear linked to persistent cross-industry differences in R&D intensity – see Figure 1. This link is sometimes interpreted as causation. However, a priori it is not clear why the *level* of industry R&D should affect industry productivity *growth*, a point that has been made by Jones (1995) for the aggregate economy. Rather, both R&D and productivity growth depend on the response of firms to deeper industry parameters.

FIGURE 1 ABOUT HERE

We develop a general equilibrium model in which both research activity and productivity growth vary endogenously across industries, to identify the factors that account for differences in each. We show that the factors that influence TFP growth also have an impact on R&D intensity. However, the converse is not true: there exist industry characteristics that affect the level of industry R&D, but not necessarily industry productivity growth rates.

The empirical literature has identified three sets of factors as potential determinants of industry variation in research intensity and productivity growth: technological opportunity (factors that affect the efficiency of research), appropriability (the extent to which R&D benefits the innovator) and demand (which influences the returns to research). These factors are implemented in the model using standard preference and technology parameters drawn from growth theory. The industry-specific factors we study are: diminishing returns to research, knowledge spillovers over time, knowledge spillovers across firms, capital intensity in the production of goods, capital intensity in the production of ideas, the elasticity of substitution across different varieties of goods within each industry, and the industry’s market size.

As our interest is in industry comparisons, we focus on equilibria where the distribution of productivity within sectors is stable and rank-preserving. We find that differences in

industry TFP growth rates depend only on factors of technological opportunity. These include the extent to which research is subject to diminishing returns, the capital share of research spending, and total knowledge spillovers. By contrast, differences in R&D intensity also depend on the extent to which knowledge spillovers accrue from the firm's *own* stock of knowledge, which we interpret as a measure of appropriability.<sup>1</sup> Product demand is fundamental in providing incentives to perform research: nevertheless, we find that *industry differences* in equilibrium TFP growth rates and R&D intensity do not depend on demand parameters that are constant over time.<sup>2</sup>

To narrow down which factors of technological opportunity best account for cross-industry comparisons in a production-function based framework, we calibrate the model using US industry data. We find that the capital intensity of research cannot account for observed industry differences in productivity growth rates. Furthermore, we find that variation in appropriability has little impact on industry variation in R&D intensity. Thus, the model indicates that variation in diminishing returns to research and in the magnitude of spillovers must jointly account for patterns of productivity growth and research activity. Finally, for each industry we select these two parameters so that the equilibrium productivity growth rate and R&D intensity level in the model match the values in the data exactly. We find that the degree of diminishing returns to R&D is the main factor behind industry variation in productivity growth rates and in R&D intensity.

In related work, Klenow (1996) studies the determinants of cross-industry differences in TFP growth and R&D intensity in a 2-sector version of the Romer (1990) model. Krusell (1998) develops a 2-sector framework to endogenize the decline in the price of capital relative to consumption goods documented by Greenwood, Hercowitz and Krusell (1997), and

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<sup>1</sup>We discuss how our notion of appropriability compares to other notions of appropriability later in the paper.

<sup>2</sup>For example we show that, while the price elasticity of demand affects the potential returns to innovation in partial equilibrium, it may not affect returns in general equilibrium when all firms are conducting research and trying to keep pace with each other. We also show that, if R&D intensity is measured using the R&D-to-sales ratio, as is common, then the price elasticity of demand may enter R&D intensity not because it affects resource allocation but by *construction*, since the denominator (sales) contains a markup reflecting this elasticity.

Vourvachaki (2007) develops a two-sector endogenous growth model to endogenize technical progress in IT: however, in these papers, there is only research in one sector, and the focus is not on the factors that determine industry TFP growth rates. In the partial equilibrium model of Nelson (1988), the extent to which knowledge spills from a firm to its competitors affects R&D intensity but not TFP growth rates, and our general equilibrium environment also yields this result. Klevorick et al (1995) and Nelson and Wolf (1997) provide evidence supporting this claim.

Section 2 provides an overview of the related literature. We do this to line up the factors we wish to embody later in our model. Section 3 describes the structure of the model and outlines the main results, and Section 4 studies its long run behavior. Section 5 uses a calibration of the model to determine the relative importance of different potential determinants of research and productivity differences. Section 6 discusses possible extensions.

## 2 Related Literature

Many studies have attempted to identify the determinants of industry variation in R&D intensity and productivity growth. While some studies assume that research *causes* productivity growth, others take our view that both are determined by deeper "fundamentals" of each industry.

The literature has focused on three sets of factors: product demand, technological opportunity, and appropriability.

*Technological opportunity* encompasses factors that lead research to be more productive in some industries than others. Opportunity has been modeled in different ways – for example, in Klenow (1996) it is a constant  $Z_i$  in the knowledge production function for industry  $i$ . Nelson (1988) interprets opportunity in terms of knowledge spillovers from different sources. Measuring opportunity is difficult: however, using surveys of R&D managers, Levin et al (1985), Cohen et al (1987) and Klevorick et al (1995) try to identify different kinds of

spillovers, relating them to R&D activity and to technical change.<sup>3</sup>

*Appropriability* relates to the extent that an innovating firm (as opposed to its competitors) benefits from its own newly generated knowledge. Cohen et al (1987), Klevorick et al (1995) and Nelson and Wol (1997) find evidence that appropriability is related to R&D intensity and, interestingly, Klevorick et al (1995) and Nelson and Wol (1997) argue that the survey data are consistent with an influence of opportunity factors on both R&D intensity and technical change, whereas appropriability is only related to R&D intensity.<sup>4</sup>

*Demand factors* affect the returns to R&D. In Schmookler (1966), larger product markets encourage innovation by offering higher returns to innovators, whereas in Kamien and Schwartz (1970) the gains from reducing production costs may be larger when demand is more elastic. The survey of Cohen and Levin (1989) suggests that the industry evidence concerning demand factors is weak. For example, Levin et al (1985) find that they lose significance in cross-industry R&D regressions when indicators of opportunity and appropriability are included. Some empirical studies of specific products or industries do find some evidence of a demand-innovation link – for example, Newell et al (1999), Popp (2002) and Acemoglu and Linn (2004). These findings underline the importance of demand in providing incentives for R&D, although it is not clear that they provide evidence relating to *industry differences* in productivity growth nor R&D intensity. Independently, several case-based and historical studies suggest that technical change is driven by scientific or engineering

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<sup>3</sup>Aside from technological opportunity, other cost-related factors could vary across industries, such as subsidy levels. We abstract from research subsidies because in the United States they are arguably very low. In the US, mostly R&D is subsidized through R&D tax credits. In practice the credit rate is about 13% of expenditures: see Wilson (2009). Only expenditures above a certain limit count towards the credit, which is 3% of sales for new firms or a 3-year moving average of past R&D spending otherwise. Wilson (2009) notes that federal R&D tax credits are in fact "recaptured" (i.e. taxed back).

<sup>4</sup>Cohen et al (1987) do find a positive link between appropriability and an indicator of innovation, also using survey data. What clouds these results is that the appropriability measure in all these papers may not distinguish sharply between appropriability and opportunity. The measure is based on the response to the question "in this line of business, how much time would a capable firm typically require to effectively duplicate and introduce a new or improved product developed by a competitor?" This may not distinguish between (a) the ease with which a competitor might access a firm's knowledge, and (b) the ease in general with which preexisting knowledge can be used to generate new knowledge. In particular, if appropriability itself is generally low, then the measure may reflect mostly differences in opportunity.

considerations rather than by demand conditions.<sup>5</sup>

The following stylized facts emerge from the empirical literature: (1) there is evidence that *opportunity* affects both statistics of interest; (2) *appropriability* is easier to relate to R&D intensity than to TFP growth rates; (3) the link between *demand factors* and research intensity (as well as rates of TFP growth) is not robust in cross industry studies, although within industry studies do provide some supporting evidence.

We wish to articulate opportunity, appropriability and demand factors within a general equilibrium growth model, based on primitives of preferences and technology drawn from the growth literature. Given the measurement difficulties inherent in studying the role of knowledge in technical progress, we use the structure of the model to guide us regarding the relationships that hold between R&D, TFP growth, and each of these factors. As a benchmark, we use a model of knowledge generation that is intentionally close to the production function approach common in both the theoretical and the empirical literature. Our model is closely related to the frameworks of Jones (1995) and Krusell (1998). The functional forms we use are necessary for balanced growth.

## 3 Economic Environment

### 3.1 Knowledge Production

There are  $z = 2$  industries. Consider a firm  $h \in [0, 1]$  in industry  $i$ , with a level of productivity that depends upon the stock  $T_{iht}$  of technical knowledge at its disposal at date  $t$ . Knowledge accumulates over time according to:

$$T_{ih,t+1} = F_{iht} + T_{iht}, \quad (1)$$

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<sup>5</sup>Nelson and Winter (1977) coin the term “natural trajectories” to describe the phenomenon that “innovation has a certain inner logic of its own [...] – particularly in industries where technological advance is very rapid, advances seem to follow advances in a way that appears somewhat ‘inevitable’ and certainly not fine tuned to the changing demand and cost conditions.” See also Rosenberg (1969). As for the case studies, Newell et al (1999) also note many energy-savings innovations that bear no apparent link to demand factors.

where  $F_{iht}$  is new knowledge.<sup>6</sup> New knowledge  $F_{iht}$  is generated by a knowledge production function, using the firm's research input and spillovers from other firms. The knowledge production function is:

$$F_{iht} = Z_i T_{iht}^{\kappa_i} T_{it}^{\sigma_i} \left( Q_{iht}^{\eta_i} L_{iht}^{1-\eta_i} \right)^{\psi_i}. \quad (2)$$

where  $\eta_i, \psi_i \in (0, 1]$ , and  $Q_{iht}$  and  $L_{iht}$  are capital and labor used in the production of knowledge. The productivity index for industry  $i$  as a whole is  $T_{it} = \int_0^1 T_{iht} dh$ , which firm  $h$  takes as given. Let  $\gamma_{iht} = T_{iht+1}/T_{iht}$  be the growth factor of  $T_{ih}$ .

Parameters  $Z_i, \kappa_i, \sigma_i, \psi_i$  and  $\eta_i$  represent technological opportunity, as they affect the productivity of research input. Parameter  $Z_i$  is an efficiency parameter for carrying out research in industry  $i$ .<sup>7</sup> It could be linked to the nature of research in the industry, or to the institutional environment. Parameter  $\kappa_i$  represents the effect of in-house knowledge on the production of new ideas, and is known in the growth literature as the intertemporal knowledge spillover. Parameter  $\sigma_i$  represents spillovers across firms within sector  $i$ . The total knowledge spillover  $\rho_i = \kappa_i + \sigma_i$  is the extent to which the production of new knowledge in sector  $i$  benefits from prior knowledge. Parameter  $\psi_i$  indicates decreasing returns to research inputs. One interpretation for  $\psi_i < 1$  is that there is duplication in research, whereby some of the knowledge created by a firm in sector  $i$  might not be new. Parameter  $\eta_i$  captures the share of capital in R&D spending.

Conditional on total knowledge spillovers, industries may differ in the importance of in-house knowledge relative to knowledge spillovers from its competitors. We define appropriability  $A_i$  as the share of total spillovers accounted for by in-house knowledge:  $A_i = \kappa_i/\rho_i$ .

This notion of appropriability is defined in technological terms: however, it is also related

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<sup>6</sup>It is common to assume that ideas depreciate. There is a distinction between physical depreciation and economic depreciation, however. For ideas to physically depreciate would imply that some share of them is exogenously forgotten. Economic depreciation, on the other hand, implies that old knowledge becomes less valuable (obsolete) as newer knowledge accumulates, and rates of economic depreciation will be endogenous in our model. See Laitner and Stolyarov (2008) for a different approach based on new knowledge sometimes reducing the value of existing knowledge to zero.

<sup>7</sup>Nelson (1988) allows  $Z_i$  grows at an exogenous rate. Since the trademark of R&D-based growth models is that technical progress is endogenous, our model does not feature exogenously growing factors other than the population.

to a more common view of appropriability in terms of the share of the profits from innovation that accrue to the innovator. For example, if  $A_i$  is small, then most of firm  $i$ 's current knowledge will spill over to other firms, who can use it to create new knowledge for use in future production to compete with firm  $i$ . On the other hand, if  $A_i = 1$ , then when a firm creates new knowledge, the spillovers only applies to this firm's future knowledge, so it is the only one to benefit and earn profits from it.

The last set of factors considered by the empirical literature relates to demand, which we present later when we close the model using standard household preferences.

### 3.2 Firm's problem

Each sector  $i \in z$  is monopolistically competitive. Firm  $h$  in sector  $i$  produces a differentiated variety  $h \in [0, 1]$  of good  $i$ . Output of variety  $h$  of good  $i$  is

$$Y_{iht} = T_{iht} K_{iht}^{\alpha_i} N_{iht}^{1-\alpha_i}, \quad \alpha_i \in (0, 1) \quad (3)$$

where  $Y_{iht}$  is output,  $K_{iht}$  and  $N_{iht}$  are capital and labor used in the production of output.

Firms are competitive in the input markets. Taking input prices  $(w_t, R_t)$  and its demand function  $p_{iht}(\cdot)$  as given, firm  $h$  in sector  $i$  chooses both production inputs  $(K_{iht}, N_{iht})$  and R&D inputs  $(Q_{iht}, L_{iht})$  to maximize the discounted stream of real profits:

$$\sum_{t=0}^{\infty} \lambda_t \frac{\Pi_{iht}}{p_{ct}}; \quad \text{where} \quad \Pi_{iht} = p_{iht} Y_{iht} - w_t (N_{iht} + L_{iht}) - R_t (K_{iht} + Q_{iht}), \quad (4)$$

where  $p_{ct}$  is the aggregate price-index for consumption goods,  $\lambda_t$  is the discount factor at time  $t$ , with  $\lambda_0 = 1$ ,  $\lambda_t = \prod_{s=1}^t \frac{1}{1+r_s}$  for  $t \geq 1$ , and  $r_t$  is the real interest rate. The transversality condition is  $\lim_{t \rightarrow \infty} \chi_{iht} T_{iht+1} = 0$ , where  $\chi_{iht}$  is the shadow price of  $T_{iht+1}$ . The complete derivation of the firm's maximization problem is given in Appendix A.

### 3.3 Equilibrium Productivity Growth

Given free mobility of inputs and competitive input markets, marginal rates of substitution are equal across activities within the firm (5), across firms within each industry (6), and also across sectors (7):

$$\frac{1 - \eta_i}{\eta_i} \frac{Q_{iht}}{L_{iht}} = \frac{1 - \alpha_i}{\alpha_i} \frac{K_{iht}}{N_{iht}}, \quad (5)$$

$$\frac{Q_{iht}}{L_{iht}} = \frac{Q_{it}}{L_{it}}; \quad \frac{K_{iht}}{N_{iht}} = \frac{K_{it}}{N_{it}}, \quad (6)$$

$$\frac{1 - \eta_i}{\eta_i} \frac{Q_{it}}{L_{it}} = \frac{1 - \eta_j}{\eta_j} \frac{Q_{jt}}{L_{jt}} = \frac{1 - \alpha_i}{\alpha_i} \frac{K_{it}}{N_{it}} = \frac{1 - \alpha_j}{\alpha_j} \frac{K_{jt}}{N_{jt}}. \quad (7)$$

It follows that the growth in capital-labor ratio for research and production are equal:

$$\frac{Q_{iht+1}/L_{iht+1}}{Q_{iht}/L_{iht}} = \frac{K_{iht+1}/N_{iht+1}}{K_{iht}/N_{iht}} = g_{kt} \quad i, h. \quad (8)$$

Using (1) and (6), the productivity growth of firm  $h$  in sector  $i$  depends on

$$\gamma_{iht} - 1 = \frac{F_{iht}}{T_{iht}} = Z_i \left( \frac{T_{iht}}{T_{it}} \right)^{\kappa_i - 1} T_{it}^{\rho_i - 1} \left( \frac{Q_{it}}{L_{it}} \right)^{\eta_i \psi_i} L_{iht}^{\psi_i}. \quad (9)$$

As our interest is in cross-industry comparisons, we focus on equilibria where the distribution of productivity within sectors is stable and *rank-preserving* i.e. in each industry  $\gamma_{iht} = \gamma_{it} \quad h$  (which includes the case of symmetric equilibria,  $T_{iht} = T_{it} \quad h$ ). Then, (9) implies  $L_{iht} = L_{it}$ . Denote  $\mathcal{N}_t$  as the total labor force, which is growing by constant factor  $g_{\mathcal{N}}$ . Let  $n_{it} = N_{it}/\mathcal{N}_t$  and  $l_{it} = L_{it}/\mathcal{N}_t$  be the fraction of labor allocated to production and research in sector  $i$ , thus  $\sum_i (l_i + n_i) = 1$ . To make meaningful comparisons across sectors, we also focus on equilibria with constant productivity growth, using (8) and (9):

**Lemma 1** *In any rank-preserving equilibrium, constant  $\gamma_i$  satisfies*

$$\gamma_i = \left[ g_{kt} \eta_i \left( \frac{l_{it+1}}{l_{it}} \right) \right]^{\frac{\psi_i}{1-\rho_i}}, \quad i. \quad (10)$$

Three terms affect cross-industry comparisons of productivity growth: (i) capital intensity of research activities  $\eta_i$ , (ii) the expression  $\frac{\psi_i}{1-\rho_i}$ , and (iii) growth in the fraction of labor allocated to research  $\left( \frac{l_{it+1}}{l_{it}} \right)$ .

We are not aware of a precedent to the first factor – *the capital intensity of research activity*. Technical improvements in the production of capital goods lead to capital deepening, and the extent to which this encourages research depends on  $\eta_i$ . Rosenberg (1969) and Nelson and Winter (1977) suggest that capital-intensive industries may enjoy inherently high TFP growth. However, equation (10) shows that what matters is not capital intensity per se, but the *capital-intensity of research activity*. The capital-intensity of production may affect the measurement of productivity, but not equilibrium rates of productivity growth.

The expression  $\frac{\psi_i}{1-\rho_i}$  is related to the historical work of Rosenberg (1969) and Nelson and Winter (1977) that underlines technological opportunity as a factor of productivity growth. Specifically, our model emphasizes the degree of decreasing returns to research input, the extent of intertemporal knowledge spillovers  $\kappa_i$ , and the magnitude of spillovers across firms  $\sigma_i$ . Interestingly, as far as spillovers are concerned, only total spillovers  $\rho_i = \kappa_i + \sigma_i$  are important, whereas the *source* of spillovers is not.

Other industry-specific factors, such as demand factors and appropriability ( $A_i = \kappa_i/\rho_i$ ), can only affect relative productivity growth rates across industries if they affect the growth rate of research labor  $\frac{l_{it+1}}{l_{it}}$  in different industries.<sup>8</sup>

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<sup>8</sup>Note that although equation (10) implies that  $\gamma_i$  depends on  $l_{it+1}/l_{it}$ , it does not require  $l_i$  to grow in order for sector  $i$  to experience positive productivity growth, as long as either there is capital deepening in research ( $g_k > 1$ ) or positive population growth ( $g_N > 1$ ).

### 3.4 Equilibrium research activity

Let  $\chi_{iht}$  be the shadow price of knowledge  $T_{iht+1}$ , which is determined by the arbitrage condition for allocating inputs across activities. In the case of capital:

$$\chi_{iht} = - \left( \frac{\lambda_t}{p_{ct}} \right) \frac{\partial \Pi_{iht} / \partial Q_{iht}}{\partial F_{iht} / \partial Q_{iht}}. \quad (11)$$

The firm's dynamic optimization condition implies that

$$\chi_{iht} = \underbrace{\left[ \frac{\lambda_{t+1}}{p_{ct+1}} \frac{\partial \Pi_{iht+1}}{\partial T_{iht+1}} \right]}_{(a) \text{ production}} + \chi_{iht+1} \underbrace{\left[ \frac{\partial F_{iht+1}}{\partial T_{iht+1}} + 1 \right]}_{(b) \text{ research} \quad (c) \text{ future knowledge}}, \quad i = z. \quad (12)$$

Equation (12) reflects three benefits to the firm of producing more knowledge: (a) more efficient *production of goods and services*, (b) more efficient *production of knowledge*, and (c) a larger stock of future knowledge.

To determine the extent to which resources are directed towards research (as opposed to production), we define research intensity as the share of research spending in total costs:

$$RND_{iht} = \frac{w_t L_{iht} + R_t Q_{iht}}{w_t (L_{iht} + N_{iht}) + R_t (Q_{iht} + K_{iht})}.$$

Using (12):

$$RND_{iht} = \left[ 1 + \frac{1}{\psi_i} \left( \frac{\frac{\chi_{iht}}{\chi_{iht+1}} - 1}{\gamma_i - 1} - \kappa_i \right) \right]^{-1}, \quad (13)$$

where using the result in (10) and the definition of  $\chi_{iht}$  in (11), It follows that in any rank-preserving equilibria with constant  $\gamma_i$ , we have

$$\frac{\chi_{iht}}{\chi_{iht+1}} = \frac{\lambda_t p_{iht} / p_{ct}}{\lambda_{t+1} p_{iht+1} / p_{ct+1}} g_{kt}^{-\alpha_i} \left( g_N \frac{l_{it+1}}{l_{it}} \right)^{-1}. \quad (14)$$

Growth in the price of  $i$  relative to consumption  $\frac{p_{iht}/p_{ct}}{p_{iht+1}/p_{ct+1}}$  requires knowledge of the demand function faced by each firm. Assume that price elasticities  $\mu_i = \frac{\partial p_{iht} / \partial Y_{iht}}{p_{iht} / Y_{iht}}$  are

sector-specific constants (i.e. identical across firm within any sector  $i$ ). Equating the value of marginal products of labor across firms, together with (6):

$$\frac{p_{iht+1}/p_{iht}}{p_{ih't+1}/p_{ih't}} = \frac{\gamma_{ih't}}{\gamma_{iht}}; \quad i, h, h', \quad (15)$$

which implies that in any rank-preserving equilibria with constant  $\gamma_i$ :

$$\frac{p_{iht+1}/p_{iht}}{p_{jht+1}/p_{jht}} = \frac{p_{it+1}/p_{it}}{p_{jt+1}/p_{jt}} = \frac{\gamma_{jt}}{\gamma_{it}} g_{kt}^{\alpha_j - \alpha_i}, \quad (16)$$

where the last equality follows from using (7) and equating the value of marginal products of labor across sectors. Substituting into (14), we have:

**Lemma 2** *If price elasticity  $\mu_i$  is a sector-specific constant then, in any rank-preserving equilibria with constant productivity growth, research intensity for any sector  $i$  satisfies:*

$$RND_{it} = \left[ 1 + \frac{1}{\psi_i} \left( \frac{\frac{\chi_{it}}{\chi_{it+1}} - 1}{\gamma_i - 1} - \kappa_i \right) \right]^{-1}; \quad i, \quad (17)$$

where

$$\frac{\chi_{it}/\chi_{it+1}}{\chi_{jt}/\chi_{jt+1}} = \left( \frac{\gamma_i}{\gamma_j} \right) \frac{l_{it}/l_{it+1}}{l_{jt}/l_{jt+1}}; \quad i. \quad (18)$$

In addition to the factors that determine  $\gamma_i$ , there are two additional terms affecting cross-industry comparisons of research intensity: (i) the degree of diminishing returns to research input  $\psi_i$ , and (ii) the effect of in-house knowledge on the production of new ideas  $\kappa_i$ .

Recall that  $\kappa_i = A_i \rho_i$ , implying that research intensity is affected by both opportunity and appropriability. Moreover, if price elasticities are sector-specific constants, industry-specific demand factors can only matter for cross-industry R&D intensity comparisons if they alter the growth rate of labor allocated to research across sectors.

### 3.5 Relating the model to the literature

Consistent with evidence reviewed in Section 2, comparisons of industry TFP growth rates depend on factors of technological opportunity (Lemma 1), whereas R&D intensity also depends upon appropriability (Lemma 2). Low appropriability reduces R&D intensity without affecting productivity growth rates, so a prediction is that there should be a *negative* relationship between measures of *intra*-industry spillovers and R&D intensity, controlling for other variables. This is exactly what Nelson and Wolf (1997) find.

Klevorick et al (1995) identify two effects of appropriability on R&D intensity. First, in their terminology, there is an "incentive effect" whereby large, un-internalized spillovers reduce R&D activity, causing the negative relationship between appropriability  $A_i$  and R&D intensity in Lemma 2. Second, there is also an "efficiency" effect, whereby larger spillovers may *encourage* R&D at other firms. The efficiency effect is seen in that, conditional on  $\kappa_i$ , a larger value of  $\sigma_i$  raises  $\rho_i$  while leaving  $A_i\rho_i$  constant, so that R&D intensity rises. However, in our model, the "efficiency" effect is related to the magnitude of spillovers, not to appropriability *per se* and, as suggested by Klevorick et al (1995), this effect disappears once opportunity is kept constant.

Lemma 1 and Lemma 2 show that industry differences in demand parameters can only affect comparisons of TFP growth rates and research intensity by affecting growth in the fraction of labor allocated to research ( $l_{it+1}/l_{it}$ ) in different industries. This is unlikely to occur for stationary demand parameters such as industry size and the price elasticity of demand in a steady state,<sup>9</sup> something that is broadly consistent with the industry-level evidence in Section 2. We return to this point after presenting the demand side of the model.

### 3.6 Closing the model: Households

We now close the model by specifying the demand side of the economy.

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<sup>9</sup>Demand parameters could matter in transition, for example if demand parameters change, something we do not explore in this paper.

There is a continuum of households, each of measure  $\mathcal{N}_t$ , and as noted earlier, it grows at constant factor  $g_{\mathcal{N}}$ . In what follows, we use lower case letters to denote per-capita variables. Goods  $i \in \{1, \dots, m-1\}$  are consumption goods while goods  $j \in \{m, \dots, z\}$  are investment goods.

The life-time utility of a household is

$$\sum_{t=0}^{\infty} (\beta g_{\mathcal{N}})^t \frac{c_t^{1-\theta} - 1}{1-\theta} \quad (19)$$

$$c_t = \prod_{i=1}^{m-1} \left( \frac{c_{it}}{\omega_i} \right)^{\omega_i}, \quad c_{it} = \left( \int_0^1 c_{iht}^{\frac{\mu_i-1}{\mu_i}} dh \right)^{\frac{\mu_i}{\mu_i-1}} \quad i \in \{1, \dots, m-1\} \quad (20)$$

where  $\beta$  is the discount factor, and  $1/\theta$  is the intertemporal elasticity of substitution. We assume that  $\beta g_{\mathcal{N}} < 1, \theta > 0, \mu_i > 1, \omega_i > 0$  and  $\sum_{i=1}^{m-1} \omega_i = 1$ . Parameters  $\mu_i$  and  $\omega_i$  capture the industry-specific demand factors considered in the literature.  $\mu_i$  is the elasticity of substitution across different varieties of good  $i$  which, in equilibrium, determines the price elasticity of demand, while  $\omega_i$  determines the spending share of each good (market size).

Each household member is endowed with one unit of labor and  $k_t$  units of capital, and receives income by renting capital and labor to firms, and by earning profits from the firms. Her budget constraint is

$$\sum_{i=1}^{m-1} \int p_{iht} c_{iht} dh + \sum_{j=m}^z \int p_{jht} x_{jht} dh = w_t + R_t k_t + \pi_t \quad (21)$$

where  $x_{jht}$  is investment in variety  $h$  of capital good  $j$ ,  $p_{iht}$  is the price of variety  $h$  of good  $i$ ,  $w_t$  and  $R_t$  are rental prices of labor and capital, and  $\mathcal{N}_t \pi_t = \sum_{i=1}^z \int_0^1 \Pi_{iht} dh$  equals total profits from firms. Her capital accumulation equation is

$$g_{\mathcal{N}} k_{t+1} = x_t + (1 - \delta_k) k_t. \quad (22)$$

The composite investment good  $x_t$  is produced using all capital types  $j$ :

$$x_t = \prod_{j=m}^z \left( \frac{x_{jt}}{\omega_j} \right)^{\omega_j}, \quad x_{jt} = \left[ \int x_{jht}^{(\mu_j-1)/\mu_j} dh \right]^{\mu_j/(\mu_j-1)} \quad j \in \{m, \dots, z\}, \quad (23)$$

where  $\mu_j > 1$ ,  $\omega_j > 0$  and  $\sum_{j=m}^z \omega_j = 1$ .<sup>10</sup> Finally, the transversality condition for capital is  $\lim_{t \rightarrow \infty} \zeta_t k_t = 0$ , where  $\zeta_t$  is the shadow price of capital. Define the price index for the consumption composite  $c_t$  and the investment composite  $x_t$  respectively as:

$$p_{ct} = \frac{\sum_{i=1}^{m-1} \int_0^1 p_{iht} c_{iht} dh}{c_t}; \quad p_{xt} = \frac{\sum_{j=m}^z \int_0^1 p_{jht} x_{jht} dh}{x_t}. \quad (24)$$

## 4 Decentralized Equilibrium

The decentralized equilibrium is standard, where the firms' and consumers' problems are defined as in Section 3. In any period  $t$ , prices must clear all goods and input markets:

$$Y_{iht} = c_{iht} \mathcal{N}_t; \quad i < m; \quad Y_{jht} = x_{jht} \mathcal{N}_t, \quad j \geq m; \quad (25)$$

$$K_t = \sum_{i=1}^z \int_0^1 (K_{iht} + Q_{iht}) dh, \quad \mathcal{N}_t = \sum_{i=1}^z \int_0^1 (N_{iht} + L_{iht}) dh. \quad (26)$$

Our aim is to compare productivity dynamics across industries, and not across different varieties of any given good. Therefore, we focus on equilibria that treat varieties within each sector  $i$  symmetrically, and suppress the firm index  $h$  henceforth.<sup>11</sup>

Full derivation of the household's utility maximization is given in the Appendix. The implied Euler condition is:

$$\frac{u'(c_t)}{\beta u'(c_{t+1})} = \frac{p_{xt+1}/p_{ct+1}}{p_{xt}/p_{ct}} \left( 1 - \delta_k + \frac{R_{t+1}}{p_{xt+1}} \right), \quad (27)$$

<sup>10</sup>Cobb-Douglas aggregation across goods allows us to derive an aggregate balanced growth path. In a multi-sector model with exogenous technological progress, Ngai and Pissarides (2007) show that Cobb-Douglas aggregation across capital goods is necessary for deriving an aggregate balanced growth path.

<sup>11</sup>In notes available upon request, we show asymmetric rank-preserving equilibria exist in which all the results of the paper hold.

which implies the real discount factor:

$$\frac{\lambda_{t+1}}{\lambda_t} = \frac{1}{1+r_{t+1}} = \frac{\beta u'(c_{t+1})}{u'(c_t)} = \frac{p_{xt}/p_{ct}}{G_{t+1}p_{xt+1}/p_{ct+1}}, \quad (28)$$

where  $G_{t+1} = 1 - \delta_k + \frac{R_{t+1}}{p_{xt+1}}$  is the gross return on capital.

## 4.1 Balanced growth

We look for a balanced growth path equilibrium (BGP), along which aggregate variables are growing at constant rates although industry TFP growth rates may be different. Such a BGP requires a constant ratio of consumption to capital:  $c/(qk)$ , where  $q$  is the relative price of capital. Define  $\Phi$  and  $\gamma_{xt}$  as:

$$\Phi = \frac{\sum_{j=m,z} \frac{\omega_j \psi_j}{1-\rho_j}}{1 - \sum_{j=m,z} \frac{\omega_j \psi_j \eta_j}{1-\rho_j} \frac{1}{1-\alpha_x}}; \quad \gamma_{xt} = \prod_{j=m,z} \gamma_{jt}^{\omega_j}; \quad \alpha_x = \sum_{j=m,z} \omega_j \alpha_j. \quad (29)$$

**Proposition 1** *Suppose there exists an equilibrium with constant  $l_i, n_i > 0$  that satisfies the transversality conditions for  $k$  and  $T_i$ ,  $i$ . If  $\Phi > 0$ , then there exists a unique balanced growth path. Along this path  $c/q$  and  $k$  grow by a constant factor  $(\gamma_x)^{1/(1-\alpha_x)}$ , where  $\gamma_x = g_N^\Phi$ , and the  $\gamma_i$  are constant and satisfy (10)  $i$ .*

The proof observes that the return to investment  $G$  is constant if  $k$  grows by a factor  $\gamma_{xt}^{1/(1-\alpha_x)}$ , which by (29) is constant if  $\gamma_i$  is constant in all capital good sectors. The restriction for constant  $\gamma_i$  follows from Section 3.3, and  $\gamma_x$  is derived from (29).<sup>12</sup>

Proposition 1 contrasts with the behavior of the one-sector model of Jones (1995). In Jones (1995),  $\Phi$  is replaced by  $\frac{\psi_1}{1-\rho_1}$ , so balanced growth path requires  $\rho_1 < 1$  (where the subscript 1 indexes the only industry in the economy). There are two important differences compared to our requirement that  $\Phi > 0$ . First, suppose  $\eta_j = 0$ , i.e. capital is not used in the production of knowledge. Then  $\Phi > 0$  is equivalent to  $\sum_{j=m}^z \frac{\omega_j \psi_j}{1-\rho_j} > 0$ , so the Jones (1995)

<sup>12</sup>The Appendix reports sufficient conditions for the existence of a BGP with R&D activity in all sectors.

restriction applies to the weighted average of  $\frac{\psi_j}{1-\rho_j}$  across capital goods in the multi-sector model.<sup>13</sup> Second, the restriction  $\sum_{j=m}^z \frac{\omega_j \psi_j}{1-\rho_j} > 0$  is not sufficient when capital is used in the production of knowledge ( $\eta_j > 0$  for some  $j \leq m$ ), as productivity improvements targeting capital goods become a factor of aggregate productivity growth by inducing capital deepening in R&D.

## 4.2 Comparing industries

In equilibrium, industries with the same level of technological opportunity (i.e. the same values of  $\psi_i, \rho_i = \kappa_i + \sigma_i$  and  $\eta_i$ ) but different appropriability  $A_i = \kappa_i/\rho_i$  display different R&D intensity, even if they have the same TFP growth rate. It follows from Lemma 1 and Lemma 2 that:

**Proposition 2** *Along the balanced growth path,*

(i) *Cross-industry comparisons of productivity growth depend only on the technological opportunity factors  $\rho_i, \psi_i$  and  $\eta_i$ .*

(ii) *In addition to these factors, cross-industry comparisons of R&D intensity depend also on appropriability  $A_i$ .*

Notice that differences in demand parameters affect neither comparisons of productivity growth rates nor of R&D intensity when  $l_i$  are constants.<sup>14</sup> General equilibrium mechanisms play a key role in this result.

In the model there are two industry demand parameters:  $\omega_i$ , the weight of good  $i$  in the utility function, and  $\mu_i$ , the elasticity of substitution across varieties of  $i$ . The spending share of each good depends on  $\omega_i$ , and the elasticity of a firm's demand function depends on  $\mu_i$ . Since  $\omega_i$  affects the level of returns to production at all dates, but not their growth rate, it

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<sup>13</sup>Note that from (10), given  $g_k, g_N \geq 1$  and  $l_i$  is constant, productivity growth in sector  $i$  is positive only if  $\rho_i < 1$ .

<sup>14</sup>Again, as shown in Lemmas 1 and 2, demand factors do not matter for industry differences if they affect only the level of  $l_i$ . In order for demand factors to matter for industry differences, they have to affect the growth rate of  $l_i$ .

does not affect the decision of whether to use resources for investment in future production (via increases in knowledge) instead of current production.

The reason  $\mu_i$  may matter in partial equilibrium is that elastic demand allows an innovator to increase market share without having to lower her output price to the same extent as the cost reduction. However, in equilibrium, all firms are performing research: R&D by the firm's *competitors* results in a commensurate fall in the relative price of *their* goods, so that this partial equilibrium benefit of research need not materialize in general equilibrium.

It is worth elaborating upon this last point. The literature on appropriability distinguishes between two channels whereby research by a firm might affect its competitors. The first is the "spillover effect" (captured by  $\sigma_i$  in our model) whereby innovations by one firm may be used by another. The second is the "business stealing" or "product rivalry" effect whereby innovations by a firm's competitors decreases its market share. In our model, the severity of this rivalry depends on  $\mu_i$ . To see this, note that  $c_{ih}$  is proportional to  $p_{ih}^{-\mu_i}$ , so that the relative market share of two firms  $h$  and  $h'$  in the same industry is:

$$\frac{p_{ih}c_{ih}}{p_{ih'}c_{ih'}} = \left(\frac{p_{ih}}{p_{ih'}}\right)^{1-\mu_i} = \left(\frac{T_{ih}}{T_{ih'}}\right)^{\mu_i-1} \quad (30)$$

where  $p_{ih}c_{ih}$  are the sales of firm  $h$ . Consider two firms that start period  $t$  with equal productivity. A given productivity improvement in one firm relative to the other will result in a larger increase in demand for higher values of  $\mu_i > 1$ .

Even though the rivalry effect is present in the model, this does not imply that  $\mu_i$  affects equilibrium TFP growth rates, as these considerations influence R&D incentives at *all* firms in the industry. In a symmetric equilibrium, firms keep pace with each other technologically so that  $\mu_i$  does not affect equilibrium research expenditure, as it does not affect equilibrium returns. The results hold in any rank-preserving equilibrium. Consistent with our results, Bloom et al (2007) estimate that the rivalry effect is quantitatively dominated by the "spillover effect" as a determinant of research activity.

The model suggests some caution in linking research intensity to demand factors empirically. The most common measure of research intensity is R&D spending divided by sales or, in terms of the model,  $RND_i^{Sales} = \frac{wL_i + RQ_i}{p_i Y_i}$ . Combined with the conditions for optimal input allocation, the R&D spending to sales ratio is:

$$RND_i^{Sales} = \left(1 - \frac{1}{\mu_i}\right) \psi_i \left[ \frac{\frac{\chi_{it}}{\chi_{it+1}} - 1}{\gamma_i - 1} - \kappa_i \right]^{-1}. \quad (31)$$

Comparing (31) to the expenditure-based measure of R&D intensity in (17), the key difference is the mark-up term. Equation (31) would appear to indicate an influence of demand parameters  $\mu_i$  on research spending in the model, and indeed Cohen et al (1987) find some indicators of industry concentration to be related to the ratio of research spending to sales. However, in an environment with imperfect competition, the volume of sales contains a markup over cost, which is not an indicator of the quantity of resources devoted to research as opposed to other activities. The denominator in this measure of research activity contains demand side variables *by construction*. Future empirical work may turn out to substantiate an economic link between R&D and markups or other demand factors: however, the model suggests caution in employing sales-based measures of R&D activity in such work.

## 5 Quantitative findings

We now calibrate our model using US industry data to identify which set of factors best accounts for observed industry differences in R&D and productivity growth. We match the model to United States data because of the rich sources of information available, because the US is arguably at the technological frontier in most industries, and because US GDP has grown at a stable rate for over a century, which is consistent with our focus on the balanced growth path of our model.<sup>15</sup>

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<sup>15</sup>The model ranking of TFP and R&D intensity is stable in a rank-preserving equilibrium. To make industry comparisons of TFP growth rates and research intensity requires those features to be stable over time in the data. We computed TFP growth rates for durable goods over non-overlapping 10-year periods,

We address the following questions:

1. The model predicts that productivity growth should be positively linked to the opportunity parameters  $\psi_i$ ,  $\rho_i$  and to  $\eta_i$ . Which of these parameters do the data suggest to be the main factor?
2. The model suggests that R&D intensity should be linked to opportunity parameters, but also to appropriability  $A_j$ . Which of these parameters do the data suggest to be the main factor?
3. What values of these parameters best account for industry variation in productivity change and research intensity in the data?

To answer these questions, we proceed as follows. We first calibrate as many parameters as possible in the model using post-war US data. Then, we ask what combinations of the remaining parameters allow the model to match industry data on both productivity growth and R&D intensity.

We do not match measured TFP growth rates directly. For example, several of the long-term rates of TFP growth estimated by Jorgenson et al (2006) are negative, and we do not believe that productivity can decline in absolute terms in the long run when it is driven by knowledge accumulation. We take seriously the view of Greenwood et al (1997) among others that quality improvements are an important source of productivity change. Thus, we calibrate TFP growth rates  $\gamma_i$  using quality-adjusted relative prices. Specifically, equation (16) implies a relationship between relative rates of price decline, capital shares, and TFP growth, and we use these to compute relative TFP growth rates.

To our knowledge, comparable quality adjusted prices are available only for durable goods. Hence, we assume that  $m = 2$ , so that there is only one sector producing non-durables. We set  $z = 15$ , so that there are 14 capital-producing industries. This partition

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using the procedure below. We found that the correlations between cross sections were always 80% or higher. Ilyina and Samaniego (2008) find that the decade-to-decade correlation of R&D intensity across US manufacturing industries is over 90%.

was the finest that allowed us to match the relative price data with the patent data we employ later to measure knowledge spillovers.

It is worth pointing out that our quantitative conclusions do not depend on the use of these particular industries. The main source of discipline on our quantitative exercise turns out to be the fact that productivity growth and R&D intensity are positively linked across industries. Figure 1 shows that this holds across manufacturing industries, and Figure 2 shows that the same is true across the capital goods industries we consider even using our different measure of productivity change  $\gamma_i$ . Calibrated values of these parameters are reported in Tables 1 and 2: more details of the calibration procedure are reported in Appendix B.

FIGURE 2 ABOUT HERE

TABLES 1, 2 ABOUT HERE

## 5.1 Opportunity and TFP growth rates

The model implies that variation in industry productivity growth rates depends on technological opportunity. We now use the data to learn about which of these factors appear quantitatively important. Using (10) and the restrictions imposed by balanced growth, industry productivity growth follows:

$$\gamma_i = (g_k^{\eta_i} g_N)^{\frac{\psi_i}{1-\rho_i}}; \quad i. \quad (32)$$

Appendix B discusses in detail how we obtain  $g_k$ ,  $\eta_i$ ,  $g_N$  and  $\gamma_i$  from the data. Our first step is to ask whether variation in  $\eta_i$  can account for industry variation in  $\gamma_i$ . To this end, we use (32) to compute  $\frac{\psi_i}{1-\rho_i}$  as a residual. Results are reported in Table 3. The correlation between  $\gamma_i$  and  $\frac{\psi_i}{1-\rho_i}$  is 0.985. There are two reasons why the contribution of  $\eta_i$  to industry

growth differences is low. First, as seen in Figure 3, the correlation between  $\eta_i$  and  $\gamma_i$  is not statistically significant (although it is positive, as implied by the model). Second, most importantly, variation in  $\eta_i$  is not of sufficient *magnitude* to generate large differences in  $\gamma_i$  on its own. To see this, we re-computed  $\gamma_i$  from (32) under the assumption that  $\frac{\psi_i}{1-\rho_i}$  was equal in all industries. When we set  $\frac{\psi_i}{1-\rho_i}$  to equal the weighted average across industries, we found that productivity growth rates ranged from 2.9 to 4.7 percent, which accounts for only about a tenth of the variation in Table 3. Thus, industry differences in productivity growth reflect significant variation in technological opportunities, as captured by  $\psi_i$  and  $\rho_i$ .

FIGURE 3 ABOUT HERE

TABLE 3 ABOUT HERE

## 5.2 Decomposing opportunity using R&D intensity

We do not have direct measures of  $\rho_i$  nor of  $\psi_i$ . Hence, to make further progress in decomposing the sources of opportunity that account for industry growth, we turn to the model predictions for research intensity.

We focus on sales-based measures of R&D intensity. We do so because this is the traditional measure in the empirical literature, but also due to data constraints.<sup>16</sup> Following the literature we measure R&D intensity as the *median ratio* of R&D expenditures to sales among firms in Compustat over the period 1950-2000. The model R&D spending to sales ratio ( $RND^{Sales}$ ) is determined by equation (31), which requires an expression for the growth rate in the shadow price of knowledge  $\frac{\chi_{it+1}}{\chi_{it}}$  along a balanced growth path. Using (14), (16) and (28) this expression is:

$$\frac{\chi_{it+1}}{\chi_{it}} = \frac{\gamma_x}{G\gamma_i} g_k^{\alpha_x} g_N = \frac{g_k g_N}{G\gamma_i}, \quad (33)$$

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<sup>16</sup>In Compustat, labor expenditures are reported by very few firms.

where the last equality follows from Proposition 1, whereby  $g_k = \gamma_x^{1/(1-\alpha_x)}$ . Recalling that  $\kappa_i = A_i \rho_i$ , the expression for  $RND^{Sales}$  in (31) becomes:

$$RND_i^{Sales} = \frac{(1 - 1/\mu_i) \psi_i}{\left(\frac{G\gamma_i}{g_k g_N} - 1\right) / (\gamma_i - 1) - A_i \rho_i}. \quad (34)$$

Thus, given data on  $RND_i^{Sales}$  and  $\gamma_i$ , we can identify  $\psi_i$  and  $\rho_i$  using our model equation (32) and (34), given values of  $(g_k, g_N, G, A_i, \mu_i)$ .

We calibrate  $\mu_i$  using industry markups, and  $G$  using the real rate of return on capital. Values of  $g_k$  and  $g_N$  have been calibrated earlier.(see Appendix B). Appropriability is related to whether spillovers across firms are a significant source of knowledge. Using the NBER patent citation database, which reports the assignee of each patent awarded since 1969, we define appropriability  $A_i$  as the share of own-industry citations that are *self-citations*. The required assumption is that  $A_i$  does not differ significantly for a given industry depending on whether or not knowledge is patented. If unpatented knowledge flows across firms more easily than patented knowledge, then the measure of spillovers implied by the patent data is an upper bound on  $A_i$ . On the other hand, if ideas that flow most easily across firms are the ones patented, then our numbers represent a lower bound on  $A_i$ . As we shall see, appropriability differences between patented and unpatented knowledge must be quite drastic to affect our results (in fact, when we assumed that  $A_i$  varied between 0 and 1 and that it was perfectly correlated with R&D intensity, our results were almost identical). Table 1 reports that appropriability  $A_i$  is generally quite low – 18.5% on average. In addition, it appears to vary little across industries, ranging in the interval [0.12, 0.34]. Thus, R&D intensity in equation (34) will be mainly determined by differences in other parameters.

We proceed by matching parameters using data on R&D intensity. As discussed in Section 5.1, we use (32) to compute  $\frac{\psi_i}{1-\rho_i}$  as a residual using data on  $\gamma_i$  and  $\eta_i$ . Note that the expression for  $\frac{\psi_i}{1-\rho_i}$  is linear in  $\psi_i$  given  $\rho_i$ . As shown in (34), R&D intensity is linear in  $\psi_i$  also (given  $\rho_i$ ), but has a different slope and intercept. Hence, there is a unique pair of

parameters that allows  $\frac{\psi_i}{1-\rho_i}$  and R&D intensity to equal two arbitrary numbers.<sup>17</sup>

## TABLES 4, 5 ABOUT HERE

Results are reported in Tables 4 and 5. Several findings stand out. First, there is very little variation in  $\rho_i$ , compared to  $\psi_i$ .<sup>18</sup> Second,  $\psi_i$  is very strongly and positively correlated with both R&D intensity and with  $\gamma_i$ , whereas  $\rho_i$  is not. Thus, the model indicates that variation in productivity growth and in R&D intensity can be mainly attributed to industry differences in  $\psi_i$ .

The average of estimates for  $\psi_i$  (weighted using  $\omega_i$ ) is 0.13. Aggregate estimates in the literature of the extent of decreasing returns to research investment (analogous to  $\psi_i$ ) vary between 0.1 and 0.6 – see Kortum (1993) and Samaniego (2007). Our average is towards the low end of this range.<sup>19</sup> The average for  $\rho_i$  is high and close to one (0.91), consistent with the aggregate values surveyed in Samaniego (2007).

### 5.3 Sensitivity

Our results indicate that variation in  $\psi_i$  is the main factor behind differences in  $\gamma_i$  and R&D intensity. To emphasize this finding, we performed two further exercises. First, we assumed that  $\psi_i = 0.13$  for all  $i$ , and computed  $\rho_i$  such that  $\psi_i/(1 - \rho_i)$  always takes on the values computed in

Table 2. In this way, the weighted average value of  $\psi_i$  is the same as in the calibrated economy, but variation in  $\rho_i$  accounts for all industry differences in productivity growth.

Figure 4 shows that the R&D intensity numbers generated by these parameters vary very

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<sup>17</sup>Let  $x_i = \psi_i/(1 - \rho_i)$ , so that  $\psi_i = f(\rho, x)$ . Using the expression for R&D intensity, it is also true that  $\psi_i = g(\rho, RND_j^{Sales})$ . Both  $f$  and  $g$  are linear and decreasing in  $\rho$ ;  $f(0, x) = 0$  and  $g(0, RND_j^{Sales}) > 0$ . Hence generically there is at most one solution for  $(\rho_i, \psi_i)$  given values of  $x_i$  and  $RND_j^{Sales}$ .

<sup>18</sup>The standard deviation of  $\rho_i$  is 0.07, whereas that of  $\psi_i$  is 0.17. Compared to their respective weighted averages, the standard error of  $\rho_i$  is 0.08, whereas that of  $\psi_i$  is 1.33.

<sup>19</sup>It is worth noting that estimates in the literature are often based on patent data, which may be biased upwards as they may reflect the values of  $\psi_i$  that correspond to the industries that patent the most (which are also the most R&D intensive).

little compared to the data, and moreover are not highly correlated with the data (the correlation is 0.15).

#### FIGURE 4 ABOUT HERE

Second, we assumed that  $\rho_i = 0.91$  for all  $i$ , and computed  $\psi_i$  such that  $\psi_i/(1 - \rho_i)$  always takes on the values computed in Table 2. The weighted average value of  $\rho_i$  is the same as in the calibrated economy, but variation in  $\psi_i$  accounts for all industry differences in productivity growth. Figure 3 shows that the R&D intensity numbers generated by these parameters vary as much as in the data, and moreover are highly correlated with the data (the correlation is 0.72). This is remarkable considering that there is nothing in this procedure to purposefully match the R&D intensity values.

Why does allowing  $\rho_i$  to account for productivity differences make it hard to account for R&D differences? When values of  $\rho_i$  are very different, and the value of  $\rho_i$  is very high for the highest-growth industries, the value of  $\psi_i$  in those industries is driven towards zero. Since  $\psi_i$  enters the R&D expression multiplicatively, this drives research intensity to zero in those industries, *caeteris paribus*. As a result, when  $\rho_i$  is very high in some industries but not others, there is no longer a monotonic relationship between  $\gamma_i$  and R&D intensity in the model. Since the data indicate that  $\gamma_i$  and R&D intensity are positively correlated, the presence of significant variation in  $\rho_i$  leads patterns of R&D intensity in the model to differ from those in the data: they become U-shaped instead of monotonic. Thus, the property of the data that imposes the most discipline on the results is the positive correlation between R&D and TFP growth, which implies that variation in  $\rho_i$  is minor.<sup>20</sup>

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<sup>20</sup>We also found that if we generated Figure 3 using higher values of  $\psi_i$  (and correspondingly lower values of  $\rho_i$ ) then R&D correlations between the model and the data were similar but the *level* of R&D intensity in the model was much higher than in the data. Thus, the level of R&D in the data is what identifies  $\psi_i$  as being generally low and  $\rho_i$  as being generally high and close to one. See Ngai and Samaniego (2009a) for details.

## 6 Discussion and extensions

As mentioned, we focus on capital goods because we think quality adjustment could be important for productivity measurement, and because (to our knowledge) comparable quality adjusted prices exist only for capital goods. The production of capital goods accounts for about 20% of US GDP (including structures), however, and it is worth thinking about the representativeness of our results. Recall that it is the positive correlation *itself* between TFP growth and R&D intensity that leads to our results, and Figure 1 shows that this property extends to manufacturing as well. This still leaves out the service sector, which is unfortunately typical of many productivity studies due to the difficulty of measuring prices and output in the service sector: at the same time, in a model of R&D-based growth, this should not be a concern since R&D is comparatively rare in the service sector – indeed, capital goods production is responsible for almost the entirety of R&D spending outside of chemicals.

We have abstracted from cross-industry spillovers to keep the mechanisms of the model transparent, but it would be interesting to include them in the model. There are two reasons why allowing them is unlikely to change our results. First, the model does not suggest that knowledge spillovers are the driving force behind industry differences in productivity:  $\psi_i$  takes center stage. Second, cross-industry spillovers appear small compared to within-industry spillovers. In the working version of this paper (Ngai and Samaniego (2009a)), we use the patent citation database to gauge the importance of cross-industry citations. This is analogous to classifying all Economics papers by field, and looking at the rates at which papers in any given field cite papers in any other given field. For all industries, we find that citations are dominated by within-industry citations, suggesting that cross-industry spillovers are relatively small.

We do not distinguish between product and process innovation, for several reasons. First, much (although by no means all) of the related empirical literature neglects the distinction. Second, it is rare that a "truly new" product is introduced. Rather, thinking of industries as

being defined at the 2- or 3-digit SIC level, both product and process innovations may result in improved (or cheaper) consumer (or capital) services of a given type. Thus, our modeling approach is consistent with our use of quality-adjusted price data. Third, although one-sector growth models that distinguish between product and process innovation sometimes have different properties (such as Young (1998)), Jones (1999) argues that these properties require a restriction on the parameter linking the rate of product innovation to the scale of the economy. Still, it would be interesting to perform our analysis in a model that allows for product innovation.

There are three ways for a firm to acquire knowledge for use in production. First, firms may produce knowledge by investing in R&D, as in our model. Second, knowledge that spills between firms may be used as an input into R&D. This activity is free in the sense that, for example, if one patent cites another, there is no requirement that any payments be made between patent holders. While our model allows for such spillovers, the knowledge production function (1) implies that a firm can only receive spillovers from other firms if it is also carrying out research, as argued by Cohen and Levinthal (1990). Third, firms may employ the knowledge produced by other firms *in production*, by means of a license payment – as in Klenow (1996). This amounts to a certain portion of the knowledge acquired by a firm from elsewhere being subject to licensing fees, as enforced by a patent system. The fact that a firm might benefit from producing knowledge not just by producing goods at lower cost but also by collecting licensing fees from its competitors might lead to higher R&D intensity. However, Arora et al (2002) find that revenues from licensing equal about 4% of R&D expenditure, suggesting that licensing is not a major incentive behind R&D activity in general, and (more importantly) that *industry variation* in licensing activity is unlikely to be sufficient to overturn our results. Aside from contributing to industry variation, one might still wonder whether allowing licensing might raise the average value of  $\psi_i$ , which (while in the empirically relevant range) is low in the calibrated model. In fact, adding a factor to the model that might increase model R&D intensity (given other parameters) implies that the

calibrated values of parameters that encourage R&D intensity (i.e.  $\psi_i$ ) are likely to *decrease* in order to match the values of R&D intensity in the data. Thus, we abstract from this third form of knowledge transfers, as the other two appear to be more quantitatively important – although the model could be extended to study patterns of patenting and licensing activity.

## 7 Concluding remarks

We develop a multi-sector, general equilibrium model of endogenous growth, incorporating a number of factors identified in the literature as potential determinants of the costs and benefits of research. We find that the main determinant of productivity growth differences across sectors are the technological opportunity parameters, especially the extent of decreasing returns to research activity. Although this parameter has not been identified as a potentially important source of cross-industry differences in the related literature, it turns out to play a pivotal role in a growth model that is consistent with stable growth over the long run. Theoretically, we find that two more factors of opportunity may be important – the extent to which new knowledge "stands on the shoulders" of prior knowledge, and the capital share of research activity – although quantitatively they do not appear to play an important role.

The fraction of total spillovers that accrues from the firm's own stock of knowledge affects cross-industry differences in research intensity but not TFP growth, whereas differences in demand factors affect neither, in line with a sense in the technology literature that technical change is primarily supply-driven. Nelson and Winter (1977) argue that innovations follow "natural trajectories" that have a technological or scientific rationale rather than being driven by movements in demand and, similarly, Rosenberg (1969) writes of innovation following a "compulsive sequence." In our model, equilibrium differences in long run productivity growth rates depend on opportunity parameters, so that long-run TFP growth rates are determined by technological factors: "natural trajectories" are an *equilibrium outcome*.

We see several directions for future work. It would be interesting to provide micro-

foundations for different factors of opportunity and appropriability. For example, could the magnitude of knowledge spillovers or the extent to which they accrue to different agents depend on the institutions that govern research, or even on organizational structure? Also, we have not used our model to explore policy implications. However, our results suggest that a "one-size fits all" R&D subsidy may not be an optimal policy when technological opportunities vary significantly across industries. We leave this topic for future work.

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## A Derivations and Proofs

### A.1 Firm's maximization

Taking the demand function  $p_{iht}(\cdot)$  and input prices  $\{w_t, R_t\}$  as given, the firm chooses  $\{N_{iht}, K_{iht}, Q_{iht}, L_{iht}\}_{t=0, \dots}$  to maximize (4) subject to (1)-(3). Optimal conditions imply:

$$w_t = p_{iht} \frac{\partial Y_{iht}}{\partial N_{iht}} \left( 1 + \frac{Y_{iht}}{p_{iht}} \frac{\partial p_{iht}}{\partial Y_{iht}} \right); \quad R_t = p_{iht} \frac{\partial Y_{iht}}{\partial K_{iht}} \left( 1 + \frac{Y_{iht}}{p_{iht}} \frac{\partial p_{iht}}{\partial Y_{iht}} \right). \quad (35)$$

Using (3), relative prices are,

$$\frac{p_{iht}}{p_{jht}} = \frac{\left( 1 + \frac{Y_{jht}}{p_{jht}} \frac{\partial p_{jht}}{\partial Y_{jht}} \right) (1 - \alpha_j) T_{jht} k_{jht}^{\alpha_j}}{\left( 1 + \frac{Y_{iht}}{p_{iht}} \frac{\partial p_{iht}}{\partial Y_{iht}} \right) (1 - \alpha_i) T_{iht} k_{iht}^{\alpha_i}}. \quad (36)$$

All firms take  $\{w_t, R_t\}$  as given, so marginal rate of substitution between capital and labor are equal across activities, firms and sectors:  $\frac{\partial Y_{iht}/\partial N_{iht}}{\partial Y_{iht}/\partial K_{iht}} = \frac{w_t}{R_t} = \frac{\partial F_{iht}/\partial L_{iht}}{\partial F_{iht}/\partial Q_{iht}}$ . Using (35), the

capital-labor ratios in equations (5)-(7) follows from (2)&(3) . Let  $k_{iht} = \frac{K_{iht}}{N_{iht}}$ , (5)-(7) imply:

$$k_{iht} = k_{it}, \quad k_{jt} = \frac{\alpha_j}{1 - \alpha_j} \frac{1 - \alpha_i}{\alpha_i} k_{it}; \quad \frac{Q_{iht}}{L_{iht}} = \frac{\eta_i}{1 - \eta_i} \frac{1 - \alpha_i}{\alpha_i} k_{it}. \quad (37)$$

So (16) follows from focusing on a rank-preserving equilibrium where  $\gamma_{iht} = \gamma_{it}$  and assuming price elasticities of demand are constants and sector-specific.

**R&D Intensity:** The firm's optimal allocation of capital across activities implies (11)

and its optimal condition for  $T_{iht+1}$  implies (12). Using (11),

$$\frac{1}{\chi_{iht+1}} \frac{\lambda_{t+1}}{p_{ct+1}} \frac{\partial \Pi_{iht+1}}{\partial T_{iht+1}} = - \frac{\partial \Pi_{iht+1} / \partial T_{iht+1}}{\partial \Pi_{iht+1} / \partial Q_{iht+1}} \frac{\partial F_{iht+1}}{\partial Q_{iht+1}}. \text{ Together with (2)-(4),}$$

$$\begin{aligned} \frac{\frac{\lambda_{t+1}}{p_{ct+1}} \frac{\partial \Pi_{iht+1}}{\partial T_{iht+1}}}{\chi_{iht+1}} &= \frac{\left(1 + \frac{Y_{iht}}{p_{iht}} \frac{\partial p_{iht}}{\partial Y_{iht}}\right) p_{iht+1} \frac{Y_{iht+1}}{T_{iht+1}} \eta_i \psi_i F_{iht+1}}{\left(1 + \frac{Y_{iht}}{p_{iht}} \frac{\partial p_{iht}}{\partial Y_{iht}}\right) \frac{\alpha_i p_{iht+1} Y_{iht+1}}{K_{iht+1}} Q_{iht+1}} \\ &= \frac{K_{iht+1}}{T_{iht+1} \alpha_i} \frac{\eta_i \psi_i F_{iht+1}}{Q_{iht+1}} = \frac{1 - \eta_i}{1 - \alpha_i} \frac{N_{iht+1}}{L_{iht+1}} \psi_i [\gamma_{iht+1} - 1]. \end{aligned}$$

where the last equality follows from (1) & (37). Rearrange (12) as:

$$\begin{aligned} \frac{\chi_{iht}}{\chi_{iht+1}} &= \frac{\frac{\lambda_{t+1}}{p_{ct+1}} \frac{\partial \Pi_{iht+1}}{\partial T_{iht+1}}}{\chi_{iht+1}} + \frac{\partial F_{iht+1}}{\partial T_{iht+1}} + 1 \\ &= \frac{1 - \eta_i}{1 - \alpha_i} \frac{N_{iht+1}}{L_{iht+1}} [\psi_i \gamma_{iht+1} - 1] + \kappa_i [\gamma_{it+1} - 1] + 1, \end{aligned}$$

where the equality follows from (1), finally,

$$\frac{n_{iht+1}}{l_{iht+1}} = \left( \frac{1 - \alpha_i}{1 - \eta_i} \right) \frac{1}{\psi_i} \left[ \frac{\frac{\chi_{iht}}{\chi_{iht+1}} - 1}{\gamma_{iht+1} - 1} - \kappa_i \right], \quad (38)$$

where  $n_{iht} = N_{iht} / \mathcal{N}_t$  and  $l_{iht} = L_{iht} / \mathcal{N}_t$ . Using (35),

$$w_t L_{iht} + R_t Q_{iht} = \left( p_{iht} + Y_{iht} \frac{\partial p_{iht}}{\partial Y_{iht}} \right) T_{iht} k_{iht}^{\alpha_i} [(1 - \alpha_i) L_{iht} + \alpha_i Q_{iht}].$$

$$\text{Use (37) : } w_t L_{iht} + R_t Q_{iht} = \left( 1 + \frac{Y_{iht}}{p_{iht}} \frac{\partial p_{iht}}{\partial Y_{iht}} \right) \frac{1 - \alpha_i}{1 - \eta_i} p_{iht} Y_{iht} \frac{L_{iht}}{N_{iht}}. \quad (39)$$

$$\text{Similarly : } w_t N_{iht} + R_t K_{iht} = \left( 1 + \frac{Y_{iht}}{p_{iht}} \frac{\partial p_{iht}}{\partial Y_{iht}} \right) p_{iht} Y_{iht}.$$

$$\text{So : } \frac{w_t N_{iht} + R_t K_{iht}}{w_t L_{iht} + R_t Q_{iht}} = \left( \frac{1 - \eta_i}{1 - \alpha_i} \right) \frac{N_{iht}}{L_{iht}} = \frac{1}{\psi_i} \left( \frac{\frac{\chi_{iht}}{\chi_{iht+1}} - 1}{\gamma_{iht+1} - 1} - \kappa_i \right),$$

where the last equality follows from (38). Substituting into the definition of  $RND_{iht}$  to obtain (13). To derive  $\chi_{iht}/\chi_{iht+1}$ , use (2) & (37):  $\frac{\partial F_{iht+1}/\partial Q_{iht+1}}{\partial F_{iht}/\partial Q_{iht}} = \gamma_{iht}^{\kappa_i} \gamma_{it}^{\sigma_i} g_{kt}^{\eta_i \psi_i - 1} \left( \frac{l_{iht+1}}{l_{iht}} \right)^{\psi_i - 1}$ .

$$\text{Use (11)&(35) : } \frac{\chi_{iht}}{\chi_{iht+1}} = \left( \frac{\lambda_t p_{iht}/p_{ct}}{\lambda_{t+1} p_{iht+1}/p_{ct+1}} \right) \gamma_{iht}^{\kappa_i - 1} \gamma_{it}^{\sigma_i} g_{kt}^{\eta_i \psi_i - \alpha_i} \left( g_N \frac{l_{iht+1}}{l_{iht}} \right)^{\psi_i - 1}. \quad (40)$$

## A.2 Household maximization

We first determine the consumer's optimal spending across goods taking as given the total per capita spending on consumption  $s_c$  and investment  $s_x$ . Omitting time subscripts:

$$\begin{aligned} \max_{\{c_{ih}\}} c \quad & s.t. \quad s_c = \sum_{i=1}^{m-1} \int_0^1 p_{ih} c_{ih} dh, \quad \text{and} \\ \max_{\{x_{jh}\}} x \quad & s.t. \quad s_x = \sum_{j=m}^z \int p_{jh} x_{jh} dh \end{aligned}$$

where  $c$  and  $x$  are defined (20) and (23). The optimal spending across varieties of consumption goods:  $(c_{ih}/c_{ih'})^{-1/\mu_i} = p_{ih}/p_{ih'} = c_{ih'} = c_{ih} (p_{ih}/p_{ih'})^{\mu_i}$ , so  $c_i = \left( \int_0^1 c_{ih'}^{\frac{\mu_i-1}{\mu_i}} dh' \right)^{\frac{\mu_i}{\mu_i-1}} = c_{ih} \left[ \int (p_{ih}/p_{ih'})^{\mu_i-1} dh' \right]^{\frac{\mu_i}{\mu_i-1}}$ . Define  $p_i = \left[ \int p_{ih} c_{ih} dh \right] / c_i = \left[ \int p_{ih}^{1-\mu_i} dh \right]^{1/(1-\mu_i)}$  to rewrite  $c_i = c_{ih} (p_{ih}/p_i)^{\mu_i}$ . Thus across good  $i$ ,  $p_i c_i / (p_j c_j) = \omega_i / \omega_j = p_i c_i = \omega_i s_c$ , together with (20),

$$c_{ih} = s_c (p_i/p_{ih})^{\mu_i} \omega_i / p_i; \quad p_c \quad s_c / c = \prod_{i=1}^{m-1} p_i^{\omega_i}. \quad (41)$$

The result follows analogously for investment,

$$x_{jh} = s_x (p_j/p_{jh})^{\mu_j} (\omega_j/p_j) \text{ and } x_j = s_x (\omega_j/p_j), \quad (42)$$

$$p_j \frac{\int p_{jh} c_{jh} dh}{x_j} = \left[ \int p_{jh}^{1-\mu_j} dh \right]^{1/(1-\mu_j)}; \quad p_x \quad s_x/x = \prod_{j=m}^z p_j^{\omega_j} \quad (43)$$

Finally, the dynamic problem is to maximize  $\sum_{t=0}^{\infty} (\beta g_N)^t u(c_t)$  by choosing  $\{c_t, x_t\}_{t=0, \dots}$  s.t.

$$p_{ct}c_t + p_{xt}x_t = w_t + R_t k_t + \pi_t \quad \text{and} \quad g_N k_{t+1} = x_t + (1 - \delta_k) k_t.$$

The solution implies (27) and (28).

### A.3 Market Equilibrium

The goods market clearing condition (25) together with the demand for goods  $ih$  in (41) and (42) imply  $\frac{Y_{iht}}{p_{iht}} \frac{\partial p_{iht}}{\partial Y_{iht}} = \frac{1}{\mu_i}$  together with (37), equations (35) and (36) become:

$$R_t = \alpha_i p_{iht} T_{iht} k_{it}^{\alpha_i - 1} (1 - 1/\mu_i); \quad w_t = (1 - \alpha_i) p_{iht} T_{iht} k_{it}^{\alpha_i} (1 - 1/\mu_i), \quad (44)$$

$$\frac{p_{iht}}{p_{jht}} = \frac{(1 - 1/\mu_j) (1 - \alpha_j) T_{jht} k_{jt}^{\alpha_j}}{(1 - 1/\mu_i) (1 - \alpha_i) T_{iht} k_{it}^{\alpha_i}}. \quad (45)$$

The capital market clearing condition (26) and (37) imply

$$k_{jht} = \left( \frac{\alpha_j}{1 - \alpha_j} \right) \frac{k_t}{\Psi_t}; \quad \Psi_t = \sum_{i,h} \left( \frac{\alpha_i}{1 - \alpha_i} n_{iht} + \frac{\eta_i}{1 - \eta_i} l_{iht} \right). \quad (46)$$

In any rank-preserving equilibrium, we know (8) and (16), using (24),

$$\begin{aligned} \frac{p_{xt+1}/p_{xt}}{p_{it+1}/p_{it+1}} &= \prod_{j=m}^z \left( \frac{p_{jt+1}/p_{jt}}{p_{it+1}/p_{it+1}} \right)^{\omega_j} = \frac{\gamma_{it}}{\gamma_{xt}} g_{kt}^{\alpha_i - \alpha_x}, \\ \frac{p_{ct+1}/p_{ct}}{p_{it+1}/p_{it+1}} &= \prod_{j=1}^{m-1} \left( \frac{p_{jt+1}/p_{jt}}{p_{it+1}/p_{it+1}} \right)^{\omega_j} = \frac{\gamma_{it}}{\gamma_{ct}} g_{kt}^{\alpha_i - \alpha_c} \end{aligned} \quad (47)$$

$$\gamma_{xt} = \prod_{j=m}^z \gamma_{jt}^{\omega_j}; \quad \gamma_{ct} = \prod_{i=1}^{m-1} \gamma_{it}^{\omega_i}; \quad \alpha_c = \sum_{i=1}^{m-1} \alpha_i \omega_i; \quad \alpha_x = \sum_{j=m}^z \alpha_j \omega_j \quad (48)$$

$$\text{So: } \frac{q_{t+1}}{q_t} = \frac{\gamma_{ct}}{\gamma_{xt}} g_{kt}^{\alpha_c - \alpha_x}. \quad (49)$$

In any rank-preserving equilibrium, substituting (28) into (40) implies

$$\frac{\chi_{iht}}{\chi_{iht+1}} = \frac{\chi_{it}}{\chi_{it+1}} = \left( \frac{G_t p_{iht} / p_{xt}}{p_{iht+1} / p_{xt+1}} \right) \gamma_{it}^{\rho_i - 1} g_{kt}^{\eta_i \psi_i - \alpha_i} \left( g_{\mathcal{N}} \frac{l_{it+1}}{l_{it}} \right)^{\psi_i - 1}. \text{ Use (47):}$$

$$\frac{\chi_{it}}{\chi_{it+1}} = \frac{G_t}{\gamma_{xt}} \gamma_{it}^{\rho_i} g_{kt}^{\eta_i \psi_i - \alpha_x} \left( g_{\mathcal{N}} \frac{l_{it+1}}{l_{it}} \right)^{\psi_i - 1}. \quad (50)$$

## Symmetric Equilibrium

We now focus on the symmetric equilibrium across firms and omit index  $h$ . Using (45)&(24),

$$\frac{p_i}{p_x} = \prod_{j=m}^z \left( \frac{p_i}{p_j} \right)^{\omega_j} = \prod_{j=m}^z \left( \frac{(1-1/\mu_j)(1-\alpha_j) T_{jt} k_{jt}^{\alpha_j}}{(1-1/\mu_i)(1-\alpha_i) T_{it} k_{it}^{\alpha_i}} \right)^{\omega_j}.$$

$$\begin{aligned} \text{Use (37)} : \quad \frac{p_{it}}{p_{xt}} &= \frac{(1-1/\mu_x) T_{xt} k_{it}^{\alpha_x - \alpha_i} \prod_{j=m}^z [\alpha_j^{\alpha_j} (1-\alpha_j)^{1-\alpha_j}]^{\omega_j}}{(1-1/\mu_i) T_{it} \alpha_i^{\alpha_x} (1-\alpha_i)^{1-\alpha_x}}. \quad (51) \\ (1-1/\mu_x) &\prod_{j=m}^z (1-1/\mu_j)^{\omega_j}, \quad T_{xt} = \prod_{j=m}^z T_{jt}^{\omega_j}. \end{aligned}$$

$$\text{Use (44)&(46)} : \frac{R_t}{p_{xt}} = T_{xt} (1-1/\mu_x) k_t^{\alpha_x - 1} \Psi_t^{1-\alpha_x} \prod_{j=m}^z [\alpha_j^{\alpha_j} (1-\alpha_j)^{1-\alpha_j}]^{\omega_j}. \quad (52)$$

Market clearing for consumption goods:  $p_{it} T_{it} k_{it}^{\alpha_i} n_{it} = \omega_i p_{ct} c_t = n_{it} = \frac{\omega_i p_{ct} c_t}{p_{it} T_{it} k_{it}^{\alpha_i}}$ .

$$\text{Use (46)&(51)} : n_i = \frac{c_t / q_t}{T_{xt} k_t^{\alpha_x}} \left( \frac{1-1/\mu_i}{1-1/\mu_x} \right) \frac{\omega_i (1-\alpha_i)}{\Psi_t \prod_{j=m}^z [\alpha_j^{\alpha_j} (1-\alpha_j)^{1-\alpha_j}]^{\omega_j}}; \quad i = 1, \dots, m-1 \quad (53)$$

Similarly, use market clearing for investment goods, (51) and (46),

$$n_{jt} = \frac{x_t}{T_{xt} k_t^{\alpha_x}} \left( \frac{1-1/\mu_j}{1-1/\mu_x} \right) \frac{\omega_j (1-\alpha_j)}{\Psi_t \prod_{j=m}^z [\alpha_j^{\alpha_j} (1-\alpha_j)^{1-\alpha_j}]^{\omega_j}}; \quad j = m, \dots, z. \quad (54)$$

## Balanced Growth Path

**Proof of Proposition 1.** We look for a balanced growth path (BGP) such that  $x, k$  and  $c$  are growing at constant rates. Household's (21) and (22) require  $x/k$  and  $c/(qk)$  to be constants. Define  $k_{et} = k_t T_{xt}^{-1/(1-\alpha_x)}$ . Let  $g_x = x_{t+1}/x_t$  for all variables  $x$ . Note when  $n_i$  and  $l_i$  are constants, (46) implies  $\Psi$  is constant, so (46) implies  $k_{it}/k_t$  is constant. Using (27),

$$\frac{1}{\beta} \left( \frac{c_{t+1}}{c_t} \right)^\theta = \frac{u'(c_t)}{\beta u'(c_{t+1})} = g_{qt} G_{t+1}, \quad (55)$$

so  $g_c$  is constant if  $g_q$  and  $G$  are constants. Using (52),  $G$  is constant if and only if  $k_e$  is constant, i.e.  $g_{kt} = \gamma_{xt}^{1/(1-\alpha_x)}$ . It follows from (53) and (54) that both  $n_i$  are constants  $i$ . Substituting (3) into (20) and (23):  $x_t = \prod_{j=m}^z \left( \frac{T_{jt} k_{jt}^{\alpha_j} n_{jt}}{\omega_j} \right)^{\omega_j}$  and  $c_t = \prod_{i=1}^{m-1} \left( \frac{T_{it} k_{it}^{\alpha_i} n_{it}}{\omega_i} \right)^{\omega_i}$ . Using (46) and (48),  $g_x = \gamma_x g_k^{\alpha_x}$ , together with (49),  $g_c = \gamma_c g_k^{\alpha_c} = g_q \gamma_x g_x^{\alpha_x}$ . Given  $g_k = \gamma_x^{1/(1-\alpha_x)}$ , both  $x/k$  and  $c/(qk)$  are constants when  $g_k$  and  $g_q$  are constants. But  $g_k$  is constant if  $\gamma_j$  are constants  $j = m, \dots, z$ . Lemma (1) implies (10) holds for  $j = m, \dots, z$ , using (48),  $\gamma_x = \prod_{i=m}^z \gamma_i^{\omega_i} = \prod_{i=m}^z (g_N g_k^{\eta_i})^{\frac{\omega_i \psi_i}{1-\rho_i}} = \prod_{i=m}^z \left( g_N \left( \gamma_x^{\eta_i/(1-\alpha_x)} \right) \right)^{\frac{\omega_i \psi_i}{1-\rho_i}}$ , which implies (29) and  $\gamma_x$  in Proposition 1. It follows from (49) that  $g_q$  is constant if and only if  $\gamma_c$  is constant, i.e.  $\gamma_i$  are constants  $i = 1, \dots, m-1$ . Given  $n_i, g_k$  and  $\gamma_i$  are constants, (50) and (38) imply  $\chi_{it+1}/\chi_{it}$  and  $n_{it}/l_{it}$  are constants  $i$ , so  $l_i$  are constants  $i$ . ■

**Corollary 1** *Let  $y = \sum \frac{p_{it}}{p_{ct}} y_{it}$ . Along BGP,  $c/y$ , real interest rate and R&D spending to GDP ratio are constants. Moreover,*

$$g_q = \gamma_c \gamma_x^{\frac{\alpha_c - 1}{1 - \alpha_x}}; \quad g_c = \gamma_c \gamma_x^{\frac{\alpha_c}{1 - \alpha_x}}; \quad \gamma_c = g_N^\Upsilon; \quad \Upsilon = \sum_{i=1}^{m-1} \left( \frac{\eta_i \Phi}{1 - \alpha_x} + 1 \right) \frac{\omega_i \psi_i}{1 - \rho_i}. \quad (56)$$

**Proof.** Given  $G$  and  $g_q$  are constants, it follows from (28) that real interest rate  $r$  is constant.

Using (51) and (46), GDP per head:

$$\begin{aligned} \sum_{i=1}^z \frac{p_{it}Y_{it}}{\mathcal{N}_t} &= p_{xt} \sum_{i=1}^z \frac{(1 - 1/\mu_x) T_{xt} k_{it}^{\alpha_x} n_i \prod_{j=m}^z [\alpha_j^{\alpha_j} (1 - \alpha_j)^{1-\alpha_j}]^{\omega_j}}{(1 - 1/\mu_i) \alpha_i^{\alpha_x} (1 - \alpha_i)^{1-\alpha_x}} \\ &= p_{xt} k_t T_{xt} k_t^{\alpha_x - 1} \sum_{i=1}^z \frac{n_i (1 - 1/\mu_x) \prod_{j=m}^z [\alpha_j^{\alpha_j} (1 - \alpha_j)^{1-\alpha_j}]^{\omega_j}}{(1 - 1/\mu_i) \Psi (1 - \alpha_i)}, \end{aligned} \quad (57)$$

so  $y/c$  is constant given  $T_{xt} k_t^{\alpha_x - 1}$  and  $c/(qk)$  are constants. Using (39) and (37),

$$\sum_{i=1}^z (L_i w_t + Q_i R) = \mathcal{N}_t R_t \frac{k_t}{\Psi} \sum_{i=1}^z \frac{l_i}{1 - \eta_i},$$

Given constant  $R_t/p_{xt}$  and (57), the R&D spending to GDP ratio is constant. For (56),  $\gamma_c$  follows from substituting (10) and  $g_k = g_N^{\Phi/(1-\alpha_x)}$  into (48). Given  $g_k = \gamma_x^{1/(1-\alpha_x)}$ ,  $g_c$  and  $g_q$  follow from (49) and constant  $c/(qk)$ . ■

**Proposition 3** *Along the BGP, the non-negativity constraints on  $l_i$  and  $n_i$  do not bind and the transversality conditions for  $T_i$  and  $k$  are satisfied if  $g_N^{(1+\frac{\eta_i \Phi}{1-\alpha_x}) \frac{\psi_i}{1-\rho_i}} > 1$ ,  $\kappa_i < 1$ ,  $i$  and  $\beta < \{g_N^{-1}, \bar{\beta}\}$ , where  $\bar{\beta} = (1/g_N)^{1+(1-\theta)(\frac{\alpha_c \Phi}{1-\alpha_x} + \Upsilon)}$  and  $\Upsilon$  is defined in (56).*

**Proof of Proposition 3.** First note that  $\beta g_N < 1$  is required for the household maximization to be well-defined. The transversality conditions (TVC) are:  $\lim_{t \rightarrow \infty} \zeta_t k_{t+1} = \lim_{t \rightarrow \infty} \chi_{it} T_{it+1} = 0$ ,  $i$ .  $\chi_{it}$  and  $\zeta_t$  are the corresponding shadow values. Using (33),

$$\frac{\chi_{it} T_{it}}{\chi_{it-1} T_{it-1}} = \frac{\gamma_x}{G} g_k^{\alpha_x} g_N = \gamma_x g_k^{\alpha_x} g_N \beta g_q g_c^{-\theta} = \beta g_N g_c^{1-\theta}, \quad (58)$$

where it uses (55),  $g_k = \gamma_x^{1/(1-\alpha_x)}$  and constant  $c/(qk)$ . Using (56),

$\lim_{t \rightarrow \infty} \chi_{it} T_{it+1} = \chi_{i0} T_{i0} \lim_{t \rightarrow \infty} \left( \beta g_N^{1+(1-\theta)(\frac{\alpha_c \Phi}{1-\alpha_x} + \Upsilon)} \right)^t$ . So TVC for  $T_i$  holds if  $\beta < \bar{\beta}$ . The shadow price for  $k$  is the discounted marginal utility,  $\zeta_t = (\beta g_N)^t \left( \frac{p_{xt}}{p_{ct}} \right) u'(c_t) = (\beta g_N)^t q_t c_t^{-\theta}$ , constant  $\frac{c}{qk}$  implies  $\frac{\zeta_t k_t}{\zeta_{t-1} k_{t-1}} = \beta g_N g_c^{1-\theta}$ , so TVC for  $k$  holds when TVC for  $T_i$  in (58) holds.

Finally, from (1),  $l_i > 0$   $\gamma_i > 1$ , so (10) implies  $g_{\mathcal{N}}^{\left(1+\frac{\eta_i \Phi}{1-\alpha_x}\right)\frac{\psi_i}{1-\rho_i}} > 1$ . From (58),  $\frac{x_{it}/x_{it+1}}{\gamma_i} = \frac{x_{it}T_{it}}{x_{it+1}T_{it+1}} = (\beta g_{\mathcal{N}} g_c^{1-\theta})^{-1} > 1$ , for  $\beta < \bar{\beta}$ . So from (38), a sufficient condition for  $n_i/l_i > 0$  (so  $n_i > 0$ ) is  $\kappa_i < 1$ . ■

## B Calibration

This section describes the detailed calibration procedure for the parameters used in Section 5.

### Calibration for $(\omega_i, \alpha_i, \mu_i, A_i)$

We compute industry spending shares  $\omega_i$  from the Bureau of Economic Analysis' capital flow tables, 1947-2007. The shares for each of our 14 capital goods sectors appear to be fairly stable during the post-war era, consistent with our Cobb-Douglas capital good aggregator (23). We compute  $\alpha_i$  from the BEA industry GDP tables (see Appendix C for details).

We calibrate the elasticity parameter  $\mu_i$  using industry markups from Oliveira, Scarpetta and Pilat (1996). These authors report markups over average cost. In the model,  $\mu_i$  is linked to the markup over *production* cost – which could be significantly larger than the markup over total cost in very research-intensive industries. In Appendix C we discuss the mapping between the reported markups and those required to calibrate  $\mu_i$ . The calibration for  $A_i$  is described in Section 5.

Values for  $(\omega_i, \alpha_i, \mu_i, A_i)$  are reported in Table 3.

### Calibration for aggregate values $(g_k, g_{\mathcal{N}}, G)$

Let  $g_y$  equal the growth factor of real output measured in units of consumption. US National Income and Product Accounts indicate that  $g_y = 1.022$  in consumption units. In the model,  $g_y$  also represents the growth of real consumption, so we can compute the growth rate of capital in quality-adjusted units  $g_k = g_y/g_q$ .

By definition,  $g_q = \prod_{j=2}^{15} \left( \frac{p_{jt+1}/p_{ct+1}}{p_{jt}/p_{ct}} \right)^{\omega_j}$ . We obtain the growth of the quality-adjusted capital price relative to consumption from Cummins and Violante (2002) for our 14 capital goods.

However, these are gross-output prices. Our model is a multi-industry value-added model, thus our prices  $p_{it}$  are value-added prices. Assuming that the share and the composition of intermediate goods in gross output are each similar across sectors, Ngai and Samaniego (2009b) show that there is a simple transformation between relative prices in a value added model and relative prices in the data. If the relative price of a good in a value added model is  $\frac{p_{it}}{p_{ct}}$ , and the relative price of good  $i$  in the data is  $\frac{\tilde{p}_{it}}{\tilde{p}_{ct}}$  (measured at the level of the good, i.e. gross output) then  $\frac{p_{it}}{p_{ct}} = \left(\frac{\tilde{p}_{it}}{\tilde{p}_{ct}}\right)^{\frac{1}{1-\zeta}}$  where  $\zeta$  is the share of intermediate goods in gross output.<sup>21</sup> We set  $\zeta = 0.45$ , and find that  $g_q = 1.052^{-1}$ , thus  $g_k = (1.022)(1.052)$ .<sup>22</sup>

The growth rate of the population is reported by the US Census Bureau,  $g_N = 1.012$ . Using the Euler condition (28), the gross return to capital,  $G = (1+r)/g_q$ . We match the real rate of return on capital to be 7% as in Greenwood et al (1997). Hence the gross return in terms of capital goods is  $G = 1.07/g_q$ .

Values for  $(g_k, g_N, G)$  are reported in Table 1.

### Calibration for $(\gamma_i, \eta_i)$

As discussed in text, we do not match measured TFP growth, instead we calibrate  $\gamma_i$  using quality-adjusted relative prices. However, this will give us mainly relative TFP growth, to pin down the level of TFP growth we use the model prediction that  $g_k = \gamma_x^{1/(1-\alpha_x)}$  from Proposition 1. By definition,  $\alpha_x = \sum_{j=2}^{15} \omega_j \alpha_j$ . Given the computed values for  $(\omega_i, \alpha_i)$ , the implied value of  $\alpha_x$  equals 0.3, thus  $\gamma_x = [(1.022)(1.052)]^{0.7} = 1.052$ .

As discussed earlier, we compute TFP growth to match the decline in the quality-adjusted relative prices for our 14 capital goods industries. This mapping is slightly complicated in our model compared to Greenwood et al (1997) because we allow input shares to differ across

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<sup>21</sup>Though their model assume identical capital intensities across sectors while we allow capital intensities to differ, it is straightforward to show that the same transformation between the relative prices in a value added model and relative prices in the data continue to hold as long as the share and composition of intermediate goods in gross output are each identical across sectors

<sup>22</sup>In the working version of the paper, however, we show that our main results are similar when we use unadjusted prices.

industries. Using (16), the definition of  $p_x$  and the calibrated value of  $\gamma_x$ , we compute  $\gamma_i$  as:

$$\frac{p_{xt+1}/p_{xt}}{p_{it+1}/p_{it+1}} = \frac{\gamma_i}{\gamma_x} g_k^{\alpha_i - \alpha_x} = \gamma_i = \gamma_x g_k^{\alpha_x - \alpha_i} g_q \left( \frac{p_{it+1}/p_{it}}{p_{ct+1}/p_{ct+1}} \right)^{-1}, \quad (59)$$

where values of  $\frac{p_{it+1}/p_{it}}{p_{ct+1}/p_{ct+1}}$  are drawn from Cummins and Violante (2002), and adjusted as discussed above.

We measure  $\eta_i$  as the capital share of research expenditures using data from the National Science Foundation Industrial Research and Development Survey. The values of  $\gamma_i$  and  $\eta_i$  are reported in Table 3. Measurement details are in the data Appendix below.

## C Data

### C.1 Patent data

The NBER patent database, described in detail in Hall et al (2001), classifies patents according to their industry of origin and type of innovation. This involves tracking the industry of origin of each patent, and of the patents that each patent cites, for 16,522,438 citation entries. While data on patents begin in 1963, citations are only available for patents granted since 1975. We place durables into 14 categories we could identify in the citation data. The industry classification in Hall et al (2001) mostly coincides with that in Table 2. The exceptions were Aircraft, Ships and Boats, Autos and Trucks, and Structures, which we put together from their finer classification, including only rubrics that we could definitively associate with the industry in question. Autos and Trucks combines classes 180, 280, 293, 278, 296, 298, 305 and 301. Structures combines classes 14 and 52 (Bridges and Static Structures). Aircraft equals class 244 (Aeronautics), and Ships and Boats is class 114 (Ships). The full list of categories may be found at [http://www.nber.org/patents/list\\_of\\_classes.txt](http://www.nber.org/patents/list_of_classes.txt)

## C.2 Capital shares

We use the 2003 edition of the NSF Industrial Research and Development Survey. The NSF does not report capital expenditures related to R&D, rather they report a value of depreciation costs. Using a perpetual inventory method and the physical depreciation rates in the model, we derive the capital stocks implied by the depreciation costs and use them to impute the values of  $\eta_i$  reported in Table 2. This requires a value of the depreciation rate for capital  $\delta_k$ : we use a value of 0.056, which we calibrate as in Greenwood et al (1997).

Values of  $\alpha_i$  come from the Bureau of Economic Analysis Industry GDP tables. Not all industries were specifically listed as the industry classification of the BEA is coarser than ours. Thus, for example, the BEA entry for "Machinery" included both our "Machinery" and our "Mining and oilfield Machinery". In this case we used the same value for both sub industries. We used tables for 1987-1997 as earlier years were even more aggregated. We followed the same procedure for  $\eta_i$ .

## C.3 Research intensity

We measure R&D intensity using the R&D to sales ratio of the median firm. The maintained assumption is that the *median firm* in Compustat is subject to weak if any financial constraints, so that its R&D behavior should reflect the "pure" technologically determined level of R&D intensity for the industry. See Rajan and Zingales (1998) and Ilyina and Samaniego (2008) on the use of median firms to detect technological characteristics. We discard the top and bottom 1 percent of observations in the sample, to reduce the influence of outliers and of possible measurement error.<sup>23</sup>

Research intensity numbers from Compustat include labor and materials costs but not capital. In the model we have removed intermediates, and we also include capital. To make the numbers comparable, first, we remove materials using the materials share of R&D in NSF

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<sup>23</sup>The medians were in fact quite close to R&D/sales numbers reported by the NSF, so we view them as accurate (NSF values were not available for all industries, which is why we did not use them directly).

data (which is small and averages around a fifth of labor spending). Then, to impute capital expenditures related to research, we use the values of  $\eta_i$  reported earlier. Finally, formal R&D spending does not necessarily reflect all the costs of conducting R&D. For example, the Bureau of Labor Statistics Occupational Employment Statistics 2007 report that, for firms in NAICS 541700 (Scientific Research and Development Services) scientists and engineers make up about 40 percent of the wage bill. Assuming that the activities of pure research firms are broadly similar to those at research units within firms that do not outsource their R&D, this suggests multiplying the Compustat R&D numbers by a factor of 2.5. The effect of the above adjustments was to increase the values of  $RND^{Sales}$  somewhat above the raw numbers in Compustat, but the results that follow were qualitatively unchanged by using the "raw" numbers from Compustat instead.

## C.4 Markups

Markups are from Oliveira, Scarpetta and Pilat (1996). Where industry values were not available for the US, we took them from Canada (or in the case of Aircraft from Italy<sup>24</sup>). These are markups over average cost. In the model calibrating  $\mu_i$  requires a measure of the markup over *production* cost – which could be significantly larger than the markup over total cost in very research-intensive industries. Let  $M$  be the markup in the model, so  $M = \frac{1}{\mu}$ ,  $X$  is the measured markup, which is the markup over average cost. Suppose  $P$  is sales,  $R$  is research cost and  $C$  is production cost. Then, the measured markup  $X = \frac{P-R-C}{R+C}$ . Let  $r$  equal R&D intensity as measured in the data (relative to sales), so that  $R = rP$ . Then, it can be shown that  $M = \frac{r(X+1)+X}{1+r(X+1)}$ , so the measured markups can be derived from those reported in the data using R&D intensity numbers. See Table 3 for these and other parameters discussed in this Appendix.

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<sup>24</sup>Aircraft values were also available for France but French markups appeared systematically larger.

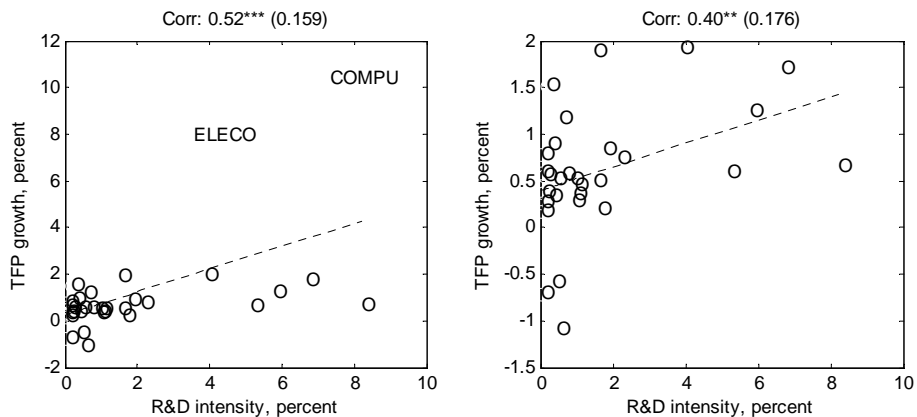


Figure 1 – Productivity growth and R&D intensity. Productivity growth rates for US industries over the post-war era are reported by Jorgenson et al (2007). R&D intensity is the median ratio of R&D expenditures to sales among firms in Compustat over the period 1950-2000. Data are for manufacturing. The dotted line represents fitted values for each graph. The right panel excludes two potential outliers, Computer equipment and electronic components. See also Terleckjy (1980) for an early survey.

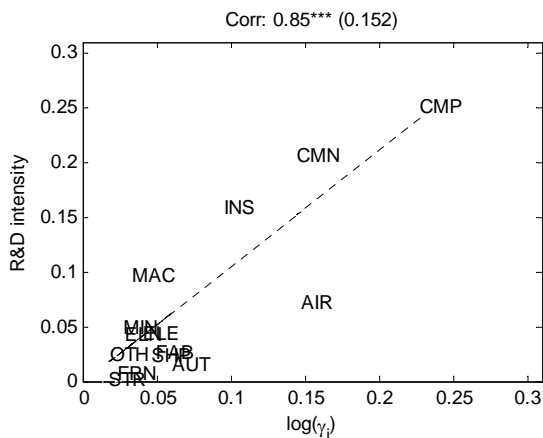


Figure 2 – Productivity growth and R&D intensity. Productivity growth rates for US industries over the post-war era are values for  $\gamma_i$  computed using the model. R&D intensity is the median ratio of R&D expenditures to sales among firms in Compustat over the period 1950-2000. Data are for the 14 capital goods sectors in the calibrated model.

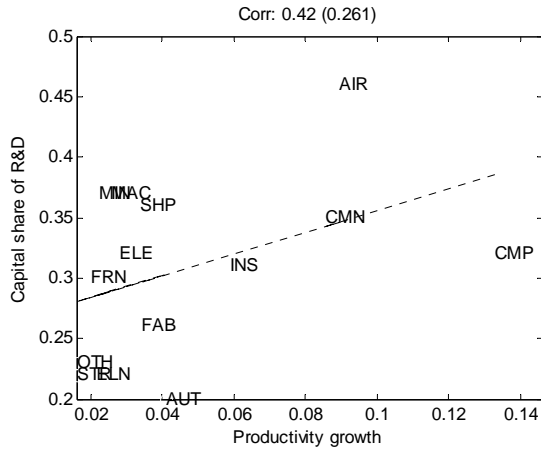


Figure 3 – Productivity growth and capital shares in research, by industry, across US capital goods. Productivity is measured using rate of decline of quality-adjusted prices relative to the consumption and services deflator. The dotted line represents an OLS regression of the two variables. Sources – Cummins and Violante (2002), Bureau of Economic Analysis, NSF.

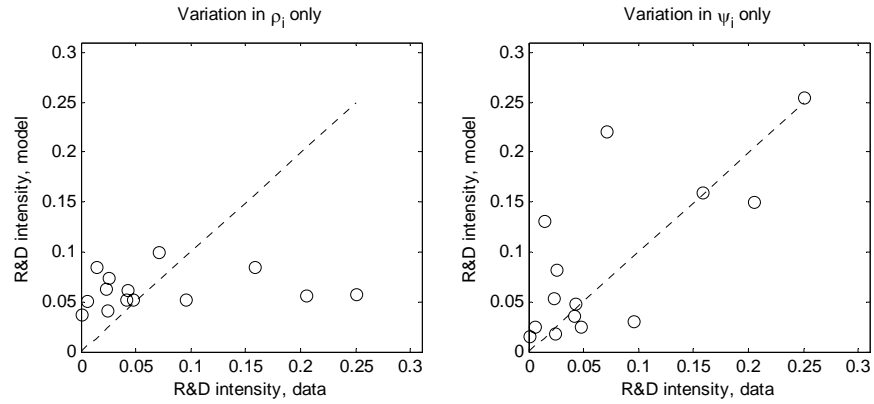


Figure 4 – R&D intensity in the model and in the data. The left panel assumes that  $\psi_i$  equals 0.13 and that  $\rho_i$  is chosen to account for all variation in  $\gamma_i$ . The right panel assumes that  $\rho_i$  is 0.91 and that  $\psi_i$  is chosen to account for all variation in  $\gamma_i$ . The dotted line is drawn at 45 degrees.

Capital good sector	$\omega_i$	$\alpha_i$	$1/\mu_i$	$A_i$
Computers and office equipment	0.049	0.24	0.55	0.16
Communication equipment	0.057	0.40	0.52	0.16
Aircraft	0.016	0.18	0.18	0.19
Instruments and photocopiers	0.042	0.35	0.20	0.17
Fabricated metal products	0.020	0.30	0.12	0.21
Autos and trucks	0.116	0.20	0.07	0.19
Elec. trans. dist. and ind. app.	0.028	0.40	0.18	0.16
Other durables	0.077	0.35	0.10	0.19
Ships and boats	0.007	0.18	0.21	0.16
Electrical equipment, n.e.c.	0.003	0.40	0.18	0.22
Machinery	0.203	0.26	0.24	0.18
Mining and oilfield machinery	0.009	0.26	0.20	0.34
Furniture and fixtures	0.028	0.26	0.06	0.13
Structures	0.346	0.32	0.17	0.12

Table 1 – Calibrated values of industry parameters. Sources and methodology are reported in the Appendix.

Variable	$g_y$	$g_q$	$g_N$	$g_k$	$G$
Value	1.022	$1.0517^{-1}$	1.012	1.075	1.125

Table 2 – Calibrated aggregate values (see text).

Capital good sector	$\log \gamma_i$	$\eta_i$	$\frac{\psi_i}{1-\rho_i}$
Computers and office equipment	0.228	0.32	6.53
Communication equipment	0.145	0.35	3.90
Aircraft	0.148	0.46	3.28
Instruments and photocopiers	0.095	0.31	2.78
Fabricated metal products	0.049	0.26	1.62
Autos and trucks	0.060	0.20	2.28
Elec. trans. dist. and ind. app.	0.040	0.32	1.16
Other durables	0.018	0.23	0.64
Ships and boats	0.047	0.36	1.24
Electrical equipment, n.e.c.	0.028	0.22	1.02
Machinery	0.034	0.37	0.88
Mining and oilfield machinery	0.028	0.37	0.72
Furniture and fixtures	0.023	0.30	0.70
Structures	0.018	0.22	0.64

Table 3 – TFP growth rates and industry parameters.  $\gamma_i$  is based on the quality-adjusted relative price of capital from Cummins and Violante (2002). The capital share of R&D spending is  $\eta_i$ . Values of  $\psi_i/(1-\rho_i)$  are computed using equation (32).

Capital good sector	R&D	$\psi_i$	$\rho_i$	$\kappa_i$	$\sigma_i$
Computers and office equipment	0.250	0.57	0.91	0.15	0.77
Communication equipment	0.205	0.47	0.88	0.14	0.74
Aircraft	0.071	0.09	0.97	0.18	0.79
Instruments and photocopiers	0.158	0.24	0.91	0.16	0.76
Fabricated metal products	0.026	0.04	0.97	0.20	0.77
Autos and trucks	0.014	0.02	0.99	0.19	0.80
Elec. trans. dist. and ind. app.	0.043	0.09	0.92	0.15	0.77
Other durables	0.024	0.07	0.88	0.17	0.72
Ships and boats	0.022	0.05	0.96	0.15	0.81
Electrical equipment, n.e.c.	0.042	0.10	0.90	0.20	0.70
Machinery	0.096	0.24	0.72	0.13	0.59
Mining and oilfield machinery	0.048	0.12	0.83	0.28	0.55
Furniture and fixtures	0.006	0.01	0.98	0.13	0.85
Structures	0.001	0.01	0.99	0.12	0.88
AVERAGE	0.057	0.13	0.91	0.14	0.77

Table 4 – R&D intensity,  $\rho_i$  and  $\psi_i$ , computed so as to match the calibrated values of  $\gamma_i$  and R&D intensity. Average values are weighted using  $\omega_i$ . Values of  $\kappa_i$  and  $\sigma_i$  may not add to  $\rho_i$  due to rounding.

Correlations	$\gamma_i$	$\psi_i$	$\rho_i$
R&D	0.85***	0.98***	-0.34
$\gamma_i$	-	0.80***	0.06
$\psi_i$	-	-	-0.42

Table 5 – Correlations between R&D intensity, productivity growth rates,  $\rho_i$  and  $\psi_i$ .