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Mapping prices into productivity in multisector growth models

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Abstract Two issues related to mapping a multi-sector model into a reduced-form value-added model are often neglected: the composition of intermediate goods, and the distinction between the productivity indices for value added and for gross output. We illustrate their significance for growth accounting using the well known model of Greenwood et al. (in Am Econ Rev 87(3):342–362, 1997), who find that about 60% of economic growth can be attributed to investment-specific technical change (ISTC). We investigate the role of intermediate goods in their framework and find that, taking into account the composition of intermediates, ISTC may well account for between 93 and 96% of post-war US growth.

Keywords Intermediate goods · Investment-specific technical change · Growth accounting · Value added · Multisector growth models

JEL Classification E13 · O30 · O41 · O47

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1 Introduction

There is growing interest in accounting for patterns of economic growth using multisector general equilibrium models.[1] These models are generally formulated in terms of value added, and are isomorphic to models that allow for intermediates. Still, the existence and structure of intermediate goods affects how value-added models should be brought to the data. We study the quantitative importance of two factors that are often neglected when mapping a multi-sector model with intermediate goods to a reduced-form one-sector model: the composition of intermediate goods, and the distinction between the productivity indices for value added and for gross output.

To see why composition matters, consider the following example. There are two industries, x^{FIN} and x^{INT}. Good x^{FIN} can be used as a final good, but some of the output of x^{INT} is also used as an intermediate in the production of x^{FIN}. Suppose the production of good x^{INT} experiences productivity improvements, whereas x^{FIN} does not. In a competitive environment,[2] productivity improvements in x^{INT} would be reflected in a decrease in the price of x^{INT}. Since x^{FIN} uses x^{INT} as an intermediate, the price of x^{FIN} would also decline. For instance, productivity improvements that lower the price of electronic components would decrease the cost of producing digital watches that embody them, even if the technology for producing watches were to remain exactly the same. A value-added model that does not account for such linkages between industries x^{FIN} and x^{INT} might understate the contribution of x^{INT} to economic growth—because one channel, its use as an intermediate, is absent from the model.

For concreteness, we conduct our analysis within the well known framework of Greenwood et al. (1997) (henceforth GHK), who find that about 60% of economic growth can be attributed to investment-specific technical change (ISTC)—technical progress in the production of investment goods. We focus on this model because it is highly tractable, and because it has motivated several other studies in which ISTC plays an important role. In practice, according to input–output tables, over half of the output of investment-good industries is used as an intermediate—for example, in the form of fabricated parts or electronic components.

To demonstrate the key implications of allowing intermediate goods (and, more importantly, of allowing equipment to be used as an intermediate), our baseline model focuses on the case in which the composition of intermediate goods is identical across sectors. We show that if the output of the equipment sector is used as an intermediate good in other sectors, then ISTC implies that the relative price of the intermediate good declines relative to the consumption good. As a result, a portion of what appears as neutral technical change in a one-sector model can be attributed to ISTC. Our gross-output model generates the same allocations of final goods as the one-sector GHK model yet, when the equipment share of intermediates is calibrated to 10%, we find that ISTC can account for over 90% of post-war US growth. GHK motivate their general equilibrium approach [and contrast it with the approach of Hulten (1992)] by observing that capital accumulation provides a channel through which the growth impact of ISTC may be amplified, so that general equilibrium growth accounting may be necessary to establish the full contribution of ISTC to growth. Accounting for the

[1] Examples include models of investment-specific technical change such as Greenwood et al. (1997), Cummins and Violante (2002) and Fisher (2006); and models of structural change such as Kongsamut et al. (2001) and Ngai and Pissarides (2008).

[2] For simplicity, we assume that production functions are identical across sectors except for sector-specific productivity. If, for example, capital shares are different across sectors, then the relationship linking relative prices and relative productivity would be more complex, as in Hornstein and Krusell (1996). Nonetheless, it would still be affected by the factors we raise.

composition of intermediate goods provides an additional general equilibrium channel that further amplifies the aggregate impact of ISTC.

To evaluate more fully the intermediate-goods channel of ISTC, we consider an extended version of our model that allows for a general input-output structure, and broaden the concept of ISTC to allow for structures-specific technical change (SSTC, as discussed in Gort et al. 1999).[3] We show that there is no longer an independent mapping between the relative price of any two given goods and the productivity of their production processes. Instead, the vector of industry productivities is a non-degenerate linear function of the input–output matrix and of the vector of prices. In this case, we find that ISTC can account for 96% of post-war US growth, even though the equipment share of the intermediates used by non-durables is barely 4.2%. One reason is that the structures' sector uses the output of the equipment sector as an intermediate good quite intensively (10.5%).

There is another reason why the distinction between value-added and gross-output matters when calibrating a multi-sector model. It is common to calibrate relative productivity growth rates in value-added models using the decline in relative output prices. However, actual prices are quoted in terms of currency per unit of gross output. In a multisector value-added model, the mapping between output prices and model productivity requires a transformation based on the intermediate share of gross output—which is large, roughly 50%. A given wedge between relative output prices turns out to reflect a much larger wedge between relative value-added productivities in the industries that produce those goods—even when the input–output structure is common across industries.

The issues we raise are well known in the productivity literature—see Hulten (1978) or Jorgensen et al. (2007). However, their relevance for quantitative general equilibrium work seems to have been overlooked. An exception is Vourvachaki (2007), who studies the contribution of Information and Communication Technology to growth through its use as an intermediate. The idea that input–output linkages are important for understanding the propagation of business cycle shocks has been well recognized in the recent literature (e.g. Basu 1995; Horvath 1998, 2000; Huang and Liu 2001). Jones (2009) studies the role of such linkages in accounting for cross-country income differences, arguing that complementarity among intermediate goods amplifies the effect of distortionary policy upon income levels. However, the importance of input–output linkages for long-run growth accounting is not explored in these papers.

Section 2 develops the model economy, and Section 3 discusses the mapping between the model of GHK and a multi-sector framework with intermediate goods. We report step-by-step derivations to ensure the mapping is clear. Section 4 extends the model to allow for a more general input–output structure. Section 5 reports quantitative results.

2 Economic environment

We present in this section a model in which the intermediate goods used by different industries have the same composition. This is the simplest way of explicitly modeling intermediate goods to illustrate their impact on general equilibrium growth accounting. However, *quantitative* results are quite sensitive to sectoral variation in the composition of intermediates. In Sect. 4 we present the general model where the composition of intermediate goods may vary across sectors.

[3] In general, investment includes both equipment and structures. The concept of ISTC in GHK refers to equipment-specific technical change as they do not view SSTC as being quantitatively important.

2.1 Households

Time is infinite and discrete. There is a representative household with the following life-time utility function:

$$E\sum_{t=0}^{\infty}\beta^{t}U\left(c_{t},l_{t}\right) \tag{1}$$

where per-period utility U is a function of contemporaneous consumption c_t and labor l_t :

$$U(c_t,l_t)=\eta\log c_t+(1-\eta)\log\left(1-l_t\right). \tag{2}$$

Households own all the capital in this economy. Capital income and labor income are subject to taxation at rates τ_k and τ_l respectively, and the proceeds of taxation are redistributed to households via a lump-sum transfer τ, so that:

$$\tau=\tau_k\left(r_{et}k_{et}+r_sk_{st}\right)+\tau_l w_t l_t \tag{3}$$

where k_{et} is equipment capital and k_{st} is structures capital.

Let u_i be the number of units of new capital goods of type $i\in\{e,s,c\}$ that are used for investment. Then, capital stocks evolve according to:

$$k_{s,t+1}=(1-\delta_s)\,k_{st}+u_{st} \tag{4}$$

$$k_{e,t+1}=(1-\delta_e)\,k_{et}+u_{et} \tag{5}$$

The household's maximization problem may be formulated recursively. Suppressing time subscripts, the household maximizes

$$V\left(k_e,k_s\right)=\max_{c,u_e,u_s,l}\left\{U\left(c,l\right)+\beta EV\left(k_e',k_s'\right)\right\} \tag{6}$$

subject to the capital accumulation equations, and also the budget constraint:

$$p_c c+p_e u_e+p_s u_s=(1-\tau_k)\left[r_e k_e+r_s k_s\right]+(1-\tau_l)\,wl+\tau \tag{7}$$

where p_i is the price of good i. r_i is the rental rate of capital of type i, and w is the wage rate. We suppress time and industry subscripts where this should not create confusion.

2.2 Three-sector model with intermediates

There are three final goods sectors: equipment, structures and consumption. In each sector $i\in\{e,s,c\}$, gross output d_i is produced with the following production function:

$$d_i=A_iF^{GO}\left(k_{ei},k_{si},m_i,l_i\right) \tag{8}$$

where $F^{\mathrm{GO}}(.)$ is Cobb–Douglas[4]:

$$F^{\mathrm{GO}}\left(k_e,k_s,m,l\right)=\left(k_e^{\alpha_e}k_s^{\alpha_s}l^{1-\alpha_e-\alpha_s}\right)^{1-\alpha_m}m^{\alpha_m} \tag{9}$$

[4] We adopt a Cobb–Douglas formulation for several reasons. First, since our paper demonstrates the implications of linkages for growth accounting, we employ a standard growth accounting framework—in particular, one close to the formulation of Greenwood et al. (1997). Even outside of general equilibrium growth accounting, the Cobb–Douglas assumption is important—see Jorgensen et al. (2007) for an extensive discussion. He and Liu (2008) find a lower contribution of ISTC to growth in transition in a model with a CES production function and no intermediates: it would of course be interesting to explore the role of linkages in transition, but, their results suggest that the order of magnitude would in any case be similar. Relaxing the Cobb–Douglas assumption also eliminates balanced growth in equilibrium, so that in the limit in their calibration with a CES production function there is no economic growth. Since their paper focuses on wage inequality this is not

The GHK model assumes that $A_s = A_c$. Their notion of *investment-specific technical change (ISTC)* is to capture the faster productivity change in the production of equipment relative to the production of consumption, i.e. the growth rate of A_e/A_c. We allow the productivity change in the production of structure and consumption to be different. In this environment, ISTC may have two components:

Definition 1 Equipment-specific technical change (ESTC): technical progress in the production of equipment at a rate more rapid than in the production of c. The rate of ESTC is the growth rate of A_e/A_c.

Definition 2 Structures-specific technical change (SSTC): technical progress in the production of equipment at a rate more rapid than in the production of c. The rate of structures-specific technical change (SSTC) is the growth rate of A_s/A_c.

The contribution to growth of ISTC is then the joint contribution of ESTC and SSTC. In the case of GHK (where $A_s = A_c$), ESTC and ISTC are the same.

Let h_i be the quantity of good i used as an intermediate. Gross output d_i is used either as a final good (u_i) or as an intermediate (h_i), so that market clearing for each sector requires:

$$d_e = u_e + h_e, \quad d_s = u_s + h_s, \tag{10}$$

$$d_c = c + h_c. \tag{11}$$

To simplify our exposition, we assume the same composite intermediate input is used in all sectors. We relax this assumption in Sect. 4. Intermediate good production is modelled as in Horvath (1998), (2000) and Ngai and Pissarides (2007). Intermediates are produced using the following technology:

$$m = \prod_{i\in\{e,s,c\}} \left(\frac{h_i}{\varphi_i}\right)^{\varphi_i}; \quad \sum_{i\in\{e,s,c\}} \varphi_i = 1, \varphi_i \geq 0. \tag{12}$$

where m is the quantity of intermediates. The market clearing condition for intermediates, capital and labor input are

$$\sum_{i\in\{e,s,c\}} m_i = m, \tag{13}$$

$$\sum_{i\in\{e,s,c\}} k_{ji} = k_j; \quad j = e, s, \tag{14}$$

$$\sum_{\in\{e,s,c\}} l_i = l. \tag{15}$$

2.2.1 Competitive equilibrium

Profit maximization for firms in each final good sector i solves:

$$\max_{k_{ei},k_{si},m_i,l_i} \{p_i d_i - r_e k_{ei} - r_s k_{si} - p_m m_i - w l_i\}. \tag{16}$$

Footnote 4 continued
central to their results, but in our model that is mainly interested in growth accounting the simplest, clearest way to demonstrate the impact of introducing intermediate groups is to use a standard analytically tractable framework with balanced growth.

The Cobb–Douglas production function (9) implies constant expenditure shares on all inputs—for instance, in the case of intermediate goods:

$$p_m m_i = \alpha_m p_i d_i. \tag{17}$$

Free mobility of inputs across sectors then implies that the capital-labor ratio and intermediate-labor ratio are equalized across sectors, so together with market clearing conditions (13) and (15), for any sector $i \in \{e, s, c\}$ we obtain:

$$\frac{m_i}{l_i} = \frac{m}{l}; \qquad \frac{k_{ji}}{l_i} = \frac{k_j}{l} \qquad j = e, s \tag{18}$$

and it follows that relative prices of gross output reflect the inverse of relative productivities in the production of gross output (8):

$$\frac{p_i}{p_j} = \frac{A_j}{A_i}. \tag{19}$$

Intermediate good producers solve:

$$\max_{h_s, h_c, h_e} \left[p_m m - \sum_{i=e,s,c} p_i h_i \right]$$

and given the intermediate goods' production function (12), optimal input use becomes:

$$p_i h_i = \varphi_i p_m m, \tag{20}$$

Together, (12) and (20) imply that the price-index for intermediate goods is

$$p_m = \prod_{i=c,e,s} p_i^{\varphi_i}, \tag{21}$$

so (19) and (21) imply that the relative price of intermediate goods is:

$$\frac{p_m}{p_c} = \prod_{i=c,e,s} \left(\frac{p_i}{p_c}\right)^{\varphi_i} = \prod_{i=c,e,s} \left(\frac{A_c}{A_i}\right)^{\varphi_i}. \tag{22}$$

2.3 Mapping into the value-added model

We now derive the mapping between the three-sector model with intermediate goods and a one-sector value-added model (as in GHK). More specifically, we show that the one-sector model has the same allocations of final goods as the three-sector model. To do this, we first derive the equivalent three-sector value added model, and then derive expression for aggregate value added in a one-sector representation.

Incorporating the optimal usage of intermediates goods in (17), the firms' problem (16) is equivalent to maximizing the profits from value added $y_i = z_i F^{VA}(k_{ei}, k_{si} l_i)$. Let p_i^y be the price index for value added. By definition,

$$p_i^y y_i \equiv p_i d_i - p_m m_i = (1 - \alpha_m) p_i d_i, \tag{23}$$

where the last equality follows from (17). Substituting optimal intermediate use (17) into the production function (9) yields:

$$d_i = \left(\frac{\alpha_m p_i}{p_m}\right)^{\alpha_m/(1-\alpha_m)} A_i^{1/(1-\alpha_m)} k_{ei}^{\alpha_e} k_{si}^{\alpha_s} l_i^{1-\alpha_e-\alpha_s}. \tag{24}$$

Together with (23), the firm's problem (16) can be re-written in terms of value added:

$$\max_{k_{ei},k_{si},l_i} \left\{ p_i^y y_i - r_e k_{ei} - r_s k_{si} - l_i w \right\}, \tag{25}$$

where the implied price index for value added is:

$$p_i^y = \left(\frac{p_i}{p_m^{\alpha_m}} \right)^{\frac{1}{1-\alpha_m}}, \tag{26}$$

the expression for real value-added is:

$$y_i = z_i k_{ei}^{\alpha_e} k_{si}^{\alpha_s} l_i^{1-\alpha_e-\alpha_s}, \tag{27}$$

and the productivity index z_i of industry value-added equals:

$$z_i \equiv (1-\alpha_m)\, \alpha_m^{\alpha_m/(1-\alpha_m)} A_i^{1/(1-\alpha_m)}. \tag{28}$$

Substituting (19) into (26) implies that the relative prices of value added reflect the inverse of relative productivity indices in the production of value added:

$$\frac{p_i^y}{p_j^y} = \left(\frac{A_j}{A_i} \right)^{1/(1-\alpha_m)} = \frac{z_j}{z_i}. \tag{29}$$

Define aggregate real value-added (in terms of consumption goods) as:

$$y \equiv \sum_{i=s,c,e} \frac{p_i^y y_i}{p_c}. \tag{30}$$

Using (27), and the results in (18) and (29),

$$p_c y = p_c^y z_c k_e^{\alpha_e} k_s^{\alpha_s} l^{1-\alpha_e-\alpha_s}. \tag{31}$$

Together with (22) and (26), this yields an expression for aggregate real value-added:

$$y = z k_e^{\alpha_e} k_s^{\alpha_s} l^{1-\alpha_e-\alpha_s} \tag{32}$$

where

$$z = (1-\alpha_m)\, \alpha_m^{\alpha_m/(1-\alpha_m)} A_c \left(\prod_{i=c,e,s} A_i^{\varphi_i} \right)^{\alpha_m/(1-\alpha_m)}. \tag{33}$$

2.4 Comparing to GHK

Greenwood et al. (1997) model investment-specific technical change (ISTC) as a falling relative price of equipment, induced by faster productivity change in the equipment-producing sector. More specifically, by spending I_{et} on equipment, households can obtain $I_{et} q_t$ units of equipment, where q_t is the price of consumption relative to equipment. Using the notation in our multi-sector model and the expression for relative prices (19), it follows that:

$$q_t = \frac{p_{ct}}{p_{et}} = \frac{A_{et}}{A_{ct}}, \tag{34}$$

which is consistent with their view that a rising q_t (a falling relative price of equipment) reflects faster productivity change in the equipment sector. However, deriving the multi-sector model with explicit intermediates allows us to note that the decline in the relative price of equipment (which is used as a measure of ISTC) is related to the growth of the relative productivity of equipment in terms of *gross output*, as shown in Eq. (34).

GHK focus on the case that the relative price of structures in terms of consumption goods is equal to one. In terms of our notation, $p_{ct} = p_{st}$, so it follows from (19) that $A_c = A_s$. The capital accumulation Eqs. (4) and (5) in the three-sector model can be reduced to their analogues in the GHK one-sector model:

$$k_{s,t+1} = (1 - \delta_s)\, k_{st} + I_{st} \tag{35}$$

$$k_{e,t+1} = (1 - \delta_e)\, k_{et} + I_{et} q_t, \tag{36}$$

where by definition, $u_{et} = q_t I_{et}$ and $u_{st} = I_{st}$ (spending on structures in GHK). Finally, we show that the market clearing conditions (10), (11), (13) and the optimal input composition (20) together imply the market clearing condition in GHK's one-sector model:

$$y = c + I_e + I_s, \tag{37}$$

which states that the final good can be used for consumption, or for investment. In other words, the one-sector model has the same allocations of final goods as the three-sector model, which completes the mapping between the three-sector gross output model and the one-sector value added model in GHK.

To derive (37) from the three-sector model, first note that the definition of aggregate value added y and of industry value added (23) imply:

$$y = (1 - \alpha_m) \sum_{i=s,c,e} \frac{p_i d_i}{p_c}. \tag{38}$$

Together with the market clearing conditions (10), (11), (13) and the optimal input composition (20),

$$y = (1 - \alpha_m) \left(I_e + I_s + c + \frac{p_m m}{p_c} \right). \tag{39}$$

Using (17), aggregate expenditure on intermediate goods is:

$$p_m m = \alpha_m \sum_{i=s,c,e} p_i d_i = \frac{\alpha_m}{1 - \alpha_m} p_c y, \tag{40}$$

where the last equality follows from (38). The market clearing condition (37) for GHK's one-sector model follows from substituting (40) into (39).

3 ISTC in the multisector model

We now underline two channels through which the quantitative implications of ISTC in a multi-sector model with intermediate goods might differ from a reduced-form one-sector value-added model. For easy comparison to GHK, we continue for now to focus on the case in which $A_c = A_s$.

3.1 Equipment's share in intermediate goods

To distinguish between different sources of technical change, we define the following.

Definition 3 The contribution of investment-specific technical change to growth is the percentage decrease in the growth rate in the model economy assuming that $A_e = A_s = A_c$. The contribution of neutral technical change is the remainder.

Definition 4 The contribution of equipment-specific technical change to growth is the percentage decrease in the growth rate in the model economy assuming that $A_e = A_c$.

Definition 5 The contribution of structures-specific technical change to growth is the percentage decrease in the growth rate in the model economy assuming that $A_s = A_c$.

In the one-sector value added model of GHK, the residual z in Eq. (32) is interpreted as an index of neutral technical change. In their paper, neutral technical change is defined in terms of technical progress that affects the goods that agents consume (sector c). Using (33), the productivity index z of aggregate real-value added derived from the multisector model is:

$$z = (1 - \alpha_m)\, \alpha_m^{\alpha_m/(1-\alpha_m)} A_c^{1/(1-\alpha_m)} \left(\frac{A_e}{A_c}\right)^{\alpha_m \varphi_e/(1-\alpha_m)}, \tag{41}$$

which includes the ISTC term A_e/A_c. Hence, if equipment is used as an intermediate good ($\varphi_e > 0$), the aggregate value added productivity index z remains influenced by technical progress specific to the equipment sector.

Let $\tilde{z} = A_c^{1/(1-\alpha_m)}$ be the value-added productivity index z net of any influence of productivity change over and above A_c in other sectors $\left(\frac{A_e}{A_c}\right)$. This is consistent with the definition of neutral productivity growth in GHK when there is no intermediate goods, and also with Definition 3. Using (41), the measure of neutral productivity growth by this definition is:[5]

$$\gamma_{\tilde{z}} = \gamma_z \gamma_q^{-\alpha_m \varphi_e/(1-\alpha_m)}. \tag{42}$$

Observation 1 A value-added model may understate the total contribution of ISTC if equipment is used as an intermediate good.

By deriving the full three-sector model, we see that the growth of the value added productivity index (γ_z) itself includes the contribution of ISTC through equipment's share as intermediate goods. Therefore the measure of *neutral* productivity growth according to Definition 3 is $\gamma_{\tilde{z}}$, which is smaller than γ_z if $A_e > A_c$.

To understand the implications of Observation 1 for growth accounting, consider the following. A significant finding of GHK, replicated in other studies,[6] is their growth accounting result that ISTC accounts for about 60% of economic growth. The contribution of ISTC to growth in GHK is determined by first calibrating their model to US data and then setting $A_e = A_c$, observing that the rate of economic growth in their calibrated model drops to 40% of the original calibrated value.

Long run growth accounting in their model yields the expression[7]:

$$\gamma_y = \gamma_z^{\frac{1}{1-\alpha_e-\alpha_s}} \gamma_q^{\frac{\alpha_e}{1-\alpha_e-\alpha_s}}. \tag{44}$$

[5] An alternative definition would be to let z be neutral productivity: however, this would imply that some portion of A_e would count as neutral technical change and so experiments that set $A_e = A_c$ would imply changes in neutral technical change. While this distinction is semantic, we prefer our nomenclature as we feel it distinguishes more clearly between different sources of technical change.

[6] See Cummins and Violante (2002) and Fisher (2006).

[7] To see this, given the aggregate value-added expression,

$$y = z k_e^{\alpha_e} k_s^{\alpha_s} l^{1-\alpha_e-\alpha_s} \Leftrightarrow \left(\frac{y}{l}\right)^{1-\alpha_e-\alpha_s} = z \left(\frac{k_e}{y}\right)^{\alpha_e} \left(\frac{k_s}{y}\right)^{\alpha_s}. \tag{43}$$

where, in their model, $q = A_e/A_c$. The value of 60% is arrived at by setting $\gamma_q = 1$, and observing that the rate of economic growth in their calibrated model drops to 40% of the observed level. In the multisector framework, this equation also holds, reflecting the influence of ISTC on growth through productivity change and through capital accumulation. However, this does not fully account for the influence of ISTC upon growth, as ISTC lowers the relative prices of intermediate goods which is implicit in the growth of the aggregate value added productivity index (γ_z). Using expression (42), Eq. (44) reduces to:

$$\gamma_y = \gamma_{\tilde{z}}^{\frac{1}{1-\alpha_e-\alpha_s}} \gamma_q^{\frac{\alpha_e+\varphi_e\alpha_m/(1-\alpha_m)}{1-\alpha_e-\alpha_s}} \quad (45)$$

Compared to (44), the exponent of γ_q has an additional term, corresponding to $\frac{\varphi_e\alpha_m/(1-\alpha_m)}{1-\alpha_e-\alpha_s}$, which equals $\frac{\varphi_e}{1-\alpha_e-\alpha_s}$ if $\alpha_m = 0.5$ as suggested by the data. Thus, given γ_q and γ_y, the contribution of ISTC to economic growth is unaffected by the presence of intermediates if $\varphi_e = 0$, but is *underestimated* if $\varphi_e > 0$. Notice that what matters is not the presence of intermediates per se but the fact that equipment is used as an intermediate.

For example, consider fabricated metals products (SIC 3400–3499), one specific group of equipment-producing industries which has experienced a significant rate of technical progress.[8] If sheet metal (SIC 3444) is installed in roofing by the construction industry (SIC 1761), then the structures sector benefits indirectly from technological improvements in the fabricated metals products industry, in the form of cheaper sheet metal. Similarly, if metal foil (SIC 3497) is used in the food services industry (SIC 5812), then the consumption and services sector benefits indirectly from technological improvements in the fabricated metals products industry, in the form of cheaper foil.

Section 5 explores how significant this underestimate might be. However, we can get a sense of its magnitude using (45) to perform some back of the envelope calculations. The impact of ISTC in a model with $\varphi_e = 0$ depends on α_e, whereas in a model with $\varphi_e > 0$ and $\alpha_m \approx 0.5$ it depends on $\alpha_e + \varphi_e$. In the calibration of GHK, a value of $\alpha_e = 0.17$ leads ISTC to account for about 60% of economic growth. Thus, even very small values of φ_e could significantly boost the contribution of ISTC to growth. For example, if $\varphi_e = 0.04$, ISTC is boosted approximately by a factor of $\frac{\alpha_e+\varphi_e}{\alpha_e} \approx 1.24$, so that ISTC would account for about 75% of economic growth. If $\varphi_e = 0.10$ then ISTC is boosted by a factor of 1.59, so that ISTC would account for almost the entirety of economic growth.[9]

There is a further implication of Eqs. (41) and (42). There is a literature that identifies investment-specific and neutral technical change by using a model isomorphic to the two-sec-

Footnote 7 continued
Along the balanced gorwth path, the value of equipment spending relative to output $\left(\frac{p_e k_e}{p_c y}\right)$ and the value of structures spending relative to output $\left(\frac{p_s k_s}{p_c y}\right)$ are each constant.

8 The quality-adjusted price of fabricated metals products relative to the the consumption-services deflator has declined by roughly 3% over the post-war era, according to the data of Cummins and Violante (2002).

9 Whelan (2003) takes a different approach to aggregating output, whereby the growth rate of GDP is the average of the real growth rates of each sector, multiplied by the share of each sector in nominal GDP. In this case, since $\gamma_c = 1.0124$ and $\gamma_k = \gamma_c\gamma_q$, and since the ratio of equipment to GDP is $\zeta \equiv 7.2\%$, γ_y as a chain weighted measure is $\gamma_c(1-\zeta) + \zeta\gamma_c\gamma_q = \gamma_c + \zeta\gamma_c(\gamma_q - 1) = 1.0147$. In this case, setting $\gamma_z = 1$ yields $\gamma_y = 1.010$, so that ISTC accounts for about 68% of growth in the case without intermediates. Whelan (2003) does not consider intermediates. However, if we decompose γ_z as above and set $\varphi_e = 0.10$, then $\gamma_c = 1.0096$, $\gamma_y = 1.0119$, and ISTC accounts for 81% of growth. Thus, this alternative approach to aggregation yields fairly similar (and somewhat magnified) results, because ζ is fairly small. The contribution of ISTC to growth in Whelan (2003) is much smaller, but (as discussed extensively in that paper) the main reason is because of the use of official rather than quality-adjusted relative price data. Later we use both.

tor value added model we discussed earlier, under the assumption that the two corresponding "shocks" are orthogonal—as in Fisher (2006), for example. Our model indicates that, in a world in which the output of the equipment sector is used as an intermediate, the assumption of orthogonality may not hold, as the object that such a 2-sector value-added model identifies as "neutral" technical change is a function of investment-specific technical change. We conjecture that, as in our paper, this likely implies that the impact of ISTC on aggregate variables in related work is *understated*.

3.2 Distinction between gross output and value-added productivity

The second channel is through the distinction between gross output and value-added productivity indexes in a multi-sector model with intermediate goods. As shown in (28), the relative productivity growth rate of gross output and the productivity growth rate in terms of value added are *not the same.*

Observation 2 The decline in the relative price of equipment (which is measured in terms of gross output prices) reflects the relative productivity growth of equipment measured in gross output—not value-added.

Under (26), the calibration of productivity growth rates in a multisector model without intermediate goods requires setting:[10]

$$\frac{\gamma_{z_e}}{\gamma_{z_c}} = \left(\frac{\gamma_{A_e}}{\gamma_{A_c}}\right)^{1/(1-\alpha_m)} = \gamma_q^{1/(1-\alpha_m)}. \tag{46}$$

This is not to say that the calibration in Greenwood et al. (1997) is in any way flawed. However, one-sector models with ISTC are often interpreted as reduced-form multisector models of value added, in which relative prices map into relative productivities. Equation (46) implies that the appropriate mapping in an explicit multi-sector model depends on the share of intermediate goods α_m.

A comment on measurement is in order. Observation 2 is made because reported prices—be they the official price indices reported in the NIPA, or the quality-adjusted prices reported in Gordon (1990)—are *gross-output prices*, the prices at which a given unit of a good is purchased. They are not *value-added prices*, which are the prices in a value-added production function, obtained after we solve for optimal intermediate-good use and substitute the solution back into the gross-output production function. In equilibrium, an improvement in

[10] Jorgensen et al. (2007) make a similar point when constructing aggregate productivity measures from industry gross output data. They define industry value-added by decomposing output growth into a weighted sum of value-added growth and intermediate input growth:

$$\frac{d_{it+1}}{d_i} = \left(\frac{y_{it+1}}{y_i}\right)^{1-\alpha_m} \left(\frac{M_{it+1}}{M_{it}}\right)^{\alpha_m}$$

Let p_{yi} be the price-index for the value-added in sector i. Since $p_i^y y_i \equiv p_i d_i - p_m m_i$, the optimal usage of intermediate goods (17) implies

$$\frac{p_{it+1}}{p_{it}} = \left(\frac{p_{it+1}^y}{p_{it}^y}\right)^{1-\alpha_m} \left(\frac{p_{mt+1}}{p_{mt}}\right)^{\alpha_m}$$

which is also a consequence of our value added price Eq. (26). Rewriting the firm's problem using this expression yields (46).

Table 1 Rate of ISTC, for γ_q and α_m

γ_q	Source	Rate of ISTC in value-added form		
		$\alpha_\mathrm{m} = 0$ (%)	$\alpha_\mathrm{m} = 0.25$ (%)	$\alpha_\mathrm{m} = 0.5$ (%)
1.008	Bureau of Economic Analysis	0.8	1.1	1.6
1.032	Greenwood et al. (1997)	3.2	4.3	6.5
1.04	Cummins and Violante (2002)	4.0	5.4	8.2

the technology of producing a given good lowers its own gross-output price and so lowers the cost of its use as an intermediate good in other sectors. Thus, the growth rate of relative value-added productivities can exceed the decline in the growth rate of relative gross-output prices (i.e. $\frac{\gamma_{z_e}}{\gamma_{z_c}} > \gamma_q$) in a world with intermediate goods.[11]

To see the potential quantitative impact of Observation 2, consider the Gordon (1990) quality-adjusted (gross-output) price series, which shows a annual decline of 3.2% in the relative price of equipment. Using this series to calibrate a multi-sector value-added model (as in Sect. 5 of GHK), one would conclude that $\gamma_{z_e}/\gamma_{z_c} = 1.032$, attributing the rest of economic growth to neutral productivity growth γ_{z_c}. Observation 2 argues that the correct estimate of the divergence in productivity across sectors in a value-added model is $\gamma_{z_e}/\gamma_{z_c} = 1.032^{1/(1-\alpha_m)}$, which is 1.065 for $\alpha_m \approx 0.5$. Thus, a given rate of divergence across industries in (gross-output) prices implies a considerably larger divergence in value-added productivity changes.

Table 1 displays the impact on the computed rate of ISTC for different measures of γ_q, varying the intermediate share α_m from zero to 50%. As can be seen, the intermediate share has a substantial impact on the rate of ISTC in value-added form (the growth rate of z_e/z_c) implied by a given growth rate of relative output prices (the growth rate of A_e/A_c).

In the model of GHK, this does not have an impact on the growth accounting exercise. However, there are contexts in which the growth rate of ISTC itself matters. For example, in the multisector value-added model of Ngai and Pissarides (2008), rates of structural change depend upon differences in sector-specific TFP growth (in value-added). Hence, a calibration of that model considering the appropriate mapping between a multisector value added model and gross output prices would imply a larger difference in sector-specific TFP growth rates—which could strengthen the ability of the model to account for observed patterns of structural change.

4 A multisector model with a general I–O matrix

We now extend the model to allow for intermediate good composition to differ across sectors. Instead of (12) assume now the intermediate good used in sector j is produced according to the function:

[11] More broadly, there is a conceptual point to make regarding the notion of "value added." The value-added concept generated by "value-added production functions" such as Eq. (27) are never directly observed in data. Industry data report gross output, and input–output tables are a way of tracking whether this output was put to final or to intermediate use. However, under certain assumptions, we show that one can still measure q and z using relative prices and the aggregate TFP residual (measured with consumption prices). A contribution of our paper is to point out that the simple multi-sector value added model relies on strong assumptions—including that the intermediate-input aggregate in the industry output production functions is the same in all industries—and also to show what might happen when we relax these assumptions.

$$m_j = \prod_{i\in\{e,s,c\}} \left(\frac{h_{ij}}{\varphi_{ij}}\right)^{\varphi_{ij}}; \quad \sum_{i\in\{e,s,c\}} \varphi_{ij} = 1, \ \varphi_{ij} \geq 0. \tag{47}$$

where h_{ij} is intermediate goods from sector j used in the production of good i. Thus, the matrix Φ with elements φ_{ij} can be mapped into the input–output table that links the flow of intermediates across the three sectors. Market clearing condition (13) is replaced by

$$\sum_{j\in\{e,s,c\}} h_{ij} = h_i; \quad i = c, e, s. \tag{48}$$

The key modification is that the price index of intermediate goods used in sector j is now different across sectors. Denote it by p_{mj}.

4.1 Relative prices and relative productivities

We next derive the corresponding expression for relative prices (19) in this general environment. The condition (17) for optimal usage of intermediate goods in sector j is modified to

$$p_{mj} m_j = \alpha_m p_j d_j. \tag{49}$$

Capital-labor ratios are still equalized across sectors as in (18) but the intermediate-labor ratios may differ across sectors due to differences in p_{mj}. More specifically, optimal intermediates and labor inputs imply that for any sectors i and j :

$$\frac{p_{mj} m_j}{l_j} = \frac{p_{mi} m_i}{l_i}. \tag{50}$$

Together with (9) and the result that capital-labor ratios are equalized across sectors, we have:

$$\frac{d_j/l_j}{d_i/l_i} = \frac{A_j}{A_i}\left(\frac{m_j/l_j}{m_i/l_i}\right)^{\alpha_m} = \frac{A_j}{A_i}\left(\frac{p_{mi}}{p_{mj}}\right)^{\alpha_m}, \tag{51}$$

Finally equate the value of the marginal product of labor across sectors to obtain relative prices:

$$\frac{p_i}{p_j} = \frac{A_j}{A_i}\left(\frac{p_{mi}}{p_{mj}}\right)^{\alpha_m}. \tag{52}$$

Comparing (52) with (19), we see that in an environment with intermediate goods relative prices p_i/p_j no longer depend solely on the relative productivity term A_j/A_i. It may also depend on the productivity of other sectors if the composition of the intermediates used by sectors i and j differs.

The optimization problem of the intermediate good producer is similar as before with p_{mj} replacing p_m. Thus, the optimal composition condition (20) is modified to

$$p_i h_{ij} = \varphi_{ij} p_{mj} m_j \quad \forall i. \tag{53}$$

Then, using (47), the price-index for intermediate goods m_j is

$$p_{mi} = \prod_k p_k^{\varphi_{ki}}. \tag{54}$$

We next derive the relationship between relative prices and relative productivities using (52) and (54). Let $q_i \equiv \frac{p_c}{p_i}$ and $a_i \equiv \frac{A_i}{A_c}$ for $i = e, s$. Then,

$$\frac{p_{mi}}{p_c} = q_e^{\varphi_{ei}} q_s^{\varphi_{si}}. \tag{55}$$

Substituting this into (52) for $i = c$ and $j = e, s$ yields:

$$q_j = a_j \left(\frac{q_e^{\varphi_{ec}} q_s^{\varphi_{sc}}}{q_e^{\varphi_{ej}} q_s^{\varphi_{sj}}} \right)^{\alpha_m} = a_j \left(q_e^{\psi_{ej}} q_s^{\psi_{sj}} \right)^{\alpha_m}, \tag{56}$$

where $\psi_{ej} \equiv \left(\varphi_{ec} - \varphi_{ej}\right)$ measures the intensity of equipment use in the consumption sector c relative to sector j, and $\psi_{sj} \equiv \left(\varphi_{sc} - \varphi_{sj}\right)$ measures the intensity of structures use in the consumption sector c relative to sector j. Taking logarithms, rewrite (56) in matrix form:

$$\begin{pmatrix} \ln a_e \\ \ln a_s \end{pmatrix} = (\mathbf{I} - \alpha_m \Psi) \begin{pmatrix} \ln q_e \\ \ln q_s \end{pmatrix}. \tag{57}$$

Thus, the equilibrium growth factors of prices and productivity indices are related via:

$$\begin{pmatrix} \ln \gamma_{a_e} \\ \ln \gamma_{a_s} \end{pmatrix} = (\mathbf{I} - \alpha_m \Psi) \begin{pmatrix} \ln \gamma_{q_e} \\ \ln \gamma_{q_s} \end{pmatrix}, \tag{58}$$

where the matrix $\Psi \equiv \begin{pmatrix} \psi_{ee} & \psi_{es} \\ \psi_{se} & \psi_{ss} \end{pmatrix}$ with elements ψ_{ij} denotes the intensity of using intermediates goods i in the consumption sector relative to sector j.

Solving the system (58) explicitly yields:

$$\gamma_{q_e}^{\theta} = \gamma_{a_e}^{1-\alpha_m \psi_{ss}} \gamma_{a_s}^{\alpha_m \psi_{se}}; \quad \gamma_{q_s}^{\theta} = \gamma_{a_s}^{1-\alpha_m \psi_{ee}} \gamma_{a_e}^{\alpha_m \psi_{es}}, \tag{59}$$

where $\theta \equiv \left[(1 - \alpha_m \psi_{ss})(1 - \alpha_m \psi_{ee}) - \alpha_m^2 \psi_{es} \psi_{se}\right]$.

Note that when the usage of intermediates is the same across sectors, i.e. $\psi_{ij} = 0, i, j = e, s$, then $\Psi = \mathbf{0}$ and $\theta = 1$. In this case, Eqs. (58) and (59) reduce to (19) in the baseline model. There are two important observations to make when comparing (58) to (19). First, the decline in the relative price of equipment (γ_{q_e}) is no longer a direct measure of γ_{a_e}. It includes also structures-specific technical change $\left(\gamma_{a_s}\right)$, to the extent that the equipment sector uses structures as an intermediate. Second, if we assume that structures experience the same productivity growth as consumption (as in the GHK model), i.e. $\gamma_{a_s} = 1$, then (59) implies that γ_{q_e} is a function of γ_{a_e} only. Even so, the decline in the relative price of equipment is still not an exact measure of ISTC. For example, if the structures sector has the same intermediate composition as the consumption sector but the equipment sector uses equipment more intensively, i.e. $\psi_{ss} = \psi_{es} = 0$ and $\psi_{ee} > 0$, we have $\theta < 1$. In this case, using the decline in the relative price of equipment as a measure of ISTC would overstate the actual contribution of ISTC to growth.

Observation 3 When the composition of intermediate goods varies across industries, the rate of decline in the price of a good i relative to good j is not a sufficient statistic for the rate of productivity growth of i relative to j: it also depends on rates of productivity growth in all industries that are used as intermediates in i and j.

4.2 Growth accounting

To derive the aggregate real value-added y, we first derive the real value-added y_i for each sector i. The procedure is similar to Sect. 2.3. Following the definition of y_i, Eq. (23) continues to hold with p_{mi} replacing p_m. Similarly, optimal intermediate use (49) implies we can rewrite d_i as in (24) with p_{mi} replacing p_m. Thus, as before, we can express the firm's problem in terms of value-added. It follows that the value-added price index p_i^y is the same as (26) with p_{mi} replacing p_m,

$$p_j^y \equiv \left(\frac{p_j}{p_{mj}^{\alpha_m}} \right)^{\frac{1}{1-\alpha_m}}, \tag{60}$$

whereas the real value-added y_i and its productivity index z_i are the same as in (27) and (28). Finally, substituting (52) into (60), the relationship between relative value-added prices, relative gross-output productivity and relative value-added productivity are the same as in (29).

Defining the aggregate real value added, y, as in (30), Eq. (31) continues to hold. Using (60),

$$y = \left(\frac{p_c}{p_{mc}} \right)^{\frac{\alpha_m}{1-\alpha_m}} z_c k_e^{\alpha_e} k_s^{\alpha_s} l^{1-\alpha_e-\alpha_s}. \tag{61}$$

where the relative gross-output price to intermediate price in the consumption sector follows from using p_{mc} in (54):

$$\frac{p_c}{p_{mc}} = \prod_{i=c,e,s} \left(\frac{p_c}{p_i} \right)^{\varphi_{ic}}. \tag{62}$$

It follows that the productivity index for aggregate value added is

$$z = (1-\alpha_m)\, \alpha_m^{\alpha_m/(1-\alpha_m)} A_c^{1/(1-\alpha_m)} \left[\prod_{i=c,e,s} \left(\frac{p_c}{p_i} \right)^{\varphi_{ic}} \right]^{\alpha_m/(1-\alpha_m)}, \tag{63}$$

so its growth is

$$\gamma_z = \gamma_c^{1/(1-\alpha_m)} \left[\gamma_{q_e}^{\varphi_{ec}} \gamma_{q_s}^{\varphi_{sc}} \right]^{\alpha_m/(1-\alpha_m)}. \tag{64}$$

The bracketed term $\left[\gamma_{q_e}^{\varphi_{ec}} \gamma_{q_s}^{\varphi_{sc}} \right]$ is a function of γ_{a_e} and γ_{a_s} [see (59)]. Thus, the aggregate productivity index z is affected by technical progress specific to the equipment sector (γ_{a_e}) and specific to the structures sector (γ_{a_s}) By definition, "neutral"productivity growth that is common to all sectors is $\gamma_{\tilde{z}} = \gamma_c^{1/(1-\alpha_m)}$. As in Observation 1, this implies that a value-added model may understate the total contribution of ISTC (including both ESTC and SSTC) if equipment and structures are used as intermediate goods in the production for consumption goods, i.e. $\varphi_{ec} > 0$ and $\varphi_{sc} > 0$.

Given the aggregate value-added expression,

$$y = z k_e^{\alpha_e} k_s^{\alpha_s} l^{1-\alpha_e-\alpha_s} \Leftrightarrow \left(\frac{y}{l} \right)^{1-\alpha_e-\alpha_s} = z \left(\frac{k_e}{y} \right)^{\alpha_e} \left(\frac{k_s}{y} \right)^{\alpha_s}. \tag{65}$$

Along the balanced growth path, the value of equipment relative to output $\left(\frac{p_e k_e}{p_c y}\right)$ and the value of structures relative to output $\left(\frac{p_s k_s}{p_c y}\right)$ are constant, thus the growth accounting equation is:

$$\gamma_y = \gamma_z^{\frac{1}{1-\alpha_e-\alpha_s}} \gamma_{q_e}^{\frac{\alpha_e}{1-\alpha_e-\alpha_s}} \gamma_{q_s}^{\frac{\alpha_s}{1-\alpha_e-\alpha_s}}. \tag{66}$$

Using (64) and the definition of $\gamma_{\tilde{z}}$,

$$\gamma_y = \gamma_{\tilde{z}}^{\frac{1}{1-\alpha_e-\alpha_s}} \gamma_{q_e}^{\frac{\alpha_e+\varphi_{ec}\alpha_m/(1-\alpha_m)}{1-\alpha_e-\alpha_s}} \gamma_{q_s}^{\frac{\alpha_s+\varphi_{sc}\alpha_m/(1-\alpha_m)}{1-\alpha_e-\alpha_s}}, \tag{67}$$

which decomposes growth into neutral productivity growth and the decline in relative prices. As noted earlier, there is no longer a one-to-one relationship between relative price changes and relative productivity changes when intermediate composition differs across sectors, so we cannot use (67) to separate the contribution of ESTC and SSTC. To derive a growth accounting equation in terms of relative productivity growth rates a_e and a_s requires Eq. (59). It follows that:

$$\gamma_y = \gamma_{\tilde{z}}^{\frac{1}{1-\alpha_e-\alpha_s}} \gamma_{a_e}^{\frac{\beta_e}{(1-\alpha_e-\alpha_s)\theta}} \gamma_{a_s}^{\frac{\beta_s}{(1-\alpha_e-\alpha_s)\theta}}, \tag{68}$$

where

$$\beta_e \equiv \left(\alpha_e + \frac{\varphi_{ec}\alpha_m}{1-\alpha_m}\right)(1-\alpha_m\psi_{ss}) + \left[\alpha_s + \frac{\varphi_{sc}\alpha_m}{1-\alpha_m}\right]\alpha_m\psi_{es},$$
$$\beta_s \equiv \left(\alpha_s + \frac{\varphi_{sc}\alpha_m}{1-\alpha_m}\right)(1-\alpha_m\psi_{ee}) + \left[\alpha_e + \frac{\varphi_{ec}\alpha_m}{1-\alpha_m}\right]\alpha_m\psi_{se}.$$

To summarize, given the observed decline in the prices of equipment and structures relative to consumption (γ_{q_e} and γ_{q_s}), we can compute the rate of ESTC $\left(\gamma_{a_e}\right)$ and SSTC $\left(\gamma_{a_s}\right)$ using (58). Finally, using the growth accounting Eq. (68) we can decompose growth into neutral productivity growth, ESTC and SSTC.

5 Quantitative results

We have shown analytically (Observation 1) that the finding of GHK (that 60% of economic growth can be attributed to ISTC) understates the contribution of ISTC due to the role of equipment as an intermediate good. We now ask whether this observation is quantitatively important, using data on the share of intermediate goods in gross output and the composition of intermediate goods. First, we assume that intermediate goods composition is common across industries, and that $A_c = A_s$. This is the specification of the model that maps most closely into the GHK framework. Then, we generalize to allow the composition of intermediates to vary across industries, and also allow $A_c \neq A_s$. This is the specification that takes account of flows of intermediate goods across sectors as measured in the input–output tables.[12]

5.1 Calibrating the baseline model

The simple baseline model in Sect. 2 assumes that (i) the composition of intermediates goods to be the same across sectors and (ii) $A_s = A_c$ as in GHK. Thus, to account for the contribu-

[12] In future work, it would be interesting to allow for a larger number of industries. In this case, an equation similar to (58) would map between prices and productivity indices for any finite number of sectors.

Table 2 Parameters used in calibration

Parameter	α_e	α_s	γ_q	γ_y	τ_k	τ_l	δ_e	δ_s
Value	0.17	0.13	1.032	1.0124	0.42	0.40	0.124	0.056

Sources: Greenwood et al. (1997) and the US Bureau of Economic Analysis

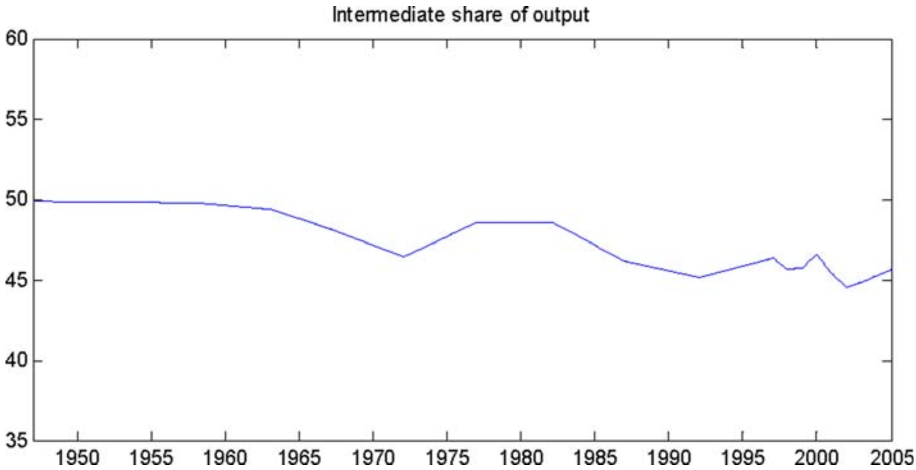

Fig. 1 Share of intermediate goods in gross output. Source: US Bureau of Economic Analysis

tion of ISTC, we need two additional parameters relative to GHK: the share of intermediate goods in gross output (α_m) and the share of equipment in intermediate goods (φ_e) .

5.1.1 The GHK calibration

To calibrate the one-sector formulation of the baseline model, we follow the same procedure as GHK, using the same values of parameters as theirs. See Table 2. The interested reader may refer to their paper for details.

5.1.2 Equipment as intermediate goods

To study the role of equipment as an intermediate good, we use the input–output tables reported by the US Bureau of Economic Analysis.[13] Figure 1 shows that the intermediate share of gross output is close to one half, consistent with the values reported in Yamano and Ahmad (2006), Vourvachaki (2007), Jones (2009) and others. Hence, we set $\alpha_m = 0.50$.

We match φ_e to the equipment share of intermediate goods. Equipment is identified with SIC codes 3400–3999.[14] This follows the definition of "durables" in Gordon (1990) and

[13] We use the Benchmark I–O tables from 1947–1997. These are generally reported every 5 years. After 1997 we use annual I–O tables. While there was a major revision of the methodology for constructing IO tables in 1997 (mainly concerning the treatment of auxiliary services), the BEA also reports tables using the methodology before revisions, and these were the tables we used.

[14] The Standard Industrial Classification (SIC) system is used by the input–output tables until 1992. Tables from 1997 onwards use the North American Industry Classification System (NAICS): these industries correspond to NAICS codes 3320–3399.

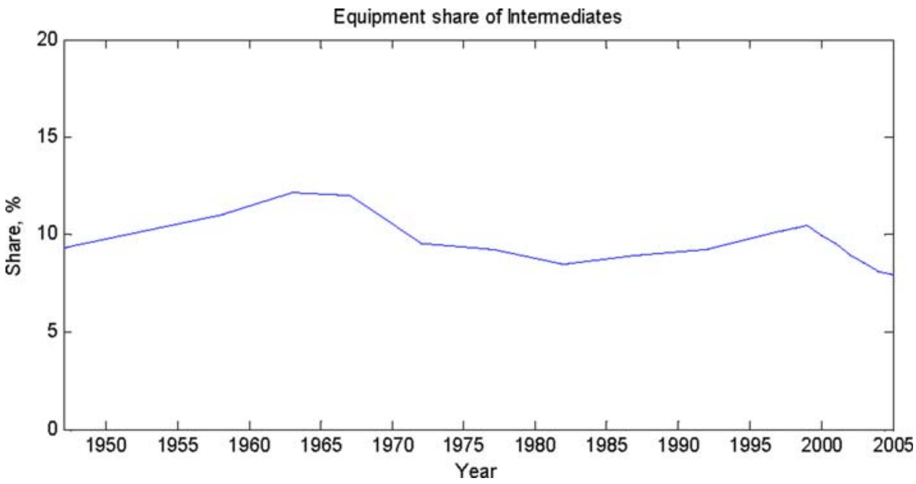

Fig. 2 Share of intermediate goods that is composed of equipment. Source: US Bureau of Economic Analysis

"equipment" in GHK. Gordon (1990) does not consider software as equipment. Hence, our value of φ_e was derived without considering software. We define structures using SIC codes 1500–1629.[15]

This definition can be applied consistently to input-output tables dating back to 1947. Our value of φ_e is thus a lower bound and, in this sense, our results are conservative.[16] Cummins and Violante (2002) do consider software as part of equipment and, although there are other differences between their method and that of GHK, they find a higher value of γ_q. A broader definition would only increase the quantitative importance of the channels we underline. Figure 2 shows that the equipment share of intermediate goods averages around 10%, and we set $\varphi_e = 0.10$.

If $\varphi_e = 0$, ISTC accounts for about 60% of economic growth, as in GHK (even in the presence of intermediate goods, i.e. $\alpha_m > 0$). However, if $\varphi_e = 10\%$, the contribution of ISTC to growth rises to 93%.[17] An equipment share of intermediates of 12% is enough for ISTC to account for the entirety of post-war US economic growth. See Fig. 3.

5.2 Sensitivity: the rate of ISTC

There is a debate regarding the appropriate empirical counterpart of q. GHK use the quality adjusted price of capital, based on the work of Gordon (1990), relative to the official deflator for consumption and services, and find that $\gamma_q = 1.032$. Using a similar method Cummins and Violante (2002) find that $\gamma_q = 1.04$, and we will examine this value. Finally, Whelan (2003) argues that Gordon (1990) and GHK overestimate q, as they assume no quality improvements in consumption and services. Hence, we also repeat the exercise using official price indices. According to official price data, $\gamma_q = 1.008$.

15 These industries correspond to NAICS codes 2300–2380.

16 Including software raises φ_e to about 11%.

17 As in GHK, $\tilde{z}_t$ rises over time until about 1947 and then declines after that to roughly 20% of its peak.

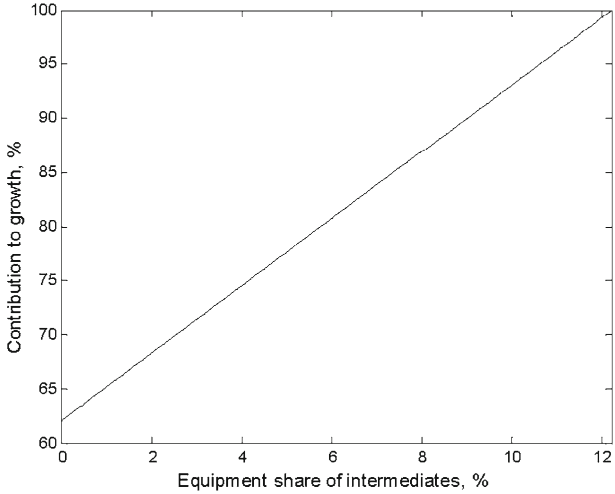

Fig. 3 Contribution of ISTC to growth for the GHK calibration and different values of φ_e

For these alternative calibrations, we again use the same parameter values as GHK.[18] Results from varying φ_e are reported in Table 4. Again, we compute the contribution of ISTC to growth using growth accounting Eq. (45), shutting down neutral technical change $(\gamma_{\tilde{z}} = 1)$, and comparing the resulting value of γ_y to the value in the data. For these alternative values of γ_q, we find that raising φ_e from zero to 4% amplifies the contribution of ISTC to growth by a quarter,[19] and raising φ_e from zero to 10% amplifies the contribution of ISTC to growth by a half. Thus, in a calibration in which γ_q is low, this amplification will not be too large in absolute terms, as the ISTC channel of growth is weak to begin with. On the other hand, if γ_q is higher as suggested by GHK and Cummins and Violante (2002), the amplification can have a significant impact on growth accounting.[20] See Table 3.

5.3 Sectorial differences in intermediate use

So far, we have assumed that the same composite intermediate input is used in all sectors. To fully take advantage of the Input-Output tables, we now allow for different compositions across sectors. Formally, intermediate goods used in sector j are produced by using technol-

[18] As shown in Ngai & Samaniego (2008), repeating the calibration procedure of GHK with different values of γ_q affects some of the other parameters of the model: however, the differences are minimal, and we abstract from them. For example, for the range $\gamma_q \in [1.008, 1.04]$, $\alpha_e \in [0.169, 0.173]$. Thus, the calibrated parameters turn out not to be too sensitive to the choice of γ_q, and a calibration with a wide range of values of γ_q is consistent with essentially the same parameters as those used by GHK.

[19] The share of equipment in the intermediates used by the consumption sector equals 4% so the results in this column are useful for later comparisons.

[20] When we use GHK's equipment price series, if $\varphi_e > 0.12$ then $\gamma_{\tilde{z}} < 1$, so that neutral technological progress experiences some regression. If we use equipment prices from Cummins and Violante (2002) then $\gamma_{\tilde{z}} < 1$ when $\varphi_e > 0.043$. This does not occur in our preferred calibration. Still, it is worth noting that, as in GHK, the growth rate of $\gamma_{\tilde{z}}$ is negative after 1974 even in the case of the GHK calibration with $\gamma_q = 1.032$ and $\varphi_e = 0.1$. Thus, the extent to which ISTC accounts for economic growth appears related to the severity of their "puzzle" of declining "neutral" productivity after 1974.

Table 3 Contribution of ISTC to growth, for different values of γ_q

Source for γ_q	Contribution of ISTC to growth			
	γ_q	$\varphi_e = 0$ (%)	$\varphi_e = 0.04$ (%)	$\varphi_e = 0.10$ (%)
Bureau of Economic Analysis	1.008	16	20	24
Greenwood et al. (1997)	1.032	62	78	93
Cummins and Violante (2002)	1.04	77	98	119

The table also reports the relative increase in this contribution when φ_e is raised from 0 to 10%

Table 4 Input-output matrix Φ for the three major sectors in GHK

	Using industry j			
	φ_{ij}	c	s	e
Supplying industry i	c	0.933	0.894	0.601
	s	0.025	0.002	0.105
	e	0.042	0.105	0.394
Total		1	1	1

Sector c represents non-durables, s is structures and e is equipment
The entry in row i column j is the share of the intermediates used by industry j composed of by the output of industry i

Table 5 Contribution of investment-specific technical change to growth

Price data		Implied tech Δ		Contrib. to growth		
Source for γ_{q_s}	γ_{q_s}	γ_{a_e}	γ_{a_s}	ESTC	SSTC	ISTC
Benchmark	1	1.0377	1.0013	.757	.022	.779
NIPA tables	1.003	1.0378	1.0042	.759	.073	.832
Gort et al. (1999)	1.01	1.0381	1.0112	.763	.193	.956

Results are reported from different values of γ_{q_s}, the rate of growth in the relative price of structures. Results assume that γ_{q_e} equals 1.032, as in GHK

ogy (47) specified in Sect. 4. Given the Cobb–Douglas structure and competitive markets, in equilibrium:

$$\varphi_{ij} = \frac{\text{value of intermediates goods } i \text{ used in sector } j}{\text{total value of intermediates goods used in sector } j}. \tag{69}$$

These values are reported in the input-output tables constructed by the BEA. The resulting values may be found in Table 4.

We also allow $A_c \neq A_s$. As discussed, this implies that there may be two forms of ISTC: *equipment*-specific technical change (ESTC) and *structures*-specific technical change (SSTC).

We require a value for γ_{q_s}, where $q_{st} \equiv p_{ct}/p_{st}$. GHK assume that $\gamma_{q_s} = 1$. However, according to the NIPA, $\gamma_{q_s} = 1.003$. Gort et al. (1999) estimate γ_{q_s} to equal 1.01 in the post-war era. We examine all three values. Results are presented in Table 5.

The contribution of ESTC to growth is robust to assumptions about γ_{q_s}—about 76% of growth. However, changes in assumptions about γ_{q_s} affect whether SSTC is important too.

Assuming that $\gamma_{q_s} = 1$ implies that SSTC has a negligible contribution to economic growth.[21] However, when $\gamma_{q_s} = 1.01$, SSTC accounts for almost 20% of economic growth. As a result, ISTC of both forms together can account for almost the entirety of post-war economic growth (96%).

It is worth noting that, in all cases, $\gamma_{a_e} > \gamma_{q_e}$ and $\gamma_{a_s} > \gamma_{q_s}$. In other words, the rate of decline of the relative price of equipment understates the rate of ESTC when we examine the full sectorial I–O matrix Φ. The same is true of SSTC. To note the quantitative importance of this distinction, suppose that $\gamma_{q_e} = 1.032$ and that $\gamma_{q_s} = 1.01$, but that there are no intermediates in the model, i.e. that $\alpha_m = 0$. Then, the contribution of ISTC to growth would be lower, at 77%, with a contribution of ESTC to growth of 62% (as in GHK) and a contribution of SSTC of 15%. The same obtains if $\alpha_m > 0$ but Φ equals the identity matrix, so that there are intermediates but no cross-sector linkages. By contrast, when Φ is measured using actual input–output data, the contribution of ISTC to growth is almost 96%.

5.4 Concluding remarks

The presence and composition of intermediate goods is important for mapping changes in relative prices into changes in rates of technical change. First, when rates of technical change differ across sectors, those sectors that experience faster technical change can contribute to economic growth by being used as intermediate goods. Second, the prices reported in the national income and product accounts, etc. are reported in terms of gross output–whereas macroeconomic models are usually formulated in terms of value added. While there exists a simple isomorphism between models with and without intermediate goods under certain assumptions, the use of gross-output prices to impute TFP growth rates in value added models does need to account for the share of intermediate goods in gross output. Moreover, when there are potentially complex cross-industry linkages, there is no longer a direct mapping between the rate of decline in the price of a good and its productivity growth rate.

We demonstrate the importance of these linkages for growth accounting using the example of Greenwood et al. (1997), a widely-cited paper that attributes a significant proportion of aggregate growth to investment-specific technical change. When our suggested mapping is used, we find that the contribution of ISTC to economic growth is even larger than their work indicates.

Finally, we find that neglecting the value added and gross output distinction underestimates the divergence of industry TFP growth rates in value added multisector models. In multi-sector models in which cross-industry resource reallocation is important, this could have a significant influence on quantitative results regarding structural change and policy, among other applications.

References

Basu, S. (1995). Intermediate goods and business cycles: Implications for productivity and welfare. *American Economic Review, 85*(3), 512–531.

Cummins, J., & Violante, G. L. (2002). Investment-specific technical change in the US (1947–2000): Measurement and macroeconomic consequences. *Review of Economic Dynamics, 5*(2), 243–284.

[21] When $\gamma_{q_s} = 1$, the relative price of structures is constant. Nonetheless, the structure of the I–O tables implies that productivity growth in structures is more rapid than in sector c (computed as $\gamma_{a_s} > 1$ in Table 5).

Fisher, J. D. M. (2006). The dynamic effects of neutral and investment-specific technology shocks. *Journal of Political Economy, 114*(3), 413–451.

Gordon, R. J. (1990). *The measurement of durable goods prices*. Chicago: University of Chicago Press.

Gort, M., Greenwood, J., & Rupert, P. (1999). Measuring the rate of technological progress in structures. *Review of Economic Dynamics, 2*(1), 207–230.

Greenwood, J., Hercowitz, Z., & Krusell, P. (1997). Long-run implications of investment-specific technological change. *American Economic Review, 87*(3), 342–362.

He, H., & Liu, Z. (2008). Investment-specific technological change, skill accumulation, and wage inequality. *Review of Economic Dynamics, 11*(2), 314–334.

Hornstein, A., & Krusell, P. (1996). Can technology improvements cause productivity slowdowns. *NBER Macroeconomics Annual, 11*, 209–259.

Horvath, M. (1998). Cyclicality and sectoral linkages: Aggregate fluctuations from independent sectoral shocks. *Review of Economic Dynamics, 1*(4), 781–808.

Horvath, M. (2000). Sectoral shocks and aggregate fluctuations. *Journal of Monetary Economics*, *45*(1), 69–106.

Huang, K. X. D., & Liu, Z. (2001). Production chains and general equilibrium aggregate dynamics. *Journal of Monetary Economics, 48*(2), 437–462.

Hulten, C. R. (1978). Growth accounting with intermediate inputs. *Review of Economic Studies, 45*(3), 511–518.

Hulten, C. R. (1992). Growth accounting when technical change is embodied in capital. *American Economic Review, 82*(4), 964–980.

Jones, C. I. (2009). *Intermediate goods and weak links: A theory of economic development*. Stanford Graduate School of Business Working Paper.

Jorgensen, D., Ho, M. S., Samuels, J., & Stiroh, K. J. (2007). Industry origins of the American productivity resurgence. *Economic Systems Research, 19*(2), 229–252.

Kongsamut, P., Rebelo, S., & Xie, D. (2001). Beyond balanced growth. *Review of Economic Studies, 68*(4), 869–882.

Ngai, L. R., & Pissarides, C. A. (2007). Structural change in a multisector model of growth. *American Economic Review, 97*(1), 429–443.

Ngai, L. R., & Pissarides, C. A. (2008). Trends in hours and economic growth. *Review of Economic Dynamics, 11*, 239–256.

Ngai, L. R., & Samaniego, R. M. (2008). *Mapping prices into productivities in multi-sector growth models*. London School of Economics, Center for Economic Performance Discussion paper 869.

Vourvachaki, E. (2007). *Multi-sector growth: The role of information and communication technologies and other intermediates in recent growth experiences*. London School of Economics, Doctoral Dissertation.

Whelan, K. (2003). A two-sector approach to modeling U.S. NIPA data. *Journal of Money, Credit and Banking, 35*(4), 627–656.

Yamano, N., & Ahmad, N. (2006). *The OECD input-output database: 2006 edition*. OECD Science, Technology and Industry Working Papers 2006/8, OECD Directorate for Science, Technology and Industry.