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EC 533 Labour Economics
Problem Set 1

1. Optimal Choice of Schooling and Returns to Schooling

   Suppose earnings are given by $y = wf(S, A)$ where S denotes years of schooling, A denotes ability, $f(.,.)$ is a human capital production function and w is the wage of a unit of human capital. An infinitely lived individual with discount rate r makes an optimal schooling choice at the beginning of her life. The individual earns nothing while in school and supplies labor inelastically thereafter. (Do this problem in continuous time.)

   (a) Derive the first order condition for the individual's maximization problem. Show that this is the schooling part of Mincer's earnings equation

   $$\frac{\partial \log y}{\partial S} = r$$

   Show that the optimal choice of schooling is independent of the wage w. Why?

   (b) Derive an expression for dS/dA. Show that a necessary and sufficient condition for schooling to rise with ability is $f_{SA} > rf_A$ where $f_x = \partial f/\partial x$ and $f_{xz} = \partial^2 f/\partial x \partial z$. Show that $dS/dA = 0$ if the human capital production function has the multiplicative form $f(S, A) = \phi(S)\delta(A)$.

   (c) Modify the individual's problem so that there is an additional flow cost of schooling c (or subsidy if c is negative).

      1. Find the first order condition for the modified problem and show that the optimal level of schooling falls with the discount rate and the flow cost of schooling and rises with the wage w for $c > 0$.
      2. Why is schooling no longer independent of the wage?
      3. Using Griliches' human capital production function $f(S, A) = \exp(\beta S + \gamma A)$ and assuming that $c < 0$, show that $dS/dA < 0$.

   (d) Return to the model without explicit schooling costs. Let the human capital production function have the form

   $$f(S, A) = \exp(AS^b)$$

   with $b < 1$ and assume that all variation in schooling is due to variation in ability A. Show that the estimated return to schooling in a simple earnings regression is

   $$\frac{cov(\log y, S)}{var(S)} = \frac{r}{b}.$$

2. Estimating Returns to Schooling

   Suppose the potential earnings of an individual with S_i years of schooling are given by

   $$y_i = \exp(\beta_0 + \beta_1 S_i + \beta_2 S_i^2 + \varepsilon_i) \qquad (1)$$

   where $\beta_1 > 0$, $\beta_2 < 0$, and ε_i represents any other factors which influence individual i's earnings potential.

(a) Suppose that individuals live forever, foregone earnings is the only cost of schooling, and the interest rate is $r < \beta_1$ for everyone. Derive the value of schooling that maximises life time wealth. Why is it not possible to estimate the returns to schooling under these assumptions by regressing the log of earnings on schooling and its square?

(b) Now suppose that individuals face different interest rates but that these differences are random (i.e. independent of unobserved earnings potential). Show that in this case OLS estimates of the parameters β_1 and β_2 will be consistent. Note that the slope parameters in a regression of y on x_1 and x_2 are

$$\begin{aligned}\text{plim } \widehat{\beta}_1 &= \frac{var(x_2)cov(y,x_1) - cov(x_1,x_2)cov(y,x_2)}{var(x_1)var(x_2) - cov(x_1,x_2)^2} \\ \text{plim } \widehat{\beta}_2 &= \frac{var(x_1)cov(y,x_2) - cov(x_1,x_2)cov(y,x_1)}{var(x_1)var(x_2) - cov(x_1,x_2)^2}.\end{aligned}$$

Why is there no ability bias in this model?

(c) Many economists believe that estimates of the returns to schooling are contaminated by ability bias that leads them to overestimate the causal effect of schooling on earnings. Propose a modification to this theoretical problem that induces a positive correlation between optimal schooling choices and unobserved earnings potential ε_i. This modifications should involve heterogeneity in the potential returns to schooling. Derive a formula for the covariance between the optimal schooling choices and ε_i in this case. Discuss the plausibility of bias from this source.

(d) Now suppose that there is heterogeneity in the interest rate again. This heterogeneity depends on a binary variable z_i and a random component η_i which change the cost of borrowing. In particular, the interest rate for individual i is

$$r_i = (1 - z_i)r_0 + z_i r_1 + \eta_i$$

and η_i is independent of z_i and ε_i. Derive the probability limit of the Wald estimator of the returns to schooling using z_i as an instrument. I.e. even though potential earnings are given by the non-linear function (1), you use z_i as an instrument in a bivariate regression of $\log y_i$ on S_i. Compare this with the population average return to schooling $E(d \log y_i / dS_i)$. Show that these two measures of the average return to schooling are the same if $P(z_i = 1) = \frac{1}{2}$ but otherwise they differ. Give some intuition for this difference.

3. Subject Choice
Start with the standard schooling model where earnings are given by $wf(S, A)$ where S denotes years of schooling, A denotes ability, $f(.,.)$ is a human capital production function, and w is the wage of a unit of human capital. Suppose an individual lives forever, has discount rate r, and makes an optimal schooling choice at the beginning of her life. There are no earnings while in school, the only costs of school are foregone earnings, and individuals supply labor inelastically.

(a) There has been a lot of interest in the returns to studying particular subjects (e.g. STEM–science, technology, engineering, or math–fields compared to humanities). Extend the model so that it can accommodate the analysis of the returns to subject choice by introducing two different subjects $s = 1, 2$. Individuals can choose either schooling S_1 or S_2 and will face a subject specifc wage w_s. Don't make any other changes to the setup of the model. Derive the optimal schooling choices.

(b) Explain why your model in part (a) is not an empirically interesting model to study subject choice. Suggest a further modification to the model which would make it more relevant. Derive the optimal schooling choices in your new model if you can or describe them in words if you can't.

(c) Drawing on your discussion of the model of optimal schooling and subject choice in the previous parts, discuss the empirical challenges of estimating the returns to studying different majors.

(d) A researcher wants to estimate the return to studying a STEM field at university compared to studying any other subject. The researcher proposes to focus on boys and use the fraction of girls in a student's high school class as an instrument for the subject choice. Describe the assumptions necessary for the IV estimator to deliver a valid estimate of this return. Discuss whether the proposal of the researcher is a promising one given your analysis of the model in the previous parts.