1. **(Training With Labor-Market Frictions):** Consider the following economy. At \( t = 0 \), the firm decides how much to invest in its employee’s general skills. The cost of an investment \( \tau \) is \( c(\tau) \), which is incurred by the firm. A worker with general skills \( \tau \) produces \( 1 + \tau \) output in period \( t = 1 \). At this point, he can also move to a different firm where his wage will be \( 1 + \tau - \theta \) where \( \theta \) is the cost of moving to a different firm. \( \theta \) is a random variable, drawn from a uniform distribution \([0, 1]\), and is the private information of the worker (i.e., the firm does not observe it). The exact sequence of events is as follows: at \( t = 0 \), the firm chooses \( \tau \) and makes a wage offer \( w \) to the worker; next, the worker, knowing her own \( \theta \), decides whether to quit or to stay.

   (a) Characterize the firm’s wage offer as a function of \( \tau \). In particular, is \( w'(\tau) \) positive, negative, zero, or ambiguously determined? Why?

   (b) Solve for the firm’s level of training and wage offer that maximize expected profit. Explain why the firm is not investing in \( \tau \)?

   (c) Suppose now that the worker can finance his own training investment. Solve for the worker’s choice of training and the firm’s wage offer. How does the worker’s choice of training compare to the first best training level?

   (d) Suppose again that the worker cannot finance her training, but that her wage, if she quits the firm, is given by \( 1 + \tau (1 - \theta) \). Explain why the mobility cost might take this form. Solve for \( \tau \) and \( w \). Why is the firm investing in training in this case? Contrast these results with those obtained in part (b). How does the training level chosen by the firm compare to the first best level?

2. **(General and Specific Training Investments):** This question asks you to think about a three-period training model. Consider the following timeline:

   In period 1, firm-specific investments in human capital are made by the worker.

   In period 2, investment in general human capital is made by the firm.

   In period 3, the firm makes a wage offer and workers decide whether to stay at the firm or work in a competitive labor market outside the firm.

   Assume that the production function has the following form:

   \[ f(\tau, s) = (1 + \tau)(1 + s) \]

   in which \( \tau \) is general human capital and \( s \) is specific human capital. The production function outside the firm is

   \[ g(\tau, s) = 1 + \tau \]

   Finally, the cost of specific human capital, incurred by the worker in period 1, is \( s^2 \), and the cost of general human capital is \( \tau^2 \) and is incurred by the firm in period 2.

   (a) What is the wage offer the firm will make to the worker in period 3? Explain.
(b) Assume that the firm cannot invest in any general human capital in period 2 (or ever). Solve for \( s \) and \( w \). Interpret.

(c) Assume instead that the worker cannot invest in specific human capital. Solve for \( \tau \) and \( w \). Interpret.

(d) Now, assume that both parties \textit{can} make investments as described above. What incentive does the firm have to invest in general human capital? What incentive does the worker have to invest in specific human capital? (HINT: use backwards induction.)

(e) Explain how and why your answer would change if \( f(\tau, s) = 1 + \tau + s \). Why is this the case?