Labor Economics
Handout on Bedard - Signalling paper

Bedard uses a signalling model with continuous ability types (θ) and three schooling groups (d, h, u). The cost of high school is $C_h(\theta)$ and the cost of university is $C_h(\theta)+ C_u(\theta)$. A fraction $1-p$ of individuals is constrained from going to university.

In a separating equilibrium, there will be two cutoff productivites θ_h and θ_u, so that individuals with ability below θ_h drop out of school, those with abilities above θ_p go to university if they are not constrained, and everybody else goes to high school. In equilibrium, wages correspond to productivities. The wage of high school graduates is given by:

$$w_h = \phi = \frac{[F(\theta_u) - F(\theta_h)]\, E[\theta|\theta_h \leq \theta < \theta_u] + (1-p)[1 - F(\theta_u)]\, E[\theta|\theta \geq \theta_u]}{[F(\theta_u) - F(\theta_h)] + (1-p)[1 - F(\theta_u)]}$$

The cutoff points are defined implictly by equating utilities of the marginal students:

$$\begin{aligned} w_d &= w_h - C_h(\theta_h) \\ w_h - C_h(\theta_u) &= w_u - C_u(\theta_u) - C_h(\theta_u) \end{aligned}$$

Using $w_d = E[\theta|\theta < \theta_h]$ and $w_u = E[\theta|\theta \geq \theta_u]$ yields

$$\begin{aligned} E[\theta|\theta < \theta_h] &= \phi - C_h(\theta_h) \\ E[\theta|\theta \geq \theta_u] &= \phi + C_u(\theta_u) \end{aligned} \tag{1}$$

We are interested in what happens to the fraction of high school drop outs as the constraint on accessing university (p) changes. Hence, we want the derivative $d\theta_h/dp$. This is found by totally differentiating the two cutoff conditions in (1):

$$\begin{aligned} \frac{\partial w_d}{\partial \theta_h} d\theta_h &= \frac{\partial \phi}{\partial p} dp + \frac{\partial \phi}{\partial \theta_h} d\theta_h + \frac{\partial \phi}{\partial \theta_u} d\theta_u - \frac{\partial C_h(\theta_h)}{\partial \theta_h} d\theta_h \\ \frac{\partial w_u}{\partial \theta_u} d\theta_u &= \frac{\partial \phi}{\partial p} dp + \frac{\partial \phi}{\partial \theta_h} d\theta_h + \frac{\partial \phi}{\partial \theta_u} d\theta_u + \frac{\partial C_u(\theta_u)}{\partial \theta_u} d\theta_u \end{aligned}$$

Look at the first equation, and collect terms

$$\underbrace{\left[\frac{\partial w_d}{\partial \theta_h} - \frac{\partial \phi}{\partial \theta_h} + \frac{\partial C_h(\theta_h)}{\partial \theta_h}\right]}_{=\gamma_h} \quad d\theta_h = \tfrac{\partial \phi}{\partial p} dp + \tfrac{\partial \phi}{\partial \theta_u} d\theta_u$$

and similarly for the second equation

$$\gamma_u d\theta_u = \frac{\partial \phi}{\partial p} dp + \frac{\partial \phi}{\partial \theta_h} d\theta_h$$

Using this to substitute for $d\theta_u$ above yields

$$\begin{aligned} \gamma_h d\theta_h &= \frac{\partial \phi}{\partial p} dp + \frac{\partial \phi}{\partial \theta_u} \frac{1}{\gamma_u} \left[\frac{\partial \phi}{\partial p} dp + \frac{\partial \phi}{\partial \theta_h} \right] d\theta_h \\ \left[\gamma_h \gamma_u - \frac{\partial \phi}{\partial \theta_h} \frac{\partial \phi}{\partial \theta_u} \right] d\theta_h &= \frac{\partial \phi}{\partial p} \left(\gamma_u + \frac{\partial \phi}{\partial \theta_u} \right) dp \\ \frac{d\theta_h}{dp} &= \frac{\frac{\partial \phi}{\partial p} \left(\gamma_u + \frac{\partial \phi}{\partial \theta_u} \right)}{\gamma_h \gamma_u - \frac{\partial \phi}{\partial \theta_h} \frac{\partial \phi}{\partial \theta_u}} \end{aligned}$$

The denominator is a stability condition and is negative. $\partial \phi / \partial p < 0$ because fewer constrained individuals will lower the average ability of the high school graduate pool. Finally

$$\gamma_u + \frac{\partial \phi}{\partial \theta_u} = \frac{\partial E \left[\theta | \theta \geq \theta_u \right]}{\partial \theta_u} - \frac{\partial C_u(\theta_u)}{\partial \theta_u} > 0$$

from the definition of γ_u. This term is positive because $\partial E \left[\theta | \theta \geq \theta_u \right] / \partial \theta_u > 0$, when fewer individuals go to university the average ability of these individuals will be higher, and $\partial C_u(\theta_u) / \partial \theta_u < 0$ by the assumption necessary for a signalling model. Hence

$$\frac{d\theta_h}{dp} > 0$$

or when the constraint is relaxed and more individuals go to university, then also more individuals will choose to drop out of high school.

In order to gain intuition on this, consider the case where we start from a situation with $p = 1$, i.e. nobody is constrained. Now lower p by a little bit. Some individuals previously attending university will now go to high school only, and the average abilty of high school graduates will rise. Therefore their wage will rise. In fact, with $p = 1$, this effect is just $\partial \phi / \partial p$. Therefore, it becomes attractive for the highest ability high school drop outs to graduate from high school in order to get the higher high school graduate wage. This will lower the both the high school drop out and high school graduate wage. Marginal university graduates will now also decide to just get a high school degree, hence increasing the high school wage. The algebra says that these second round effects cannot dominate, and even with $p < 1$, some high school drop outs will be induced to graduate as p falls.