A Note on Bias in Just Identified IV with Weak Instruments

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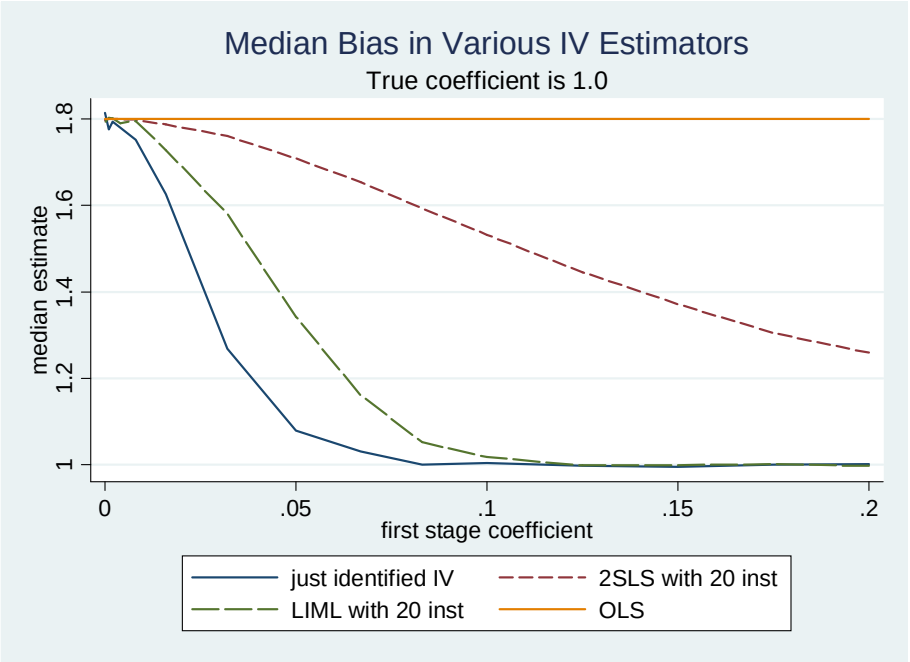
On p. 209 we discuss the bias of 2SLS with weak instruments and we say "just-identified 2SLS [...] is approximately *unbiased.*" A point of possible confusion here has to do with the word *approximately.* How good an approximation is it to say that just-identified IV is unbiased? After all, conventional asymptotic theory is an approximation too. The answer is in Figure 4.6.1, which shows that just identified IV with a weak instrument is centered at the truth while overidentified 2SLS is biased towards OLS. On the other hand, if the first stage for a just-identified model is really weak, say no first stage at all, just-identified IV has the same bias as OLS. So it isn't right to say, without qualification, "just-identified IV is unbiased." But bias with a just-identified model is not usually worth worrying about because if the instruments are so weak that just-identified IV is seriously biased, then you'll easily see the cosmic weakness of your first stage in such cases by virtue of large second-stage standard errors.

As in Figures 4.6.1-4.6.3, we can make this point with a small Monte Carlo experiment.

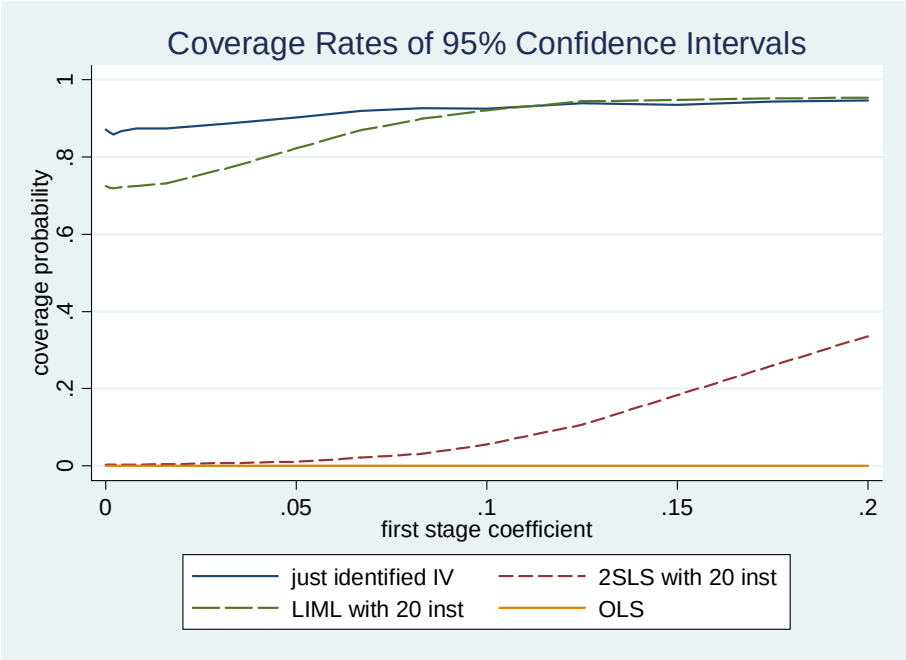
The figure on the next page extends the results in the book. The setup of the Monte Carlo results is the same as that described on p. 210. However, the figure reports only the medians of the simulated distributions. Results for four estimators are shown: OLS, just identified IV with one instrument, overidentified 2SLS with 20 instruments, and LIML with 20 instruments. As before, only the first instrument has a positive first stage coefficient. In figures 4.6.1 and 4.6.2 in the book, this coefficient is set to 0.1, and in figure 4.6.3 it is zero. Here, we vary this first stage coefficient between zero and 0.2.

The coefficient to be estimated is equal to 1. OLS is badly biased with a sampling distribution centered around 1.8. Overidentified 2SLS is substantially biased for all the first stage values used in the figure: even



for a value of 0.2 the 2SLS estimator is centered around 1.25, away from the truth of 1. Just identified IV and LIML perform much better. For first stage coefficients between 0.1 and 0.2 they are centered at the truth of 1. For this range, our statement that these estimates are approximately median unbiased is correct. The population first stage F-statistic for the just identified model in this range is above 10. The F-statistic relevant for the overidentified LIML in the same range is much lower, it is between 0.5 and 2.

With a really low first stage coefficient, less than 0.1, just identified IV and LIML are also biased. LIML with 20 instruments performs worse than just identified IV. For example, for a first stage coefficient of 0.05, overidentified LIML is centered around 1.4 while just identified IV is centered around 1.1. As the first stage coefficient approaches zero, the median for all estimators approaches that of OLS. But the first stage in these cases is so weak (the first stage t-statistic is 1.6 or less in the just identified model) that it seems unlikely anyone would think the project is worth pursuing. With a first-stage t so low, it's very unlikely anything useful can be learned from the second stage.



An important distinction between overidentified 2SLS and just identified IV (or LIML for that matter) is that the sampling distribution of the 2SLS estimator with weak instruments is not very spread out. While the IV and LIML distributions also tend to be too tight, for these estimators things are not as bad as for overidentified 2SLS. For practical purposes this means even though just identified IV with weak instruments may be biased you will likely detect the problem because your standard errors are bound to be large in this case (although still somewhat smaller than what they should be).

The relative saftety of just-identified IV (and LIML) is reflected the coverage rate. The coverage rate is the probability that a confidence interval includes the true parameter. For a 95% confidence interval this should be 0.95. Actual coverage rates will be lower if an estimator is biased and/or if the estimated standard error is too small. As a result, coverage rates for OLS and 2SLS are very poor, as can be seen in the second figure on this page. The 2SLS coverage rate remains below 0.35 for all values of the first stage coefficient in the figure. Things are very different for LIML and just identified IV. For first stage coeffients of 0.1 or above, coverage rates for

these estimators are near perfect. Even for lower first stage coefficients, the coverage rates never drop below 0.7 for LIML or 0.85 for IV, even when the first stage is exactly zero. Hence coverage rates of these estimators are still decent even in the region where first stage F-statistics are very low. Taking all the indications for weak instruments (low first stage F, large confidence intervals), just identified IV and LIML are unlikely to lead researchers astray in actual applications.

References for further reading

Davidson, Russel and James G. MacKinnon (2006) "The Case Against JIVE," *Journal of Applied Econometrics*, 21, 827-833, and the discussion following their paper.

Mariano, Roberto S. (2001): "Simultaneous Equation Model Estimators: Statistical Properties and Practical Implications," chapter 6 in *A Companion to Theoretical Econometrics*, ed. Badi H. Baltagi, Blackwell Publishing, 122-143.

Nelson, Charles R. and Richard Startz (1990): "Some Further Results on the Exact Small Sample Properties of the Instrumental Variable Estimator," *Econometrica*, 58, 967-976.