# Centralized versus Decentralized Provision of Local Public Goods: A Political Economy Analysis*


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Revised May 2000


**Abstract**

This paper takes a fresh look at the trade-off between centralized and decentralized provision of local public goods. The point of departure is to model a centralized system as one in which public spending is financed by general taxation, but districts can receive *different* levels of local public goods. In a world of benevolent governments, the disadvantages of centralization stressed in the existing literature disappear, suggesting that the case for decentralization must be driven by political economy considerations. Our political economy analysis assumes that under decentralization public goods are selected by locally elected representatives, while under a centralized system policy choices are determined by a legislature consisting of elected representatives from each district. We then study the role of taste heterogeneity, spillovers and legislative behavior in determining the case for centralization.



*The authors thank Bob Inman for many useful discussions, and Roger Gordon, Jim Hines, Ben Lockwood, Tom Nechyba, Torsten Persson and seminar participants at NBER, NYU, LSE, UNC, Oxford, Mannheim, Michigan and Virginia for comments. The Suntory Toyota International Centres for Economics and Related Disciplines provided valuable financial support.

# 1 Introduction

Which tier of government should be responsible for particular taxing and spending decisions? From Philadelphia to Maastricht, this question has vexed constitution designers. Yet still the issues are unresolved. Witness the recent debate in the U.S. over whether the States or Federal government should take responsibility for welfare policy.[1] In Europe, the principle of subsidiarity dictates that functions should be decentralized where possible, without any clearly defined criteria for centralization to be desirable.

This paper takes a fresh look at the trade off between centralized and decentralized provision of local public goods. Our analysis departs from the existing literature in emphasizing the *politics* of decision making. Centralization requires a system of governance that balances regional interests. Accordingly, the decision making unit typically incorporates a legislature consisting of representatives from each member district. The behavior of the legislature is then key in determining the performance of centralized systems. Legislatures that produce minimum winning coalitions expose members of federations to the risk of expropriation. More universalistic legislatures offer insurance against this. However, they are open to manipulation as citizens use the political process to exploit the budgetary externality that centralization creates. While influential commentators, such as Inman and Rubinfeld (1997a) and (1997b), have stressed their importance for the performance of federal systems, such issues have yet to be formally incorporated into models of fiscal federalism. Here we develop an analysis which integrates them with the more traditional concerns, emphasized in Oates (1972), of achieving the right balance between respecting heterogeneous tastes and internalizing spillovers.

The existing literature has typically modelled centralization as a system in which public spending is financed by general taxation and all districts receive a uniform level of the local public good. By contrast, a decentralized system is one in which local public goods are financed by local taxation and each district is free to choose its own level. The drawback with a decentralized system is that it is susceptible to free-rider problems, while centralized decision-making produces a "one size fits all" outcome that does not reflect local needs. This logic underpins Oates' (1972) *Decentralization Theorem* stating that, in the absence of spillovers, decentralization is preferable. When spillovers are present, the appropriate level of government depends on weighing the benefits of internalizing externalities with the costs of uniformity.

The usual assumption of uniform provision of local public goods like roads, parks and airports, is very hard to justify empirically.[2] This paper, therefore,

[1] See Inman and Rubinfeld (1997a) for discussion of this.

[2] The case of federal highway spending in the U.S. serves to illustrate the point. A significant fraction of funds in the Federal Highway Aid Program are earmarked by legislators for specific projects in their districts. Moreover, while the remaining funds are allocated according to a formula, this formula is manipulated to target spending to particular favored states. In the words of Senator Patrick Moynihan, "You don't have a formula here, you have 50 negotiated numbers" (Washington Post, May 23, 1998 - cited in Knight (2000)). See Knight (2000) for further analysis of the allocation of funds in the Federal Highway Aid Program.

allows a centralized system to allocate different levels of local public goods to different districts. In this framework, decision making by benevolent governments makes centralization the preferred system — it respects the preferences of citizens at the district level, while optimally accounting for cross-border externalities. The case for decentralization must now follow from political economy considerations, specifically the operation of centralized systems.

We assume that, in a decentralized system, local public goods are selected by locally elected representatives, while in a centralized system policy choices are made by a legislature consisting of elected representatives from each district. Two specifications of legislative behavior are considered. Our benchmark case assumes that spending on local public goods in the legislature is determined by a "minimum winning coalition" of representatives as suggested, for example, by Buchanan and Tullock (1962), Riker (1962), Ferejohn, Fiorina and McKelvey (1987) and Baron and Ferejohn (1989). We then contrast this with a specification in which representatives determine public spending in a more inclusive way along the lines suggested by Weingast (1979) among others. The election of representatives draws on the citizen-candidate model of representative democracy (Osborne and Slivinsky (1996), Besley and Coate (1997)), in particular the extension to legislative elections due to Coate (1997).

The remainder of the paper is as organized as follows. Section 2 briefly reviews some related literature while section 3 outlines the framework for our analysis. As background, section 4 provides a brief review of the implications of Oates' analysis in our framework. Section 5 then presents our political economy analysis for the benchmark of a non-cooperative legislature. Section 6 considers the case where the legislature acts on a more cooperative basis. Section 7 discusses the significance of our maintained assumption of uniform financing and concluding remarks appear in section 8.

# 2 Related Literature

In addition to the work of Oates (1972), there is a significant body of literature comparing centralized and decentralized policy-making. One important strand of literature, dating back to Tiebout (1956), sees the advantages of decentralization as stemming from the *mobility* of citizens across local jurisdictions. This invokes the argument that the ability of citizens to vote with their feet sharpens the constraints faced by local policy-makers (see, for example, Courant, Gramlich and Rubinfeld (1979), Epple and Zelenitz (1981) and Oates (1985)). This results in decentralized policies more closely reflecting citizens' preferences. Underpinning this is the notion that unconstrained choices by politicians would not reflect their constituents' interests. The idea that decentralization may more effectively deal with political agency problems also motivates the recent contributions of Bardhan and Mookherjee (1998) and Tommasi and Weinschelbaum (1999).

In a series of recent papers, Persson and Tabellini have used political economy analysis to shed light on the difference in policy outcomes emerging from centralized and decentralized systems. Persson and Tabellini (1994) ask whether a more centralized system of government will tend to lead to a larger government sector. Like us, they model policy choices under centralization as emerging from a legislature consisting of representatives from each district. However, they do not consider elections to such a legislature. Persson and Tabellini (1996a,b) contrast risk sharing by centralized and decentralized governments. They focus on the trade-off between improved risk sharing under centralization with the increased moral hazard due to regions taking on more risk. They also consider how different federal constitutions shape regional transfers in political equilibrium and which type of constitutional arrangement performs better.

A number of recent papers have focused on the positive political economy question of when a society will choose centralized over decentralized policy-making. For example, Bolton and Roland (1997) analyze when a federation would be likely to break up. They work with the assumption that policies are uniform under centralization. In addition, they assume exogenously given efficiency gains from centralization so that their trade-off is then principally between reaping the benefits of these against the ability to tailor policies to individual districts' tastes. Alesina and Spoalare (1997) consider the optimal and equilibrium number of districts in a model that trades off scale economies against preference diversity. Ellingsen (1998) considers the positive and normative economics of centralization of a pure public good.[3] The median voter of the larger community created by centralization selects policy. If the districts are identical, then centralization is attractive since there is scope for cost sharing and cross-district free-rider problems are eliminated. However, heterogeneity undermines the case for centralization.

The closest paper to ours is the independent work of Lockwood (1998). Like us, he is critical of the assumption that centralization implies uniformity in public spending across districts and develops a political economy analysis of decentralization versus centralization of public good provision. He also assumes that a centralized system forms policy in a legislature comprising of elected representatives from each district. He specifies an extensive form bargaining game for the legislature which predicts that spillovers affect the nature of the legislative outcome. However, in contrast to this paper, he assumes that the local public good in each district is discrete and that citizens are homogeneous. The latter assumption makes legislative elections straightforward. Overall, Lockwood's focus is complementary with ours, paying greater attention to legislative processes and less attention to election outcomes.

# 3 The Model

The economy is divided into 2 geographically distinct districts indexed by $i \in \{1, 2\}$. Each district has a continuum of citizens with a mass of unity.[4] There are

[3] His analysis briefly considers the extension to a local public good.

[4] We ignore issues of mobility in this analysis. While such considerations are obviously important, incorporating them is sufficiently difficult that they are best left for a separate

3 goods in the economy; a single private good, $x$, and two local public goods, $g_1$ and $g_2$, each one associated with a particular district. The latter can be thought of as roads or parks. Each citizen is endowed with some of the private good. To produce one unit of either of the public goods, requires $p$ units of the private good.

Each citizen in district $i$ is characterized by a public good preference parameter $\lambda$. The preferences of a type $\lambda$ citizen in district $i$ are

$$x + \lambda[(1-\kappa)b(g_i) + \kappa b(g_{-i})],$$

where $b(\cdot)$ is a twice continuously differentiable, increasing, and strictly concave function satisfying $b(0) = 0$. The parameter $\kappa \in [0, 1/2]$ indexes the degree of *spillovers*; when $\kappa = 0$ citizens care only about the public good in their own district, while when $\kappa = 1/2$ they care equally about the public goods in both districts. While spillovers are the same for all citizens, those with higher $\lambda$'s value public goods more highly.

In each district, the range of preference types is $[0, \overline{\lambda}]$. The *mean type* in district $i$ is denoted by $m_i$ and we assume throughout that this equals the median type.[5] We assume that the mean type is at least as high in district 1 ($m_1 \geq m_2$); that $m_2 > p/b'(0)$ and that $2m_1 < \overline{\lambda}$. The role of the latter two conditions will become apparent below.

Under a *decentralized system*, the level of public good in each district is chosen by the government of that district and public expenditures are financed by a uniform head tax on local residents. Thus, if district $i$ chooses a public good level $g_i$, each citizen in district $i$ pays a tax of $pg_i$.[6] Under a *centralized system*, the levels of both public goods are determined by a government that represents both districts, with spending being financed by a uniform head tax on all citizens. Thus, public goods levels $(g_1, g_2)$, result in a head tax of $\frac{p}{2}(g_1+g_2)$.[7]

Our criterion for comparing the performance of centralized and decentralized systems will be aggregate public good surplus. With public good levels $(g_1, g_2)$, this is

$$S(g_1, g_2) = [m_1(1-\kappa) + m_2\kappa]b(g_1) + [m_2(1-\kappa) + m_1\kappa]b(g_2) - p(g_1 + g_2).$$

Our assumption that $m_1 \geq m_2 > p/b'(0)$, implies that the surplus maximizing public good levels are positive for all spillover levels and satisfy the *Samuelson conditions*

$$[m_i(1-\kappa) + m_{-i}\kappa]b'(g_i) = p, \ i \in \{1, 2\}.$$

paper.

[5] This is by no means necessary to undertake the analysis. It just serves to simplify the results. The interested reader should have no trouble understanding how relaxing the assumption alters the results.

[6] We will assume throughout that citizens endowments are large enough to meet their tax obligations.

[7] Section 7 discusses the significance of common financing for our results.

Letting $f(\alpha) = \arg\max\{\alpha b(f) - pf : f \geq 0\}$, the surplus maximizing public good levels can be written as

$$(g_1, g_2) = (f(m_1(1-\kappa) + m_2\kappa), f(m_2(1-\kappa) + m_1\kappa)).$$

Note that when $m_1 > m_2$, the surplus maximizing level is higher in district 1 for all $\kappa < 1/2$.

# 4 Oates' Analysis: A Review

Many public finance economists' views on the relative merits of decentralization and centralization have been shaped by Oates (1972). To provide the background for our analysis, we briefly review Oates' analysis in our framework. Oates' supposes that, in a decentralized system, each district's policy is chosen independently by a social planner whose objective is to maximize public goods surplus in the district. Accordingly, we look for a pair of expenditure levels $(g_1^d, g_2^d)$ that satisfy

$$g_i^d = \arg\max_{g_i}\{m_i[(1-\kappa)b(g_i) + \kappa b(g_{-i}^d)] - pg_i\},\ i \in \{1, 2\}.$$

Taking the first order conditions, yields:

$$(g_1^d, g_2^d) = (f(m_1(1-\kappa)), f(m_2(1-\kappa))).$$

Each district's planner only takes account of the benefits received by citizens in his district and, accordingly, local public good decisions are only surplus maximizing when there are no spillovers. With spillovers, public goods are underprovided in both districts and this underprovision is increasing in the extent of spillovers.

In a centralized system, Oates assumes that a planner chooses a uniform level of the public good to maximize aggregate public goods surplus. This level, denoted $g^c$, satisfies

$$g^c = \arg\max_{g}\{[m_1 + m_2]b(g) - 2pg\},$$

yielding

$$g^c = f(\frac{m_1 + m_2}{2}).$$

The common level of public good is independent of the level of spillovers and equals the surplus maximizing level when the districts are identical. However, when $m_1 > m_2$, centralization under-provides public goods to district 1 and over-provides them to district 2 except when $\kappa = \frac{1}{2}$.

It is clear from these results that, when districts are homogeneous, centralization dominates decentralization whenever spillovers are present. Moreover, when districts are not identical, decentralization dominates when there are no spillovers while centralization is better when spillovers are maximal. It can also be shown that surplus under decentralization is decreasing in spillovers, so that there is a critical level of spillovers above which centralization dominates. This yields the following proposition.[8]

**Proposition 1** *(i) Under Oates' assumptions, if the districts are identical and spillovers are present ($\kappa > 0$), a centralized system produces a higher level of surplus than does decentralization. If there are no spillovers ($\kappa = 0$), the two systems generate the same level of surplus. (ii) If the districts are not identical, there is a critical value of $\kappa$, greater than $0$ but less than $\frac{1}{2}$, such that a centralized system produces a higher level of surplus if and only if $\kappa$ exceeds this critical level.*

Thus, without spillovers, a decentralized system is superior - a result often referred to as *Oates' Decentralization Theorem.* With spillovers and identical districts, a centralized system is preferred. With spillovers and non-identical districts, it is necessary to compare the magnitude of the two effects. Centralization is desirable if and only if spillovers are sufficiently large.[9]

The trade-off identified by Oates relies critically on the assumption that expenditures under centralization are uniform across districts. If, under cen-

[8] The proof of this and the subsequent results may be found in the Appendix.

[9] While Oates' analysis is typically interpreted as suggesting that heterogeneity hurts the case for centralization, this does not follow logically from his assumptions. To establish such a result, it would be necessary to show that as districts became more heterogeneous, the critical level of spillovers increased. Proposition 1 only tells us that the critical level of spillovers is higher for an economy with heterogeneous districts than for an economy with identical districts. In fact, there is no guarantee that the critical level is decreasing in heterogeneity. This is because heterogeneity can worsen the social costs of under-provision and hence has an ambiguous effect on surplus under decentralization. The issue may be analyzed by letting $S^d(\gamma, \kappa)$ denote surplus under decentralization with spillovers $\kappa$, when $(m_1, m_2) = (\gamma\xi, (1-\gamma)\xi)$. Then, since surplus under centralization is independent of both $\gamma$ and $\kappa$, the critical value of $\kappa$, denoted $\kappa^*(\gamma)$, is uniquely defined by the equation: $S^d(\gamma, \kappa^*) = S^c_o$. To show that $\kappa^*$ is an increasing function of $\gamma$, it is necessary to show that for all $\gamma$, $\partial S^d(\gamma, \kappa^*)/\partial\gamma > 0$. Differentiating, we have:

$$\begin{aligned}\frac{\partial S^d(\gamma,\kappa)}{\partial\gamma} &= \xi(1-2\kappa)[b(f(\xi\gamma(1-\kappa))) - b(f(\xi(1-\gamma)(1-\kappa)))] \\ &\quad + \xi\kappa p\{\frac{1-\gamma}{\gamma}f'(\xi\gamma(1-\kappa)) - \frac{\gamma}{1-\gamma}f'(\xi(1-\gamma)(1-\kappa))\}.\end{aligned}$$

The first term is the direct effect of increasing heterogeneity and is positive. It reflects the fact that, under decentralization, district 1 has a higher level of public goods than district 2, and greater heterogeneity concentrates citizens with higher tastes for public goods in the high public good district. The second term reflects the indirect effect of heterogeneity, which works through changes in the public good levels of the two districts. As the districts become more heterogeneous, the first best level of local public goods rises for district 1 and falls for district 2. In the equilibrium, district 1 allocates more resources to public goods and district 2 allocates less. Since local public goods are under-provided in equilibrium, the first effect raises surplus and the latter lowers it. The second term reflects these two effects, the first part the benefits from increasing provision in district 1 and the second part the costs from reducing provision in district 2. Since $\gamma \geq 1/2$, this second term is negative if $f''(\alpha) \leq 0$. It is clear that the first term goes to zero as spillovers increase, and hence it is possible that $\partial S^d(\gamma,\kappa)/\partial\gamma < 0$. This fact makes it possible to construct examples in which the critical value of spillovers is actually decreasing in heterogeneity.

tralization, the planner could choose different levels of public goods for the two districts, he would choose the surplus maximizing level for each district. Thus, a *centralized system would always produce at least as much surplus as a decentralized system and strictly more in the presence of spillovers.* Hence, since the reason for imposing the uniformity constraint seems unclear on both empirical and theoretical grounds, Oates' analysis is suspect.

# 5 A Political Economy Analysis

## 5.1 Policy determination under decentralization

Under decentralization, we assume that each district elects a representative to choose policy.[10] Following the citizen-candidate approach, this representative is a citizen from the district in question. Accordingly, representatives are characterized by their public good preferences $\lambda$. There is no *ex ante* policy commitment, so that these preferences determine their policy choices if they win office.

The policy determination process has two stages. First, elections determine which citizens are selected to represent the two districts. Second, policies are chosen simultaneously by the elected representative in each district. Working backwards, let the types of the representatives in districts 1 and 2 be $\lambda_1$ and $\lambda_2$. Then the policy outcome $(g_1(\lambda_1), g_2(\lambda_2))$ satisfies

$$g_i(\lambda_i) = \arg\max_{g_i}\{\lambda_i[(1-\kappa)b(g_i) + \kappa b(g_{-i}(\lambda_{-i}))] - pg_i\} \text{ for } i \in \{1,2\}\,.$$

Solving this yields

$$(g_1(\lambda_1), g_2(\lambda_2)) = (f(\lambda_1(1-\kappa)), f(\lambda_2(1-\kappa))).$$

Each district's spending is higher the stronger is the public goods preference of its representative and lower the higher the level of spillovers.

If the representatives in districts 1 and 2 are of types $\lambda_1$ and $\lambda_2$, a citizen of type $\lambda$ in district $i$ will enjoy a public goods surplus

$$\lambda[(1-\kappa)b(g_i(\lambda_i)) + \kappa b(g_{-i}(\lambda_{-i}))] - pg_i(\lambda_i).$$

These preferences over types determine citizens' voting decisions. A pair of representative types $(\lambda_1^*, \lambda_2^*)$ is majority preferred under decentralization if, in each district $i$, a majority of citizens prefer the type of their representative to any other type $\lambda \in [0, \overline{\lambda}]$, given the type of the other district's representative $\lambda_{-i}^*$.

[10] We recognize that, in the real world, decisions in decentralized systems are typically made by legislatures consisting of elected representatives of each of the sub-districts of the districts. Our assumption that decisions are made by a single representative is simply trying to capture the reality that there will be a greater commonality of interest across these sub-districts than across districts. In the extreme, when all sub-districts in a district are homogeneous and all impacted by the public good in the same way, then they will all wish to elect the same type of representative and the district legislature will act as if it is a single individual.

We assume that the elected representatives in the two districts will be of these majority preferred types. Thus, if the majority preferred types are $(\lambda_1^*, \lambda_2^*)$, the policy outcome under decentralization will be $(g_1(\lambda_1^*), g_2(\lambda_2^*))$. There are two possible justifications. First, there is an equilibrium of the citizen-candidate model in which a candidate of the majority preferred type from each district runs and is elected unopposed.[11] Second, if in each district, two Downsian parties compete for office by selecting candidates, equilibrium will involve both parties in each district selecting candidates of the majority preferred type.

The optimal type of representative for a citizen of type $\lambda$ in district $i$ maximizes $\lambda(1-\kappa)b(g_i(\lambda_i)) - pg_i(\lambda_i)$. Since a type $\lambda$ candidate chooses the public good level that solves this problem, each citizen prefers a candidate of his own type. Citizens' preferences over types are *single-peaked*[12] implying that a pair of representative types is majority preferred under decentralization if and only if it is a median pair; i.e., $(\lambda_1^*, \lambda_2^*) = (m_1, m_2)$. Thus we have:

**Lemma 1** *The policy outcome under decentralization is*

$$(g_1, g_2) = (f(m_1(1-\kappa)), f(m_2(1-\kappa))).$$

This has a conventional flavor since local public good provision respects the preferences of the median voter within a district. This is well known for pure local public goods ($\kappa = 0$), and has served as the workhorse predictive model for local public finance — see Rubinfeld (1987). It also holds in our model when $\kappa > 0$.[13] Under our assumption that the median taste in each district equals the mean, our model agrees with Oates' analysis of decentralization.

## 5.2 Policy determination under centralization

The policy determination process under centralization also has an election and a policy selection stage. One citizen from each district is elected to serve in a legislature. The legislature then determines public spending in each district. A key issue is how to approach decision making in the legislature. There is no standard model in the literature although a number of different approaches have been suggested.[14] Our benchmark model captures the *minimum winning coalition* view of distributive policy-making.[15] Under this view, a coalition of

[11] This assumes that the costs of running are small and that no public good would be provided if no-body ran. The logic is basically that in Proposition 2 of Besley and Coate (1997). If there are perquisites of office, then multiple candidates of the majority-preferred type might run.

[12] Given any two types $\lambda_i$ and $\lambda_i'$ such that $\lambda_i < \lambda_i' < \lambda$ or $\lambda < \lambda_i' < \lambda_i$, type $\lambda$ citizens always prefer type $\lambda_i'$ citizens.

[13] The assumption that preferences are additive is critical for this result. It would no longer be optimal for the median voter to elect a median citizen with $\kappa > 0$, if the public goods were complements or substitutes. For example, substitutes give an incentive for the median types to elect a citizen below the median to represent them, thus accentuating the free-rider problem.

[14] See Collie (1988) for a review.

[15] Distributive policies are those, like centrally financed local public goods, that primarily benefit the constituents of one district but whose costs are borne collectively.

51% of the representatives forms to share the benefits of public spending among their districts. Districts whose representatives do not belong to this coalition are only allocated spending to the extent that this benefits coalition members. The logic is that, in a majority rule legislature, if there were any more than 51% of the representatives in the coalition supporting the spending bill, the majority of coalition members would benefit from expelling the surplus members and further concentrating spending on their own districts. Since there are many possible minimum winning coalitions, this view suggests that there will be uncertainty concerning the identity of the coalition that coalesces to determine spending.[16]

In our two district model, each representative can be thought of as a minimum winning coalition, so we may capture this view by assuming that each district's representative is selected to choose policy with equal probability. Thus, if the representatives are of types $\lambda_1$ and $\lambda_2$, the policy outcome will be $(g_1^1(\lambda_1), g_2^1(\lambda_1))$ with probability 1/2 and $(g_1^2(\lambda_2), g_2^2(\lambda_2))$ with probability 1/2 where $(g_1^i(\lambda_i), g_2^i(\lambda_i))$ is the optimal choice of district $i$'s representative; that is,

$$(g_1^i(\lambda_i), g_2^i(\lambda_i)) = \arg \max_{(g_i, g_{-i})} \{\lambda_i[(1-\kappa)b(g_i) + \kappa b(g_{-i})] - \frac{p}{2}(g_i + g_{-i})\}.$$

It is easily checked that

$$(g_i^i(\lambda_i), g_{-i}^i(\lambda_i)) = (f(2\lambda_i(1-\kappa)), f(2\lambda_i\kappa)),\ i \in \{1, 2\}.$$

We refer to this legislative behavior as *non-cooperative* as it leaves open the possibility of gains to cooperation among representatives. Below, we consider the implications of assuming that the legislators capture these gains by some form of bargaining.

If the representatives' types are $\lambda_1$ and $\lambda_2$, a citizen of type $\lambda$ in district $i$ obtains an expected public goods' surplus of

$$\frac{1}{2} \sum_{k \in \{1,2\}} \{\lambda[(1-\kappa)b(g_i^k(\lambda_k)) + \kappa b(g_{-i}^k(\lambda_k))] - \frac{p(g_1^k(\lambda_k) + g_2^k(\lambda_k))}{2}\}.$$

A pair of representative types $(\lambda_1^*, \lambda_2^*)$ is majority preferred if, in each district a majority of citizens prefer the type of their representative to any other type, given the other district's representative type. As above, we assume that the elected representatives in the two districts will be of the majority preferred types.[17] Thus, if the majority preferred representative types are $(\lambda_1^*, \lambda_2^*)$, the

[16] In some theories, the identity of the minimum winning coalition depends on the policy preferences of the legislators (Ferejohn, Fiorina and McKelvey (1987) and Baron (1991)). This creates incentives for voters to select representatives whose preferences are such that they are included in the minimum winning coalition (Niou and Ordeshook (1985), Chari, Jones and Marimon (1997) and Coate (1997)). In equilibrium, therefore, the set of elected representatives is such that the identity of the minimum winning coalition is uncertain. While our specification of non-cooperative legislative behavior abstracts from such considerations, they only serve to reinforce the basic thrust of our arguments.

[17] The two justifications given in the decentralized case remain valid. See Coate (1997) for more discussion of the citizen-candidate approach to legislative elections.

policy outcome will be $(g_1^1(\lambda_1^*), g_2^1(\lambda_1^*))$ with probability 1/2 and $(g_1^2(\lambda_2^*), g_2^2(\lambda_2^*))$ with probability 1/2.

This model of the legislature implies that each district $i$'s representative only affects the outcome if he is selected to choose policy and then he selects his utility maximizing policy. This implies that each citizen prefers a candidate of his own type. Thus, a pair of representative types is majority preferred if and only if it is a median pair; i.e., $(\lambda_1^*, \lambda_2^*) = (m_1, m_2)$. This establishes:

**Lemma 2** *The policy outcome under centralization with a non-cooperative legislature is* $(g_1, g_2) = (f(2m_1(1-\kappa)), f(2m_1\kappa))$ *with probability* $1/2$ *and* $(g_1, g_2) = (f(2m_2(1-\kappa)), f(2m_2\kappa))$ *with probability* $1/2$.

This result makes clear the drawbacks of centralization when policies are determined by minimum winning coalitions. The first problem is that each district faces uncertainty as to the amount of public good that it will receive, reflecting the uncertainty in the identity of the minimum winning coalition. The second problem concerns the levels of the public goods chosen by each minimum winning coalition. For low levels of spillovers, public goods are over-provided to districts in the minimum winning coalition and under-provided to those outside the coalition, reflecting the budgetary externality created by common financing. While higher spillovers lead those in the minimum winning coalition to allocate public goods to districts outside the coalition, the chosen levels still only reflect the preferences of those in the coalition. When $m_1 > m_2$, district 1's representative always over-provides local public goods to his own district, while district 2's representative always under-provides local public goods to district 1. It is only when the districts are identical and spillovers are complete in the sense that $\kappa = 1/2$, that centralization produces the surplus maximizing level of local public goods.

## 5.3 Decentralization versus centralization

We have already seen that public goods levels are surplus maximizing under decentralization in the absence of spillovers. Centralization, on the other hand, produces the surplus maximizing public goods levels only if the districts are identical and spillovers are complete. Thus, with identical districts, decentralization dominates when spillovers are small and centralization dominates when spillovers are large. With non-identical districts, only the former statement applies. We can also show that, in the case of identical districts, the performance of centralization is increasing in $\kappa$. Increasing spillovers causes each district's representative to reduce his own district's public good level and increase that of the other district (assuming it is positive). This raises surplus since local public goods are over-provided in the decisive representative's district and under-provided in the other district. It follows that there exists a critical value of spillovers above which centralization dominates when districts are identical.

**Proposition 2** *(i) With a non-cooperative legislature, if the districts are identical, there exists a critical value of* $\kappa$, *strictly greater than* $0$ *but less than* $\frac{1}{2}$, *such that a centralized system produces a higher level of surplus if and only if $\kappa$ exceeds this critical level. (ii) If the districts are not identical, a decentralized system produces a higher level of surplus when spillovers are sufficiently small.*

Comparing this with Proposition 1, there are two main differences. First, Oates' analysis suggests that with identical districts, centralization is better than decentralization for *all* spillover levels. However, the political economy analysis reverses the conclusion for small spillover levels. With a non-cooperative legislature, public good decisions reflect only the preferences of those representatives in the minimum winning coalition. For small spillover levels, members of this coalition have no incentive to provide public goods to those outside. This biases public spending level towards a particular set of districts. This, coupled with the random identity of the minimum winning coalition, creates a loss of surplus relative to decentralization. As spillovers increase, members of the minimum winning coalition allocate more spending to districts whose representatives are outside the coalition increasing aggregate surplus and strengthening the case for centralization. At the same time, decentralization is less attractive as locally elected leaders free-ride on each other's policies.

The second difference with Proposition 1 arises when districts are not identical. While Oates' analysis shows that centralization must dominate decentralization for sufficiently large spillovers, Proposition 2 offers no such assurance. Indeed, no such assurance is possible, as our next Proposition will demonstrate. It shows that, for sufficiently diverse districts, decentralization is better than centralization when spillovers are maximal for an example where $b(g) = \ln(1+g)$. To show this, we use the following simple way of representing the degree of heterogeneity between the two districts. Let $(m_1, m_2) = (\xi\gamma, \xi(1-\gamma))$ for some $\xi > 2p/b'(0)$ where $\gamma$ measures the degree of cross-district heterogeneity. Districts are identical if $\gamma = 1/2$, and become more heterogeneous as $\gamma$ increases. Since a value of $\gamma$ equal to $1 - p/\xi b'(0)$ implies that $m_2 = p/b'(0)$, consistency with our assumptions implies that $\gamma < 1 - p/\xi b'(0)$.

**Proposition 3** *Suppose that $b(g) = \ln(1+g)$ and that $(m_1, m_2) = (\xi\gamma, \xi(1-\gamma))$ for some $\xi > 4p$ and $\gamma \in [\frac{1}{2}, 1 - p/\xi)$. Then, for sufficiently large $\gamma$, decentralization produces a higher level of surplus than centralization with a non-cooperative legislature when spillovers are maximal.*

To understand the result, consider what happens when spillovers are maximal and $\gamma$ is close to $1-p/\xi b'(0)$. The surplus maximizing public goods levels are then $f(\frac{\xi}{2})$ for both districts. Under decentralization, district 2 provides no local public goods while district 1 provides $f(\frac{\xi}{2}\gamma)$. Under centralization, district 2's representative provides almost no public goods for either district if he is selected while district 1's representative provides $f(\xi\gamma)$ for both districts. Given that each district's representative is selected with equal probability, this is almost equivalent to one district having no public goods and the other having $f(\xi\gamma)$. Hence, the only real difference between the two systems is that, under decentralization, the district with public goods has $f(\frac{\xi}{2}\gamma)$ while, under centralization, it has $f(\xi\gamma)$. Comparing this with the surplus maximizing level, decentralization produces too little of the public good and centralization too much. However, if $p/\xi b'(0)$ is sufficiently small, then the public goods level $f(\frac{\xi}{2}\gamma)$ is preferred to $f(\xi\gamma)$ when $\gamma$ is near its upper bound of $1 - p/\xi b'(0)$. In the logarithmic case, $b'(0) = 1$ and the assumption that $p/\xi < 1/4$ is sufficient to yield the result.

While decentralization can dominate centralization for all spillover levels when districts are not identical, is it still the case that the *relative* performance of centralization is necessarily increasing in spillovers? This is a difficult question in general, but we have been able to find conditions under which it is true.

**Proposition 4** *With a non-cooperative legislature, the relative performance of centralization is increasing in the degree of spillovers if (i)* $m_2 > 2p/b'(0)$ *and (ii) for all* $\alpha \geq p/b'(0)$, $f''(\alpha) \leq 0$ *and* $\frac{-f''(\alpha)\alpha}{f'(\alpha)} < \frac{m_2}{m_1 - m_2}$.

What makes the problem difficult is that we can no longer guarantee that increasing spillovers will move all the public good levels in a surplus maximizing direction under centralization when districts differ. Since $m_1 > m_2$, district 1's representative will always oversupply public goods to his own district and hence increasing spillovers will move district 1's public good level in the right direction. However, if $m_2(1-\kappa) < m_1\kappa$, he will also oversupply public goods to district 2 and increasing spillovers will move district 2's public good level in the wrong direction. The benefits of moving district 1's public good level in the right direction must therefore be weighed up against the losses from moving district 2's public good levels in the wrong direction. Similarly, if $m_2(1-\kappa) < m_1\kappa$, district 2's representative will undersupply public goods to his own district and hence increasing spillovers will move district 2's public good level in the wrong direction. The benefits of moving district 1's public good level in the right direction must therefore be weighed up against the losses from moving district 2's public good levels in the wrong direction. Thus, it is clear that increasing spillovers induces a beneficial change in public good levels for $\kappa \leq \frac{m_2}{m_1+m_2}$ but for higher levels of $\kappa$, it is not clear. The conditions on $f''$ serve to guarantee that the net change is always beneficial. They are satisfied, for example, in the logarithmic case $b(g) = \ln(1+g)$, in which case $f''(\alpha) = 0$.

# 6 Centralization with a Cooperative Legislature

Under the minimum winning coalition view of legislative decision-making, policy outcomes are ex ante Pareto inefficient from the viewpoint of the representatives. Since the number of legislators is typically relatively small, Coasian logic suggests that legislators should find their way around the inefficiency created by majoritarian decision criteria (Wittman (1989)). This theoretical observation, coupled with the empirical observation that (at least in the United States) minimum winning coalitions for this type of spending seem the exception rather than rule, has led many to abandon the minimum winning coalition view of legislative behavior in favor of more cooperative approaches. In this section, we study whether our basic conclusions of the previous section hold with more cooperative legislative behavior.

### 6.1 Policy determination

The fact that there are gains from cooperation does not lead to a clear alternative for predicting legislative choices, since there are many pairs of public spending levels that are both efficient from the viewpoint of the representatives and that *ex ante* Pareto dominate minimum winning coalition outcomes. Here we assume that legislators' behavior can be described by the *utilitarian bargaining solution*; that is, they agree to the public goods allocation that maximizes their joint public goods surplus. This solution would seem to offer centralization the best chance of dominating decentralization given our welfare criterion. This provides a stiff test of the robustness of the results offered in the last section.

Our utilitarian solution can also be motivated by the literature on universalism in legislatures. Based on study of the U.S. Congress, Weingast (1979) suggested that legislators avoid the problems associated with minimum winning coalitions by developing a norm of universalism that allows each representative to participate in decision making. On this view, each representative chooses the spending he would like for his own district and the legislature passes an omnibus bill consisting of all these spending levels. Unfortunately, however, this produces a vector of spending levels that are inefficient for the representatives, which undermines the justification for the theory (see, for example, Schwartz (1994)). An alternative formalization, is that the norm also requires representatives to take account of the costs and benefits to their colleagues (Inman and Fitts (1990)) and our utilitarian solution is a natural way of capturing this.

Under this specification, if the representatives are of types $\lambda_1$ and $\lambda_2$, the policy outcome, $(g_1(\lambda_1,\lambda_2), g_2(\lambda_1,\lambda_2))$, will maximize

$$\sum_{i\in\{1,2\}} \{\lambda_i[(1-\kappa)b(g_i)+\kappa b(g_{-i})] - \frac{p}{2}(g_i+g_{-i})\}.$$

It is straightforward to verify that

$$(g_1(\lambda_1,\lambda_2), g_2(\lambda_1,\lambda_2)) = (f(\lambda_1(1-\kappa)+\lambda_2\kappa), f(\lambda_1\kappa+\lambda_2(1-\kappa))).$$

Accordingly, when the representative types are $\lambda_1$ and $\lambda_2$, a citizen of type $\lambda$ in district $i$ obtains public goods surplus

$$\lambda[(1-\kappa)b(g_i(\lambda_1,\lambda_2))+\kappa b(g_{-i}(\lambda_1,\lambda_2))] - \frac{p(g_1(\lambda_1,\lambda_2)+g_2(\lambda_1,\lambda_2))}{2}.$$

A pair of majority preferred representative types is defined in the by now familiar way. A public goods pair $(g_1, g_2)$ is a policy outcome under centralization with a cooperative legislature if there exists a majority preferred pair $(\lambda_1^*, \lambda_2^*)$ such that $(g_1, g_2) = (g_1(\lambda_1^*, \lambda_2^*), g_2(\lambda_1^*, \lambda_2^*))$.

The main additional complication introduced by cooperative legislatures lies in finding the majority preferred types. This is because the public good level for each district depends on the type of the legislator in *both* districts and, thereby, generates incentives for citizens in each district to delegate policy making strategically to a representative with different tastes than their own.[18]

To begin the task of finding the majority preferred types, note that a pair of representative types $(\lambda_1^*, \lambda_2^*)$ is majority preferred if and only if in each district $i$ the median type prefers $\lambda_i^*$ to any other type $\lambda \in [0, \overline{\lambda}]$, given the other district's representative type $\lambda_{-i}^*$.[19] This means that $(\lambda_1^*, \lambda_2^*)$ is majority preferred if and only if $(\lambda_1^*, \lambda_2^*)$ is a Nash equilibrium of the two player game in which each player has strategy set $[0, \overline{\lambda}]$ and player $i \in \{1, 2\}$ has payoff function

$$U_i(\lambda_1, \lambda_2, m_i) = m_i[(1-\kappa)b(g_i(\cdot)) + \kappa b(g_{-i}(\cdot))] - \frac{p(g_1(\cdot) + g_2(\cdot))}{2}.$$

In this game, the district $i$ median citizen tries to manipulate $\lambda_i$ so that he obtains something close to his preferred policy outcome anticipating the subsequent working of the legislature. Since he only has one degree of freedom, $\lambda_i$, and two objectives $(g_1, g_2)$, this instrument is rather blunt. While raising $\lambda_i$ always leads to an increase in district $i$'s level of public goods, if $\kappa > 0$, it also raises the public goods level in the other district.

In order to obtain an explicit solution for the equilibrium public goods choices when the districts are not identical, we adopt the logarithmic specification of public good benefits from Proposition 3; i.e., $b(g) = \ln(1+g)$. This specification implies that $f(\alpha)$ is a linear function of $\alpha$, which allows the computation of closed form solutions. To state the equilibrium policy choices, define $\widehat{\kappa}$ as the solution to

$$\frac{[\widehat{\kappa}^3 + (1-\widehat{\kappa})^3]}{\widehat{\kappa}(1-\widehat{\kappa})} = \frac{m_1}{m_2}.$$

In the case of identical districts, $\widehat{\kappa} = 1/2$. In the non-identical districts case, $\widehat{\kappa} < 1/2$. Now we have:

**Lemma 3:** *(i) Suppose that the legislature is cooperative and that the districts are identical ($m_1 = m_2 = m$). Then, if $f''(\alpha) \leq 0$ for all $\alpha \geq p/b'(0)$, the policy outcome under centralization is*

$$g_1 = g_2 = f(2m[(1-\kappa)^2 + \kappa^2]).$$

*(ii) If the districts are not identical and $b(g) = \ln(1+g)$, the policy outcome is*

$$(g_1, g_2) = \left(f\left(\frac{2m_1[(1-\kappa)^4 - \kappa^4]}{(1-\kappa)^2 - \frac{m_1}{m_2}\kappa^2}\right), f\left(\frac{2m_1[(1-\kappa)^4 - \kappa^4]}{\frac{m_1}{m_2}(1-\kappa)^2 - \kappa^2}\right)\right)$$

[18] This is reminiscent of Persson and Tabellini (1992) who consider strategic delegation in a bargaining context to model European integration.

[19] Observe that if district $i$ elects a citizen of a higher type, then it receives more of both public goods. It follows that if citizens of type $\lambda$ prefer a type $\lambda_i'$ candidate to a type $\widetilde{\lambda}_i$, where $\lambda_i' < \widetilde{\lambda}_i$ ($\lambda_i' > \widetilde{\lambda}_i$), then so must all citizens of types lower (higher) than $\lambda$. This implies that a majority of citizens in district $i$ prefer a type $\lambda_i'$ candidate to a type $\widetilde{\lambda}_i$ candidate if and only if the median type prefers a type $\lambda_i'$ candidate to a type $\widetilde{\lambda}_i$ candidate.

*if* $\kappa < \widehat{\kappa}$, *and*

$$(g_1, g_2) = (f(2m_1(1-\kappa)), f(2m_1\kappa)),$$

*if* $\kappa \geq \widehat{\kappa}$.

Recall that with identical districts, the surplus maximizing level of public goods is $f(m)$. Thus, part (i) of the Lemma tells us that local public goods are provided at too high a level for all $\kappa < 1/2$. The extent of overprovision is decreasing in the degree of spillovers, with the surplus maximum achieved at $\kappa = 1/2$. The mechanism underlying this over-provision is *strategic delegation* as each district's median voter delegates policy-making to a representative with a higher preference for public goods.[20]

The incentives to strategically delegate can be seen most clearly in the case of zero spillovers. Then, the optimal spending levels for district 1's median voter are $(g_1, g_2) = (f(2m), 0)$. Assuming that district 2 elects a representative with the median preference, electing a representative of type $m$ would produce a public goods outcome $(g_1, g_2) = (f(m), f(m))$. Electing a representative with a stronger taste for public spending raises district 1's public goods allocation, with no impact on district 2's. Each district is thus drawn to elect a type $2m$ representative.

As spillovers increase, the optimal spending levels in the two districts for each district's median voter converge. Moreover, electing a representative with a higher taste for public goods increases spending in the other district. Thus, the districts elect representatives with preferences closer to their own. In the limiting case of complete spillovers, each district elects a representative of the median type and local public goods are provided optimally.

Heterogeneity creates an additional conflict over the *level* of public spending. This can be seen most clearly in the case of maximal spillovers. If $\kappa = \frac{1}{2}$ and each district elects a representative of the median type, the public goods levels are $g_1 = g_2 = f(\frac{m_1+m_2}{2})$. This common level is too high for district 2's median voter and too low for district 1's. This gives district 2's median voter an incentive to have a lower representative type to reduce public goods spending, while district 1's median voter desires a representative with a higher valuation. They pull in opposite directions until one or both districts has put in their most extreme type. Our assumption that $2m_1 < \overline{\lambda}$ implies that district 1 can obtain its preferred level of public goods when district 2 has put in its most extreme type so that district 1's median voter ends up getting his preferred outcome of $g_1 = g_2 = f(m_1)$.

This additional conflict of interest creates a complex relationship between spillovers and public goods levels. Analyzing the solutions described in part (ii) of the Lemma, we can show that district 1's public goods level is decreasing in the level of spillovers for $\kappa$ sufficiently small and $\kappa > \widehat{\kappa}$. However, it is increasing in spillovers for $\kappa$ sufficiently close to but less than $\widehat{\kappa}$.[21] This appears puzzling

[20] The strategic incentive to elect representatives with strong preferences for local public spending also arises in the analysis of Chari, Jones and Marimon (1997).

[21] This and the other claims concerning the public good levels described in Lemma 3 are established in the Appendix.

as district 1's median voter's preferred public goods level is actually decreasing in spillovers. The result reflects the conflict over spending levels that emerges as spillovers increase. To prevent district 2 from pulling down spending in both districts, district 1's median voter elects a representative with a higher preference for public goods, raising district 1's public good level. District 2's public goods level is decreasing in the level of spillovers for $\kappa \leq \widehat{\kappa}$ and increasing thereafter. It increases for spillover levels in excess of $\widehat{\kappa}$, because it is now effectively controlled by district 1's median voter.

Comparing these outcomes with the surplus maximizing levels of public spending, district 1's public good level is always too high. The level provide to district 2 is too high for small $\kappa$ and when $\kappa$ is sufficiently large. However, it is less than the surplus maximizing level for $\kappa$ sufficiently close to $\widehat{\kappa}$. This under-provision is in sharp contrast to the over-provision results for the case of identical districts.

## 6.2 Decentralization versus centralization

From the above discussion, it is clear that centralization produces the surplus maximizing public goods levels only when the districts are identical and spillovers are complete. Thus, with identical districts, decentralization dominates when spillovers are small and centralization is better when spillovers are large. With identical districts, the performance of centralization improves as $\kappa$ increases and a critical value of spillovers exists above which centralization is welfare superior. With non-identical districts, decentralization continues to dominate when spillovers are small. However, unlike the non-cooperative case, centralization dominates when spillovers are large enough. These findings are laid out in:

**Proposition 5** *(i) With a cooperative legislature and identical districts, if $f''(\alpha) \leq 0$ for all $\alpha \geq p/b'(0)$, there is a critical value of $\kappa$, strictly greater than $0$ but less than $\frac{1}{2}$, such that a centralized system produces a higher level of surplus if and only if $\kappa$ exceeds this critical level. (ii) If the districts are not identical and $b(g) = \ln(1+g)$, then a decentralized system produces a higher level of surplus when spillovers are sufficiently small, while a centralized system produces a higher level when spillovers are sufficiently large.*

Thus, the basic lessons of Proposition 2 generalize to the case of a cooperative legislature. Decentralization dominates centralization for low levels of spillovers in both the identical and non-identical districts cases, while centralization dominates for high levels of spillovers in the homogenous case. The only real difference is that centralization dominates with sufficiently high spillovers in the case where the districts are not identical.

The findings are interesting because this model of legislative behavior assumes that legislators behave in a surplus maximizing way. Despite this, centralization is strictly inferior to decentralization when spillovers are small, even in the homogenous case. The shared financing of public goods under centralization leads voters to delegate strategically to representatives who provide excessive levels of public goods. Moreover, even when spillovers are large and districts share an interest in each other's public goods, the conflict of interest over the level of public spending means that centralization yields policy outcomes that are far from the surplus maximizing ideal. While the Proposition shows that centralization dominates decentralization when spillovers are high, there should be no general presumption that the relative performance of centralization is increasing in spillovers. In fact surplus under centralization is decreasing in $\kappa$ for $\kappa$ sufficiently close to $\widehat{\kappa}$. In this range, increasing spillovers, both increases district 1's public good which is over-provided and decreases district 2's public good which is under-provided.

# 7 Non-uniform Taxing and Spending

Throughout, we have assumed that, under centralization, financing decisions are uniform, while expenditure decisions are not. This is justified on empirical grounds since most centralized systems of government appear to operate (approximately) according to such rules.[22] However, investigating what would happen without uniform financing lends insight into the role played by this assumption in generating the results.

Suppose then that taxes can be different in each district, so that the legislature decides on a pair of local public goods levels $(g_1, g_2)$ and a pair of district-specific taxes $(T_1, T_2)$. Suppose that the highest tax level that can be imposed on district $i$ is $\overline{T}_i$ and that $\overline{T}_i > p[f(m_1(1-\kappa)) + f(m_1\kappa)]$ for each district $i$. Further suppose that $T_i$ can be negative, so that taxes can serve two roles — raising revenue for public spending and cross-district redistribution.

Consider first the case of a non-cooperative legislature. Citizens will continue to desire a representative who shares their type, implying that the majority preferred representative types will be $(m_1, m_2)$. District $i$'s representative will wish to extract as many resources as it can from members of the other districts and thus will set $T_{-i} = \overline{T}_{-i}$ and $T_i = p[g_i + g_{-i}] - \overline{T}_{-i}$. His optimal public good levels will, therefore, solve:

$$Max_{(g_i, g_{-i})}\{m_i[(1-\kappa)b(g_i) + \kappa b(g_{-i})] - p[g_i + g_{-i}]\},$$

which yields $(g_i, g_{-i}) = (f(m_i(1-\kappa)), f(m_i\kappa))$. Thus, the policy outcome is $(g_1, g_2) = (f(m_1(1-\kappa)), f(m_1\kappa))$ with probability 1/2 and $(g_1, g_2) = (f(m_2(1-\kappa)), f(m_2\kappa))$ with probability 1/2.

[22] We do not mean that tax burdens are common across districts. However, it is typical to have a common tax *code*. In the U.S., it would probably be unconstitutional for the federal government to tax incomes at different rates across states. That said, the federal government can offer tax credits for things like charitable contributions or research and development which may have a differential impact across jurisdictions.

Compared to the case of uniform financing (see Lemma 2), it is evident that public goods levels are always lower. Since the winning coalition can extract wealth from the other districts in the form of tax financed transfers, coalition members effectively pay the whole cost of public spending. The budgetary externality found under uniform financing is therefore eliminated, which reduces the incentive to provide public goods. This eliminates the case for centralization, as we now demonstrate:

**Proposition 6** *Suppose that the legislature behaves non-cooperatively and that there is non-uniform taxation. Then, for all levels of spillovers, surplus under decentralization is at least as high as that under centralization if $f''(\alpha) \leq 0$ for all $\alpha \geq p/b'(0)$.*

This result is easily understood in the case of identical districts ($m_1 = m_2 = m$). With non-uniform taxation, the public goods levels are $(f(m(1-\kappa)), f(m\kappa))$ if district 1's representative forms the minimum winning coalition and $(f(m\kappa), f(m(1-\kappa)))$ otherwise. Thus, public spending is the same as under decentralization in the district whose representative forms the minimum winning coalition and less in the other district. This is always further from the social optimum than under decentralization!

This result implies that uniform financing actually makes the case for centralization. The budgetary externality provides a needed boost to public spending levels. While it leads to over-provision when spillovers are small, it appropriately subsidizes provision when spillovers are high. This gives a rather different spin on the normal discussion of budgetary externalities in the literature (for example in Weingast, Shepsle and Johnsen (1981) and Chari, Jones and Marimon (1997)). This is because the literature focuses on the case where $\kappa = 0$ when, as we have shown, centralization is not a good idea. In the presence of spillovers, providing the appropriate marginal budgetary subsidy is necessary to provide an appropriate boost to spending. For small spillovers a large budgetary externality is likely to be too distortionary so that the preference for decentralization is maintained. However as $\kappa$ becomes large, the virtues of this financing externality come to dominate.

The implications of non-uniform taxation in the cooperative legislature case are more difficult to anticipate because our model of decision making yields no prediction about the allocation of taxes between districts. This is because the representatives' utilities are linear in income and hence the sum of their payoffs is independent of the distribution of income. In particular, therefore, it is possible that they agree on a uniform sharing rule, in which case the analysis of the previous section applies.

Since the representatives are indifferent over all sharing rules, one can ask whether a sharing rule exists that would eliminate strategic delegation. In the identical districts case, the tax system

$$(T_1, T_2) = ((1-\kappa) g_1 + \kappa g_2, \kappa g_1 + (1-\kappa) g_2)$$

serves this purpose. Under this tax system, each district elects a representative of type $m$ and the policy choices maximize aggregate surplus.[23] As in Oates' analysis, centralization dominates decentralization for all levels of spillovers. However, the result does not carry over to where districts differ. Here, the fact that the two districts disagree over both the level of spending and its allocation, again gives rise to strategic delegation.

# 8 Conclusion

The relative merits of decentralized and centralized systems of public spending have long been of interest to public economists. The conventional wisdom sees the main drawback with decentralized decision making as the under-provision of local public goods in the presence of spillovers. This is because local decision makers will neglect benefits going to other districts. Centralized decision making, on the other hand, has the limitation of "one size fits all" policy outcomes that are insensitive to the preferences of localities. It is this concern that we have taken issue with here, since the assumption of uniform provision of local public goods does not seem a reasonable characterization of how centralized systems work. Indeed, a large political science literature is devoted precisely to understanding the allocation of local public goods across districts in centralized systems.

This paper has developed an alternative vision of the drawbacks of centralization — that sharing costs of local public spending in a centralized system creates a conflict of interest between the citizens in different jurisdictions. Citizens from different districts may be expected to disagree both about the level of public spending and its allocation across districts. If, as is typically the case, policy decisions are made by a legislature consisting of representatives from each district, this conflict of interest will play out in the legislature. If decisions on local public goods are made by a minimum winning coalition of representatives, the allocation of public goods will be characterized by uncertainty and inequity. If decisions are made in a more cooperative way, then strategic delegation via elections will produce excessive public spending.

Understanding how this conflict of interest is affected by spillovers and differences between districts sheds light on the relative performance of centralization. Our results suggest rather different conclusions concerning the circumstances under which centralized decision making will be desirable. In particular, neither homogeneity of districts nor complete spillovers imply a case for centralization as Propositions 2 and 3 demonstrate. All that can be said in general is that there is

[23] To see why this tax system induces the median citizen to want a representative of his own type, suppose that district 2's representative is of type $m$ and let $v(\lambda_1) = U_1(\lambda_1, m, m)$ denote the payoff of district 1's median voter when he selects a representative of type $\lambda_1$. Then, we may write:

$$v(\lambda_1) = (1-\kappa)[mb(f(1)) - pf(1)] + \kappa[mb(f(2)) - pf(2)],$$

where $f(1) = f(\lambda_1(1-\kappa)+m\kappa)$ and $f(2) = f(m(1-\kappa)+\lambda_1\kappa)$. Clearly, this payoff is optimized when $\lambda_1(1-\kappa) + m\kappa = m(1-\kappa) + \lambda_1\kappa = m$, which implies that $\lambda_1 = m$.

a strong case for decentralization when spillovers are small and a strong case for centralization when districts are similar and spillovers are large. When districts are heterogeneous and spillovers are large, then which system is superior will depend on how a centralized legislature is expected to behave. Moreover, the familiar presumption that higher spillovers help the case for centralization does not emerge cleanly from our political economy analysis. Overall, our analysis suggests a weaker case for centralization than does the conventional view.

Much remains to be done to develop our understanding of the decentralization versus centralization question from this political economy perspective. While our analysis has allowed us to parametrize the key dimensions of the problem and compute closed form solutions for equilibrium levels of public spending, a more general treatment of the issues addressed here (if feasible) would be useful. A further weakness is that we have assumed that the only function of decentralized and centralized governments was providing the public goods in question. In reality, both levels of government determine numerous issues and hence the political consequences of transferring responsibility for one area of spending are likely to be much more subtle than our analysis suggests.

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## Appendix

**Proof of Proposition 1:** Part (i) of the proposition follows immediately from the relationship between the public goods levels under the two systems and the surplus maximizing levels. For part (ii), note that aggregate public goods surplus under decentralization is given by:

$$\begin{aligned} S^d(\kappa) &= [m_1(1-\kappa)+m_2\kappa]b(f(m_1(1-\kappa))) + [m_2(1-\kappa)+m_1\kappa]b(f(m_2(1-\kappa))) \\ &\quad -p[f(m_1(1-\kappa))+f(m_2(1-\kappa))], \end{aligned}$$

while surplus under centralization is:

$$S^c_o(\kappa) = [m_1+m_2]b(f(\frac{m_1+m_2}{2})) - 2pf(\frac{m_1+m_2}{2}).$$

From our earlier discussion, we know that when $m_1 \neq m_2$, $S^c_o(0) < S^d(0)$ and $S^c_o(\frac{1}{2}) > S^d(\frac{1}{2})$. It is also clear that surplus under centralization is independent of $\kappa$. To prove the result we will show that surplus under decentralization is non-increasing in $\kappa$ and decreasing at the point at which $S^c_o(\kappa) = S^d(\kappa)$.

Investigating how surplus under decentralization depends on $\kappa$ is marginally complicated by the possibility of non-interior solutions for each district's public good choice. There are three cases to consider: (i) $m_2 \geq 2p/b'(0)$, (ii) $m_2 < 2p/b'(0) \leq m_1$, and (iii) $m_1 < 2p/b'(0)$. In case (i), for each district $i$, $f(m_i(1-\kappa))$ satisfies the first order condition $m_i(1-\kappa)b'(f(m_i(1-\kappa))) = p$ for all $\kappa$. This means that for all $\kappa$

$$\begin{aligned} \frac{dS^d}{d\kappa} &= (m_2-m_1)[b(f(m_1(1-\kappa))) - b(f(m_2(1-\kappa)))] \\ &\quad -m_2\kappa p\frac{f'(m_1(1-\kappa))}{1-\kappa} - m_1\kappa p\frac{f'(m_2(1-\kappa))}{1-\kappa}, \end{aligned}$$

where $f'(\alpha) = -b'(f(\alpha))/\alpha b''(f(\alpha))$. Since this is negative, surplus under decentralization is decreasing in this case.

In case (ii), for district 1, $f(m_1(1-\kappa))$ satisfies the first order condition $m_1(1-\kappa)b'(f(m_1(1-\kappa))) = p$ for all $\kappa$. However, for district 2, $f(m_2(1-\kappa))$ satisfies the first order condition $m_2(1-\kappa)b'(f(m_2(1-\kappa))) = p$ for all $\kappa \leq 1-p/m_2b'(0)$, while $f(m_2(1-\kappa)) = 0$ for all $\kappa > 1-p/m_2b'(0)$. Thus, for all $\kappa < 1-p/m_2b'(0)$

$$\begin{aligned} \frac{dS^d}{d\kappa} &= (m_2-m_1)[b(f(m_1(1-\kappa))) - b(f(m_2(1-\kappa)))] \\ &\quad -m_2\kappa p\frac{f'(m_1(1-\kappa))}{1-\kappa} - m_1\kappa p\frac{f'(m_2(1-\kappa))}{1-\kappa}, \end{aligned}$$

and for all $\kappa > 1-p/m_2b'(0)$

$$\frac{dS^d}{d\kappa} = (m_2-m_1)b(f(m_1(1-\kappa))) - m_2\kappa p\frac{f'(m_1(1-\kappa))}{1-\kappa}.$$

Since both these derivatives are negative, surplus under decentralization is again decreasing.

In case (iii), for each district $i$, $f(m_i(1-\kappa))$ satisfies the first order condition $m_i(1-\kappa)b'(f(m_i(1-\kappa))) = p$ for all $\kappa \leq 1 - p/m_2 b'(0)$. For $\kappa \in (1-p/m_2b'(0), 1-p/m_1b'(0)]$, $f(m_1(1-\kappa))$ satisfies the first order condition $m_1(1-\kappa)b'(f(m_1(1-\kappa))) = p$ while $f(m_2(1-\kappa)) = 0$. For $\kappa > 1 - p/m_1b'(0)$, for each district $i$, $f(m_i(1-\kappa)) = 0$. Thus, for all $\kappa < 1 - p/m_2b'(0)$

$$\begin{aligned}\frac{dS^d}{d\kappa} &= (m_2 - m_1)[b(f(m_1(1-\kappa))) - b(f(m_2(1-\kappa)))] \\ &\quad -m_2\kappa p\frac{f'(m_1(1-\kappa))}{1-\kappa} - m_1\kappa p\frac{f'(m_2(1-\kappa))}{1-\kappa};\end{aligned}$$

for all $\kappa \in (1-p/m_2b'(0), 1-p/m_1b'(0))$,

$$\frac{dS^d}{d\kappa} = (m_2-m_1)b(f(m_1(1-\kappa))) - m_2\kappa p\frac{f'(m_1(1-\kappa))}{1-\kappa},$$

and for all $\kappa > 1 - p/m_1b'(0)$

$$\frac{dS^d}{d\kappa} = 0.$$

It follows that surplus under decentralization is decreasing for all $\kappa < 1 - p/m_1b'(0)$ and constant thereafter. Notice that $S^d(\kappa) = S^d(\frac{1}{2})$ for all $\kappa > 1 - p/m_1b'(0)$ and hence that $S^d(\kappa) = S^d(\frac{1}{2}) < S^c_o(\frac{1}{2}) = S^c_o(\kappa)$ for all $\kappa > 1 - p/m_1b'(0)$. ∎

**Proof of Proposition 2:** Aggregate public goods surplus under decentralization is as for Oates analysis, while surplus under centralization is:

$$\begin{aligned}S^c_n(\kappa) &= \frac{1}{2}\sum_{i\in\{1,2\}}\{[m_i(1-\kappa)+m_{-i}\kappa]b(f(2m_i(1-\kappa))) + [m_{-i}(1-\kappa)+m_i\kappa]b(f(2m_i\kappa)) \\ &\quad -p[f(2m_i(1-\kappa)) + f(2m_i\kappa)]\}.\end{aligned}$$

For part (i), we know from the earlier discussion that when $m_1 = m_2$, $S^c_n(0) < S^d(0)$ and $S^c_o(\frac{1}{2}) > S^d(\frac{1}{2})$. We also know from the proof of Proposition 1, that surplus is non-increasing in spillovers under decentralization. Thus, it suffices to show that surplus is increasing in spillovers under centralization.

When $m_2 = m_1 = m$, $f(2m\kappa)$ satisfies the first order condition $2m\kappa b'(f(2m\kappa)) = p$ for all $\kappa \geq p/2mb'(0)$ while $f(2m\kappa) = 0$ for all $\kappa < p/2mb'(0)$. Thus, for all $\kappa < p/2mb'(0)$

$$\frac{\partial S^c_n}{\partial\kappa} = m(1-2\kappa)p\frac{f'(2m(1-\kappa))}{1-\kappa},$$

and for all $\kappa > p/2mb'(0)$

$$\frac{\partial S^c_n}{\partial\kappa} = m(1-2\kappa)p\{\frac{f'(2m(1-\kappa))}{1-\kappa} + \frac{f'(2m\kappa)}{\kappa}\}.$$

Both derivatives are positive, as required.

For part (ii), note that when $m_1 \neq m_2$, $S_n^c(0) < S^d(0)$. Since both surplus functions are continuous functions of $\kappa$, for each $(m_1, m_2)$ there exists $\varepsilon > 0$ such that $S_n^c(\kappa) < S^d(\kappa)$ for all $\kappa < \varepsilon$. ■

**Proof of Proposition 3:** Let $S^d(\gamma, \kappa)$ and $S_n^c(\gamma, \kappa)$ denote surplus under decentralization and centralization when $m_1 = \xi\gamma$; $m_2 = \xi(1-\gamma)$; and $b(g) = \ln(1+g)$. What we need to show is that there exists $\widehat{\gamma} \in [\frac{1}{2}, 1 - p/\xi)$ with the property that for all $\gamma \in (\widehat{\gamma}, 1 - p/\xi)$, $S^d(\gamma, \frac{1}{2}) > S_n^c(\gamma, \frac{1}{2})$. Evaluating the surplus expressions, we have that for $\gamma > 1 - 2p/\xi$,

$$S^d(\gamma, \frac{1}{2}) = \frac{\xi}{2} \ln \frac{\xi\gamma}{2p} - \frac{\xi\gamma}{2} + p,$$

and

$$S_n^c(\gamma, \frac{1}{2}) = \frac{\xi}{2}\{\ln \frac{\xi\gamma}{p} + \ln \frac{\xi(1-\gamma)}{p}\} - \xi + 2p.$$

Thus, taking limits we obtain

$$\lim_{\gamma \to 1-p/\xi} S^d(\gamma, \frac{1}{2}) = \frac{\xi}{2}\{\ln(\frac{\xi}{2p}(1-p/\xi)) - (1 - \frac{3p}{\xi})\},$$

and

$$\lim_{\gamma \to 1-p/\xi} S_n^c(\gamma, \frac{1}{2}) = \frac{\xi}{2}\{\ln(\frac{\xi}{p}(1-p/\xi)) - (2 - \frac{4p}{\xi})\}.$$

Thus, we have that

$$\begin{aligned}
\lim_{\gamma \to 1-p/\xi} S^d(\gamma, \frac{1}{2}) - \lim_{\gamma \to 1-p/\xi} S_n^c(\gamma, \frac{1}{2}) &= \frac{\xi}{2}\{1 - p/\xi - \ln 2\} \\
&> \frac{\xi}{2}\{0.3 - p/\xi\} \\
&> \frac{\xi}{2}\{0.3 - 0.25\} > 0.
\end{aligned}$$

It follows that there must exist $\widehat{\gamma} \in [\frac{1}{2}, 1 - p/\xi)$ with the property that for all $\gamma \in (\widehat{\gamma}, 1 - p/\xi)$, $S^d(\gamma, \frac{1}{2}) > S_n^c(\gamma, \frac{1}{2})$. ■

**Proof of Proposition 4:** We will show that, under the stated conditions, $S_n^c(\kappa) - S^d(\kappa)$ is an increasing function of $\kappa$. We begin by obtaining expressions for the derivatives $\frac{dS^d}{d\kappa}$ and $\frac{dS_n^c}{d\kappa}$. In the proof of Proposition 1, we showed that, under the assumption that $m_2 > 2p/b'(0)$, for each district $i$, $f(m_i(1-\kappa))$ satisfies the first order condition $m_i(1-\kappa)b'(f(m_i(1-\kappa))) = p$ for all $\kappa$, implying that for all $\kappa$

$$\begin{aligned}
\frac{dS^d}{d\kappa} &= (m_2 - m_1)[b(f(m_1(1-\kappa))) - b(f(m_2(1-\kappa)))] \\
&\quad - m_2\kappa p \frac{f'(m_1(1-\kappa))}{1-\kappa} - m_1\kappa p \frac{f'(m_2(1-\kappa))}{1-\kappa}.
\end{aligned}$$

Turning to centralization, for all $\kappa < p/2m_1b'(0)$, $f(2m_i\kappa) = 0$ for each district $i$ and hence

$$\begin{aligned}\frac{dS_n^c}{d\kappa} &= \frac{1}{2}\{(m_2-m_1)[b(f(2m_1(1-\kappa))) - b(f(2m_2(1-\kappa))]\\ &+[m_1(1-\kappa)-m_2\kappa]\frac{pf'(2m_1(1-\kappa))}{1-\kappa} + [m_2(1-\kappa)-m_1\kappa]\frac{pf'(2m_2(1-\kappa))}{1-\kappa}\}.\end{aligned}$$

For all $\kappa \in (p/2m_1b'(0), p/2m_2b'(0))$, we know that $f(2m_1\kappa)$ satisfies the first order condition $2m_1\kappa b'(f(2m_1\kappa)) = p$, while $f(2m_2\kappa) = 0$. Thus,

$$\begin{aligned}\frac{dS_n^c}{d\kappa} &= \frac{1}{2}\{(m_2-m_1)[b(f(2m_1(1-\kappa))) - b(f(2m_1\kappa)) - b(f(2m_2(1-\kappa))]\\ &+[m_1(1-\kappa)-m_2\kappa]\frac{pf'(2m_1(1-\kappa))}{1-\kappa}\\ &+[m_2(1-\kappa)-m_1\kappa]\frac{pf'(2m_1\kappa)}{\kappa} + [m_2(1-\kappa)-m_1\kappa]\frac{pf'(2m_2(1-\kappa))}{1-\kappa}\}.\end{aligned}$$

For all $\kappa > p/2m_2b'(0)$, we know that $f(2m_i\kappa)$ satisfies the first order condition $2m_i\kappa b'(f(2m_i\kappa)) = p$ for each district $i$. Thus,

$$\begin{aligned}\frac{dS_n^c}{d\kappa} &= \frac{1}{2}\{(m_2-m_1)[b(f(2m_1(1-\kappa))) - b(f(2m_1\kappa)) + b(f(2m_2\kappa))) - b(f(2m_2(1-\kappa))]\\ &+[m_1(1-\kappa)-m_2\kappa]\frac{pf'(2m_1(1-\kappa))}{1-\kappa} + [m_2(1-\kappa)-m_1\kappa]\frac{pf'(2m_1\kappa)}{\kappa}\\ &+[m_1(1-\kappa)-m_2\kappa]\frac{pf'(2m_2\kappa)}{\kappa} + [m_2(1-\kappa)-m_1\kappa]\frac{pf'(2m_2(1-\kappa))}{1-\kappa}\}.\end{aligned}$$

We now establish the following two claims.

**Claim 1:** For all $\kappa \in (p/2m_1b'(0), p/2m_2b'(0))$,

$$\begin{aligned}\frac{dS_n^c}{d\kappa} &\geq \frac{1}{2}\{(m_2-m_1)[b(f(2m_1(1-\kappa))) - b(f(2m_2(1-\kappa))] +\\ &[m_1(1-\kappa)-m_2\kappa]\frac{pf'(2m_1(1-\kappa))}{1-\kappa} + [m_2(1-\kappa)-m_1\kappa]\frac{pf'(2m_2(1-\kappa))}{1-\kappa}\}.\end{aligned}$$

To prove this, it is enough to show that

$$(m_1-m_2)b(f(2m_1\kappa)) \geq [m_1\kappa - m_2(1-\kappa)]\frac{pf'(2m_1\kappa)}{\kappa}.$$

The inequality holds if $m_1\kappa \leq m_2(1-\kappa)$ so we may assume that $m_1\kappa > m_2(1-\kappa)$. By assumption, $b(\cdot)$ is a strictly concave function satisfying $b(0) = 0$. Moreover, for $\alpha \geq p/b'(0)$, we have that $f$ is increasing and concave. It follows that the composite function $b(f(\cdot))$ is a concave function on the interval $[p/b'(0), \infty)$. Thus,

$$\begin{aligned}b(f(2m_1\kappa)) &= b(f(2m_1\kappa)) - b(0) \geq b'(f(2m_1\kappa))f'(2m_1\kappa)2m_1\kappa\\ &= pf'(2m_1\kappa).\end{aligned}$$

It is therefore enough to show that

$$m_1 - m_2 \geq [m_1\kappa - m_2(1-\kappa)]\frac{1}{\kappa} = m_1 - m_2\frac{(1-\kappa)}{\kappa}$$

This follows from the fact that $\kappa \leq \frac{1}{2}$.

**Claim 2:** For all $\kappa > p/2m_2 b'(0)$,

$$\begin{aligned}\frac{dS_n^c}{d\kappa} \quad &\geq \quad \frac{1}{2}\{(m_2 - m_1)[b(f(2m_1(1-\kappa))) - b(f(2m_2(1-\kappa))] \\ &\quad + [m_1(1-\kappa) - m_2\kappa]\frac{pf'(2m_1(1-\kappa))}{1-\kappa} \\ &\quad + [m_2(1-\kappa) - m_1\kappa]\frac{pf'(2m_2(1-\kappa))}{1-\kappa}\}.\end{aligned}$$

To prove this, it is enough to show that

$$[m_1(1-\kappa) - m_2\kappa]f'(2m_2\kappa) \geq [m_1\kappa - m_2(1-\kappa)]f'(2m_1\kappa).$$

This is clearly true when $m_1\kappa \leq m_2(1-\kappa)$. But when $m_1\kappa > m_2(1-\kappa)$, the result follows from the assumption that $f''(\alpha) \leq 0$ for all $\alpha \geq p/b'(0)$.

To compare how surplus changes with spillovers under the two regimes, we will need the following two claims.

**Claim 3:** For all $\kappa$,

$$\frac{1}{2}[b(f(2m_1(1-\kappa))) - b(f(2m_2(1-\kappa))] \leq b(f(m_1(1-\kappa))) - b(f(m_2(1-\kappa))).$$

Defining the function

$$\phi(\lambda) = b(f(\lambda m_1(1-\kappa))) - b(f(\lambda m_2(1-\kappa))),$$

it is enough to show that $\phi(2) \leq \phi(1)$. This must be the case if $\phi'(\lambda) \leq 0$ for all $\lambda \in [1,2]$. Differentiating, we obtain

$$\phi'(\lambda) = [f'(\lambda m_1(1-\kappa)) - f'(\lambda m_2(1-\kappa))]/\lambda$$

This is negative, since $f''(\alpha) \leq 0$ for all $\alpha \geq p/b'(0)$.

**Claim 4:** For all $\kappa$,

$$[m_1(1-\kappa) - m_2\kappa]f'(2m_1(1-\kappa)) > [m_1\kappa - m_2(1-\kappa)]f'(2m_2(1-\kappa)).$$

The result is obvious if $m_1\kappa \leq m_2(1-\kappa)$, so suppose that $m_1\kappa > m_2(1-\kappa)$. Defining the function

$$\phi(\lambda) = [m_1(1-\lambda) - m_2\lambda]f'(2m_1(1-\lambda)),$$

we need to show that $\phi(\kappa) > \phi(1-\kappa)$. Hence, it is enough to show that $\phi'(\lambda) < 0$ for $\lambda \in [\kappa, 1-\kappa]$. Differentiating, we obtain

$$\phi(\lambda) = -[m_1 + m_2]f'(2m_1(1-\lambda)) - [m_1(1-\lambda) - m_2\lambda]f''(2m_1(1-\lambda))2m_1.$$

Thus, we require that

$$-[m_1(1-\lambda) - m_2\lambda]f''(2m_1(1-\lambda))2m_1 < [m_1 + m_2]f'(2m_1(1-\lambda))$$

or, equivalently, that

$$-\frac{f''(2m_1(1-\lambda))2m_1(1-\lambda)}{f'(2m_1(1-\lambda))} < \frac{[m_1 + m_2]}{[m_1 - m_2\frac{\lambda}{(1-\lambda)}]}$$

But, by hypothesis, we know that:

$$-\frac{f''(2m_1(1-\lambda))2m_1(1-\lambda)}{f'(2m_1(1-\lambda))} < \frac{m_2}{m_1 - m_2}.$$

Thus, it is enough to show that

$$\frac{m_2}{m_1 - m_2} < \frac{[m_1 + m_2]}{[m_1 - m_2\frac{\lambda}{(1-\lambda)}]}.$$

We know that $\frac{\lambda}{1-\lambda} \geq \frac{\kappa}{1-\kappa} > \frac{m_2}{m_1}$. Thus, it is enough to show that

$$\frac{m_2}{m_1 - m_2} < \frac{m_1[m_1 + m_2]}{[m_1^2 - m_2^2]} = \frac{m_1}{m_1 - m_2}.$$

This follows from the fact that $m_1 > m_2$. (maybe should change the assumption here)

Combining these four claims, we conclude that for all $\kappa$ at which $\frac{dS_n^c}{d\kappa}$ exists, we have that

$$\frac{dS_n^c}{d\kappa} \geq (m_2 - m_1)[b(f(m_1(1-\kappa))) - b(f(m_2(1-\kappa)))]$$

and hence that $\frac{dS_n^c}{d\kappa} > \frac{dS^d}{d\kappa}$. It follows that the relative performance of centralization is increasing in $\kappa$. ■

**Proof of Lemma 3:** As observed in the text, $(\lambda_1^*, \lambda_2^*)$ is majority preferred under centralization with a cooperative legislature if and only if $(\lambda_1^*, \lambda_2^*)$ is a Nash equilibrium of the two player game in which each player has strategy set $[0, \overline{\lambda}]$ and player $i \in \{1, 2\}$ has payoff function $U_i(\lambda_1, \lambda_2, m_i)$. We prove the Lemma by calculating the set of equilibria of this game and computing the associated policy outcomes.

(i) We establish part (i) of the Lemma via a series of claims.

**Claim 1:** For all $\kappa$

$$\lambda_1^* = \lambda_2^* = 2m[(1-\kappa)^2 + \kappa^2]$$

is an equilibrium.

We need to show that if one district's median voter is electing a type $2m[(1-\kappa)^2+\kappa^2]$ candidate, then the other district's median voter wishes to do the same. Let $\upsilon(\lambda_1) = U_1(\lambda_1, \lambda_2^*, m)$ denote the payoff of district 1's median voter when he selects a representative of type $\lambda_1$ and district 2's representative is of type $\lambda_2^* = 2m[(1-\kappa)^2 + \kappa^2]$. Then, we have that

$$\upsilon(\lambda_1) = m[(1-\kappa)b(f(1)) + \kappa b(f(2))] - \frac{p}{2}(f(1) + f(2)),$$

where $f(1) = f(\lambda_1(1-\kappa) + \lambda_2^*\kappa)$ and $f(2) = f(\lambda_2^*(1-\kappa) + \lambda_1\kappa)$.

It is straightforward to show that district 1's median voter will always want to choose a representative type such that $f(1) > 0$. Thus, we may assume that $\lambda_1(1-\kappa) + \lambda_2^*\kappa > b'(0)/p$. Then, differentiating, we obtain

$$\upsilon'(\lambda_1) = [m(1-\kappa)^2 b'(f(1)) - \frac{p}{2}(1-\kappa)]f'(1) - [\frac{p}{2}\kappa - m\kappa^2 b'(f(2))]f'(2).$$

where $f'(1) = f'(\lambda_1(1-\kappa) + \lambda_2^*\kappa)$ and $f'(2) = f'(\lambda_2^*(1-\kappa) + \lambda_1\kappa)$. Using the fact that $b'(f(\alpha)) = p/\alpha$, we may write:

$$\begin{aligned}\upsilon'(\lambda_1) &= [\frac{m(1-\kappa)^2}{\lambda_1(1-\kappa)+\lambda_2^*\kappa} - \frac{(1-\kappa)}{2}]pf'(1) - [\frac{\kappa}{2} - \frac{m\kappa^2}{\lambda_2^*(1-\kappa)+\lambda_1\kappa}]pf'(2).\\ &= [\frac{2m(1-\kappa)^2 - (\lambda_1(1-\kappa)^2 + \lambda_2^*\kappa^2)}{2(\lambda_1(1-\kappa)+\lambda_2^*\kappa)}]pf'(1) - [\frac{\lambda_2^*(1-\kappa)\kappa + \lambda_1\kappa^2 - 2m\kappa^2}{2(\lambda_2^*(1-\kappa)+\lambda_1\kappa)}]pf'(2).\end{aligned}$$

Observe that, if $\lambda_1 < \lambda_2^*$, then $\lambda_1(1-\kappa) + \lambda_2^*\kappa \leq \lambda_2^*(1-\kappa) + \lambda_1\kappa$ and $f'(1) \geq f'(2)$. Moreover,

$$\begin{aligned}& 2m(1-\kappa)^2 - (\lambda_1(1-\kappa)^2 + \lambda_2^*\kappa^2) - [\lambda_2^*(1-\kappa)\kappa + \lambda_1\kappa^2 - 2m\kappa^2]\\ =\ & 2m[(1-\kappa)^2 + \kappa^2] - \lambda_1[(1-\kappa)^2 + \kappa^2] - \lambda_2^*[(1-\kappa)\kappa + \kappa^2]\\ >\ & 2m[(1-\kappa)^2 + \kappa^2][1 - (1-\kappa)^2 - \kappa^2 - (1-\kappa)\kappa - \kappa^2] = 0.\end{aligned}$$

This implies that $\upsilon'(\lambda_1) > 0$. A similar argument implies that if $\lambda_1 > \lambda_2^*$, then $\upsilon'(\lambda_1) < 0$. Thus, $\lambda_1 = \lambda_2^*$ is the choice of representative that maximizes the district 1 median voter's payoff.

**Claim 2:** If $\kappa < 1/2$ and $(\lambda_1^*, \lambda_2^*)$ is an equilibrium then

$$\lambda_1^* = \lambda_2^* = 2m[(1-\kappa)^2 + \kappa^2].$$

It is straightforward to show that if $(\lambda_1^*, \lambda_2^*)$ is an equilibrium then $f(\lambda_1^*(1-\kappa) + \lambda_2^*\kappa) > 0$ and $f(\lambda_2^*(1-\kappa) + \lambda_1^*\kappa) > 0$, so we assume this in what follows.

The next point to note is that $\lambda_1^* = \lambda_2^*$. To see this, suppose, to the contrary, that $\lambda_1^* > \lambda_2^*$. Then, the Kuhn Tucker conditions imply that

$$\frac{\partial U_1(\lambda_1^*, \lambda_2^*, m)}{\partial \lambda_1} \geq 0 \geq \frac{\partial U_2(\lambda_1^*, \lambda_2^*, m)}{\partial \lambda_2}.$$

We may write

$$\frac{\partial U_1}{\partial \lambda_1} = [\frac{2m(1-\kappa)^2 - (\lambda_1^*(1-\kappa)^2 + \lambda_2^*\kappa^2)}{2(\lambda_1^*(1-\kappa) + \lambda_2^*\kappa)}]pf'(1) - [\frac{\lambda_2^*(1-\kappa)\kappa + \lambda_1^*\kappa^2 - 2m\kappa^2}{2(\lambda_2^*(1-\kappa) + \lambda_1^*\kappa)}]pf'(2)$$

and

$$\frac{\partial U_2}{\partial \lambda_2} = [\frac{2m(1-\kappa)^2 - (\lambda_2^*(1-\kappa)^2 + \lambda_1^*\kappa^2)}{2(\lambda_2^*(1-\kappa) + \lambda_1^*\kappa)}]pf'(2) - [\frac{\lambda_1^*(1-\kappa)\kappa + \lambda_2^*\kappa^2 - 2m\kappa^2}{2(\lambda_1^*(1-\kappa) + \lambda_2^*\kappa)}]pf'(1).$$

Since $f'(2) > f'(1)$, the Kuhn Tucker conditions and these expressions imply that

$$\frac{[2m(1-\kappa)^2 - (\lambda_1^*(1-\kappa)^2 + \lambda_2^*\kappa^2)](\lambda_2^*(1-\kappa) + \lambda_1^*\kappa)}{[\lambda_2^*(1-\kappa)\kappa + \lambda_1^*\kappa^2 - 2m\kappa^2](\lambda_1^*(1-\kappa) + \lambda_2^*\kappa)} \geq \frac{f'(2)}{f'(1)} > 1,$$

and

$$\frac{[\lambda_1^*(1-\kappa)\kappa + \lambda_2^*\kappa^2 - 2m\kappa^2](\lambda_2^*(1-\kappa) + \lambda_1^*\kappa)}{[2m(1-\kappa)^2 - (\lambda_2^*(1-\kappa)^2 + \lambda_1^*\kappa^2)](\lambda_1^*(1-\kappa) + \lambda_2^*\kappa)} \geq \frac{f'(2)}{f'(1)} > 1.$$

Since

$$\frac{\lambda_2^*(1-\kappa) + \lambda_1^*\kappa}{\lambda_1^*(1-\kappa) + \lambda_2^*\kappa} < 1,$$

these two inequalities imply that

$$\frac{2m(1-\kappa)^2 - (\lambda_1^*(1-\kappa)^2 + \lambda_2^*\kappa^2)}{\lambda_2^*(1-\kappa)\kappa + \lambda_1^*\kappa^2 - 2m\kappa^2} > 1$$

and

$$\frac{\lambda_1^*(1-\kappa)\kappa + \lambda_2^*\kappa^2 - 2m\kappa^2}{2m(1-\kappa)^2 - (\lambda_2^*(1-\kappa)^2 + \lambda_1^*\kappa^2)} > 1.$$

These two inequalities, in turn, imply that

$$2m[(1-\kappa)^2 + \kappa^2] > \lambda_1^*[(1-\kappa)^2 + \kappa^2] + \lambda_2^*[(1-\kappa)\kappa + \kappa^2]$$

and

$$2m[(1-\kappa)^2 + \kappa^2] < \lambda_2^*[(1-\kappa)^2 + \kappa^2] + \lambda_1^*[(1-\kappa)\kappa + \kappa^2].$$

But this is inconsistent with the hypothesis that $\lambda_1^* > \lambda_2^*$.

Thus, we know that $\lambda_1^* = \lambda_2^* = \lambda^*$. It is straightforward to show that $\lambda^* < 2m$ and thus $\frac{\partial U_1(\lambda^*,\lambda^*,m)}{\partial \lambda_1} = 0$. This equation implies that

$$2m(1-\kappa)^2 - \lambda^*[(1-\kappa)^2 + \kappa^2] = \lambda^*[(1-\kappa)\kappa + \kappa^2] - 2m\kappa^2.$$

Rearranging and solving, yields

$$\lambda^* = 2m[(1-\kappa)^2 + \kappa^2].$$

**Claim 3:** If $\kappa = 1/2$ and $(\lambda_1^*, \lambda_2^*)$ is an equilibrium then

$$\lambda_1^* + \lambda_2^* = 2m.$$

To see this, simply note that when $\kappa = 1/2$, we have that

$$\frac{\partial U_1(\lambda_1, \lambda_2, m)}{\partial \lambda_1} = [\frac{m - (\frac{\lambda_1+\lambda_2}{2})}{\lambda_1 + \lambda_2}]pf'(\frac{\lambda_1+\lambda_2}{2}).$$

It follows from these claims that when $\kappa < 1/2$ there is a unique majority preferred pair that generates the policy outcome $g_1 = g_2 = f(2m[(1-\kappa)^2+\kappa^2])$. When $\kappa = 1/2$ there are multiple majority preferred pairs, but they all generate the policy outcome $g_1 = g_2 = f(m)$. This proves part (i) of the Lemma.

(ii) We now turn to part (ii) of the Lemma. Note first that if $b(g) = \ln(g+1)$, then for $\alpha \geq p$, $f(\alpha) = \frac{\alpha}{p} - 1$. When $f''(\alpha) = 0$, each player's payoff function is a twice continuously differentiable and strictly concave function of his strategy and each player's strategy set is compact and convex. Thus, the set of equilibria is non-empty. Moreover, $\frac{\partial^2 U_1}{\partial \lambda_1 \partial \lambda_2} < 0$ and $\frac{\partial^2 U_2}{\partial \lambda_2 \partial \lambda_1} < 0$, implying that types are strategic substitutes.

For $i = 1, 2$, let $r_i : [0, \overline{\lambda}] \to [0, \overline{\lambda}]$ denote the district $i$ median voter's *reaction function.* By definition, for all $\lambda_2 \in [0, \overline{\lambda}]$,

$$r_1(\lambda_2) = \arg\max\{U_1(r_1, \lambda_2, m_1) : r_1 \in [0, \overline{\lambda}]\},$$

and for all $\lambda_1 \in [0, \overline{\lambda}]$,

$$r_2(\lambda_1) = \arg\max\{U_2(\lambda_1, r_2, m_2) : r_2 \in [0, \overline{\lambda}]\}.$$

Then, $(\lambda_1^*, \lambda_2^*)$ is an equilibrium of the game if and only if $(\lambda_1^*, \lambda_2^*) = (r_1(\lambda_2^*), r_2(\lambda_1^*))$.

Some general features of the reaction functions follow from the properties of the payoff functions. The fact that each player's payoff is a strictly concave and differentiable function of his strategy implies (i) that $r_1(\lambda_2) = 0$ if $\partial U_1(0, \lambda_2, m_1)/\partial \lambda_1 < 0$; (ii) that $r_1(\lambda_2) = \overline{\lambda}$ if $\partial U_1(\overline{\lambda}, \lambda_2, m_1)/\partial \lambda_1 > 0$; and (iii) that otherwise $r_1(\lambda_2)$ is implicitly defined by the first order condition $\partial U_1(r_1(\lambda_2), \lambda_2, m_1)/\partial \lambda_1 = 0$. In addition, the fact that types are strategic substitutes implies that $r_1(\lambda_2)$ is non-decreasing. Similar remarks apply to the district 2 median voter's reaction function

It remains therefore to determine the details of each player's reaction function. Let $\overline{\lambda}_1(\overline{\lambda}_2)$ denote the level of $\lambda_1(\lambda_2)$ beyond which district 2's median voter (district 1's median voter) would like a type 0 representative. These levels are implicitly defined by the equalities

$$\partial U_1(0, \overline{\lambda}_2, m_1)/\partial \lambda_1 = 0,$$

and

$$\partial U_2(\overline{\lambda}_1, 0, m_2)/\partial \lambda_2 = 0.$$

Using the facts that

$$\frac{\partial U_1}{\partial \lambda_1} = p\{m_1[\frac{(1-\kappa)^2}{\lambda_1(1-\kappa)+\lambda_2\kappa} + \frac{\kappa^2}{\lambda_2(1-\kappa)+\lambda_1\kappa}] - \frac{1}{2}\},$$

and

$$\frac{\partial U_2}{\partial \lambda_2} = p\{m_2[\frac{(1-\kappa)^2}{\lambda_2(1-\kappa)+\lambda_1\kappa} + \frac{\kappa^2}{\lambda_1(1-\kappa)+\lambda_2\kappa}] - \frac{1}{2}\},$$

we obtain

$$\overline{\lambda}_1 = 2m_2\{\frac{(1-\kappa)^3+\kappa^3}{\kappa(1-\kappa)}\}$$

and

$$\overline{\lambda}_2 = 2m_1\{\frac{(1-\kappa)^3+\kappa^3}{\kappa(1-\kappa)}\},$$

Observe that $\frac{(1-\kappa)^3+\kappa^3}{\kappa(1-\kappa)}$ is decreasing in $\kappa$, takes on the value 1 when $\kappa = 1/2$ and tends to infinity as $\kappa$ goes to zero. This implies that $\overline{\lambda}_1 \geq 2m_2$ and $\overline{\lambda}_2 \geq 2m_1$.

Next we characterize the highest type representative each district's median voter would want. It is straightforward to show that

$$\frac{\partial U_1(2m_1, 0, m_1)}{\partial \lambda_1} = 0$$

and

$$\frac{\partial U_2(0, 2m_2, m_2)}{\partial \lambda_2} = 0,$$

which implies that district $i$'s median voter desires a type $2m_i$ candidate when the other district selects a type 0 candidate. By assumption, $2m_i < \overline{\lambda}$, so that the upper bound constraint on type choice is not binding here. It follows that for both districts $i = 1, 2$, $r_i(0) = 2m_i$.

We may conclude from the above that for all $\lambda_2 \in [0, \min\{\overline{\lambda}_2, \overline{\lambda}\}]$, $r_1(\lambda_2)$ is implicitly defined by the first order condition

$$\frac{\partial U_1(r_1(\lambda_2), \lambda_2, m_1)}{\partial \lambda_1} = 0$$

and for all $\lambda_2 \in (\min\{\overline{\lambda}_2, \overline{\lambda}\}, \overline{\lambda}]$,

$$r_1(\lambda_2) = 0.$$

Further, we know that $r_1(0) = 2m_1$ and that $r_1(\lambda_2)$ is downward sloping on $[0, \min\{\overline{\lambda}_2, \overline{\lambda}\}]$. Similarly, for all $\lambda_1 \in [0, \min\{\overline{\lambda}_1, \overline{\lambda}\}]$, $r_2(\lambda_1)$ is implicitly defined by the first order condition

$$\frac{\partial U_2(\lambda_1, r_2(\lambda_1), m_2)}{\partial \lambda_2} = 0$$

and for all $\lambda_1 \in (\min\{\overline{\lambda}_1, \overline{\lambda}\}, \overline{\lambda}]$,

$$r_2(\lambda_1) = 0.$$

Further, we know that $r_2(0) = 2m_2$ and that $r_2(\lambda_1)$ is downward sloping on $[0, \min\{\overline{\lambda}_1, \overline{\lambda}\}]$.

We can now prove the lemma. If $\kappa < \widehat{\kappa}$, it follows from the definition of $\widehat{\kappa}$ in the text that $\frac{(1-\kappa)^3+\kappa^3}{\kappa(1-\kappa)} > \frac{m_1}{m_2}$. This in turn implies that

$$\overline{\lambda}_1 = 2m_2\{\frac{(1-\kappa)^3 + \kappa^3}{\kappa(1-\kappa)}\} > 2m_1.$$

This inequality implies that there exist no boundary equilibria in which $\lambda_i^* = 0$ for one or more districts. If $\lambda_2^* = 0$, then $\lambda_1^* = r_1(0) = 2m_1$, but since $2m_1 < \overline{\lambda}_1$ we know that $r_2(2m_1) > 0$ which contradicts the fact that $\lambda_2^* = 0$. If $\lambda_1^* = 0$, then $\lambda_2^* = r_2(0) = 2m_2$, but since $2m_2 < \overline{\lambda}_2$ we know that $r_1(2m_2) > 0$ which contradicts the fact that $\lambda_1^* = 0$. Since $\max r_i(\lambda_{-i}) < \overline{\lambda}$, it is apparent that there can be no boundary equilibria in which $\lambda_i^* = \overline{\lambda}$ for one or more districts.

It follows that there must exist an interior equilibrium. Any such equilibrium $(\lambda_1^*, \lambda_2^*)$ must satisfy the first order conditions $\frac{\partial U_i(\lambda_1^*, \lambda_2^*, m_i)}{\partial \lambda_i} = 0$ for $i \in \{1, 2\}$. Using the expressions for $\frac{\partial U_i}{\partial \lambda_i}$, $i \in \{1, 2\}$ from above, we may write these first order conditions as:

$$m_1[\frac{(1-\kappa)^2}{\lambda_1^*(1-\kappa) + \lambda_2^*\kappa} + \frac{\kappa^2}{\lambda_2^*(1-\kappa) + \lambda_1^*\kappa}] = \frac{1}{2},$$

and

$$m_2[\frac{(1-\kappa)^2}{\lambda_2^*(1-\kappa) + \lambda_1^*\kappa} + \frac{\kappa^2}{\lambda_1^*(1-\kappa) + \lambda_2^*\kappa}] = \frac{1}{2}.$$

Combining the two first order conditions, we obtain

$$\lambda_1^*(1-\kappa)+\lambda_2^*\kappa=[\lambda_2^*(1-\kappa)+\lambda_1^*\kappa]\frac{m_1(1-\kappa)^2-m_2\kappa^2}{m_2(1-\kappa)^2-m_1\kappa^2}.$$

Using this and the first order conditions for $\lambda_1^*$ and $\lambda_2^*$ respectively yields:

$$\lambda_1^*(1-\kappa)+\lambda_2^*\kappa=\frac{2m_1m_2[(1-\kappa)^4-\kappa^4]}{m_2(1-\kappa)^2-m_1\kappa^2}$$

and

$$\lambda_2^*(1-\kappa)+\lambda_1^*\kappa=\frac{2m_1m_2[(1-\kappa)^4-\kappa^4]}{m_1(1-\kappa)^2-m_2\kappa^2}.$$

Thus, as claimed, the policy outcome is

$$(g_1,g_2)=(f(\frac{2m_1m_2[(1-\kappa)^4-\kappa^4]}{m_2(1-\kappa)^2-m_1\kappa^2}),f(\frac{2m_1m_2[(1-\kappa)^4-\kappa^4]}{m_1(1-\kappa)^2-m_2\kappa^2})).$$

If $\kappa\geq\widehat{\kappa}$, it follows that $\frac{(1-\kappa)^3+\kappa^3}{\kappa(1-\kappa)}\leq\frac{m_1}{m_2}$, which in turn implies that

$$\overline{\lambda}_1=2m_2\{\frac{(1-\kappa)^3+\kappa^3}{\kappa(1-\kappa)}\}\leq 2m_1.$$

This inequality implies that there exists a boundary equilibrium in which $(\lambda_1^*,\lambda_2^*)=(2m_1,0)$. This is because $r_2(2m_1)=0$ and $r_1(0)=2m_1$. The same arguments from above imply that there exist no other boundary equilibria. We also claim that there are no interior equilibria. Any such equilibrium $(\lambda_1^*,\lambda_2^*)$ must satisfy the first order conditions $\frac{\partial U_i(\lambda_1^*,\lambda_2^*,m_i)}{\partial\lambda_i}=0$ for $i\in\{1,2\}$. These first order conditions imply that

$$\begin{aligned}&m_1[\lambda_2^*(1-\kappa)^3+\lambda_1^*\kappa(1-\kappa)^2+\lambda_1^*(1-\kappa)\kappa^2+\lambda_2^*\kappa^3]\\ =\ &m_2[\lambda_1^*(1-\kappa)^3+\lambda_2^*\kappa(1-\kappa)^2+\lambda_2^*(1-\kappa)\kappa^2+\lambda_1^*\kappa^3].\end{aligned}$$

This means that

$$\lambda_2^*=\frac{[m_2((1-\kappa)^3+\kappa^3)-m_1\kappa(1-\kappa)]}{[m_1((1-\kappa)^3+\kappa^3)-m_2\kappa(1-\kappa)]}\lambda_1^*$$

But the assumption that $\kappa\geq\widehat{\kappa}$ implies that $\lambda_2^*\leq 0$ if $\lambda_1^*>0$,which, in turn, is inconsistent with the hypothesis that $(\lambda_1^*,\lambda_2^*)>(0,0)$. Thus, the only equilibrium is that $(\lambda_1^*,\lambda_2^*)=(2m_1,0)$ which implies that

$$(g_1,g_2)=(f(2m_1(1-\kappa)),f(2m_1\kappa)),$$

as required. ■

**Fact** *Let $(g_1^c(\kappa),g_2^c(\kappa))$ be the public good levels described in Lemma* 3 *and assume that $m_1>m_2$. Then* (i) *$g_1^c(\kappa)$ is decreasing for $\kappa$ sufficiently small and*

*$\kappa > \widehat{\kappa}$, but increasing for $\kappa$ sufficiently close to but less than $\widehat{\kappa}$; (ii) $g_2^c(\kappa)$ is decreasing for $\kappa \leq \widehat{\kappa}$ and increasing thereafter; (iii) $g_1^c(\kappa)$ is greater than the surplus maximizing level; and (iv) $g_2^c(\kappa)$ exceeds the surplus maximizing level for $\kappa$ sufficiently small and $\kappa$ sufficiently large, but it is less than the surplus maximizing level for $\kappa$ sufficiently close to $\widehat{\kappa}$.*

**Proof:** (i) For all $\kappa \leq \widehat{\kappa}$, we have that

$$g_1^c(\kappa) = f(\frac{2m_1[(1-\kappa)^4 - \kappa^4]}{(1-\kappa)^2 - \frac{m_1}{m_2}\kappa^2}).$$

Letting

$$\varphi(\kappa) = \ln 2m_1[(1-\kappa)^4 - \kappa^4] - \ln[(1-\kappa)^2 - \frac{m_1}{m_2}\kappa^2],$$

we will show that $\varphi'(0) < 0$ and $\varphi'(\widehat{\kappa}) > 0$. The derivative of this expression is

$$\varphi'(\kappa) = 2\{\frac{[(1-\kappa) + \frac{m_1}{m_2}\kappa]}{[(1-\kappa)^2 - \frac{m_1}{m_2}\kappa^2]} - \frac{2[(1-\kappa)^3 + \kappa^3]}{[(1-\kappa)^4 - \kappa^4]}\}.$$

Note first that $\varphi'(0) = -2$ as required. At $\kappa = \widehat{\kappa}$, we have that

$$\frac{(1-\widehat{\kappa})^3 + \widehat{\kappa}^3}{\widehat{\kappa}(1-\widehat{\kappa})} = \frac{m_1}{m_2}.$$

Thus,

$$\varphi'(\widehat{\kappa}) = 2\{\frac{[(1-\widehat{\kappa})^2 + (1-\widehat{\kappa})^3 + \widehat{\kappa}^3]}{[(1-\widehat{\kappa})^4 - \widehat{\kappa}^4]} - \frac{2[(1-\widehat{\kappa})^3 + \widehat{\kappa}^3]}{[(1-\widehat{\kappa})^4 - \widehat{\kappa}^4]}\}.$$

It follows that $\varphi'(\widehat{\kappa}) > 0$ if

$$(1-\widehat{\kappa})^2 > (1-\widehat{\kappa})^3 + \widehat{\kappa}^3 = (1-\widehat{\kappa})^2(1-\widehat{\kappa}) + \widehat{\kappa}^3,$$

or, equivalently, if

$$(1-\widehat{\kappa})^2 > \widehat{\kappa}^2$$

which is true.

For all $\kappa > \widehat{\kappa}$, we have that

$$g_1^c(\kappa) = f(2m_1(1-\kappa)),$$

which is obviously decreasing in $\kappa$.

(ii) For all $\kappa \leq \widehat{\kappa}$, we have that

$$g_2^c(\kappa) = f(\frac{2m_1[(1-\kappa)^4 - \kappa^4]}{\frac{m_1}{m_2}(1-\kappa)^2 - \kappa^2}).$$

Thus, we need to show that $\frac{2m_1[(1-\kappa)^4-\kappa^4]}{\frac{m_1}{m_2}(1-\kappa)^2-\kappa^2}$ is decreasing in $\kappa$ or, equivalently that

$$\ln 2m_1[(1-\kappa)^4-\kappa^4] - \ln[\frac{m_1}{m_2}(1-\kappa)^2-\kappa^2]$$

is decreasing in $\kappa$. The derivative of this expression is

$$2\{\frac{[\frac{m_1}{m_2}(1-\kappa)+\kappa]}{[\frac{m_1}{m_2}(1-\kappa)^2-\kappa^2]} - \frac{2[(1-\kappa)^3+\kappa^3]}{[(1-\kappa)^4-\kappa^4]}\},$$

and hence we need to show that for all $\kappa \leq \widehat{\kappa}$

$$\frac{[\frac{m_1}{m_2}(1-\kappa)+\kappa]}{[\frac{m_1}{m_2}(1-\kappa)^2-\kappa^2]} < \frac{2[(1-\kappa)^3+\kappa^3]}{[(1-\kappa)^4-\kappa^4]}$$

Observe that the expression on the left hand side is decreasing in $\frac{m_1}{m_2}$. Thus, if the inequality holds for $\frac{m_1}{m_2}=1$, it holds for all $\frac{m_1}{m_2}$. Thus, it suffices to show that

$$\frac{1}{[(1-\kappa)^2-\kappa^2]} < \frac{2[(1-\kappa)^3+\kappa^3]}{[(1-\kappa)^4-\kappa^4]}.$$

We know that $(1-\kappa)^4-\kappa^4=[(1-\kappa)^2+\kappa^2][(1-\kappa)^2-\kappa^2]$ and hence the above inequality is equivalent to

$$[(1-\kappa)^2+\kappa^2] < 2[(1-\kappa)^3+\kappa^3] = 2[(1-\kappa)^2(1-\kappa)+\kappa^2\kappa].$$

This holds if

$$(1-2\kappa)(1-\kappa)^2 > (1-2\kappa)\kappa^2,$$

which is true.

For all $\kappa > \widehat{\kappa}$, we have that

$$g_2^c = f(2m_1\kappa),$$

which is obviously increasing in $\kappa$.

(iii) Suppose first that $\kappa \leq \widehat{\kappa}$. Then, since the surplus maximizing public good level for district 1 is $f(m_1(1-\kappa)+m_2\kappa)$, we must show that

$$f(\frac{2m_1[(1-\kappa)^4-\kappa^4]}{(1-\kappa)^2-\frac{m_1}{m_2}\kappa^2}) > f(m_1(1-\kappa)+m_2\kappa),$$

or, equivalently, that

$$\frac{2m_1[(1-\kappa)^4-\kappa^4]}{(1-\kappa)^2-\frac{m_1}{m_2}\kappa^2} > m_1(1-\kappa)+m_2\kappa.$$

Rearranging, we require that

$$2m_1[(1-\kappa)^4 - \kappa^4] > \{(1-\kappa)^2 - \frac{m_1}{m_2}\kappa^2\}\{m_1(1-\kappa) + m_2\kappa\}.$$

Since $\frac{m_1}{m_2} > 1$, it is enough to show that

$$2[(1-\kappa)^4 - \kappa^4] > \{(1-\kappa)^2 - \kappa^2\}$$

or, equivalently, that

$$(1-\kappa)^2 + \kappa^2 > 1/2.$$

This is true for all $\kappa$ in the relevant range.

If $\kappa > \widehat{\kappa}$, then we must show that

$$f(2m_1(1-\kappa)) > f(m_1(1-\kappa) + m_2\kappa),$$

which is obviously true.

(iv) The surplus maximizing public good level for district 2 is $f(m_2(1-\kappa) + m_1\kappa)$. If $\kappa \leq \widehat{\kappa}$, then

$$g_2^c(\kappa) = f(\frac{2m_1[(1-\kappa)^4 - \kappa^4]}{\frac{m_1}{m_2}(1-\kappa)^2 - \kappa^2})$$

while if $\kappa > \widehat{\kappa}$, then

$$g_2^c(\kappa) = f(2m_1\kappa).$$

It is obvious that $g_2^c(\kappa)$ exceeds the surplus maximizing level for $\kappa$ sufficiently close to 0 and $\kappa$ sufficiently close to 1/2. To show that $g_2^c(\kappa)$ is less than the surplus maximizing level when $\kappa$ is sufficiently close to $\widehat{\kappa}$, it is enough to demonstrate that

$$\frac{2[(1-\widehat{\kappa})^4 - \widehat{\kappa}^4]}{\frac{m_1}{m_2}(1-\widehat{\kappa})^2 - \widehat{\kappa}^2} < \frac{m_2}{m_1}(1-\widehat{\kappa}) + \widehat{\kappa}.$$

Recalling that, by definition,

$$\frac{(1-\widehat{\kappa})^3 + \widehat{\kappa}^3}{\widehat{\kappa}(1-\widehat{\kappa})} = \frac{m_1}{m_2},$$

this expression becomes:

$$(1-\widehat{\kappa})^3 + \widehat{\kappa}^3 < (1-\widehat{\kappa})^2,$$

or, equivalently,

$$\widehat{\kappa}^2 < (1-\widehat{\kappa})^2$$

which is true. ■

**Proof of Proposition 5:** Letting $(g_1^c(\kappa), g_2^c(\kappa))$ be the policy outcome under centralization with a cooperative legislature and spillovers $\kappa$, surplus under centralization with a cooperative legislature is given by

$$\begin{aligned} S_c^c(\kappa) &= [m_1(1-\kappa)+m_2\kappa]b(g_1^c(\kappa)) + [m_2(1-\kappa)+m_1\kappa]b(g_2^c(\kappa)) \\ &\quad -p(g_1^c(\kappa)+g_2^c(\kappa)). \end{aligned}$$

For part (i), we know from the earlier discussion that when $m_1 = m_2 = m$, $S_n^c(0) < S^d(0)$ and $S_o^c(\frac{1}{2}) > S^d(\frac{1}{2})$. We also know from the proof of Proposition 1, that surplus is non-increasing in spillovers under decentralization. Thus, it suffices to show that surplus is increasing in spillovers under centralization.

When $m_2 = m_1 = m$,

$$g_1^c(\kappa) = g_2^c(\kappa) = f(2m[(1-\kappa)^2+\kappa^2])$$

Since $m > p/b'(0)$, we know that $f(2m[(1-\kappa)^2+\kappa^2])$ satisfies the first order condition $2m[(1-\kappa)^2+\kappa^2]b'(f(2m\kappa)) = p$ for all $\kappa$. Thus, for all $\kappa$

$$\frac{dS_c^c}{d\kappa} = 2m(1-2\kappa)pf'(2m[(1-\kappa)^2+\kappa^2])\frac{1-4\kappa(1-\kappa)}{[(1-\kappa)^2+\kappa^2]} > 0.$$

Intuitively, increasing spillovers causes each district to select a representative with a type closer to the median which reduces the over provision of local public goods.

For the first part of (ii), note that when $m_1 \neq m_2$, $S_c^c(0) < S^d(0)$. Since both surplus functions are continuous functions of $\kappa$, for each $(m_1, m_2)$ there exists $\varepsilon > 0$ such that $S_c^c(\kappa) < S^d(\kappa)$ for all $\kappa < \varepsilon$. Similar logic establishes the second part of (ii), if $S_c^c(\frac{1}{2}) > S^d(\frac{1}{2})$. Thus, it remains to establish this inequality. Let $(m_1, m_2)$ be given and suppose that $m_1 > m_2$. We can find $\xi$ and $\gamma$ so that $(m_1, m_2) = (\xi\gamma, \xi(1-\gamma))$ for some $\xi > 2p$ and $\gamma \in [\frac{1}{2}, 1-p/\xi)$. In addition, since $\widehat{\kappa} < 1/2$, we have that

$$g_1^c(\frac{1}{2}) = g_2^c(\frac{1}{2}) = \frac{\xi\gamma}{p} - 1.$$

This implies that

$$S_c^c(\gamma, \frac{1}{2}) = \frac{\xi}{2}\{\ln(\frac{\xi\gamma}{p}) + \ln(\frac{\xi\gamma}{p})\} - 2p[\frac{\xi\gamma}{p} - 1].$$

Under decentralization, if $\xi(1-\gamma) \geq 2p$,

$$(g_1, g_2) = (\frac{\xi\gamma}{2p} - 1, \frac{\xi(1-\gamma)}{2p} - 1)$$

and surplus is given by:

$$S^d(\gamma, \frac{1}{2}) = \frac{\xi}{2}\{\ln\frac{\xi}{2p}\gamma + \ln\frac{\xi}{2p}(1-\gamma)\} - p[\frac{\xi\gamma}{2p} - 1 + \frac{\xi(1-\gamma)}{2p} - 1].$$

If $\xi(1-\gamma) < 2p$,

$$(g_1, g_2) = (\frac{\xi\gamma}{2p} - 1, 0)$$

and surplus is

$$S^d(\gamma, \frac{1}{2}) = \frac{\xi}{2}\{\ln \frac{\xi}{2p}\gamma\} - p[\frac{\xi\gamma}{2p} - 1].$$

Taking differences, we have that if $\xi(1-\gamma) \geq 2p$

$$S_c^c(\gamma, \frac{1}{2}) - S^d(\gamma, \frac{1}{2}) = \frac{\xi}{2}\{\ln 2 + \ln(\frac{2\gamma}{1-\gamma})\} - \frac{3\xi\gamma}{2} + \frac{\xi(1-\gamma)}{2}$$

While for $\xi(1-\gamma) < 2p$,

$$S_c^c(\gamma, \frac{1}{2}) - S^d(\gamma, \frac{1}{2}) = \frac{\xi}{2}\{\ln(2) + \ln(\frac{\xi\gamma}{p})\} - \frac{3\xi\gamma}{2} + p.$$

Differentiating the former difference with respect to $\gamma$ yields

$$\begin{aligned} \frac{d[S_c^c(\gamma, \frac{1}{2}) - S^d(\gamma, \frac{1}{2})]}{d\gamma} &= \frac{\xi}{2\gamma(1-\gamma)} - 2\xi \\ &= \xi\frac{1 - 4\gamma(1-\gamma)}{2\gamma(1-\gamma)} \geq 0. \end{aligned}$$

Thus, this difference is non-decreasing in $\gamma$. Accordingly, if $S_c^c(\gamma, \frac{1}{2}) > S^d(\gamma, \frac{1}{2})$ at $\gamma = 1/2$, then the inequality holds for all $\gamma$ in the relevant range. But $\gamma = 1/2$ corresponds to the symmetric case and we indeed know that surplus under centralization is higher than decentralization then. Differentiating the latter difference with respect to $\gamma$ yields

$$\frac{d[S_c^c(\gamma, \frac{1}{2}) - S^d(\gamma, \frac{1}{2})]}{d\gamma} = \frac{\xi}{2}\{\frac{1}{\gamma} - 3\} < 0$$

which implies that the surplus difference is decreasing in $\gamma$. Thus, if we can show that $S_c^c(\gamma, \frac{1}{2}) > S^d(\gamma, \frac{1}{2})$ at $\gamma = 1 - p/\xi$, then the inequality holds for all $\gamma$ in the relevant range. Evaluating, we have

$$S_c^c(1 - p/\xi, \frac{1}{2}) - S^d(1 - p/\xi, \frac{1}{2}) == \frac{\xi}{2}\{\ln(2) + \ln(\frac{\xi - p}{p}) - 3 + \frac{5p}{\xi}\}.$$

This expression is increasing in $\xi$. Thus, it suffices to show it is positive at $\xi = 2p$. Evaluating, yields

$$S_c^c(\frac{1}{2}, \frac{1}{2}) - S^d(\frac{1}{2}, \frac{1}{2}) == p\{\ln(2) - \frac{1}{2}\} > 0.$$

■

**Proof of Proposition 6:** Aggregate surplus under centralization with non-uniform financing is given by:

$$\begin{aligned} S_n^c(\kappa) &= \frac{1}{2}[(m_1(1-\kappa)+m_2\kappa)\,b\,(f(m_1(1-\kappa)))+(m_2(1-\kappa)+m_1\kappa)\,b\,(f(m_1\kappa)) \\ &\quad -pf(m_1(1-\kappa))-pf(m_1\kappa)]+ \\ &\quad \frac{1}{2}[(m_2(1-\kappa)+m_1\kappa)\,b\,(f(m_2(1-\kappa)))+(m_1(1-\kappa)+m_2\kappa)\,b\,(f(m_2\kappa)) \\ &\quad -pf(m_2(1-\kappa))-pf(m_2\kappa)]. \end{aligned}$$

We must show that $S_n^c(\kappa) \leq S^d(\kappa)$ for all $\kappa$, where $S^d(\kappa)$ is as defined in the proof of Proposition 1. We divide the proof into sequence of three Claims.

**Claim 1:** If $\kappa \leq \max\{\frac{m_2}{m_1+m_2}, \frac{p}{m_1 b'(0)}\}$, then $S_n^c(\kappa) \leq S^d(\kappa)$.

To see this note that, since $b(\cdot)$ is concave, we have that

$$\begin{aligned} S_n^c(\kappa) &\leq (m_1(1-\kappa)+m_2\kappa)\,b\left(\frac{f(m_1(1-\kappa))+f(m_2\kappa)}{2}\right)-p\frac{f(m_1(1-\kappa))+f(m_2\kappa)}{2} \\ &\quad +(m_2(1-\kappa)+m_1\kappa)\,b\left(\frac{f(m_2(1-\kappa))+f(m_1\kappa)}{2}\right)-p\frac{f(m_2(1-\kappa))+f(m_1\kappa)}{2}. \end{aligned}$$

Notice that the expression on the right hand side is similar to the expression for surplus under decentralization, except that the public good levels are $\frac{f(m_1(1-\kappa))+f(m_2\kappa)}{2}$ and $\frac{f(m_2(1-\kappa))+f(m_1\kappa)}{2}$ instead of $f(m_1(1-\kappa))$ and $f(m_2(1-\kappa))$. The functions $(m_1(1-\kappa)+m_2\kappa)\,b(g)-pg$ and $(m_2(1-\kappa)+m_1\kappa)\,b(g)-pg$ are clearly increasing on the intervals $[0, f(m_1(1-\kappa))]$ and $[0, f(m_2(1-\kappa))]$ respectively. The desired inequality then follows from the facts that $\frac{f(m_1(1-\kappa))+f(m_2\kappa)}{2} \leq f(m_1(1-\kappa))$ and, if $\kappa \leq \max\{\frac{m_2}{m_1+m_2}, \frac{p}{m_1 b'(0)}\}$, that $\frac{f(m_2(1-\kappa))+f(m_1\kappa)}{2} \leq f(m_2(1-\kappa))$.

**Claim 2:** If $\kappa \geq \min\{\frac{1}{2}, 1-\frac{p}{m_2 b'(0)}\}$, then $S_n^c(\kappa) \leq S^d(\kappa)$.

If $\frac{1}{2} \leq 1-\frac{p}{m_2 b'(0)}$, the result follows immediately from the fact that $S_n^c\left(\frac{1}{2}\right) = S^d\left(\frac{1}{2}\right)$. So suppose that $\frac{1}{2} > 1-\frac{p}{m_2 b'(0)}$ and let $\kappa \in (1-\frac{p}{m_2 b'(0)}, \frac{1}{2}]$. Then, $f(m_2(1-\kappa)) = f(m_2\kappa) = 0$. It follows that

$$\begin{aligned} S_n^c(\kappa) &= \frac{1}{2}[(m_1(1-\kappa)+m_2\kappa)\,b\,(f(m_1(1-\kappa)))+(m_2(1-\kappa)+m_1\kappa)\,b\,(f(m_1\kappa)) \\ &\quad -pf(m_1(1-\kappa))-pf(m_1\kappa)] \\ &\leq (m_1(1-\kappa)+m_2\kappa)\,b\left(\frac{f(m_1(1-\kappa))+f(m_1\kappa)}{2}\right)-p\frac{f(m_1(1-\kappa))+f(m_1\kappa)}{2} \\ &\leq (m_1(1-\kappa)+m_2\kappa)\,b\,(f(m_1(1-\kappa)))-pf(m_1(1-\kappa)) \\ &\leq S^d(\kappa). \end{aligned}$$

**Claim 3:** If $\max\{\frac{m_2}{m_1+m_2}, \frac{p}{m_1 b'(0)}\} < \kappa < \min\{\frac{1}{2}, 1 - \frac{p}{m_2 b'(0)}\}$, then $S_n^c(\kappa) \leq S^d(\kappa)$.

If not, there exists $\widehat{\kappa} \in (\max\{\frac{m_2}{m_1+m_2}, \frac{p}{m_1 b'(0)}\}, \min\{\frac{1}{2}, 1 - \frac{p}{m_2 b'(0)}\})$ such that $S_n^c(\widehat{\kappa}) > S^d(\widehat{\kappa})$. We will show that the function $S_n^c(\kappa) - S^d(\kappa)$ is increasing on $[\widehat{\kappa}, \min\{\frac{1}{2}, 1 - \frac{p}{m_2 b'(0)}\})$ which, since $S_n^c(\kappa) - S^d(\kappa)$ is continuous, contradicts Claim 2. If $\kappa \in [\widehat{\kappa}, \min\{\frac{1}{2}, 1 - \frac{p}{m_2 b'(0)}\})$, there are two possibilities: either $\kappa < \frac{p}{m_2 b'(0)}$ in which case $f(m_2\kappa) = 0$ or $\kappa > \frac{p}{m_2 b'(0)}$ in which case $f(m_2\kappa) = 0$. In the former case, we have that:

$$\begin{aligned}\frac{dS_n^c(\kappa)}{d\kappa} &= (m_2 - m_1)\frac{1}{2}\left[b(f(m_1(1-\kappa))) - b(f(m_2(1-\kappa))) - b(f(m_1\kappa))\right] \\ &\quad - m_2\kappa p\frac{f'(m_1(1-\kappa))}{2(1-\kappa)} - m_1\kappa p\frac{f'(m_2(1-\kappa))}{2(1-\kappa)} + m_2(1-\kappa)p\frac{f'(m_1\kappa)}{2(1-\kappa)}.\end{aligned}$$

In the latter,

$$\begin{aligned}\frac{dS_n^c(\kappa)}{d\kappa} &= (m_2 - m_1)\frac{1}{2}\left[b(f(m_1(1-\kappa))) + b(f(m_2\kappa)) - b(f(m_2(1-\kappa))) - b(f(m_1\kappa))\right] \\ &\quad - m_2\kappa p\frac{f'(m_1(1-\kappa))}{2(1-\kappa)} - m_1\kappa p\frac{f'(m_2(1-\kappa))}{2(1-\kappa)} \\ &\quad + m_1(1-\kappa)p\frac{f'(m_2\kappa)}{2(1-\kappa)} + m_2(1-\kappa)p\frac{f'(m_1\kappa)}{2(1-\kappa)}.\end{aligned}$$

Now note from the proof of Proposition 1 that since $\kappa < 1 - \frac{p}{m_2 b'(0)}$, in both cases we have that

$$\begin{aligned}\frac{dS^d(\kappa)}{d\kappa} &= (m_2 - m_1)[b(f(m_1(1-\kappa))) - b(f(m_2(1-\kappa)))] \\ &\quad - m_2\kappa p\frac{f'(m_1(1-\kappa))}{1-\kappa} - m_1\kappa p\frac{f'(m_2(1-\kappa))}{1-\kappa}.\end{aligned}$$

It is immediate that $\frac{dS^d(\kappa)}{d\kappa} < \frac{dS_n^c(\kappa)}{d\kappa}$, which implies that $S_n^c(\kappa) - S^d(\kappa)$ is increasing on $[\widehat{\kappa}, \min\{\frac{1}{2}, 1 - \frac{p}{m_2 b'(0)}\})$ as required. ■