London School of Economics
Department of Economics

Vassilis Hajivassiliou
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MSc Econometrics (Ec402)
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Problem Set #9

Instructions: To be discussed in week 11. **Please prepare answers to parts (a) and (b) of this question in PDF format, and submit via Moodle to receive feedback from your Class Teacher.**
NB: There is no need for you to attempt to answer part (c) — that will be discussed in the classes next week (11).

Each class teacher has prepared a portal on Moodle. Please submit your PDF by Friday, 6 December (end of week 10). That way your class teacher will have time to review your submissions and return comments at the class of week 11.

1. Consider a linear regression model with time-series data (y, X) of sample size T. The k regressors are grouped in two parts, X_A and x_B, of dimensions $T \times (k-1)$ and $T \times 1$ respectively. In other words, the second regressor group consists of a single regressor.

   Suppose that the model satisfies the following assumptions:

$$\begin{array}{rc} A1: & rank(X) = k < T \\ A2: & y = X\beta + \epsilon = X_A\beta_A + x_B\beta_B + \epsilon \quad \text{with } E\epsilon = 0 \\ A3Rmi.X_A: & E(\epsilon|X_A) = E\epsilon \\ A3.x_B & \epsilon \text{ and } x_B \text{ correlated for all periods } t \\ A4\Omega: & E(\epsilon\epsilon'|X) = c^2\Omega \\ A5G: & \epsilon_t|X \sim N(0, \sigma^2) \end{array}$$

   In other words, regressor x_B is an endogenous regressor. We are particularly interested in the true coefficients of the X_A variables, β_A.

   (a) Suppose that the $T \times T$ matrix Ω is fully known. Define the Ordinary Least Squares (OLS) and Ideal Generalized Least Squares (IGLS) estimators of the whole β vector in this case. Explain carefully whether or not the OLS and/or IGLS can be unbiased and consistent for the true β.

   (b) Now suppose that the matrix Ω is known to equal the identity matrix I_S. In view of the endogeneity of x_B, a colleague proposes Instrumental Variables (IV) Estimation, defined by:

$$\hat{\beta}_{IV} = (W'X)^{-1}W'y$$

   The colleague explains that the matrix W should be of the same dimension as X. She further explains that W should be constructed using only exogenous variables, implying that X_A can be used. Since this disallows the use of the

endogenous regressor x_B, the colleague adds that we must find k_z additional "instrument" variables ($k_z \geq 1$) to construct W. Explain this method and describe the properties that all instrument variables used to construct W should possess.

(c) Define the terms 'instrument validity' and 'instrument relevance'.

The colleague proposes the following instrument variables. Using this terminology, discuss the following variables as possible instruments:

i. Variable z_1 is the sum of the first three regressor variables from the X_A group, i.e., $z_{t1} = x_{A,t1} + x_{A,t2} + x_{A,t3}$

ii. Variable z_2 is the square of the fourth regressor variable from the X_A group, i.e., $z_{t2} = x_{A,t5}^2$

iii. Variable z_3 is a measure of how many sunspots occured on the surface of the sun in period t.
[Note: Sunspots are dark, planet-size regions of strong magnetic fields on the surface of the sun.]

Using the properties for good instrument variables you discussed in (b), discuss whether these three proposed instrument variables are appropriate.