JOURNAL OF APPLIED ECONOMETRICS
J. Appl. Econ. **22**: 479–510 (2007)
Published online in Wiley InterScience
(www.interscience.wiley.com) DOI: 10.1002/jae.953

UNEMPLOYMENT AND LIQUIDITY CONSTRAINTS

VASSILIS A. HAJIVASSILIOU[a]* AND YANNIS M. IOANNIDES[b]

[a] *Department of Economics, London School of Economics, London, UK*
[b] *Department of Economics, Tufts University, Medford, Massachusetts, USA*

SUMMARY

We present a dynamic framework for the interaction between borrowing (liquidity) constraints and deviations of actual hours from desired hours, both measured by discrete-valued indicators, and estimate it as a system of dynamic binary and ordered probit models with panel data from the Panel Study of Income Dynamics. We analyze a household's propensity to be liquidity constrained by means of a dynamic binary probit model. We analyze qualitative aspects of the conditions of employment, namely whether the household head is involuntarily overemployed, voluntarily employed, or involuntarily underemployed or unemployed, by means of a dynamic ordered probit model. We focus on the possible interaction between the two types of constraints. We estimate these models jointly using maximum simulated likelihood, where we allow for individual random effects along with an autoregressive process for the general error term in each equation. A novel feature of our method is that it allows for the random effects to be correlated with regressors in a time-invariant fashion. Our results provide strong support for the basic theory of constrained behavior and the interaction between liquidity constraints and exogenous constraints on labor supply. Copyright © 2007 John Wiley & Sons, Ltd.

Received 10 March 2000; Revised 28 March 2006

1. INTRODUCTION

The present paper uses panel data on households to address empirically the interaction between liquidity constraints and exogenous restrictions on labor supply decisions. Our techniques allow us to estimate with panel data general dynamic limited dependent variable models with a flexible dynamic structure. The presence of constraints is taken as an institutional datum. Whether and when they bind for particular individuals in a given population are the endogenous variables of interest.

We take as a starting point that capital market imperfections may prevent individuals from borrowing against their future income without collateral.[1] Intuitively, households are most likely to be liquidity constrained at times of events that are closely related to labor market conditions (e.g., unemployment) or other events, such as ill health, that have direct consequences for labor supply behavior. When labor supply is jointly considered with food consumption,[2] some serious analytical difficulties emerge. These stem from the fact that observed hours of work (or employment) are not necessarily the outcome of free choice in the same way as food consumption

* Correspondence to: Vassilis A. Hajivassiliou, Department of Economics, London School of Economics, Houghton Street, London WC2A 2AE, UK. E-mail: vassilis@econ.lse.ac.uk
Contract/grant sponsor: National Science Foundation; Contract/grant number: SES-9000200; SES-9211913.
[1] See Hall and Mishkin (1982); Flavin (1985); Zeldes (1989); Ball (1990).
[2] Food and housing are the only major components of the consumption bundle for which data are consistently available in the Panel Study of Income Dynamics (PSID).

is. Specifically, individuals may be involuntarily unemployed, underemployed, or overemployed. For such individuals, the unconstrained model of fluctuations in employment and hours worked may not be appropriate. We address here such qualitative aspects of employment jointly with liquidity constraints.

Our treatment of the endogeneity of regime switching and of the possible dependence between liquidity constraints and restrictions on labor supply behavior goes further than previous work. Typically, the past literature has only considered agents who were thought to be either liquidity constrained or not constrained, but remained so throughout the period of observation. For example, Ball (1990) restricts his sample to those who have never been constrained in the labor market. Casual empiricism suggests, and the data confirm, that switches in the state of households do occur. Households are most likely to be constrained early in their lifetimes, or at times of major purchases, changes in employment conditions, or other unforeseen events (death, catastrophic illnesses, etc.), while business cycle conditions regularly force them to update their decisions. The evolution over time of a household's socioeconomic circumstances makes it all the more important to allow for endogenous constraints with a dynamic structure.

Allowing for the coexistence of exogenous restrictions on labor supply and liquidity constraints is a novel feature of the present work. It is firmly rooted in the modern life cycle theory of labor supply, while at the same time it encompasses a dynamic generalization of the approach, pioneered by Ashenfelter (1980), that studies unemployment as a 'constraint on choice rather than a result of it,' the latter being the hallmark of neoclassical theory of freely chosen labor supply. Our results provide strong support for the basic theory of constrained behavior and the interaction between liquidity constraints and constraints on labor supply that we propose in this paper. Our work thus complements important previous research on hours constraints by Ham (1982, 1986), Ham and Reilly (2002), and Kahn and Lang (1992).

Our econometric models may be estimated in their full generality only by simulation estimation methods. In this paper we apply the method of maximum smoothly simulated likelihood (MSSL) developed in Börsch-Supan and Hajivassiliou (1993), Hajivassiliou *et al*. (1996), and Hajivassiliou and McFadden (1998). See also Hajivassiliou (2004) for a detailed development of MSSL for general panel limited dependent variable models with simultaneity.

The paper is organized as follows. Section 2 discusses some important aspects of the data which help motivate our model. Section 3 presents a rudimentary life cycle optimization model and derives a dynamic discrete choice model for liquidity constraints and quantity constraints on labor supply. Section 4 discusses the econometric specification of the model and Section 5 presents the empirical results, reviews diagnostic tests performed on the estimated models, and contrasts with the previous literature. These results pertain to dynamic models for the discrete events of whether or not a household is liquidity constrained and whether or not household heads are subject to quantity restrictions in their labor supply behavior. Section 6 concludes. Appendix A provides technical details on the method of MSSL.[3]

2. QUALITATIVE ASPECTS OF EMPLOYMENT AND LIQUIDITY CONSTRAINTS: EVIDENCE FROM THE PANEL STUDY OF INCOME DYNAMICS

Our primary data source is the Panel Study of Income Dynamics (Hill, 1992), PSID for short. We originally worked with two different samples: all heads and male heads. The sample of all

[3] Appendix B, available on the Data Archive journal website, discusses additional details on the recoding of the data.

heads contains 46,031 observations on 3206 separate household spells. The sample of male heads contains 32,408 observations on 2410 separate household spells. We have chosen to focus on the sample of male heads because it is substantially more homogeneous than that of all heads. We report summary statistics for key variables in Table I. Tables II–V report additional aspects of the data, which we discuss in further detail below. Even within such a homogeneous sample, all key dynamic aspects of the data that pertain to regime switching display a fair amount of hitherto unexplored richness.[4]

An overview of the pattern of transitions and the underlying dynamics of regime switching observed in the data may be obtained by looking at cross-tabulations for the transitions from being constrained to unconstrained and vice versa, given in Tables II–V. A household is classified as liquidity constrained in a particular time period if its total wealth (the sum of reported housing wealth and calculated non-housing wealth) is low relative to its reported typical disposable income.

Table I. Descriptive statistics. Number of household spells: 2410

Variable	Nobs	Mean	SD	Med.	Mode	Min.	Max.	InterQuart
county unempl. rate	32870	6.5878	2.8626	6	5	1	34	3
hd disabled?	35860	0.1271	0.3331	0	0	0	1	0
out of lab. force?	36963	0.1232	0.3287	0	0	0	1	0
overemployed?	36963	0.0484	0.2146	0	0	0	1	0
underemployed?	36963	0.1723	0.3776	0	0	0	1	0
unemployed?	36963	0.0218	0.146	0	0	0	1	0
vol. employed?	36963	0.6341	0.4816	1	1	0	1	1
education hd	34631	4.9438	1.8144	5	4	0	8	2
food needs	36963	1054.895	417.5646	1016	669	337	9999	555
family size	35917	3.1169	1.4472	3	2	1	14	2
growth food needs	35913	−0.0134	0.2296	0	0	−2.7044	2.7044	0.0249
hd age	34828	41.5437	15.0652	38	29	17	92	23
tenure hd (months)	32654	82.1139	96.3621	39	0	0	960	156
live in north-centr?	36961	0.3159	0.4649	0	0	0	1	1
live in north-east?	36961	0.1985	0.3989	0	0	0	1	0
live in south?	36961	0.3055	0.4606	0	0	0	1	1
hd married?	34828	0.8773	0.3279	1	1	0	1	0
num. child. 0–17 yrs	34828	1.0314	1.2217	1	0	0	8	2
num. child. 0–5 yrs	27951	0.3731	0.6815	0	0	0	4	1
occup. unempl. rate	28737	5.8824	3.6405	4.6999	3	1.4	17.0999	4.8000
race of hd: black?	36954	0.0523	0.2226	0	0	0	1	0
race of hd: white?	36954	0.8972	0.3036	1	1	0	1	0
real disposable inc.	35793	10588.7	8535.672	9535.164	0	−223144	530110.5	6680.785
hd cath./eastorthdx?	32846	0.1354	0.3422	0	0	0	1	0
hd no religion/DK?	36963	0.4800	0.4996	0	0	0	1	1
hd ‘protestant’?	32846	0.4234	0.4941	0	0	0	1	1
real int. rate aft. tx	33088	0.0242	0.0241	0.0231	0.0024	−0.0357	0.0946	0.0368
real total asset inc.	34767	861.1424	4227.124	7.0049	0	−4085.33	466999.5	404.4456
spouse age	34820	34.6510	18.5908	33	0	0	87	23
unempld in $(t-1)$?	35917	0.1146	0.3186	0	0	0	1	0
liquidity constrained?[a]	34563	0.2724	0.4452	1	1	0	1	1

[a] zdumc2 = 1 if total asset income relative to average income over last two periods is less than 1/6. See Section 8.1 for details.

[4] The above number of 32,408 observations on 2410 household spells with male heads used in the estimations includes observations with missing values filled in; continuous variables were filled in by individual time-means and discrete ones by most likely individual values.

DOI: 10.1002/jae

Table II. One-period transitions in liquidity indicator S_t: male heads

	$S_t = 1$ liq. constrained	$S_t = 0$ not liq. constrained	Row percent
$S(t-1) = 1$ liq. constrained	8.7	21.1	29.8
$S(t-1) = 0$ not liq. constrained	16.9	53.3	70.2
Column percent	25.6	74.4	100.00

Table III. Dynamic transition counts: male heads

Number of transitions	ΔS_t		ΔE_t			
	Frequency	Cumulative	Frequency		Cumulative	
			4 cells	5 cells	4 cells	5 cells
0	15.4	15.4	2.1	2.0	2.1	2.0
1	9.7	25.1	5.0	4.6	7.2	6.7
2	9.4	34.5	6.5	6.2	13.6	12.9
3	8.2	42.7	7.2	7.1	20.9	20.0
4	9.3	52.0	8.3	8.1	29.1	28.1
5	7.9	59.1	8.7	8.8	37.9	37.0
6	7.1	67.0	7.5	7.2	45.4	44.1
7	6.9	74.0	7.4	7.7	52.8	51.8
8	5.1	79.1	7.2	7.4	60.1	59.1
9	4.1	83.2	6.2	6.0	66.3	65.1
10	3.3	86.5	5.4	5.6	71.7	70.7
11	3.2	89.6	4.8	4.8	76.5	75.6
12–19	10.3	100.0	23.5	24.4	100.0	100.0

Table IV. One-period transitions in employment indicator E_t: male heads

	−1 over/ed	0	1 under/- or un/ed	99 out-of-the-labor-force	Row percent
−1 overemployed	0.23	3.08	0.81	0.66	4.78
0	3.01	38.06	10.60	7.30	58.96
1 under/unemployed	0.87	11.03	4.36	2.21	18.48
99 out-of-the-labor-force	0.84	11.00	3.42	2.53	17.78
Column percent	4.94	63.16	19.19	12.71	100.00

Table V. Cross-tabulation of S_t vs. E_t: male heads

	$E_t = -1$ over/ed	$E_t = 0$	$E_t = 1$ under/- or un/ed	$E_t = 99$ out-of-the-labor-force	Row percent
$S_t = 1$ liq. constrained	1.15	15.17	8.69	2.22	27.24
$S_t = 0$ not liq. constrained	3.73	47.34	11.15	10.54	72.76
Column percent	4.88	62.51	19.85	12.76	100.00

The construction of the labor constraint indicators with the aid of flowcharts appears in the full version of the paper, available on our websites.

Under the definitions adopted, in the sample of male heads approximately 74.4% of the observations are associated with unconstrained households and the remainder are constrained. As

DOI: 10.1002/jae

reported in Table II, of the households with male heads approximately 53.3% remain unconstrained in two successive periods, 21.1% move from constrained to unconstrained, and 16.9% move from unconstrained to constrained.

Table III shows that about 84.6% of household observations in the sample of male heads exhibit a switch to a different liquidity constraint regime at least once during the period of observation, and nearly 15.8% switch 10 times or more. Furthermore, more than 97.9% of the sample changes employment state at least once, and about 33.7% exhibit 10 or more such transitions. These numbers bolster our argument that the dynamics of regime switching need to be investigated properly when working with long panel data sets.

We have also found a rich pattern in dynamics that characterizes transitions over different states of qualitative aspects of employment. Table IV reports one-period transitions in terms of four categories (cells) of qualitative aspects of employment. About 63% of households with male heads are voluntarily employed in a given period and more than half of this fraction (38% overall) remain voluntarily employed in the subsequent period. An additional 4.9% are classified as overemployed, and the remainder are underemployed or unemployed (19.2%), and out of the labor force (12.7%).

Cross-tabulations between labor supply status and liquidity constraint regime, reported in Table V, strongly suggest substantial contemporaneous correlation between the respective indicators. According to Table V, only less than a quarter (15.17 of 62.51), of voluntarily employed households face a binding liquidity constraint, and only slightly less than a fifth (1.15 of 4.88) of overemployed ones, are constrained on the liquidity side. In sharp contrast, 42% (8.69 of 19.85) of underemployed and 58% of unemployed individuals are so constrained.

We conclude that the data do imply a potential joint dependence of being liquidity constrained upon the qualitative state of employment and of the qualitative state of employment upon being liquidity constrained. The presence of unemployment, contemporaneously or in earlier years, may accentuate, in and of its own, the propensity of a worker to be liquidity constrained, as suggested by the cross-tabulations of Table V. Hence, we turn to a model of such behavior.

3. LIFE CYCLE OPTIMIZATION WITH LIQUIDITY AND OTHER QUANTITY CONSTRAINTS

We develop a behavioral model where time is discrete, lifetime horizon is of finite length T, and lifetime utility is additively separable across periods. Utility per period depends on consumption and leisure. Let h_t denote hours worked per year and the endowment of leisure normalized to 1, $\overline{L}_t = 1$, W_t denote the hourly wage rate, G_t consumption (other than leisure), P_{G_t} its price, and P_t the full price vector, $P_t = (W_t, P_{G_t})$. Direct utility per period is written as $u(h, G)$. Extension to the case of a vector of consumption goods is obvious. We assume both consumption good and leisure to be normal. There exists a single riskless asset with a constant rate of return r, $r \geq -1$. We simplify further by setting $P_{G_t} = 1$, and by letting the real wage, W_t, be the sole source of uncertainty, and to be independently and identically distributed over time.

To the direct utility per period function $u(h, G)$ there corresponds an indirect utility function $v(b; W)$, where b denotes asset decumulation, $b = G - Wh$. Let $\{\mathcal{N}_t, t = 0, 1, \ldots\}$ denote uncertainty, in the form of a stochastic process with well-defined transition probabilities; $\mathcal{N}_t$ denotes new information the household receives at time t. Define $\mathcal{N}^t = \{\mathcal{N}_0, \mathcal{N}_1, \ldots, \mathcal{N}_t\}$ to be the information state as of time t, which comprises the set of past realizations of all of exogenous state variables and with respect to which the expectation is taken in (1) below, in this case just W_t, and of the

DOI: 10.1002/jae

endogenous (but predetermined) state variables, in this case beginning of period t assets, A_t. A standard statement of the consumer lifetime optimization problem[5] is from period t on is:

$$\max_{\{h_t\mathbf{G}_t;\ldots\}} u[h_t, G_t|\mathcal{N}_t] + E_t\left\{\sum_{k=t+1}^{T}\frac{1}{(1+\rho)^{k-t}}u_k, G_k|\mathcal{N}]|\mathcal{N}^t\right\} \tag{1}$$

subject to the constraint

$$b_t = G_{\mathrm{t}} - W_t h_t \tag{2}$$

and $A_{t+1} = (1+r)(A_t - b_t)$. Period t decisions are made after W_t has been observed.

3.1. Liquidity Constraints

Unlike the classic treatment (Deaton, 1991) of the liquidity-constrained problem with beginning of period t assets A_t as the single decision variable, we may fix ideas for our model by using two state variables $(A_t, W_t) : W_t$ is an exogenous state variable; A_t is an endogenous one.

We introduce a liquidity constraint, that is, individuals may not hold negative financial wealth at the end of period t, in a 'canonical' form:[6]

$$A_t - b_t \geq 0, \quad t = 1, \ldots, T \tag{3}$$

It follows that relative to Deaton (1991) the presence of leisure in the utility function implies, *ceteris paribus*, that the optimal decision is a function of assets and the real wage. This is a special case of the problem handled by Hajivassiliou and Ioannides (1996), who show that the optimal solution of problem (1) subject to the liquidity constraint (3) satisfies

$$\frac{\partial v(b_t; W_t}{\partial b_t} = \max\left\{\frac{\partial v(A_t; W_t)}{\partial b_t}, \frac{1}{1+\rho}E_t\left\{(1+r)\frac{\partial v(b_{t+1}; W_t)}{\partial b_{t+1}}\right\}\right\} \tag{4}$$

That is, marginal utility is a supermartingale (with a drift). In the infinite horizon case, the solution is of the form $b = b(A; W)$, which is associated with a threshold value of A_t, $\tilde{A}(W_t)$, such that the optimal net asset decumulation has the form

$$b_t = A_t, \qquad A_t < \tilde{A}(W_t) \tag{5}$$

$$b_t = B(A_t; W_t), \quad A_t \geq \tilde{A}(W_t) \tag{6}$$

Equations (5) and (6) define a threshold value of assets as the value of $A, \tilde{A}(W)$, for which the two terms in the right-hand side of (4) are equal to one another. Assets above this value imply that the individual is unconstrained; otherwise, the individual is constrained (Hajivassiliou and Ioannides, 1996).

[5] This statement of the problem follows MaCurdy (1983) and constitutes a multidimensional version of the problem addressed by Altonji (1986), Browning *et al*. (1985), and MaCurdy (1983). Our estimation approach adopts elements of Blundell and Walker (1986).

[6] See Clarida (1987) and Zeldes (1989). Deaton (1991) includes in the definition the value of the endowment of leisure in period t.

DOI: 10.1002/jae

It is straightforward to extend this model so as to define G_t as expenditure on a vector of consumption goods other than leisure with a price vector $\mathbf{P}_{G_t}$. In that case, indirect utility per period reflects the additional determinants, $v[b_t; W_t, \mathbf{P}_{G_t}|\mathcal{N}_t]$. With additive time separability, the problem admits a two-stage budgeting structure (*cf.* Blundell and Walker, 1986). Once b_t is known, labor supply and commodity demands in period t are obtained using Roy's identity.

Our econometric analysis handles liquidity constraints by means of a *liquidity constraint indicator*, a single endogenous variable representing the discrete event of whether or not an individual is liquidity constrained:

$$S_t \equiv S(A_t; W_t, \mathbf{P}_{G_t}; \mathcal{N}^t) = \mathbf{1}[\tilde{A}(W_t, \mathbf{P}_{G_t}; \mathcal{N}^t) - A_t \geq 0] \tag{7}$$

where the *indicator function* $\mathbf{1}[\mathcal{C}]$ is equal to 1 if condition $\mathcal{C}$ is true and to 0 otherwise.

3.2. Quantity Constraints on Labor Supply

We extend formally the life cycle optimization problem (1) subject to (2) and (3), so as to allow for exogenous restrictions on labor supply. Such an extension may be interpreted as a dynamic generalization of Ashenfelter (1980). It is motivated by the availability, within the PSID data, of answers to a number of questions that we interpret as pertaining to voluntary versus involuntary aspects of employment.[7]

Let us consider, in particular, that the consumer believes his labor supply must satisfy a sequence of constraints

$$h_t \leq h_{RU_t}, \quad t = 0, 1, \ldots, T \tag{8}$$

$$h_{RO_t} \leq h_t, \quad t = 0, 1, \ldots, T \tag{9}$$

with probability one. We think of h_{RU_t} and h_{RO_t} as representing demand for an individual's labor in his local labor market. Likely determinants of the constraining quantities are various cyclical factors and, in addition, such factors as the local unemployment rate, the difference between the number of applicants and vacancies in an individual's labor market, the unemployment rate in an individual's (one-digit) occupation, and regional dummies, all variables that are available in the PSID. Quantity constraint (8) may be used to represent *involuntary* unemployment or underemployment. Quantity constraint (9) may be used to represent, symmetrically, *involuntary* overemployment. We abstract from the labor force participation decision, which of course would introduce an additional qualitative employment state. When compared to liquidity constraints (3), quantity constraints (8)–(9) may have an even better claim to possessing a strong 'Keynesian' flavor.[8]

---

[7] Appendix B, available on the Data Archive journal website, provides details on how we recoded the PSID information in order to measure unemployment, underemployment, or overemployment.

[8] Ham (1986) notes that the effect on a worker of demand shocks to an industry or a region may depend on his characteristics and various human capital variables, which, following others, we include in the model as determinants of labor supply behavior. Card (1994), however, argues that Keynesian-style labor market constraints are not indispensable for rationalizing Ham's findings on the importance of demand factors. He suggests instead that individuals may decide on their labor supply at a higher frequency time unit than the year (for which data are available) and that there may be significant fixed costs on either the worker's side or the employer's side of the labor market. We do not test for such effects.

DOI: 10.1002/jae

We denote the solution for the unconstrained (*notional*) labor supply from problem (1), subject to all constraints, conditionally upon S_t, by $h_t = H(A_t; W_t, \mathbf{P}_{G_t}; \mathcal{N}^t|S_t)$. As this is a function of assets, following MaCurdy (1983) we may refer to it as the *pseudo* labor supply function. An *employment state indicator* may now be defined in terms of the pseudo labor supply function as:

$$E_{\mathrm{t}} = E(A_{\mathrm{t}}; W_{\mathrm{t}}, \mathbf{P}_t; \mathcal{N}^t|S_t) = -1, \quad \text{if } h_t = h_{RO_t} \geq H(A_t; W_t, \mathbf{P}_{G_t}; \mathcal{N}^t|S_t) \tag{10}$$

$$E_{\mathrm{t}} = E(A_{\mathrm{t}}; W_{\mathrm{t}}, \mathbf{P}_t; \mathcal{N}^t|S_t) = 0, \quad \text{if } h_{RO_t} < H(A_t; W_t, \mathbf{P}_{G_t}; \mathcal{N}^t|S_t) < h_{RU_t} \tag{11}$$

$$E_{\mathrm{t}} = E(A_{\mathrm{t}}; W_{\mathrm{t}}, \mathbf{P}_t; \mathcal{N}^t|S_t) = 1, \quad \text{if } h_t = h_{RU_t} \leq H(A_t; W_t, \mathbf{P}_{G_t}; \mathcal{N}^t|S_t) \tag{12}$$

It readily follows from its definition that E_t lends itself to an *ordered* discrete-choice formulation.

It is helpful to try and visualize the determination of the employment state indicator in a static-equivalent setting. We note that once the period t net asset decumulation b_t has been determined, we may refer to a standard consumption–leisure choice diagram, such as in Figure 1. Given prices and net asset decumulation, the position of the 'budget line' is determined. Furthermore, given parameters and values for the observables and unobservables, a particular individual who is in the labor force may be in one of three categories. An individual may be of type V, in which case employment is determined according to point V_T, and the individual is voluntarily employed. We note that in this case $h_{RO} \leq h \leq h_{RU}$. Alternatively, an individual may be of type U; i.e., one who wishes to work according to point U_T. He may not, however, work as much as he wishes because of the underemployment constraint h_{RU}. In such a case, employment is determined according to point U_R, and the individual is involuntarily underemployed (or unemployed) working h_{RU} hours. Finally, an individual may be of type O; i.e., one who wishes to work according to point O_T. Such an individual may not, however, be able to work as little as he wishes because of the overemployment constraint h_{RO}. In such a case employment is determined according to point O_R, and the individual is involuntarily overemployed, working h_{RO} hours.[9] An appropriate analytical representation of this choice problem requires that it always be the case that $h_{RU_t} \geq h_{RO_t}$, which we impose econometrically. The economic intuition of this assumption is straightforward. The maximum amount an individual is allowed to work must not be less than the minimum.

In view of the discussion of the determinants of h_{RU_t} and h_{RO_t} above, we would expect that an upturn in the business cycle would increase the magnitudes of both of the constraining quantities. This would cause the overemployment constraint to become tighter and the underemployment one to be relaxed. Both those outcomes accord with economic intuition.

The general problem of dynamic consumption decisions subject to quantity constraints belongs to a class of decision problems with mixed discrete–continuous decisions the estimation of which has been discussed by Pakes (1994) and Rust (1994). However, it is important to note that even though in the present paper we are interested only in the estimation of a discrete dynamic decision problem, the original problem is not reducible in terms of discrete decisions only, and a statement of the full dynamic programming problem is called for. We eschew, for reasons of brevity, additional details of the problem, and refer to our earlier working paper Hajivassiliou and Ioannides (1994). We instead propose an estimation model for the vector of endogenous variables (S_t, E_t), defined in (7) and (10)–(12). Our approach admits as special cases some of the problems examined by several

[9] If it may be assumed that the notional labor supply function is locally monotonic with no backward bending portion, the definition of E_t may be alternatively stated in terms of wage comparisons. In fact, such a definition may be more appropriate, given that constraints h_{RU} and h_{RO} are actually not observed.

DOI: 10.1002/jae

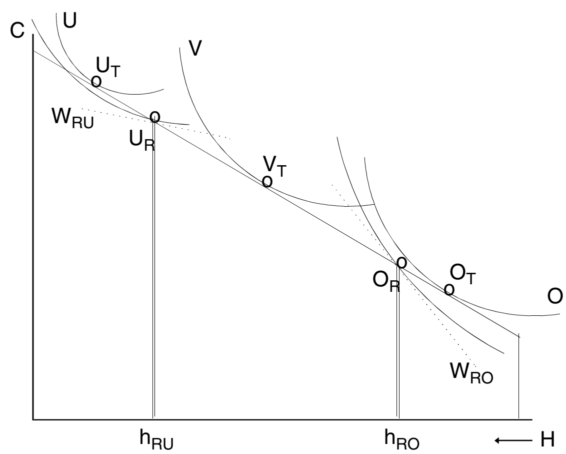

Figure 1. Labor constraints

previous researchers, including in particular Ball (1990) and Zeldes (1989), whose contributions we discuss in detail at the beginning of Section 5.3 below.

In lieu of a complete treatment, a number of remarks are in order. First, if an individual in a particular period is unconstrained with respect to either liquidity or employment, anticipation of constraints' possibly binding some time in the future are reflected in current decisions through the *conditional value functions* $V_t^{s,e}$, defined as the optimal value of remaining lifetime utility, conditional on $\{s, e\}$.[10] Intuitively, to the extent that constraints (3), (8) and (9) ever bind, they would affect total lifetime resources and utility outcomes.

Second, in spite of considerable research efforts during the last few years, structural estimation of a general mixed discrete–continuous model like ours has run up against insurmountable (at present) computational difficulties.[11] It is for this reason that we pursue estimation of approximate reduced form aspects of the problem.

Third, the functional form of the optimal solution for b_t as a function of state variables does depend upon whether or not the individual is constrained with respect to either liquidity or employment or both. This dependence is, in turn, transmitted to commodity demands and to labor supply, a fact that we exploit in specifying our estimation models in Section 4 below.

4. ECONOMETRIC MODELS

4.1. Simultaneous Determination of Liquidity and Employment Constraint Indicators

We shall aim in this paper at estimating the parameters of the stochastic structures determining two discrete endogenous variables, defined by (7) and (10)–(12), as functions of observable characteristics of the decision maker and his environment, while allowing for dynamics. We do so

[10] For the usefulness of the conditional valuation function, see Hotz and Miller (1993) and Rust (1988, 1994). They allow the dynamic discrete choice problem to be transformed to an equivalent but static one; the conditional valuation functions play the role of values of a static utility associated with discrete alternative courses of action.

[11] There are no breakthroughs in the mixed discrete–continuous decision problems comparable to Rust (1988) and to Hotz and Miller (1993) for purely discrete decisions. See Pakes (1994) and Rust (1994).

DOI: 10.1002/jae

by introducing state dependence, via the dependent variable's own lagged values as regressors. In addition, we allow contemporaneous spillover and lagged spillover effects from one to the other endogenous variable. All of our models are jointly estimated as systems of discrete decisions that allow for unobservable persistent heterogeneity in the stochastic structures, while imposing so-called 'coherency conditions' required for logical and statistical validity of our models. Even though the individual decisions may be estimated as reduced forms, the model of Section 3 allows them to be construed in quasi-structural forms also, by means of the conditional value functions as we shall see shortly. That is, the employment indicator E_t may be defined conditionally on S_t, which makes the model consistent with the two-stage budgeting setting of Blundell and Walker (1986), and vice versa for the liquidity constraint indicator, conditional on the employment indicator. To the extent that the discrete indicators may not be perfectly observed in our data, the stochastic shocks in the respective models may be interpreted as consisting partly of an observation error. The implications of this are discussed below.

4.2. General Unordered Reduced Forms

We assume for simplicity linear functional forms for the conditional valuation functions for individual i at time t:

$$V_{it}^{s,e} = \Psi_{it}^{s,e}\beta_{s,e} + \varepsilon_{it}^{s,e},\ s \in \{0, 1\},\ e \in \{-1, 0, 1\} \tag{13}$$

where $\Psi_{it}^{s,e}$ are vectors of polynomial functions of explanatory variables, which might include lagged values of endogenous variables, $\beta_{s,e}$ is a corresponding vector of parameters, and $\varepsilon_{it}^{s,e}$ are random variables that correspond to the unobserved components of utility at time t.

Once we have assumed a particular stochastic structure for the $\varepsilon_{it}^{s,e}$ s, we may use (13) to estimate the model. This specification yields a hexanomial model, as becomes evident immediately below. As an example, the probability that an individual is observed voluntarily employed and liquidity unconstrained in period t, defined as the probability of $[V_{it}^{0,0} \geq V_{it}^{0,1}, V_{it}^{0,0} \geq V_{it}^{0,-1}]$ is given by

$$\begin{aligned}\text{prob}\ [S = 0, E = 0|\Psi_{it}^{s,e}] = \text{prob}\ \Big[&\varepsilon_{it}^{0,0} - \varepsilon_{it}^{0,-1} \geq \Psi_{it}^{0,-1}\beta_{0,-1} - \Psi_{it}^{0,0}\beta_{0,0},\\ &\varepsilon_{it}^{0,0} - \varepsilon_{it}^{0,1} \geq \Psi_{it}^{0,1}\beta_{0,1} - \Psi_{it}^{0,0}\beta_{0,0}, \varepsilon_{it}^{0,0} - \varepsilon_{it}^{1,0} \geq \Psi_{it}^{1,0}\beta_{1,0} - \Psi_{it}^{0,0}\beta_{0,0},\\ &\varepsilon_{it}^{0,0} - \varepsilon_{it}^{1,1} \geq \Psi_{it}^{1,1}\beta_{1,1} - \Psi_{it}^{0,0}\beta_{0,0}, \varepsilon_{it}^{0,0} - \varepsilon_{it}^{1,-1} \geq \Psi_{it}^{1,-1}\beta_{1,-1} - \Psi_{it}^{0,0}\beta_{0,0}\Big]\end{aligned} \tag{14}$$

The likelihood of this event may be written in terms of the probability distribution functions of the $\varepsilon_{it}^{s,e}$ s, while allowing for the presence of lagged endogenous variables among the Ψs. Since the $\varepsilon_{it}^{s,e}$ s are unobserved components of the state vector (Rust, 1988), it is appropriate to treat them as unobservable random shocks, which may reflect individual heterogeneity. Given the state of the art in estimating dynamic discrete choice models, a fairly general assumption we can make is to treat them as random effects with a time-invariant component and an $AR(1)$ component to the general error term.

Specifically, we assume the $\varepsilon_{it}^{s,e}$ s are of the form

$$\varepsilon_{it}^{s,e} = \eta_i^{s,e} + \varsigma_{it}^{s,e} \tag{15}$$

DOI: 10.1002/jae

where the $\eta_i^{s,e}$ s are time-invariant random individual effects and the $\varsigma_{it}^{s,e}$ s obey the $AR(1)$ structure:

$$\varsigma_{it}^{s,e} = \rho_{AR}^{s,e}\varsigma_{it-1}^{s,e} + \xi_{it}^{s,e} \tag{16}$$

where the $\xi_{it}^{s,e}$ s are random variables independently and identically distributed (i.i.d.) over time with means equal to zero, and a 6×6 variance–covariance matrix. This covariance matrix captures the contemporaneous correlation over the six random variables indexed by $\{\{0, 1\} \times \{-1, 0, 1\}\}$.

This implies a $T_i \times 6$-dimensional correlated vector for observation i. In general, because the limited dependent variables in this model are purely discrete, to achieve identification one needs to normalize the conditional valuation functions of one of the six outcomes to zero. Hence, the parameters that can be estimated are as follows: five of the six parameter vectors β_{se} in (13), 14 ($= 5 \times 6/2 - 1$) elements of the contemporaneous variance–covariance matrix of the $\xi_{it}^{s,e}$ s in (16), 15 ($= 5 \times 6/2$) elements of the contemporaneous variance–covariance matrix of the $\eta_i^{s,e}$ random effects in (15), and five of the autoregressive coefficients $\rho_{AR}^{s,e}$ in (16).

Thus, consideration of all possible liquidity and labor supply constraints leads to switching regressions, with switching occurring in two dimensions: one, on account of liquidity constraints; two, on account of quantity constraints on labor supply. The introduction of exogenous constraints on labor supply augments the number of possible regimes in a given period from two, in the case of liquidity constraints alone, to six. Thus, the number of possible outcomes corresponds to the six possibilities defined by $\{\{0, 1\} \times \{1, 0, -1\}\}$.[12] This may be handled as a system of simultaneous discrete response models, corresponding to the discrete events (S_t, E_t).

4.3. Quasi-Structural Form Models with Ordering

The model we developed above does suggest a more specific (and thus testable) stochastic structure, that is, one involving two discrete endogenous variables that jointly generate six regimes with a set of implied restrictions, namely that the employment state indicator is naturally *ordered.* Of those endogenous variables, the liquidity constraint indicator, S_t, introduced in (7), may be handled by means of dynamic probit model. The employment state indicator E_t, defined by (10)–(12), suggests that it be modeled as an ordered probit model. Section 5.1 below describes the binary probit part of our model, which assumes that a binary regime indicator for S_t is perfectly observable for every household in every period. The second part of our model, which assumes a perfectly observed employment state indicator E_t is available, is a dynamic ordered probit model and is discussed in Section 5.2. Joint estimation of these two models, discussed in Section 5.3, allows for interactions between liquidity-constrained behavior and qualitative aspects of employment behavior and combines the above dynamic probit and ordered probit sides. Specifically, the likelihood of unemployment is allowed to be affected by an individual's being constrained in the labor market.

We highlight the fact that the ordered probit model may be nested in the classical sense into the general unrestricted hexanomial model introduced in Section 4.2 above. See also Weeks and Orme (1998) for an independent approach to a similar issue in comparing bivariate and multinomial choice models. It is simpler to show this if we concentrate on the labor employment indicator E_t and drop the time subscript. We then have that

$$\text{prob }[E = 0] = \text{prob }[\varepsilon_0 - \varepsilon_{-1} \geq \Psi_{-1}\beta_{-1} - \Psi_0\beta_0, \varepsilon_0 - \varepsilon_1 \geq \Psi_1\beta_1 - \Psi_0\beta_0]$$

[12] If the status of being out of the labor force (voluntarily unemployed) is included and underemployment is distinguished from unemployment we would have 10 states.

DOI: 10.1002/jae

By defining $\varepsilon'_1 \equiv \varepsilon_1 - \varepsilon_0$ and $\varepsilon'_{-1} \equiv \varepsilon_{-1} - \varepsilon_0$, the above probability may be written in terms of the bivariate distribution function: prob $[E = 0]$ = prob $[\varepsilon'_{-1} \leq \Psi_{-1,1}\beta_{-1,1}, \varepsilon'_1 \leq \Psi_{1,1}\beta_{1,1}]$. It follows that this set-up is equivalent to an ordered probit model in terms of a single underlying random variable, ε'_{-1}, if and only if $\varepsilon'_{-1} \equiv -\varepsilon'_1$ (which implies $\varepsilon_1 \equiv \varepsilon_{-1}$), and provided that, in addition, the following conditions are satisfied: first, the variable components of $\Psi_{-1,1}\beta_{-1,1}$ and $\Psi_{1,1}\beta_{1,1}$ have coefficients which are opposite to one another (i.e., their variable components sum to 0); and second, their intercepts differ. These testable restrictions are discussed in Section 5.4 below.

There is a simple way to view the hexanomial model, defined by the outcomes $\{\{0, 1\} \times \{1, 0, -1\}\}$, in relation to the simultaneous system composed of the binary probit and the ordered probit model. The hexanomial model is assumed to determine which of the six regimes prevails by comparing functions of regressors and parameters against draws of *six* latent variables, without reference to ordering. In contrast, the simultaneous system composed of the binary probit and the ordered probit model orders the outcomes on the employment margin and describes them by *two* underlying latent variables. A further difference is that the 6-regime unrestricted version of our model exhibits jointness of the two margins of constrained behavior S vs. E that is non-separable in terms of observables since each regime models directly an (S, E) pair. In contrast, the restricted version models the S and E sides in a separable fashion, giving the econometrician the freedom of including or not including observable spillover effects on each side.

4.4. The Problem of Imperfections in the Constraint Indicators

Our assumption that the binary regime indicator for liquidity constraints S_t and the ordered employment indicator E_t are perfectly observable, while serving well to illustrate our basic approach, is problematic in general, especially with respect to S_t. A particular threshold amount of financial assets $\tilde{A}_{it}$, which depends on individual characteristics as well as market variables but is not observed directly, determines switching of regimes for S_{it}. That is, holding assets A_{it} exceeding the threshold level signifies that the household is not subject to a borrowing constraint in a period t.

One could consider generalizing our econometric model to allow for an imperfect indicator, J_{it}, specifying whether or not liquidity constraints are binding, based on $I_{ai_{t+1}}$, the observed value of asset income for household i at the beginning of period $t + 1$. Since typically assets vary in their liquidity characteristics, which are unobservable, the procedure we (and others before us) have used to impute asset stocks is at best imperfect. It is therefore important to account for implied imperfections in the regime indicators and thus allow for misclassification (cf. Lee and Porter, 1984). One approach would be to allow for random *coding errors* in the equations defining the regime indicators. This model, in contrast to the Lee and Porter (1984) formulation, allows the probability of misclassification to vary endogenously and to be determined by economic fundamentals. Provided such coding errors are i.i.d., they would not affect the consistency up to scale of the discrete estimation procedures we adopt in this paper.

5. MAIN ESTIMATION MODELS AND EMPIRICAL RESULTS

Since households must adapt their behavior to the presence of constraints on asset holdings and on labor supply, the path of the regime indicators $[S_t, E_t]$ is endogenous. Zeldes (1989), who works with liquidity constraints and food consumption only, does not deal with switching; neither do

DOI: 10.1002/jae

Altonji (1986) or Ball (1990), who work with food consumption and labor supply data,[13] nor Ham (1986), who uses only labor supply data.[14] We exploit the substantial time variation associated with qualitative employment status categories. This is one of the reasons for which this paper may be considered a generalization of Ball's. Given specific assumptions about the distribution of the unobservables, this endogeneity can be analyzed by maximum simulated likelihood estimators.

It is instructive to highlight the interaction between the liquidity and labor supply constraint indicators, S_t and E_t, by considering structural forms for the pair of two endogenous variables $[S_t, E_t]$ as a system. Consider first models for $[S_t, E_t]$ symmetrically defined with dummy endogenous variables and general state dependence as follows:

$$S_{it} = \mathrm{BP}(S_{it}^*) \equiv \mathrm{BP}(\gamma_{11}S_{i,t-1} + \gamma_{12}S_{i,t-2} + \delta_0 E_{it} + \delta_1 E_{i,t-1} + \delta_2 E_{i,t-2} + X_{it}\beta^{bp} + \varepsilon_{it}^{bp}) \quad (17)$$

$$E_{it} = \mathrm{OP}(E_{it}^*) \equiv \mathrm{OP}(\gamma_{21}E_{i,t-1} + \gamma_{22}E_{i,t-2} + \kappa_0 S_{it} + \kappa_1 S_{i,t-1} + \kappa_2 S_{i,t-2} + X_{it}\beta^{op} + \varepsilon_{it}^{op}) \quad (18)$$

where BP and OP denote *binary probit* and *ordered probit* functions, respectively. In a static version of our setting, coherency conditions (Schmidt, 1981) reduce to conditions that the model be recursive; that is, the coefficients δ_0 and κ_0 in (17) and (18) satisfy $\delta_0 \cdot \kappa_0 = 0$. See Appendix A, Section A.3 for details. Note that the correlation between the errors ε_{it}^{bp} and ε_{it}^{op} in (17) and (18) is of particular interest, because the presence of unemployment may accentuate the propensity of an individual to be liquidity constrained even after conditioning on all observable information.

In the remainder of the paper we report and discuss estimation results for the system of quasi-structural forms (17) and (18). We specify their stochastic structure as follows:

$$\varepsilon_{it}^{bp} = \eta_i^{bp} + \varsigma_{it}^{bp}, \quad \varsigma_{it}^{bp} = \rho_{AR}^{bp}\varsigma_{it-1}^{bp} + \xi_{it}^{bp}, \quad |\rho^{bp}| < 1 \quad (19)$$

$$\varepsilon_{it}^{op} = \eta_i^{op} + \varsigma_{it}^{op}, \quad \varsigma_{it}^{op} = \rho_{AR}^{op}\varsigma_{it-1}^{op} + \xi_{it}^{op}, \quad |\rho^{op}| < 1 \quad (20)$$

where η_i^{bp}, η_i^{op} are time-invariant unobservable characteristics of household i assumed to be Gaussian i.i.d. over the sample with zero means, standard deviations $\sigma_{\eta_i}^{bp}$, $\sigma_{\eta_i}^{op}$, respectively; and ς_{it}^{bp}, ς_{it}^{op} are stationary $AR(1)$ random processes with autocorrelation coefficients ρ_{AR}^{bp}, ρ_{AR}^{op}, and i.i.d. innovations ξ_{it}^{bp}, ξ_{it}^{op}, respectively. The latter are assumed to be contemporaneously correlated, conditional on all explanatory variables, including lagged dependent variables, with correlation coefficient $\mathrm{corr}(\xi_i^{bp}, \xi_i^{op})$. The variances of the i.i.d. shocks are normalized to 1. We also assume that the innovations ξ_{it}^{bp}, ξ_{it}^{op} are mean independent of the explanatory variables X_{it}, while the time-invariant effects η_i^{bp} and η_i^{op} are allowed to be correlated with the regressors X_{it} in a time-invariant fashion. See Hajivassiliou (2003) on this point, which follows Chamberlain (1984) and models the dependence of η_i^{bp} and η_i^{op} on regressors as

$$E(\eta_i^{bp}|X_{i1}, \ldots, X_{iT_i}) = \overline{X}_{i.}\theta^{bp} \text{ and } E(\eta_i^{op}|X_{i1}, \ldots, X_{iT_i}) = \overline{X}_{i.}\theta^{op}.$$

[13] Zeldes (1989) assumes that regimes are perfectly observable and uses only data for the unconstrained group in the estimations. If, as expected, regimes are endogenously determined, his procedure will give unreliable inferences. Altonji (1986) excludes constrained individuals. Ball's approach differs from Zeldes' only in his using jointly food consumption and labor supply data. Ball (1990) uses data from the PSID for 1968–1981 and classifies a worker as constrained if he either experiences a spell of unemployment or cannot work as many hours as he wants in any sample year. Biddle (1988) uses PSID data for 1976–1980 and a scheme similar to Ball's to classify workers as constrained.

[14] Ham (1986)'s use of dummy endogenous variables to account for the impact of constraints is less general than ours, but his separation of underemployment from unemployment is noteworthy, especially in view of his finding that business cycle variables are good instruments for unemployment but not for underemployment.

DOI: 10.1002/jae

This device introduces individual time-averaged sample means $\overline{X}_{i.}$ as additional regressors in S_{it}^* and E_{it}^* in (17) and (18) and hence results in random persistent heterogeneity effects η_i^{bp} and η_i^{op} in (19) and (20), that are uncorrelated with the regressors. Assuming that the errors have a non-scalar variance–covariance structure conditional on all explanatory variables, including lagged dependent variables, is often done to express coexistence of state dependence and heterogeneity, as in Heckman (1981a), or to express impact of habits, as in Hotz *et al.* (1988). The full stochastic structure we assume here implies that we do not need to instrument for the lagged dependent variables, but do need to specify the distribution of the initial conditions. Our MSSL/GHK estimation procedure incorporates fully these features. See Appendix A, Section A.5 for more details.[15]

Next, we summarize the models that we estimate. It is pretty clear that univariate probit models for $[S_{it}, E_{it}]$ are fully dominated by the bivariate ones. We do not report the univariate results for reasons of brevity, but present diagnostics to that effect. Column (a) of Table VI reports liquidity constraint and employment indicators, according to (17) and (18), under the restrictions $\delta_0 = \delta_1 = \delta_2 = 0$ and $\kappa_0 = \kappa_1 = \kappa_2 = 0$, respectively, namely that neither contemporaneous nor lagged spillover effects are included from the other constraint side of the model. Column (b) of Table VI reports a similar joint estimation after we further augment the dynamic structure by allowing for lagged spillover effects but not contemporaneous ones, that is, in terms of the system (17)–(18), under the restrictions $\delta_0 = 0$ and $\kappa_0 = 0$, respectively. Finally, column (c) of Table VI reports joint estimations of the full set of quasi-structural forms that include both contemporaneous and lagged spillover effects, but always making sure to guarantee the coherency conditions. That is, column (c) of Table VI reports the results for quasi-structural form (17) for the liquidity constraint equation, estimated jointly with a model for the employment constraints equation like

Table VI. Liquidity constraint equation: parameter estimates male heads, in-the-labor-force; dependent variable: zdumc2 (S = 1 if liquidity constrained, 0 if not.)

Liquidity constraint equation Jointly with empl. eq.	Version VI(a) Version VII(a)		Version VI(b) Version VII(b)		Version VI(c) Version VII(b)[a]	
Log-likelihood	−29428.97		−29422.74		−29401.40	
16 $\overline{X}i \cdot$ LR	192.16		189.77		181.69	
$\hat{P}_1(\overline{X})$	0.18		0.18		0.18	
$\hat{P}_1$	0.27		0.27		0.27	
$\mathbf{1}(X_{it}\hat{\beta} > 0)$	0.26		0.26		0.26	
Correct predictions	0.88		0.88		0.88	
Parameter	Estimate	*t*-statistic	Estimate	*t*-statistic	Estimate	*t*-statistic
$\mathrm{corr}(\xi_{it}^{bp}, \xi_{it}^{op})$	0.43	18.2	0.38	15.3	0.34	7.89
$\sigma_{\eta_i}^{bp}$	0.85	31.2	0.85	31.2	0.85	31.1
ρ_{AR}^{bp}	0.68	11.8	0.68	14.3	0.68	13.8

[15] In such a setting, the presence of a lagged dependent variable among the regressors does not necessarily imply a contemporaneous correlation in every period. For example, as Heckman (1981a) explains, it is still possible to assume that conditional on the RHS variables the only residual correlation is through the random effect plus its $AR(1)$ structure, as we have assumed. Our approach is considerably more general than Heckman's in that it explicitly allows for the unobservable heterogeneity effects to be correlated with regressors in a time-invariant fashion.

Table VI. (*Continued*)

Regressor	Estimate	t-statistic	Estimate	t-statistic	Estimate	t-statistic
intercept	94.4	2.55	95.7	2.59	94.5	2.55
liq. cons. binding at $t-1$?	1.12	38.9	1.12	38.8	1.12	38.9
liq. cons. binding at $t-2$?	0.15	5.04	0.15	4.95	0.15	4.98
overemployed?	—	—	—	—	0.006	0.11
overemployed at $t-1$?	—	—	0.01	0.27	0.02	0.35
overemployed at $t-2$?	—	—	−0.04	−0.67	−0.03	−0.65
unemployed?	—	—	—	—	0.12	3.85
unemployed at $t-1$?	—	—	0.03	1.04	0.02	0.56
unemployed at $t-2$?	—	—	0.04	1.34	0.04	1.20
county unmpl. rate	−0.0003	−0.06	−0.0006	−0.10	−0.001	−0.21
head disabled?	−0.04	−0.78	−0.04	−0.79	−0.05	−0.86
education head	−0.06	−5.03	−0.06	−4.83	−0.06	−4.61
year = 1976–1979	0.89	2.78	0.89	2.76	0.88	2.73
year = 1980–1983	0.86	3.02	0.86	3.02	0.85	3.00
year = 1984–1987	0.59	2.61	0.58	2.58	0.58	2.57
food needs	−0.0002	−3.70	−0.0002	−3.72	−0.0002	−3.76
growth food needs	−0.25	−5.47	−0.25	−5.51	−0.25	−5.51
head age	−0.41	−10.26	−0.41	−10.23	−0.40	−10.06
head age cubed	−0.00005	−6.73	−0.00005	−6.71	−0.00005	−6.57
head age squared	0.008	7.80	0.008	7.78	0.008	7.63
tenure head (months)	−0.001	−2.77	−0.001	−2.69	−0.001	−2.47
tenure head squared	1.06e−06	0.88	1.04e−06	0.86	8.9e−07	0.73
unempl. insur. head	9.16e−06	0.45	6.66e−06	0.33	−2.02e−06	−0.10
labr market state	0.01	0.70	0.01	0.70	0.01	0.66
live in north-centr.?	−0.12	−1.97	−0.12	−1.94	−0.12	−1.93
live in other regions?	0.43	2.80	0.43	2.79	0.43	2.73
live in south?	0.12	1.97	0.12	1.99	0.12	2.00
live in west?	0.08	1.18	0.08	1.22	0.09	1.26
head single?	0.70	14.02	0.70	13.97	0.69	13.92
num. chldrn age 0–5	−0.05	−2.35	−0.05	−2.39	−0.05	−2.37
occupational unempl.	0.02	3.36	0.02	3.26	0.02	3.16
head black?	0.61	6.27	0.60	6.22	0.59	6.19
head other race?	0.18	1.60	0.17	1.55	0.17	1.50
head relig. chr./east.orthodox?	0.09	1.58	0.09	1.61	0.09	1.57
head relig. Jewish?	0.16	1.45	0.17	1.45	0.16	1.42
head relig. Protestant?	0.16	3.75	0.16	3.76	0.16	3.77
real interest rate	14.25	7.64	14.1	7.57	13.6	7.31
head union member?	−0.03	−0.73	−0.03	−0.77	−0.03	−0.86
plus 16 time-averages	See text, p. 491		See text, p. 491		See text, p. 491	

[a] Joint estimation with employment version 7(c) would have violated the coherency condition discussed above.

the one reported in column (b) of Table VII, that is, with the quasi-structural form (18) under the coherency condition $\kappa_0 = 0$. Column (c) of Table VII reports the results for quasi-structural form (18) for the employment constraints equation, estimated jointly with a model for the liquidity constraint equation like the one reported in column (b) of Table VI, or stated alternatively, with the quasi-structural form (17) under the coherency condition $\delta_0 = 0$.

5.1. Empirical Results: The Liquidity Constraint Side of the Model

Consider the results reported in Table VI. These report estimations for the liquidity constraint indicator S_{it} model, according to (17), jointly estimated with different versions of the employment

DOI: 10.1002/jae

constraint indicator E_{it} model, according to (18). The dependent variable S_{it} is measured by a dummy variable identical to Zeldes' 'total wealth split' of the data into constrained ($S = 1$) and unconstrained ($S = 0$) households (see Appendix B on the journal website). The estimations of the employment constraints models are discussed in Section 5.2 below.

The variance of the i.i.d. component in the $AR(1)$ shock is normalized to 1. The presence of the random effect structure is statistically very significant. The coefficients of most explanatory variables are also very significant and generally have the expected sign. The importance of the panel structure is confirmed by comparing estimations of a homogeneous probit model with an identical set of explanatory variables restricted to have an i.i.d. error structure (i.e., $\sigma_{\eta}^{bp} = 0$ and $\rho_{AR}^{bp} = 0$). This is the starting point for our estimations, but we do not report them here for reasons of brevity, except to note that a number of key coefficients, e.g., that of the real rate of interest, have the wrong sign when the panel structure is ignored.

The results highlight the importance of the dynamic structure. The lagged values of all endogenous variables are always very significant and imply substantial state dependence. The autoregressive correlation coefficient ρ_{AR}^{bp} is estimated to be 0.68 with an asymptotic t-statistic of 11.8. The standard deviation of the random effect η_i^{bp}, $\sigma_{\eta_i}^{bp}$, is also statistically significant, with a t-statistic of 31.2, and so is $\mathrm{corr}(\xi_{it}^{bp}, \xi_{it}^{op})$, whose estimate is 0.43 and its t-statistic is 18.2. The estimated coefficients for the two lags of the endogenous variable $S_{i,t-1}$ and $S_{i,t-2}$, which are included in the regression, are 1.12 and 0.15, and their t-statistics are 38.9 and 5.04, respectively. Both those effects and the sign of the autocorrelation coefficient suggest a high degree of persistence in the likelihood of being liquidity constrained.

The lagged endogenous variables for overemployment and for under- or unemployment in the previous two periods, respectively, are not included in the regression we report in column (a) but are included in those reported in columns (b) and (c). They do not appear significant and their estimated coefficients are numerically small, suggesting no substantial role for lagged spillovers from the employment side.

Table VII. Employment constraints equation: parameter estimates male heads, in-the-labor-force; dependent variable: LabCon3 (E = −1 if overemployed, 0 if voluntarily, +1 if under-/unemployed)

Employment constraints equation	Version VII(a)		Version VII(b)		Version VII(c)	
Estimated jointly with liquid. eq.	Version VI(a)		Version VI(b)		Version VI(b)[a]	
Log-likelihood	−29428.97		−29422.74		−29406.32	
16 $\overline{X}i \cdot$ LR	241.72		236.85		222.69	
$\hat{P}_{\cdot}(\overline{X})$	overE:0.03	unE:0.19	overE:0.03	unE:0.19	overE:0.03	unE:0.19
$\overline{\hat{P}_{\cdot}}$	overE:0.06	unE:0.22	overE:0.06	unE:0.22	overE:0.06	unE:0.22
Correct predictions	overE:0.94	unE:0.81	overE:0.94	unE:0.81	overE:0.94	unE:0.81
Parameter	Estimate	t-statistic	Estimate	t-statistic	Estimate	t-statistic
$\mathrm{corr}(\xi_{it}^{bp}, \xi_{it}^{op})$	0.43	18.26	0.38	15.3	0.34	7.89
$\sigma_{\eta_i}^{op}$	0.55	22.4	0.52	21.8	0.49	20.4
ρ_{AR}^{op}	0.45	7.47	0.43	8.23	0.40	7.40
θ^{-}	−2.72	−4.25	−2.72	−4.26	−2.72	−4.26
θ^{+}	Normalized at 0		Normalized at 0		Normalized at 0	

Table VII. (*Continued*)

Regressor	Estimate	t-statistic	Estimate	t-statistic	Estimate	t-statistic
intercept	33.7	24.3	31.6	24.3	31.9	24.3
overemployed at $t-1$?	−0.68	−21.3	−0.68	−21.3	−0.68	−21.3
overemployed at $t-2$?	−0.32	−10.1	−0.32	−10.1	−0.32	−10.1
unemployed at $t-1$?	0.70	36.3	0.69	36.1	0.69	36.1
unemployed at $t-2$?	0.37	19.3	0.36	19.1	0.36	19.1
liq. const. binds	—	—	—	—	0.12	5.07
liq. cons. at $t-1$	—	—	0.05	1.99	−0.01	−0.49
liq. cons. at $t-2$	—	—	0.01	0.51	−0.002	−0.10
county unmpl. rate	0.007	2.11	0.007	2.13	0.007	2.15
head disabled?	0.04	1.62	0.04	1.59	0.04	1.61
education head	−0.03	−5.78	−0.03	−5.62	−0.03	−5.52
year = 1976–1979	0.11	0.54	0.12	0.57	0.10	0.49
year = 1980–1983	0.26	1.41	0.26	1.44	0.25	1.34
year = 1984–1987	0.19	1.31	0.20	1.34	0.18	1.23
food needs	0.0001	4.40	0.0001	4.44	0.00001	4.39
growth food needs	−0.09	−2.68	−0.10	−3.00	−0.08	−2.54
head age	−0.09	−5.16	−0.08	−4.90	−0.07	−4.37
head age cubed	−0.00001	−4.19	−0.00001	−4.05	−0.00001	−3.67
head age squared	0.002	4.47	0.002	4.29	0.002	3.85
tenure head (months)	−0.002	−8.19	−0.002	−8.07	−0.002	−7.96
tenure head squared	3.2e−06	5.29	3.2e−06	5.22	3.2e−06	5.16
unempl. insur. head	0.0003	16.63	0.0003	16.63	0.0003	16.62
imputed wage	0.002	2.45	0.002	2.52	0.003	2.63
labr market state	0.02	1.93	0.02	1.90	0.02	1.88
in north-centr.?	−0.05	−2.32	−0.05	−2.25	−0.05	−2.16
in other regions?	0.05	0.60	0.05	0.54	0.04	0.46
in south?	−0.02	−0.92	−0.02	−0.95	−0.02	−0.98
in west?	−0.15	−6.08	−0.15	−6.08	−0.15	−6.08
head single?	0.05	1.92	0.04	1.51	0.03	0.96
num. chldrn age 0–5	0.01	0.99	0.01	1.00	0.01	1.03
occupational unempl.	0.01	5.06	0.01	4.96	0.01	4.88
head black?	0.15	4.34	0.14	4.04	0.13	3.79
head other race?	0.25	5.24	0.25	5.24	0.25	5.20
head relig. chr./east.orthodox?	0.03	1.19	0.03	1.24	0.03	1.20
head relig. Jewish?	0.05	1.06	0.05	1.05	0.05	0.98
head relig. Protestant?	−0.01	−0.57	−0.01	−0.54	−0.01	−0.63
real interest rate	13.2	12.83	12.9	12.54	12.6	12.22
head union member?	0.08	4.35	0.08	4.43	0.08	4.45
plus 16 time averages	see text, p. 491		see text, p. 491		see text, p. 491	

[a] Joint estimation with liquidity constraint version 6(c) would have violated the coherency condition.

The other regressors, denoted by X_{it}, include, roughly speaking, preferences, labor supply variables, and labor demand variables. Specifically, education, food needs (a PSID variable measuring household composition, a weighted sum of the current ages of family members adjusted for total family size), age, race, religion, marital status and the real rate of interest are included in the X_{it} group. Several of these variables have also been used by Zeldes (1989). Adding to the list, we include such labor demand and supply variables as county unemployment rate, local labor market conditions, unemployment rate in the household head's occupation, and labor supply variables such as job tenure, number of children below the age of five (in order to account for additional effects from the presence of young children, over and above what is accounted for in food needs), union membership and being disabled. Several of these variables were used by Ham

DOI: 10.1002/jae

(1982). Also included are geographical dummies and three grouped wave dummies (summarizing the years 1976–9, 1980–3, and 1984–7). A cubic structure for age is very significant, implying a highly nonlinear negative effect of age upon the probability of being liquidity constrained. A higher real rate of interest is associated with a higher probability of being constrained, exactly as expected. A household head's being black has a positive and very significant effect on that probability, and being married and highly educated has very significant negative effects. All these results accord with intuition. Also included in these regressions are 16 time averages of all time-varying regressors as discussed on pp. 491–492 above. Their inclusion is very significant according to the χ^2 statistics reported at the top of Table VI.[16] We elaborate on the consequences of this finding in Section 5.4 below.

5.2. The Labor Constraints Side of the Model

Table VII reports estimation results for the ordered probit side of the model for an employment indicator E_{it} as the dependent variable according to equations (17) and (18) for the sample of male heads. This variable corresponds to the definition (10)–(12) for members of the labor force only and its construction is discussed in detail in Appendix B of the website version of the paper. We present in column (a) estimation results for the quasi-structural form of the ordered probit side, while ignoring all spillovers from the liquidity constraint side, that is $\kappa_0 = \kappa_1 = \kappa_2 = 0$. The next two columns, (b) and (c), include lagged liquidity constraint spillovers; while column (b) excludes and column (c) includes contemporaneous spillovers. The employment constraints equation is estimated jointly with the liquidity constraints equation: column (b), Table VII, is estimated jointly with column (b), Table VI, and column (c), Table VII, is estimated jointly with the Table VI(b) version so as to ensure the coherence condition holds.

We use data for members of the labor force only and do not distinguish econometrically the cases of underemployment and unemployment. These closely reflect the ordering of outcomes according to our theoretical model. The ordered probit side of the model with panel data is given by

$$\begin{aligned} E_{it} &= -1, \quad \text{if } E_{it}^* < \theta^-, \quad \text{overemployment} \\ E_{it} &= 0, \quad \theta^- \leq \text{ if } E_{it}^* \leq \theta^+, \quad \text{voluntary employment} \\ E_{it} &= 1, \quad \theta^+ < \text{ if } E_{it}^*, \quad \text{under/unemployment} \end{aligned} \tag{21}$$

where E_{it}^* is defined in (18). This part of the model estimates an intercept, a vector of unknown coefficients, and a stochastic structure defined by (20) that includes the lower threshold θ^-, standard deviation of the time-invariant component, $\sigma_{\eta_i}^{op}$, the autocorrelation coefficient ρ_{AR}^{op}, and corr$(\xi_{it}^{bp}, \xi_{it}^{op})$, the correlation coefficient between ξ_{it}^{bp} and ξ_{it}^{op}, the i.i.d. components of the stochastic structure of the binary probit and the ordered probit equations (19) and (20). The upper threshold, θ^+, is normalized at 0. We note that, analogously to η_i^{bp}, we allow for the individual effect η_i^{op} to be correlated with the explanatory variables in a time-invariant fashion.

An interesting result that may be highlighted here is that the estimates of σ_η^{bp} appear relatively better determined than those of σ_η^{op}. The reason may be due to the fact that information on the

[16] The time-invariant regressors are not included for obvious reasons; neither are the wave dummies, which do not exhibit much variability.

DOI: 10.1002/jae

liquidity constraint is bound to be less precise compared to that on the labor constraint, the former being imputed as opposed to being directly based on questionnaire responses. Consequently, the specification improvements in moving from column (a) to (b) and (c) in the labor side of our models in Table VII should be expected to be more marked than the respective ones of the liquidity side in Table VI. Thus, the more precise information in the dependent variable of Table VII allows greater improvements in the identification of σ_η^{op} relative to the overall error, leading to less stability in the σ_η^{op} estimates across the three columns.

Column (a) of Table VII reports results for the counterpart for the employment constraints equation of the liquidity constraint equation that is reported in column (a), Table VI, when those two equations are jointly considered. The panel structure is very significant, as we have already discussed in Section 5.1 above: the models reported in the respective columns of Tables VI and VII share the same panel structures.

Two lagged values of the indicator that a household head is involuntarily unemployed are both very significant, with estimated coefficients of 0.70 and 0.37, and t-statistics of 36.3 and 19.3, respectively. Thus, being involuntarily unemployed makes one more likely to be so again in the future. Dummies for being overemployed in the past have the opposite effect and are also very significant. Also very significant in Table VII is the threshold θ^- associated with involuntary under- or unemployment relative to voluntary employment. This is negative, as it should be, given that the upper threshold is normalized at 0. These findings strengthen an earlier but somewhat tentative result by Clark and Summers (1982) on the importance of persistence elements in explaining cyclical behavior in labor supply. These results imply a rich dynamic structure for the labor constraints indicator. The model reported in Table VII, column (b), differs from that of column (a) only on account of the inclusion of the two lags for the liquidity constraint indicator, one of which is marginally significant, although their inclusion is jointly significant according to the χ^2 test.

The remaining explanatory variables included in the regression coincide with those used by Ham (1982). Of the regional dummies, the one indicating residence in the western USA is highly significant. A cubic effect for age is significant and implies that age reduces the probability of being underemployed. Similar and even more significant is the effect of job tenure on the likelihood of being underemployed. Race and religion are significant. Having a disabling health condition, being male, a union member, and having many children all have very strong and statistically important positive effects. Collecting unemployment insurance and the imputed wage both have numerically very small but statistically significant effects. Being married and being educated both have very significant and negative effects. A set of variables representing demand effects all have very significant coefficients. Higher values of the unemployment rate in the county of residence and in the occupation of the household head imply higher values for the likelihood of underemployment or unemployment. With a few exceptions, these results accord with intuition. They do imply a persistent and possibly 'trapping' effect caused by past unemployment and underemployment. Similarly to the liquidity constraint model, also included in these regressions are 16 time averages of all time-varying regressors. Their inclusion is significant according to the χ^2 statistics reported at the top of Table VII and we discuss the consequences of this finding in Section 5.4 below.

5.3. Quasi-structural Form Models for Liquidity and Labor Supply Constraints

Let us now focus on columns (c) of Tables VI and VII, which report joint estimation results for quasi-structural forms for S_{it} and E_{it} *as a system* taking into account the full possibility of the

DOI: 10.1002/jae

lagged and contemporaneous spillover effects across the two sides of the models, while always imposing the coherency conditions discussed above. Column (c), Table VI, reports results for equation (17) estimated jointly with (18) with the restriction $\kappa_0 = 0$ imposed. Intuitively speaking, column (b), Table VII, reports the results for an equation determining the marginal probability for E_{it}; column (c), Table VI, reports results for an equation determining the probability for S_{it}, conditional on E_{it}. Using analogous intuition, column (c), Table VII, reports results for equation (18) estimated jointly with (17), with the coherency condition $\delta_0 = 0$, and whose results are reported in column (b), Table VI. In like manner, column (b), Table VI, reports the results for an equation determining the marginal probability for S_{it}; and column (c), Table VII, reports results for an equation determining the probability for E_{it}, conditional on S_{it}.

Inclusion of contemporaneous spillover effects, that is, inclusion of E_{it} in the liquidity constraint equation for Sit, and alternatively of S_{it} in the employment constraints equation for E_{it}, is statistically significant according to the likelihood ratio test.[17] It is crucial to remember that the contemporaneous spillovers are included *in turn* and *not simultaneously*, since the latter would have violated the coherency of the model. The models reported in columns (c), Table VI, and (b), Table VII, have a joint log-likelihood function of −29,401.4 versus −29,422.7 for columns (b), Table VII, and (b), Table VI. Similarly, the models reported in columns (c), Table VII, and (b), Table VI, have a joint log-likelihood function of −29,406.3 versus −29,422.7 for columns (b), Table VII, and (b), Table VI, jointly. Thus the simultaneous equations system passes the likelihood ratio tests in terms of significant improvement in the overall likelihood.

This approach accounts for the joint determination of S_{it} and E_{it} while imposing the coherency condition, $\kappa_0 = 0$ and $\delta_0 = 0$, respectively, for the two models. In Appendix A, Section A.2, we explain how we handle the presence of endogenous variables on the right-hand side in these specifications through the use of maximum simulated likelihood in conjunction with the GHK simulator.[18] Particularly noteworthy is our estimation of the autoregressive structure and contemporaneous correlations in $(\varepsilon_{it}^{bp}, \varepsilon_{it}^{op})$, the error structure of equations (17) and (18), detailed in (19) and (20), as well as allowance for individual effects and regressors being possibly correlated in a time-invariant fashion.

All the components of the stochastic panel structure are estimated to be very significant for both models. Interestingly, the standard deviation for the random effect $\sigma_{\eta_i}^{bp}$ in the liquidity constraint equation varies imperceptibly across the various models but $\sigma_{\eta_i}^{op}$ does vary in the case of the structural form. The respective correlation coefficient is significantly smaller in the case of the structural form. Similar is the case with the autocorrelation coefficients ρ_{AR}^{bp} and ρ_{AR}^{op}, and the estimate of the former is much larger than the latter. The estimated correlation coefficient declines as we move to the right on each table. In moving from columns (a), Tables VI and VII, to columns (b) of Tables VI and VII, the lagged dependent variables of the employment constraints indicator are added to the liquidity constraint equation and those of the liquidity constraint indicator to the employment equation. This reduces the contemporaneous correlation as expected. And similarly moving from columns (b), Tables VI and VII, to columns (c) of the same tables, contemporaneous spillover effects are added *in turn* as required to guarantee the coherency conditions.

[17] The E_{it} spillover effect was decomposed into its two constrained parts, overemployment ($E_{it} = -1$) and under/unemployment ($E_{it} = 1$).

[18] The exclusion restrictions for the liquidity constraint model follow Zeldes (1989), except that we include in addition quadratic and cubic effects for the age variable, marital status, geographical dummies, race and religion. This list follows quite closely results that Zeldes discusses but does not report in his paper.

DOI: 10.1002/jae

We see from column (c), Table VI, that being unemployed has a strong positive and significant effect on the likelihood of being liquidity constrained. Being overemployed is not significant. The lagged values of both those variables are actually not statistically significant. Most of the determinants of being liquidity constrained remain significant in the structural form too, as do the own lagged dependent variables. Being black is associated with higher likelihood of being constrained, while being other nonwhite, e.g., Asian, a lower one. It should be noted that, with the exception of the spillover terms, the selection of regressors tries to encompass fully the existing literature.

Turning now to the likelihood of being underemployed or unemployed,[19] we see that being liquidity constrained has a very significant positive effect, as do the own lagged values of the variables expressing being overemployed, which have negative effects, and unemployed, which have positive effects. Most of the determinants of the likelihood of being underemployed retain their significance. Unemployment rate in the county of residence and in the occupation of the head of household, and tightness of local labor market conditions,[20] are all very significant and with signs in accord with intuition. Being nonwhite is associated with higher likelihood of being underemployed or unemployed.

Finally, we note that the inclusion of the endogenous variable expressing the employment constraints indicator as an explanatory variable for the liquidity constraint indicator and of the endogenous variable expressing the liquidity constraint indicator as an explanatory variable for the employment constraints indicator are each significant in terms of the likelihood ratio test. The respective differences from columns (b) and (c) in both Tables VI and VII are statistically significant according to the standard χ^2 test.

A comparison with earlier research is appropriate at this point. Ham (1982) explores the qualitative aspects of labor supply in detail while using a single cross-section of 835 workers from the PSID for 1971. A worker is unconstrained if he/she is neither underemployed nor unemployed, and three categories of constrained workers are distinguished: unemployed but not underemployed, underemployed but not unemployed, and underemployed and unemployed. Ham ignores the possibly constraining effect of overemployment by arguing that it is relatively unimportant. Ham (1982) uses univariate and bivariate probit models for underemployment and unemployment as distinct selection rules to correct for sample selection bias affecting labor supply behavior. He finds that unemployment and underemployment reflect constraints on behavior. He notes that different factors may determine those states; e.g., business cycle variables are important for unemployment but not for underemployment.

In examining the data, we have also replicated Ham's criteria and confirmed the consistency of our selection with his. The difference of his selection from our labor constraint indicator E_t is that his is not *ordered* and does not distinguish overemployment (which, however, is not numerically very important).

Kahn and Lang (1992) argue hours constraints may be motivated by contract theory. They employ a static ordered probit model of discrete events which are roughly comparable to ours. Their tests of specific features of labor contract theory with data from the 1981 wave of the PSID largely reject such explanations of hours constraints.

[19] The exclusion restrictions for this model follow Ham (1982).

[20] This categorical variable measures tightness of the local labor market for unskilled workers, with values ranging from 1, for good conditions, to 5, for bad conditions.

DOI: 10.1002/jae

The present paper with its emphasis on possibly time-varying discrete events in panel data is more closely related to Ham (1986), who uses PSID data for 473 individuals from 1971 to 1979. His experiments with dummy variables for underemployment and unemployment, defining them identically to his earlier work (Ham, 1982) as time-varying right-hand side endogenous variables, is an improvement over Ball's notion of time-invariant constraints. In view of the endogeneity of these dummy variables, Ham (1986) instruments them by means of a set of exogenous variables chosen to proxy the labor market conditions facing a worker. However, those events are inherently discrete, and Ham's econometric procedures do not handle them as such.

Hyslop (1999) studies the intertemporal labor force participation behavior of married women within a dynamic search framework using panel data. He estimates multiperiod linear probability and probit models, allowing for a rich dynamic structure. He finds very significant state dependence, unobserved heterogeneity, and serial correlation. In line with our findings here, he reports a crucial role for lagged state dependence and temporal correlation in the unobservables in such dynamic discrete models of employment behavior. In contrast to our study here, he places emphasis on the linear probability models, whereas we focus solely on probit ones. More importantly, his approach does not allow for the random persistent heterogeneity effects to be correlated with the regressors, whereas our results are more robust in this dimension.

Ham and Reilly (2002) extend the Lucas–Rapping model of equilibrium labor supply by means of the implicit contracts model as an equilibrium model and of the hours restrictions as a disequilibrium model, and test their models using PSID for 1972–1992, and Current Expenditure Survey data for 1984–1992. They reject the Lucas–Rapping predictions of intertemporal substitution in labor supply. While their model is clearly more closely grounded in economic theory, the dynamics in the stochastic structure of our model are much richer than theirs, which are restricted to time dummies only and rely on instrumenting to account for dynamic elements in the stochastic structure.

5.4. Diagnostics

We report at the top of each column of Tables VI and VII various probability predictions and data proportions of selected regimes. In Table VI, the binary probit side estimates are used to construct the predicted probability of being liquidity constrained at the sample means, $\hat{P}_{lc}(\overline{X})$, the average predicted probability $\overline{\hat{P}_{lc}}$, and the percentage of observations that have positive predicted latent values for the liquidity constraint indicator $\overline{\mathbf{1}}(x_{it}\hat{\beta}^{bp} > 0)$. Finally, we give the percentage of observations correctly predicted by our models (in terms of the predicted indicator $\overline{\mathbf{1}}(\cdot)$) matching the observed liquidity constraint indicator.

Our estimation results suggest remarkably good fits. Specifically, the percentages of correctly predicted values are 88% for the $S_{it} = 1$ event, 81% for $E_{it} = -1$, and 94% for $E_{it} = 1$, while the mean predicted values match almost exactly their respective observed sample means.

Additional information on how well our models fit the data is provided by Figures 2 and 3, where we have plotted predicted probabilities over time using our model estimates. Calculations based on columns (a) and (b) of Tables VI and VII are contrasted to those based on columns (c). We note that the year-by-year predictions vary cyclically and conform rather well to the historical economic facts business cycle timing of the US economy for the period under study.

More specifically, Figure 2 compares the time variation in the predicted probabilities of being liquidity constrained based on version (a) of the model (neither lagged nor contemporaneous

DOI: 10.1002/jae

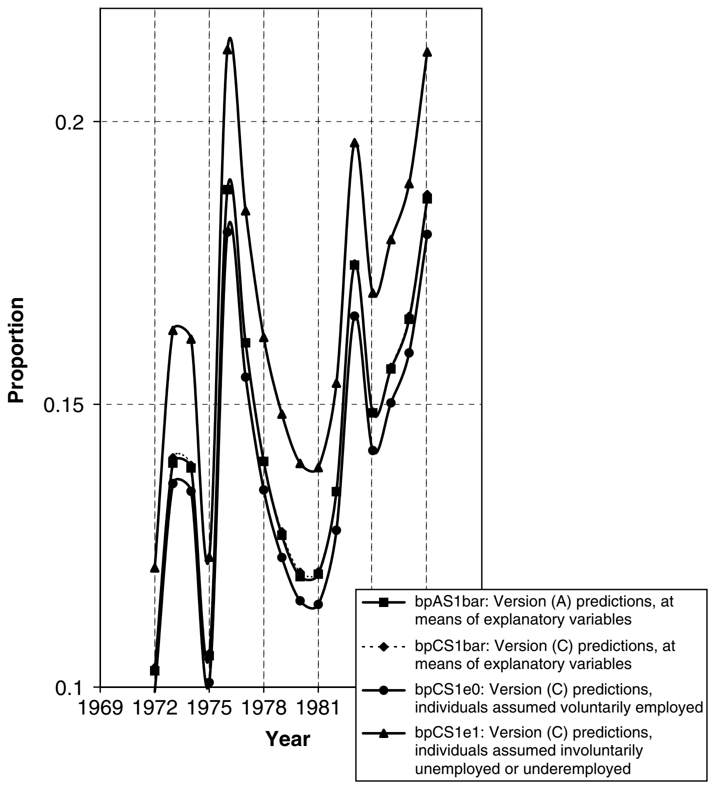

Figure 2. Prob(binding liquidity constraint)

spillover effects from the employment side), to those obtained from version (c) (with full employment spillovers). The version (c) estimates allow us to obtain predictions for the hypothetical case that all individuals were involuntarily under- or unemployed, suggesting that in such a case the probability of a binding liquidity constraint would rise by an additional 10%. Figure 3 presents the results of performing an analogous exercise for the employment constraints side of the model. In that case, the impact of a binding liquidity constraint spilling over to the employment constraints side is slightly more modest: the predicted probabilities of being voluntarily constrained drop by about 7–8%, while those of being under- or unemployed rise by a similar amount.

As should be evident from equations (17) and (18), the joint 6-regime discrete response model we estimate has the specific binary/ordered structure we described above. A test of this specification readily follows from the theoretical model: it is to estimate the model as an unrestricted, i.e., *unordered*, hexanomial probit and test the overidentifying restrictions. Such an estimation is feasible using the simulated maximum likelihood method we employ in this paper.[21]

[21] The model we derive from our theory is clearly nested in the classical sense in such a standard hexanomial probit model, which makes this testing approach have good asymptotic power properties. The correct distribution theory required for these tests is complicated by the fact that the null hypothesis involves restrictions on the boundary of the parameter space. Weeks and Orme (1998) circumvent this difficulty by a score test.

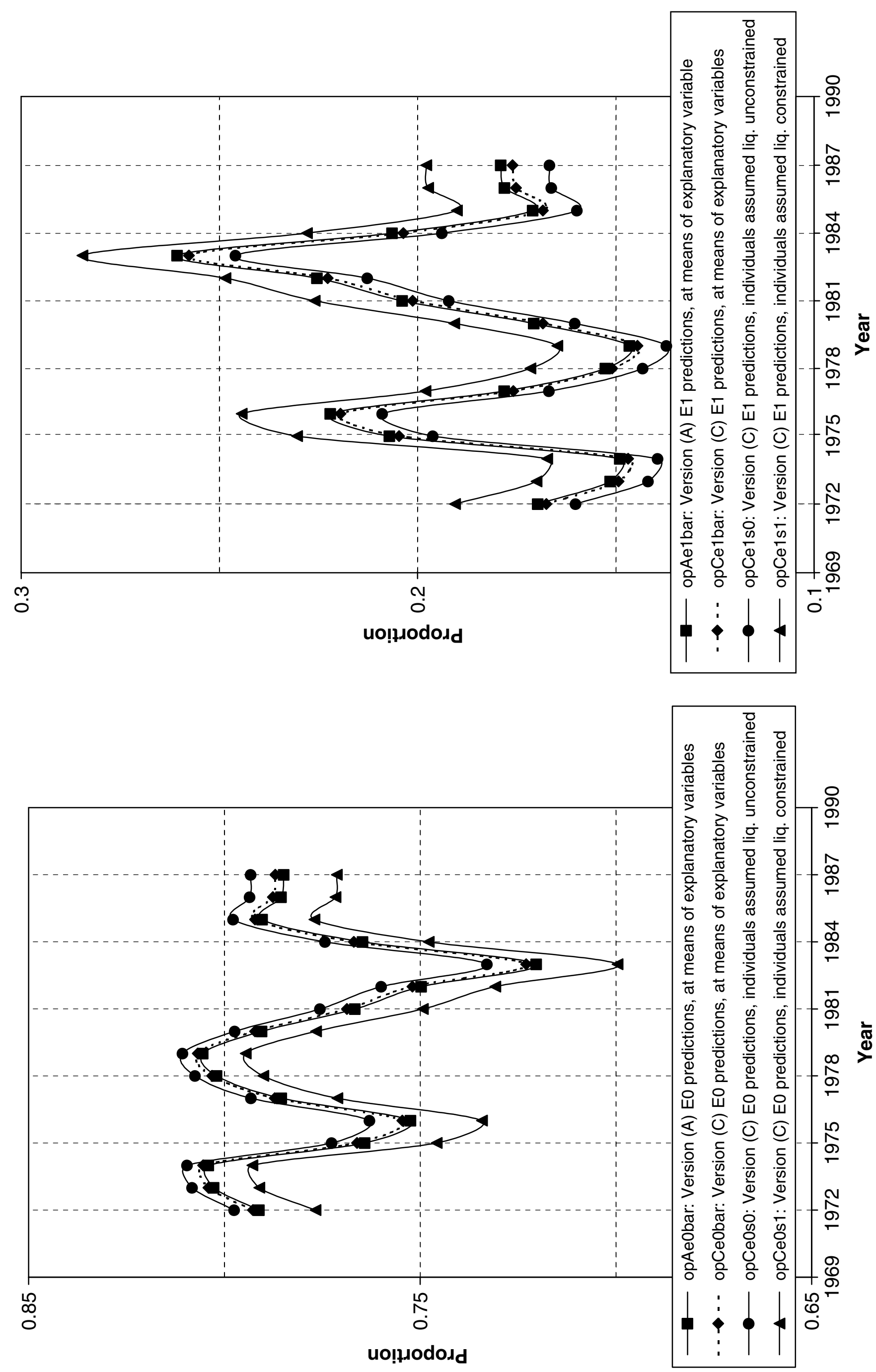

Figure 3. (a) Prob(VolEmpl); (b) Prob(Unempl)

DOI: 10.1002/jae

We discussed in Section 3.1 above that the unordered hexanomial probit model involves a staggering increase in the number of parameters to be estimated relative to the ordered bivariate model. For example, the slope parameters of the valuation functions amount to 180, since 5β vectors are estimated for each explanatory variable. In order to conduct such a test by means of state-of-the-art technology we have to restrict ourselves to a subset of the data. We estimated an unrestricted trinomial probit model for the labor constraint indicator and an unrestricted hexanomial probit model for the full model and compared them with the respective restricted ones. We refrain from reporting all of our estimation results because of the number of parameters involved. We are happy to note that key aspects of the overidentifying restrictions are not rejected. In particular, referring to the discussion on p. 490 above, we note that the estimated correlation coefficient between the i.i.d. terms of the $AR(1)$ components of the errors for the unrestricted trinomial model is nearly -1, exactly as predicted by the ordered model. Similarly, the most highly significant of the components of the parameter vectors of the indicator functions are quite near the theoretical prediction that they sum to zero. We take these results as powerful evidence in favor of our theoretical structure.

Finally, we discuss a novel test of the validity of the assumption made typically that the random effects are uncorrelated with the independent variables of the model. We noted above that we have estimated both models by introducing the time means of those of the independent variables which are time varying as separate regressors (Chamberlain, 1984). These are the results that are reported in Tables VI and VII, as we indicate in both tables. Exclusion of the time means is statistically rejected according to the likelihood ratio test (χ^2 values in excess of 220 with 16 degrees of freedom, rising to over 240 in the more restrictive column (a) versions). The models that we report lend themselves to a more intuitive interpretation in that the estimated coefficients relate to the effect of a variable's deviation from its time average. The fact that exclusion of the time means is drastically rejected and that the estimates do not differ very much from those obtained without the time averages implies that the assumption that the random effects are uncorrelated with the regressors in our model is rejected. The model without the time averages would be inconsistent and hence including the time means is important in soaking such correlations.

6. CONCLUSION

We explore in this paper empirical implications of a theory of labor supply and consumption decisions that goes further than previous research in allowing for a role of such institutional constraints as limited access to borrowing and involuntary unemployment and overemployment. We report estimations for discrete dependent variables with two simultaneous dynamic probit models. The first describes a household's propensity to be constrained in borrowing, while the second, a dynamic ordered probit model for a labor constraint indicator, describes qualitative aspects of the conditions of employment, that is, whether the household head is involuntarily overemployed, voluntarily employed, or involuntarily underemployed or unemployed. These models are estimated, separately and jointly, as well as in ordered and in unordered quasi-structural form versions. We believe that the dynamic labor constraint model has not been considered before in the literature, nor has a panel model with as general a structure for the unobservables. The quasi-structural forms we estimated capture state dependence and spillovers among the underlying decisions, while the panel structure of the unobservables allows for correlation between the time-invariant components

DOI: 10.1002/jae

of the random effects in the two equations and for an autoregressive component. Our diagnostics suggest that our estimation models exhibit remarkably good fits.

In terms of its structure and empirical objectives, the paper may be considered as an integration of two separate strands of the empirical literature. One strand highlights the equilibrium/disequilibrium dichotomy (Ashenfelter, 1980; Ham, 1986), and the other the interaction between labor supply and consumption decisions (Altonji, 1986; Ball, 1990).

Individuals may face restrictions on the amount of work they can supply to their employers as well as restrictions on borrowing against their future incomes. Although they may resent such restrictions, they still adapt their lifetime plans to them and in the light of the best information they have about the presence of such constraints in the future. The assumption that is made sometimes, namely that all fluctuations in employment status and hours worked over time is voluntary, is an undue restriction that may therefore lead to inconsistent estimation and misinterpretation of the data. These problems can be overcome when information is utilized, as in this paper, about the voluntary/involuntary nature of changes in employment over time.

From among the unexplored areas of research that our approach has opened up, we note the possibility of estimating and testing the extent of the dependence of the structural form for each of the endogenous variables conditionally on the regime characterizing the other. In view of the difficulty of estimating life cycle consistent dynamic models, we note that the simulation methods that we employ here may be combined fruitfully in the future with non-parametric methods (Magnac and Thesmar, 2002). These issues deserve attention in future research.

APPENDIX A: ECONOMETRIC METHODOLOGY

A.1. Maximum Smoothly Simulated Likelihood based on the GHK Simulator

In this paper we employ the method of maximum smoothly simulated likelihood (MSSL) in conjunction with the Geweke–Hajivassiliou–Keane (GHK) simulator in order to overcome the well-known computation intractabilities of the multiperiod (panel) limited-dependent-variable models presented in Section 4. The MSSL approach was developed in Börsch-Supan and Hajivassiliou (1993), while its theoretical properties were derived rigorously in Hajivassiliou and McFadden (1998). See also Hajivassiliou (1993). The GHK simulator has become the leading simulator for multivariate normal rectangle probabilities of the form encountered in ML estimation of LDV models under Gaussian distributional assumptions. Extensive Monte-Carlo evidence in Hajivassiliou *et al.* (1996) shows this simulator to dominate all other known simulators for this problem. To outline this method, define $q(u, a, b) \equiv \Phi^{-1}(\Phi(a) \cdot (1-u) + \Phi(b) \cdot u)$, where $0 < u < 1$ and $-\infty \leq a < b \leq \infty$. Then for u, a uniform (0, 1) random variate, q will be a standard normal truncated on $[a, b]$.

Proposition 1: Consider the multivariate normal $M \times 1$ random vector $Y \sim N(X\beta, \Omega)$ with Ω positive definite, the linear transformation $Z = FY \sim N(FX\beta, \Sigma)$, with F non-singular and $\Sigma = F\Omega F'$, and the event $\mathbf{B} \equiv \{a^* \leq Z = FY \leq b^*\}$, with $-\infty \leq a^* < b^* \leq +\infty$. Define $P \equiv \int_B n(z; FX\beta, \Sigma)\mathrm{d}z$, $a \equiv a^* - FX\beta$, $b \equiv b^* - FX\beta$, and let L denote the lower-triangular Cholesky factor of Σ. Let $(u_1, \ldots, u_M)$ be a vector of independent uniform (0, 1) random variates. Define recursively for $j = 1, \ldots, M$:

$$e_j = q(u_j, (a_j - L_{j1}e_1 - \ldots - L_{j,j-1}e_{j-1})/L_{jj}, (b_j - L_{j1}e_1 - \ldots - L_{j,j-1}e_{j-1})/L_{jj}) \qquad (22)$$

DOI: 10.1002/jae

$$Q_j \equiv \Phi((b_j - L_{j1}e_1 - \ldots - L_{j,j-1}e_{j-1})/L_{jj}) - \Phi((a_i - L_{j1}e_1 - \ldots - L_{j,j-1}e_{j-1})/L_{jj}) \quad (23)$$

Define $e \equiv (e_1, \ldots, e_M)'$, $\tilde{Y} \equiv X\beta + F^{-1}Le$, and $Q(e) \equiv Q_1 \ldots Q_M$. Then $\tilde{Y}$ is a random vector on **B**, and the ratio of the densities of $\tilde{Y}$ and Y at $y = X\beta + F^{-1}Le$, where e is any vector satisfying $a \leq Le \leq b$, is $P/Q(e)$. See Börsch-Supan and Hajivassiliou (1993) for proof.

A.2. Simultaneous Determination of the Liquidity and Employment Constraint Indicators

For a typical household spell i (assumed to be independently distributed from other household spells) and dropping the i index for simplicity, the MSSL method allows us to take fully into account the simultaneity in the determination of the liquidity (S_t) and the employment constraint (E_t) indicators. Let us define two latent dependent variables $y_{1t}^* \equiv S_t^*$ and $y_{2t}^* \equiv E_t^*$ that are the underpinnings of S_t and E_t according to the LDV models given by equations (19) and (20), namely:

$$S_t = \begin{cases} 1 & \text{if } S_t^* > 0 \\ 0 & S_t^* \leq 0 \end{cases}, \qquad E_t = \begin{cases} -1 & \text{if } E_t^* < \theta^- \\ 0 & \text{if } \theta^- \leq E_t^* < \theta^+ \\ 1 & \text{if } \theta^+ \leq E_t^* \end{cases}$$

Also dropping the t subscript for ease of notation, we consider the model with spillover effects on both sides, i.e., the one exhibiting full simultaneity:

$$\begin{aligned} y_1^* &\equiv S^* = \mathbf{1}(y_2^* < \theta^-)\delta_{01} + \mathbf{1}(y_2^* > \theta^+)\delta_{02} + x_1\beta_1 + \varepsilon_1 \\ y_2^* &\equiv E^* = \mathbf{1}(y_1^* > 0)\kappa_0 + x_2\beta_2 + \varepsilon_2 \end{aligned}$$

Note that we have decomposed the contemporaneous spillover effect $\delta_0 E$ on the right-hand side of S^* into $\delta_{01}\mathbf{1}(E = -1) + \delta_{02}\mathbf{1}(E = 1)$, i.e., into separate terms for the overemployment and the under/unemployment indicators.

Since (S, E) lie in $\{0, 1\} \times \{-1, 0, 1\}$, the six possible configurations may be enumerated as follows:

S	E	$y_1^* \equiv S^*$	$y_2^* \equiv E^*$
0	-1	$\delta_{01} + x_1\beta_1 + \varepsilon_1 < 0,$	$x_2\beta_2 + \varepsilon_2 < \theta^-$
0	0	$x_1\beta_1 + \varepsilon_1 < 0,$	$\theta^- < x_2\beta_2 + \varepsilon_2 < \theta_+$
0	1	$\delta_{02} + x_1\beta_1 + \varepsilon_1 < 0,$	$\theta^+ < x_2\beta_2 + \varepsilon_2$
1	-1	$\delta_{01} + x_1\beta_1 + \varepsilon_1 > 0,$	$\kappa_0 + x_2\beta_2 + \varepsilon_2 < \theta^-$
1	0	$x_1\beta_1 + \varepsilon_1 > 0,$	$\theta^- < \kappa_0 + x_2\beta_2 + \varepsilon_2 < \theta^+$
1	1	$\delta_{02} + x_1\beta_1 + \varepsilon_1 > 0,$	$\theta^+ < \kappa_0 + x_2\beta_2 + \varepsilon_2$

In terms of the GHK simulator described above, the probability of a pair (S, E) is equivalent to the probability

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} < \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix} < \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

DOI: 10.1002/jae

where $(\varepsilon_1, \varepsilon_2)' \sim N((\mu_1, \mu_2)', \Sigma_\varepsilon)$, and a and b are:

S	E	a_1	a_2	b_1	b_2
0	−1	$-\infty$	$-\infty$	$-(\delta_{01} + x_1\beta_1)$	$\theta^- - x_2\beta_2$
0	0	$-\infty$	$\theta^- - x_2\beta_2$	$-x_1\beta_1$	$\theta^+ - x_2\beta_2$
0	1	$-\infty$	$\theta^+ - x_2\beta_2$	$-(\delta_{02} + x_1\beta_1)$	$+\infty$
1	−1	$-(\delta_{01} + x_1\beta_1)$	$-\infty$	$+\infty$	$\theta^- - \kappa_0 - x_2\beta_2$
1	0	$-x_1\beta_1$	$\theta^- - \kappa_0 - x_2\beta_2$	$+\infty$	$\theta^+ - \kappa_0 - x_2\beta_2$
1	1	$-(\delta_{02} + x_1\beta_1)$	$\theta^+ - \kappa_0 - x_2\beta_2$	$+\infty$	$+\infty$

A.3. Coherency Conditions

To maintain the logical consistency of the model (known in the literature as 'statistical coherency'), S_t^* should not depend on E_t^*, if E_t^* depends on S_t^*, and vice versa. Formally, the coherency conditions in terms of the above notation are: $(\delta_{01} + \delta_{02})\kappa_0 = 0$ and $\delta_{01}\delta_{02}\kappa_0 = 0$. In other words, either $\kappa_0 = 0$, in which case δ_{01}, δ_{02} are free to differ from 0, or $\kappa_0 \neq 0$, in which case both δ_{01} and δ_{02} must be zero.

To verify this requirement, suppose $(S, E) = (0, 0)$. This rules out $(S, E) = (0, -1)$ because $x_2\beta_2 + \varepsilon_2 > \theta^-$, and rules out $(S, E) = (1, 0)$ because $x_1\beta_1 + \varepsilon_1 < 0$. But $(1, -1)$ is not ruled out if the coherency conditions do not hold, since δ_{01} could be sufficiently negative and κ_0 sufficiently positive to imply the $(1, -1)$ conditions. Similarly, the $(1, 1)$ possibility cannot be ruled out in the absence of the coherency conditions, since δ_{02} and κ_0 can be sufficiently positive. Such logical inconsistencies are clearly ruled out if either (a) $\kappa_0 = 0$ or (b) δ_{01} and δ_{02} are simultaneously 0. See Hajivassiliou (2003) for novel ways of approaching coherency conditions in LDV models with simultaneity.

A.4. Treatment of Flexible Serial and Contemporaneous Correlations

We have described in Section A.2 how the probability of a pair (S_{it}, E_{it}) can be represented in terms of the GHK implementation through the linear inequality

$$\begin{pmatrix} a_{1it} \\ a_{2it} \end{pmatrix} < \begin{pmatrix} \varepsilon_{1it} \\ \varepsilon_{2it} \end{pmatrix} < \begin{pmatrix} b_{1it} \\ b_{2it} \end{pmatrix}$$

Define the 2×1 vectors a_{it}, b_{it}, and ε_{it}. Stacking all the T_i periods of observation for individual i gives the $2 \cdot T_i \times 1$ vectors a_i, b_i, and ε_i, where ε_i has the $2 \cdot T_i \times 2 \cdot T_i$ variance–covariance matrix with structure characterized by the precise serial correlation assumptions made on the ε_{it}s. In particular, one-factor random effect assumptions will imply an equicorrelated block structure on Σ_ε, while our most general assumption of one-factor random effects *combined with* an $AR(1)$ process for each error implies that Σ_ε combines equicorrelated and Toeplitz-matrix features.

Through this representation, the probability of a complete sequence of the observable (S, E) behavior for individual household i, conditionally on the initial conditions S_{i0} and E_{i0}, is given by $P(S_1, \ldots, S_{T_i}, E_1, \ldots, E_{T_i}) = \text{prob}(a_i < \varepsilon_i < b_i)$. Consequently, our approach incorporates fully: (a) the contemporaneous correlations in ε_{it}; (b) the one-factor plus AR(1) serial correlations in ε_i;

DOI: 10.1002/jae

and (c) the dependency of S_{it} on E_{it}; and vice versa. The possible endogeneity of S_{i0} and E_{i0} is handled by the approach described next.

A.5. Econometric Treatment of Endogeneity of Initial Conditions

The importance of allowing for endogenous initial conditions in our estimation may be illustrated by considering the probability of an individual making a binary choice in each of three consecutive periods, in a model with Markov state dependence:

$$y_3 = \mathbf{1}(y_3^* > 0) = \mathbf{1}(\gamma_3 y_2 + x_3'\beta_3 + \varepsilon_3 > 0) \tag{24}$$

$$y_2 = \mathbf{1}(y_2^* > 0) = \mathbf{1}(\gamma_2 y_1 + x_2'\beta_2 + \varepsilon_2 > 0) \tag{25}$$

$$y_1 = \mathbf{1}(y_1^* > 0) = \mathbf{1}(\gamma_2 y_0 + x_1'\beta_1 + \varepsilon_1 > 0) \tag{26}$$

Since information is only available for periods 1–3, equation (26) cannot be used in the estimation since y_0 is missing. Pretending that y_1 is exogenous and working with probability $\text{prob}(y_3, y_2|y_1, x_1, x_2, x_3)$ is, of course, incorrect and can lead to serious inconsistency, especially in short panels, compared to the correct likelihood contribution $\text{prob}(y_3, y_2, y_1|x_1, x_2, x_3)$ that incorporates the endogeneity of y_1. For example, in the case $(y_1, y_2, y_3) = (1, 1, 1)$:

$$\text{prob }(y_3 = 1, y_2 = 1|y_1 = 1, x_1, x_2, x_3) = \int_{-\gamma - x_3\beta_3}^{\infty} \int_{-\gamma - x_2\beta_2}^{\infty} f_{\varepsilon_2,\varepsilon_3}(\varepsilon_2, \varepsilon_3)\mathrm{d}\varepsilon_2\mathrm{d}\varepsilon_3 \tag{27}$$

while

$$\text{prob }(y_3 = 1, y_2 = 1, y_1 = 1|x_1, x_2, x_3) = \int_{a_1}^{\infty} \int_{-\gamma - x_3\beta_3}^{\infty} \int_{-\gamma - x_2\beta_2}^{\infty} f_{\varepsilon_2,\varepsilon_3,\varepsilon_1}(\varepsilon_2, \varepsilon_3, \varepsilon_1)\mathrm{d}\varepsilon_2\mathrm{d}\varepsilon_3\mathrm{d}\varepsilon_1 \tag{28}$$

where a_1 depends on y_0 and other unavailable earlier information. The solution we adopt considers the marginal probit model

$$y_1 = \mathbf{1}(x_1'\varsigma_1 + x_2'\varsigma_2 + x_3'\varsigma_3 + u_1 > 0) \tag{29}$$

and estimates the model combining (29) with (24) and (25) through the probability approximation

$$\text{prob }(y_3, y_2, y_1|x_1, x_2, x_3) = \int_{-x_1\varsigma_1 - x_2\varsigma_2 - x_3\varsigma_3}^{\infty} \int_{-\gamma - x_3\beta_3}^{\infty} \int_{-\gamma - x_2\beta_2}^{\infty} f_{\varepsilon_2,\varepsilon_3,u_1}(\varepsilon_2, \varepsilon_3, u_1)\mathrm{d}\varepsilon_2\mathrm{d}\varepsilon_3\mathrm{d}u_1 \tag{30}$$

This is the nonlinear analogue of the solution proposed by Barghava and Sargan (1982) for the linear dynamic model and uses the best linear projection for the latent variable y_1^* by using *all* data for *all* periods available to the econometrician, which of course was not available to the decision-maker at the time t. This approach implies a new error term u_1 that is different from ε_1 of (26) and hence we need to allow it to be flexibly correlated with ε_2 and ε_3. As Heckman (1981b) explains, in general u_1 does not have the same distribution as the ε s (assumed here to be Gaussian), so our treating (29) as a probit is an approximation. Such approximations are shown by Heckman's Monte Carlo evidence not to be too critical when working with panel data with a

DOI: 10.1002/jae

moderately large time dimension (about 8 or higher). This makes us confident in the quality of our approximate solution in view of the relatively large number of time periods available for each individual household (up to 18) in our data set.

ACKNOWLEDGEMENTS

Financial support by the National Science Foundation under grants SES-9000200 and SES-9211913 is gratefully acknowledged. Ioannides thanks the John D. and Catherine T. MacArthur Foundation for support through the Research Network on Social Interactions and Economic Disparities. We are grateful to two anonymous referees and the editor, Melvyn Weeks, for generous and very insightful comments that allowed us to improve the paper. We thank Steven Zeldes for kindly making his PSID data available to us for cross-checking against ours, Larry Ball for helpful suggestions with the data, Fumio Hayashi for supplying us with a user-friendly program for reading the PSID data, and Kevin Lee, Steve Nickell, Ariel Pakes, Steven Stern, Jonathan Thomas, and Guglielmo Weber for useful discussions and suggestions. We also thank Steve Bergantino, Raja Chakir, Stelios Corres, Kamhon Kan, Anne Royalty, and Ann-Margret Westin for comments and expert research assistance. We benefited from comments made during presentations at the 1992 National Science Foundation/National Center for Supercomputing Applications Conference on 'Large Scale Computation in Economics' in the University of Illinois, the 1992 ASSET Congress in Toulouse, the University of Uppsala, the University of Guelph, Columbia University, the Congress of the Society for Economic Dynamics and Control in Nafplion, Yale University, VPI&SU, UCLA, LSE, the Department of Applied Economics at Cambridge University, and the Conference on 'International Perspectives on the Macroeconomic and Microeconomic Implications of Financing Constraints,' University of Bergamo and CEPR, October 1994.

REFERENCES

Altonji JG. 1986. Intertemporal substitution in labor supply: evidence from micro data. *Journal of Political Economy* **94**: S176–S215.

Ashenfelter O. 1980. Unemployment as disequilibrium in a model of aggregate labor supply. *Econometrica* **48**: 547–564.

Ball L. 1990. Intertemporal substitution and constraints on labor supply: evidence from panel data. *Economic Inquiry* **28**: 706–724.

Barghava A, Sargan D. 1982. Estimating dynamic random effects models from panel data covering short time periods. *Econometrica* **51**: 1635–1660.

Biddle JE. 1988. Intertemporal substitution and hours restrictions. *Review of Economics and Statistics* **70**: 347–351.

Blundell R, Walker I. 1986. A life-cycle consistent empirical model of family labor supply using cross-section data. *Review of Economic Studies* **53**: 539–558.

Börsch-Supan A, Hajivassiliou VA. 1993. Smooth unbiased multivariate probability simulators for maximum likelihood estimation of limited dependent variable models. *Journal of Econometrics* **58**(4): 347–368.

Browning M, Deaton A, Irish M. 1985. A profitable approach to labor supply and commodity demands over the life cycle. *Econometrica* **53**: 503–550.

Card D. 1994. Intertemporal labor supply: an assessment. In *Advances in Econometrics: Sixth World Congress of the Econometric Society*, Vol. I, *Barcelona*, 1990, Sims CA (ed.). Cambridge University Press: Cambridge, UK; 49–78.

Chamberlain G. 1984. Panel data. In *Handbook of Econometrics*, Vol. 2, Griliches Z, Intrilligator MD (eds). North-Holland: Amsterdam; 1247–1318.

DOI: 10.1002/jae

Clarida RH. 1987. Consumption, liquidity constraints and asset accumulation in the presence of random income fluctuations. *International Economic Review* **28**: 339–351.
Clark KB, Summers LH. 1982. Labor force participation: timing and persistence. *Review of Economic Studies* **54**: 825–844.
Deaton AS. 1991. Saving and liquidity constraints. *Econometrica* **59**: 1221–1248.
Flavin MA. 1985. Excess sensitivity of consumption to current income: liquidity constraints or myopia. *Canadian Journal of Economics* **18**: 117–136.
Hajivassiliou VA. 1993. Simulation estimation methods for limited dependent variable models. In *Handbook of Statistics*, Vol. 11: *Econometrics*, Maddala GS, Rao CR, Vinod HD (eds). Amsterdam: North-Holland; 519–543.
Hajivassiliou VA. 2003. A reevaluation of panel data estimators in the presence of regressors correlated with unobservables. Working paper, London School of Economics.
Hajivassiliou VA. 2004. The method of maximum smoothly simulated likelihood for LDV models with simultaneity. Working paper, London School of Economics.
Hajivassiliou VA, Ioannides YM. 1994. Unemployment and liquidity constraints. Discussion paper no. 243, Centre for Economic Performance, London School of Economics.
Hajivassiliou VA, Ioannides YM. 1996. Duality and liquidity constraints under uncertainty. *Journal of Economic Dynamics and Control* **20**: 1177–1192.
Hajivassiliou VA, McFadden DL. 1998. The method of simulated scores for the estimation of LDV models. *Econometrica* **66**: 863–896.
Hajivassiliou VA, McFadden DL, Ruud P. 1996. Simulation of multivariate orthant probabilities: methods and programs. *Journal of Econometrics* **72**: 85–134.
Hall RE, Mishkin FS. 1982. The sensitivity of consumption to transitory income: estimates from panel data on households. *Econometrica* **50**: 461–480.
Ham JC. 1982. Estimation of a labour supply model with censoring due to unemployment and underemployment. *Review of Economic Studies* **49**: 335–354.
Ham JC. 1986. Testing whether unemployment represents intertemporal labour supply behaviour. *Review of Economic Studies* **53**: 559–578.
Ham JC, Reilly KT. 2002. Testing intertemporal substitution, implicit contracts, and hours restriction models of the labor market using micro data. *American Economic Review* **92**: 905–927.
Heckman J. 1981a. Statistical models for discrete panel data. In *Structural Analysis of Discrete Data with Econometric Applications*, Manski CF, McFadden DL (eds). MIT Press: Cambridge, MA; 114–178.
Heckman J. 1981b. Dynamic discrete models. In *Structural Analysis of Discrete Data with Econometric Applications*, Manski CF, McFadden DL (eds). MIT Press: Cambridge, MA; 179–195.
Hotz VJ, Miller RA. 1993. Conditional choice probabilities and the estimation of dynamic models. *Review of Economic Studies* **60**: 497–529.
Hotz VJ, Kydland FE, Sedlacek GL. 1988. Intertemporal preferences and labor supply. *Econometrica* **56**: 335–360.
Hill MS. 1992. *The Panel Study of Income Dynamics: A User's Guide.* Sage: Newbury Park, CA.
Hyslop DR. 1999. State dependence, serial correlation and heterogeneity in intertemporal labor force participation of married women. *Econometrica* **67**(6): 1255–1294.
Kahn S, Lang K. 1992. Constraints on the choice of work hours. *Journal of Human Resources* **27**: 661–678.
Lee LF, Porter RH. 1984. Switching regression models with imperfect sample separation information with an application on cartel stability. *Econometrica* **52**: 391–418.
MaCurdy TE. 1983. A simple scheme for estimating an intertemporal model of labor supply and consumption in the presence of taxes and uncertainty. *International Economic Review* **24**: 265–289.
Magnac T, Thesmar D. 2002. Identifying dynamic discrete decision processes. *Econometrica* **70**: 801–816.
Pakes A. 1994. Dynamic structural models: problems and prospects. Part II: Mixed continuous discrete controls and market interactions. In *Advances in Econometrics: Sixth World Congress of the Econometric Society*, Vol. I, *Barcelona*, 1990, Sims CA (ed.). Cambridge University Press: Cambridge, UK; 171–259.
Rust J. 1988. Maximum likelihood estimation of discrete choice processes. *SIAM Journal of Control and Optimization* **26**: 1006–1024.
Rust J. 1994. Estimation of dynamic structural models: problems and prospects. Part I: Discrete decision processes. In *Advances in Econometrics: Sixth World Congress of the Econometric Society*, Vol. I, *Barcelona*, 1990, Sims CA (ed.). Cambridge University Press: Cambridge, UK; 119–170.

DOI: 10.1002/jae

Schmidt P. 1981. Constraints on the parameters in simultaneous tobit and probit models. In *Structural Analysis of Discrete Data and Econometric Applications*, Manski CF, McFadden DL (eds). MIT Press: Cambridge, MA; 422–434.
Weeks M, Orme C. 1998. The statistical relationship between bivariate and multinomial probit. Working paper, Department of Applied Economics, University of Cambridge.
Zeldes S. 1989. Consumption and liquidity constraints: an empirical investigation. *Journal of Political Economy* **97**: 305–346.

