

Inefficient continuation decisions, entry costs, and the cost of fluctuations

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Abstract

Fluctuations in firms' revenues reduce firms' survival chances and are costly from a social welfare point of view *even* when agents are risk neutral if (i) the decision to continue operating a firm is not efficient so that fluctuations lead to inefficient reductions in firms' life expectancy and (ii) the shortening of firms' life expectancy reduces firm entry due to (for example) the presence of entry costs. Conservative estimates for the per-period costs of moderate fluctuations, like business cycles, are between 0.18% and 2.12% of GDP, but some calculations result in estimates exceeding 30% of GDP.

Keywords: business cycles, frictions, inefficiencies

JEL Classification: E24, E32

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1 Introduction

Fluctuations are a fact of life. They come in many varieties such as idiosyncratic, sectoral, regional, and aggregate fluctuations. This paper documents that even modest fluctuations, like business cycles, are quite costly in a very simple framework with risk neutral agents and the following quite standard features. First, creating a project requires a fixed startup cost. Second, projects are not all the same. Here projects have different productivity levels and different startup costs, but our mechanism would also operate for alternative forms of heterogeneity. Third, the decision to start or continue operating an existing project is subject to inefficiencies, that is, "frictions" prevent some profitable projects from operating. Fourth, the fluctuations affect the severity of the inefficiency, either positively or negatively. Using this framework, we show that fluctuations are costly because they deter entry and lower the *average* level of output produced. Whereas it has been difficult to develop models in which moderate fluctuations have non-negligible per capita costs, the costs of business cycles in our framework correspond to a permanent drop in output that can easily exceed several percentage points and possibly could be substantially higher. In contrast, the classic Lucas (1987) paper reports an estimate for the costs of business cycles that is less than one tenth of a percentage point of consumption when the coefficient of risk aversion is equal to 10.

Before providing intuition for the mechanism, we motivate the key underlying features of our framework. Starting a "project", whether it is a company, a plant, or a job, is almost never costless and entry/startup costs are part of many economic models. Regarding the inefficient decision to start or continue operating a project, one can think of the inability to obtain financing,¹ the inability to motivate workers or prevent them from shirking,² the inability to write contracts that prevent the employer from exploiting the employee,³ or the inability to adjust real wages sufficiently downward. Finally, it is a natural feature of models with inefficiencies that the impact of the inefficiency fluctuates over time. This feature can be illustrated using the net worth channel. In a recession, firms' net worth levels drop. At these lower equity levels, firms are more likely to exploit the convexity (due to limited liability) in the payoffs and increase the amount of risk undertaken. Consequently, it will be more difficult for firms to obtain credit. Similarly, the ability for financial intermediaries to channel funds from savers to firms may very well be weakened during a recession due to an increase in (the impact of) frictions.

The reason why fluctuations are costly in our framework is quite intuitive. There is a stochastic aggregate variable Φ_p that affects firms' productivity levels', and firms' revenues determine to what extent projects are affected by the inefficiency. Consider projects whose characteristics are such that they are at risk of inefficiently being discontinued. Positive (negative) movements in Φ_p decrease (increase) the number of projects that make inefficient continuation decisions. If there are no entry costs, then there is no robust reason why the positive effects of an increase in Φ_p would not offset the negative effects

¹As for example in Townsend (1979), Bernanke and Gertler (1989) Kiyotaki and Moore (1997), Carlstrom and Fuerst (1998), and Bernanke, Gertler, and Gilchrist (1999).

²See Shapiro and Stiglitz (1984).

³See Ramey and Watson (1997).

of a decrease in Φ_p .⁴ With entry costs, however, this is no longer true. The reason is that fluctuations in Φ_p reduce projects' expected lifetime; low surplus projects are now terminated when the economy enters a recession. This means that startup costs have to be paid more often. More importantly, fluctuations in Φ_p bring about a reduction in the number of created projects. These firms are marginal in terms of being able to overcome the inefficiency, but have positive value from a social welfare point of view. Consequently, their disappearance is welfare reducing.

Our model does not rely on high risk aversion to explain why moderate fluctuations like business cycles are costly. In fact, we assume that agents are risk neutral. As pointed out in Lucas (2003), if agents are highly risk averse, then the question arises why high risk aversion does not show up in, for example, the diversification of individual portfolios, the level of insurance deductibles, or the wage premiums of jobs with high earnings risk.

The framework used is simple and contains only a small set of structural parameters and for most of them it is not difficult to consider a set of plausible values. One important ingredient in our quantitative assessment is the mass of projects that are not created in a world with business cycles, but are created in a world without business cycles. Since these projects are not observed in the actual world with business cycles, we have to find a way to estimate this mass. Our identification procedure consists of two elements. First, economic theory pins down exactly the productivity levels and startup costs at which entry would occur, also in the world without business cycles that we do not observe. Second, in a way that will be made more precise below, we determine the mass of these projects basically by assuming that there are no sudden changes in how different types of projects are distributed in the relevant area.

Our paper fits into a line of research that investigates the effect of uncertainty on the level or growth rate of output, which Lucas (2003) refers to as "*... a promising frontier on which there is much to be done*".⁵ Besides the assumption of linear utility, our framework differs from related papers in that we focus on different characteristics to generate the relationship between volatility and the level of real activity, namely entry costs and an inefficiency in the decision to operate a project; two simple features often found in the literature.

The rest of this paper is organized as follows. In section 2, we develop our framework. In section 3, we highlight a key discontinuity in our framework, namely that business cycles of arbitrarily small magnitude have a large impact on some projects. In section 4, we discuss the impact of business cycles of ordinary magnitude. In section 5, we discuss which elements are necessary for our mechanism to work. In section 6, we discuss our approach to obtain quantitative estimates for the costs of business cycles. The results are presented in section 7. In section 8, we discuss modifications of the model that lead to larger costs of business cycles. The literature is discussed in section 9 and the last section concludes.

⁴As in Caballero and Hammour (2005), the inefficiencies would make recessions more contractionary and booms more expansionary. Our paper, however, is about the costs of business cycles, *not* about the costs of recessions.

⁵In section 9, we discuss the related theoretical literature and provide references for papers that establish empirical support for the view that business cycles do not leave the long-run growth path unchanged.

2 Model

In this section, we present a generic model that is characterized by startup costs and an inefficiency in the decision to operate and continue the project. The model developed here is very simple and abstract, which makes it easy to explain why fluctuations are costly if there are both startup costs and inefficient continuation decisions. Although the model is abstract, we think that the highlighted interaction between startup costs and inefficient continuation decisions is so fundamental that it will play a role in many fully specified models with these two features. Support for this assertion is provided in Appendix A in which we discuss two different models that incorporate an agency problem into a business cycle model.

Agents. The economy is inhabited by utility-maximizing risk-neutral agents. We assume that agents are risk neutral to accentuate that business cycles in our model are costly *even* if agents are risk neutral.

Productivity of active and inactive projects. There is a continuum of projects indexed by i . Projects are owned by a single agent. We make this last assumption to make clear that the presence of the highlighted mechanism does not rely on inefficiencies in how the costs and/or revenues of the projects are shared between different agents. Although not important for the qualitative results, such additional inefficiencies can make the mechanism *quantitatively* more important. This is documented in section 8.

Project i is characterized by a startup cost, $\phi_c(i)$, and a productivity parameter, $\phi_p(i)$; both are assumed to be constant through time. A project can be active or inactive. To produce market output the project has to be activated, which requires paying the startup costs. Each period, individual active projects could be afflicted by an exogenous shock that will inactivate the project. This shock occurs with probability $1 - \rho$. The project can be reactivated by paying the startup costs again. Production of active project i , $y_t(i)$, is given by

$$y_t(i) = \phi_p(i)\Phi_{p,t}, \quad (1)$$

where $\Phi_{p,t}$ is aggregate productivity. From now on, we suppress the i index, but the reader should keep in mind that ϕ_p and ϕ_c vary across projects and all the other variables, including $\Phi_{p,t}$, do not.

When a project is not active, then it does not generate any market production. The project's revenues are in this case equal to μ . We assume that there are no transfers, such as subsidies, to agents with an inactive project. This means that μ only consists of benefits such as home production or the value of leisure. We do not include transfers in the analysis, because this additional inefficiency would only distract attention from the main mechanism.

Aggregate fluctuations. Two different assumptions about $\Phi_{p,t}$ are considered. Under the first assumption, $\Phi_{p,t}$ is constant through time and equal to 1. In this case, the projects are heterogeneous, but face an unchanging macroeconomic environment. Under the second

assumption, $\Phi_{p,t}$ is a stochastic variable that varies across time with an unconditional mean equal to 1. The most common interpretation of $\Phi_{p,t}$ is that it is an aggregate shock that is common to all projects. In this case, fluctuations in $\Phi_{p,t}$ correspond to business cycle fluctuations. For simplicity, we assume that $\Phi_{p,t}$ can take on only two values, Φ_+ in a boom and Φ_- in a recession. The probability of transitioning out of a boom, $1 - \pi$, is equal to the probability of transitioning out of a recession. This implies that the expected durations of staying in a boom and a recession are equal to each other.⁶ Since $E[\Phi_{p,t}] = 1$, it also implies that $\Phi_+ - 1 = 1 - \Phi_- = \Delta_{\Phi_p}$.

Inefficiencies and the operating/continuation decision. Two decisions have to be made before production can take place. First, the decision has to be made whether to *activate* the project by paying the startup cost ϕ_c . Second, the decision has to be made whether to *operate* the project and *continue* to the next period.⁷ We start with the second decision.

The key aspect of the model is that operating the project is hampered by an inefficiency. The inefficiency is that a project can only continue operating if the period t revenues are sufficiently high. In particular, we require that

$$\phi_p \Phi_{p,t} \geq \chi_t. \quad (2)$$

We will refer to the requirement given in equation (2) as the efficiency requirement. Projects that do not satisfy this condition cannot operate and generate μ instead of $\phi_p \Phi_{p,t}$. The value of χ_t could be a constant, but we allow for the possibility that χ_t varies with $\Phi_{p,t}$. That is, $\chi_t = \chi(\Phi_{p,t})$. When $\Delta_{\Phi_p} = 0$, then the value of χ_t is equal to $\chi(1)$. With some abuse of notation we will refer to $\chi(1)$ as χ .

Assumption 1

$$\mu < \min \{ \chi(\Phi_-), \chi(\Phi_+) \}. \quad (3)$$

This assumption ensures that the efficiency requirement plays a role in both a boom and a recession.

It is important to realize that the efficiency requirement is *not* part of technology, but is capturing a private inefficiency.⁸ An inefficiency is a private inefficiency if the

⁶NBER recessions are shorter than NBER expansions. But the classification used by the NBER is not symmetric. The reason is that the NBER does not classify a recession as a period when observed growth is below a long-run average, but as a period when observed economic activity is *sufficiently* bad. For our purpose, it makes more sense to consider HP-filtered residuals. Within our sample from 1947Q1 to 2010Q4, we find that from 1949Q1 to 2008Q3 there are 16 complete recessions (periods with negative HP-filtered residuals surrounded by positive residuals) and 16 complete booms (periods with positive HP-filtered residuals surrounded by negative values). The average durations are equal to 7.1 and 7.9 quarters for recessions and booms, respectively. This corresponds roughly to the assumption adopted here that the expected duration of a boom is roughly equal to the expected duration of a recession.

⁷Our model shares these two decisions with labor market matching models. In fact, our setup is similar to the one in Robin (2010). We differ from this literature in that we simplify the analysis by setting the matching probability equal to one and we introduce an inefficiency in the continuation decision.

⁸In Section 5, we explain why the inefficiency has to be a private inefficiency.

participant(s) in the project is (are) worse off if the project does not remain active. The following examples clarify what is and what is not a private inefficiency. Suppose that the owner of an inactive project not only receives μ , the value generated by an inactive project, but also receives a transfer from the government equal to $\chi - \mu > 0$. Then it would be socially inefficient for a project with $\phi_p = \chi > \mu$ to become inactive, since the government has to finance the transfer to this inactive project, while the inactive project is only producing μ . But it is not *privately* inefficient to stop operating, since the owner of the project is not worse off. Now suppose that the government without good reason prohibits projects to operate if χ exceeds ϕ_p and $\chi > \mu$. Then projects with $\mu < \phi_p < \chi$ would face a private inefficiency. The reason is that the inefficiency makes the beneficiaries of these projects worse off, since it causes these projects to generate μ instead of the higher ϕ_p .

Interpretation of the inefficiency. To understand the model, the reader may want to hold on to this last simple interpretation of the efficiency requirement. But the mechanism we highlight in this paper does not depend on this particular interpretation. In fact, the mechanism manifests itself in the presence of several different types of inefficiencies. The reason is that models with inefficiencies typically imply a condition like our efficiency requirement. That is, if the firms' own internal resources are not high enough, then the firm cannot overcome inefficiencies due to, for example, moral hazard problems.

In Appendix A, we describe two explicit dynamic models with agency problems in which agents carefully consider both the current and future consequences of their actions. In both models, the inefficiency leads to a condition equal to or similar to our efficiency requirement. It is easy to think of other types of inefficiencies. For example, another obvious reason for private inefficiencies is the presence of sticky wages, either because of social norms, inefficient bargaining, or efficiency wages.

Inefficiencies and the cut-off level for ϕ_p . Let $\tilde{\phi}_{p,bc}$ be the value of ϕ_p such that equation (2) holds with equality when there are business cycles, that is, when $\Delta_{\Phi_p} > 0$. Thus,

$$\tilde{\phi}_{p,bc} \Phi_{p,t} = \chi_t \text{ or } \tilde{\phi}_{p,bc} = \frac{\chi_t}{\Phi_{p,t}}.$$

In most business cycle models with inefficiencies, the value of χ_t would be less cyclical than $\Phi_{p,t}$. The reason for this property is that the value of χ_t is typically affected by the value of the alternatives to producing market output. For example, this could be the value of the project when resources are diverted from the regular production process. The value of these alternatives is typically assumed to be less sensitive to $\Phi_{p,t}$ than market production, i.e., $\phi_p \Phi_{p,t}$. The value of $\tilde{\phi}_{p,t}$ would then be *countercyclical*. In this case, the fraction of agents that is affected by the inefficiency decreases in a boom and increases in a recession. Our mechanism does not depend on this property. It also operates if χ_t is more procyclical than $\Phi_{p,t}$, i.e., when the fraction of agents that is affected by the inefficiency is *procyclical*. All that is needed for our channel to be operative is that $\tilde{\phi}_{p,t}$ is cyclical, either procyclical or countercyclical. The only case that has to be ruled out is that χ_t is proportional to $\Phi_{p,t}$ as stated in the following assumption.

Assumption 2 *One of the following two conditions holds:*

$$(i) \quad \tilde{\phi}_{p,bc}(\Phi_{p,t}) = \frac{\chi_t}{\Phi_{p,t}} = \frac{\chi(\Phi_{p,t})}{\Phi_{p,t}} \text{ decreases with } \Phi_{p,t} \text{ or} \quad (4)$$

$$(ii) \quad \tilde{\phi}_{p,bc}(\Phi_{p,t}) = \frac{\chi_t}{\Phi_{p,t}} = \frac{\chi(\Phi_{p,t})}{\Phi_{p,t}} \text{ increases with } \Phi_{p,t} \quad (5)$$

This assumption is trivially satisfied if χ_t is constant. To focus the discussion, we describe the framework and the results under the assumption that $\tilde{\phi}_{p,bc}$ is decreasing with $\Phi_{p,t}$. The impact of frictions then increases during economic downturns, which we feel is the more natural possibility. Formal results are stated, however, using the more general condition given above.

Activation decision. Projects can be activated instantaneously by paying a start-up cost, ϕ_c . When the project has been idle for some time, then ϕ_c would have to be paid once more to restart it. That is, one cannot simply mothball the project and restart it as if there had been no interruption. One possible reason for this is that it may take some time and effort before the project is operating at its potential productivity of ϕ_p again.⁹

For a project with productivity level ϕ_p and startup costs ϕ_c , activation will occur if

$$N_{bc}(\phi_c, \phi_p, 1, \Phi_{p,t}) - \phi_c \geq \mu + \beta E_t [N_{bc}(\phi_c, \phi_p, 0, \Phi_{p,t+1})], \quad (6)$$

where E_t is the expectation conditional on period- t information, $N_{bc}(\phi_c, \phi_p, 1, \Phi_{p,t})$ is the discounted value of the project's current and future earnings when the startup costs have been paid, and $N_{bc}(\phi_c, \phi_p, 0, \Phi_{p,t})$ is the discounted value of the project's earnings when the startup costs have not been paid.¹⁰ When the startup costs are not paid, then it remains possible to start the project at a future date.

Assumption 3

$$(i) \quad 0 < \beta < 1, 0 < \rho < 1, 0 < \pi < 1, \phi_c \geq 0$$

$$(ii) \quad \frac{(1+\Delta_{\Phi_p})\chi^{-\mu}}{1-\beta\rho\pi} < \frac{\chi^{-\mu}}{1-\beta\rho} \quad (7)$$

$$(iii) \quad \Delta_{\Phi_p} < \frac{\chi^{-\mu}}{\chi}$$

The first part of the assumption simply ensures that parameters do not take on nonsensical values. The second part is less trivial, but is also a weak assumption. It ensures that the NPV of the sequence of surplus values a project earns during one single stretch of high

⁹For some projects, the value of ϕ_c may be low. For example, in the US car industry it is not uncommon to leave capital idle or underutilized for some time and recall former workers when economic conditions improve. In our benchmark calibration, we set the lower bound of the distribution of ϕ_c equal to zero, which clearly would accommodate the possibility of low (re)starting costs.

¹⁰That is, $N_{bc}(\phi_p, \phi_c, 0, \Phi_{p,t})$ is equal to $N_{bc}(\phi_p, \phi_c, 1, \Phi_{p,t}) - \phi_c$ when entry is optimal and is equal to $\mu + \beta E_t N_{bc}(\phi_p, \phi_c, 0, \Phi_{p,t+1})$ when entry is not optimal.

Φ_+ values does not exceed the NPV of the sequence of surplus values this project earns over its complete natural life if there are no business cycles to possibly end it prematurely. To see that this is a weak assumption, suppose that $\beta = 0.99$, $\rho = 0.9$, and $\pi = 0.5$. Then this condition is satisfied as long as the surplus in a boom is not more than 409% above the surplus value in a world without business cycles. If the second part of assumption 3 does not hold, then *one* particular negative consequence of business cycles is no longer present. It is not *necessary*, however, for business cycles to be costly. The third part of this assumption also restricts the magnitude of aggregate business cycles. It is made for convenience and means that we have to consider less types of projects in proposition 9.

Interpretation of a project. The two key aspects of a project are that (i) a startup cost has to be paid to activate the project and (ii) inefficiencies make it impossible to continue to operate the project if the project does not generate enough revenues.

There are many undertakings with these two criteria. For example, a job is characterized by these two criteria. First, to create a new job requires some investment, both to create the job itself and to find a suitable worker. Second, inefficiencies could occur both in terms of ensuring cooperation between the employer and the employee and in terms of securing financing. When the project is a job, then μ would capture the value of leisure and/or home production.

But there are other ways to interpret projects, both on a smaller and on a larger scale. An example of a possible interpretation on a smaller scale would be an additional task for an existing worker. If creation of the task requires an up-front investment and introduces agency problems, then it would fit the description of a project in our model. Alternatively, one can think of much bigger projects such as the opening of a mine or starting a company.

Calibration. The value of β is set equal to the standard value of 0.99. We follow Krusell and Smith (1998) and set π equal to 0.875, which means that the expected duration of a boom and a recession is equal to 8 quarters.¹¹ The parameters μ indicates the value generated by an inactive project. Shimer (2005) uses a value of leisure that is equal to 40% of market production. But his measure refers to all benefits that an unemployed worker receives, whereas here μ indicates the actual value generated by an inactive project. In case of a job it should exclude any type of transfer such as unemployment benefits; it should only include the value of home production and possibly the utility gain if a worker does not work. As our benchmark, we assume that half of the number used by Shimer (2005) consists of actual net benefits generated by an unemployed worker. The value used by Shimer (2005) is considered to be too low by some.¹² Hall (2006) estimates the flow value of leisure forgone to be equal to 43%, and we consider this as an alternative estimate.¹³

The parameter ρ controls the rate of exogenous destruction and $1/(1-\rho)$ is the expected duration of the project if the duration is not affected by business cycle considerations.

¹¹As pointed out in footnote 6, this corresponds roughly to observed durations.

¹²See Mortensen and Nagypál (2007) for a discussion.

¹³Personally we value our leisure a lot less, but maybe we are overestimating our market production levels.

Obviously, there are many types of projects and different expected lifetimes. If the project is a company, a plant, a mine, or a ship, then the expected duration could easily exceed 10 years or 40 quarters, which would correspond to a value of ρ equal to 0.975. But there are also projects with much shorter durations. The values of ρ considered are equal to 0.875, 0.9167, and 0.975, which correspond with expected durations of respectively 2, 3, and 10 years.

These are the only parameter values we need for the results reported in the next section that focuses on the impact of arbitrarily small fluctuations in $\Phi_{p,t}$. In particular, we do not have to take a stand on the numerical value of χ , we only have to assume that the value is such that assumptions 1 and 3 are satisfied.

3 The impact of arbitrarily small aggregate fluctuations

In this section, we first describe a slightly more general version of the efficiency requirement and introduce some terminology. Next, we describe the economy when there are no fluctuations in $\Phi_{p,t}$, i.e., when $\Delta_{\Phi_p} = 0$. Finally, we discuss the impact on the economy of *arbitrarily small* fluctuations in $\Phi_{p,t}$.

3.1 Preliminaries

The ultimate purpose of this section is to analyze the impact on the economy when Δ_{Φ_p} increases from 0 to an arbitrarily small number. By considering small changes in Δ_{Φ_p} , we highlight the property of the model that small business cycles have large effects on some borderline projects. The inequality in equation (2) is a weak inequality, but there is no reason why it should not be a strict inequality. Whether it is a weak or a strict inequality does not matter when Δ_{Φ_p} is not arbitrarily small.¹⁴ But it does matter when we consider the case of arbitrarily small values for Δ_{Φ_p} . The following assumption removes the ambiguity and ensures that the analysis of arbitrarily small fluctuations captures all aspects of regular fluctuations.¹⁵

Assumption 4 *For each combination of (ϕ_c, ϕ_p) , there is one project that faces the requirement that*

$$\phi_p \Phi_{p,t} \geq \chi_t \tag{8}$$

and one project that faces the requirement that

$$\phi_p \Phi_{p,t} > \chi_t. \tag{9}$$

Definition 5 *Projects that face the efficiency requirement given in equation (8) are referred to as "weak-inequality" projects and projects that face the efficiency requirement given in equation (9) are referred to as "strict-inequality" projects.*

¹⁴Unless there happens to be point mass *exactly* at the cut-off points.

¹⁵We only need this assumption for the analysis of arbitrarily small fluctuations and then only for projects with a value of ϕ_p equal to $\tilde{\phi}_{p,bc}$.

Since agents are risk neutral, business cycles only affect agents' utility if aggregate fluctuations lead to different decisions. If decisions are not affected, then business cycles just make revenues more volatile, which is not important for risk-neutral agents. There are two types of projects that are affected by business cycles as made precise in the following two definitions.

Definition 6 *Cyclical projects have the property that they can overcome inefficiencies in a boom, but not in a recession. Thus, cyclical projects are projects for which¹⁶*

$$\frac{\chi_t}{1 + \Delta_{\Phi_p}} \leq \phi_p < \frac{\chi_t}{1 - \Delta_{\Phi_p}} \text{ if the project is a weak-inequality project}$$

and

$$\frac{\chi_t}{1 + \Delta_{\Phi_p}} < \phi_p \leq \frac{\chi_t}{1 - \Delta_{\Phi_p}} \text{ if the project is a strict-inequality project.}$$

Definition 7 *Timed-entry projects are projects such that (i) the NPV of activating the project is higher than the NPV of not activating in a boom and (ii) the NPV of activating the project is lower than the NPV of not activating in a recession. Thus, timed-entry projects are projects for which the following holds:*

$$N_{bc}(\phi_c, \phi_p, 1, 1 + \Delta_{\Phi_p}) - \phi_c \geq \mu + \beta E_t [N_{bc}(\phi_c, \phi_p, 0, \Phi_{p,t+1})]$$

and

$$N_{bc}(\phi_c, \phi_p, 1, 1 - \Delta_{\Phi_p}) - \phi_c < \mu + \beta E_t [N_{bc}(\phi_c, \phi_p, 0, \Phi_{p,t+1})].$$

3.2 Economy with no aggregate fluctuations

In this subsection, we assume that $\Delta_{\Phi_p} = 0$, which implies that $\Phi_{p,t}$ is constant through time, which in turn implies that χ_t is constant as well (and equal to χ). If $\Phi_{p,t}$ is constant, then a firm either *always* overcomes the existing inefficiencies or *never* overcomes them. That is, the cut-off level for ϕ_p is constant and given by

$$\tilde{\phi}_{p,\text{no-bc}} = \chi. \tag{10}$$

Projects with $\phi_p < \tilde{\phi}_{p,\text{no-bc}}$ do not enter, since they can never overcome the inefficiency. Projects with $\phi_p \geq \tilde{\phi}_{p,\text{no-bc}}$ will enter as long as the startup costs are low enough, where projects with a higher value for ϕ_p allow for activation at higher values of ϕ_c .

This version of the model is graphically described in figure 1. Projects in the shaded area are activated and produce market output, since their value of ϕ_p exceeds χ and their startup costs are low enough. We refer to the cut-off level for ϕ_c in the world without business cycles as $\tilde{\phi}_{c,\text{no-bc}}$. If productivity is high enough to satisfy the efficiency requirement, then $\tilde{\phi}_{c,\text{no-bc}}$ is equal to the difference between the NPV of the revenues of

¹⁶The definition is given for the case that part (i) of assumption 2 holds. If part (ii) holds instead, then one would simply have to change the position of $1 + \Delta_{\Phi_p}$ and $1 - \Delta_{\Phi_p}$ in this definition.

the activated project and the NPV of the revenues of the not activated project. Thus, if $\phi_p > \chi$ ($\phi_p \geq \chi$) and the firm is a strict(weak)-inequality project, then

$$\tilde{\phi}_{c,\text{no-bc}}(\phi_p) = \frac{(\phi_p - \mu)}{1 - \beta\rho}. \quad (11)$$

If the efficiency requirement is not satisfied, then entry will not occur, no matter how low the entry costs are. To economize on notation, we will typically drop the argument and simply write $\tilde{\phi}_{c,\text{no-bc}}$.

The top panel of figure 2 plots the NPV (before the startup cost has been paid) as a function of ϕ_c for projects with $\phi_p = \chi$ and when $\Delta_{\Phi_p} = 0$. Strict-inequality projects with $\phi_p = \chi$ just do not meet the efficiency requirement. Consequently, their NPV is equal to $\mu/(1 - \beta)$ independent of the project's value for ϕ_c . Weak-inequality projects with $\phi_p = \chi$ just do meet the efficiency requirement. Their value decreases with ϕ_c until ϕ_c is so high that the value of activating is equal to the value of not activating, $\mu/(1 - \beta)$.

Now consider a weak-inequality project with $\phi_p = \chi$ and $\phi_c = \tilde{\phi}_{c,\text{no-bc}}$. The location of this project is identified with the letter A in the figure. At $\phi_c = \tilde{\phi}_{c,\text{no-bc}}$, the owner of this project would be (just) indifferent between activating and not activating the project in a world without business cycles. Thus, the NPV of this weak-inequality project is equal to the NPV of a strict-inequality project with the same values of ϕ_p and ϕ_c , which does not enter in a world without business cycles. As the value of ϕ_c drops, it begins to matter whether a project with $\phi_p = \chi$ is a weak or a strict-inequality project and the NPV of the weak-inequality project increases whereas the NPV of the strict-inequality project remains the same. This is an important observation. The crux of the matter is that two projects that are identical except that one *just* can and one *just* cannot overcome the efficiencies can have very different NPV values.

3.3 Quantitative impact of arbitrarily small fluctuations

When considering arbitrarily small fluctuations, we only have to consider projects with values of ϕ_p and/or ϕ_c that are equal to the cut-off points, i.e., projects with either $\phi_p = \tilde{\phi}_{p,\text{no-bc}} = \chi$ or $\phi_c = \tilde{\phi}_{c,\text{no-bc}}$. Projects with $\phi_p = \tilde{\phi}_{p,\text{no-bc}} = \chi$ are cyclical projects and projects with $\phi_c = \tilde{\phi}_{c,\text{no-bc}}$ are timed-entry projects.¹⁷ The impact on cyclical projects is discussed in section 3.3.1 and the impact on timed-entry projects is discussed in section 3.3.2.

3.3.1 Impact of arbitrarily small business cycles on cyclical projects

We have to distinguish between three different types of cyclical projects. First, some projects are *permanently* driven out of existence by business cycles. This happens even for arbitrarily small values of Δ_{Φ_p} . Second, some projects *temporarily* stop producing because of business cycles. Third, some projects that never produce in a world without business cycles, will at times produce in a world with business cycles. We now discuss these three types of cyclical projects.

¹⁷Both the weak and the strict-inequality projects with $\phi_p = \chi$ are cyclical projects.

Permanent stop of market production when ϕ_c is high. We start the analysis by considering the weak-inequality project with $\phi_p = \chi$ and $\phi_c = \tilde{\phi}_{c,\text{no-bc}}$. A weak-inequality project with $\phi_p = \chi$ is productive enough to satisfy the efficiency requirement. Since $\phi_c = \tilde{\phi}_{c,\text{no-bc}}$, the NPV of this project is by definition equal to the NPV of a project that is not activated. The owner would be indifferent between activating and not activating. Now suppose that there are business cycles, but that the size of the fluctuations, i.e., the value of Δ_{Φ_p} , is arbitrarily small. If the economy is in a recession, i.e., $\Phi_{p,t} = 1 - \Delta_{\Phi_p}$, then the project cannot overcome the inefficiency and it clearly would not make sense to pay the startup costs and activate the project. Now suppose that the economy is in a boom, i.e., $\Phi_{p,t} = 1 + \Delta_{\Phi_p}$. As long as the economy is in a boom, then the project can overcome the inefficiencies and produce χ which strictly exceeds μ . However, the project cannot remain active when the economy gets into a recession, since the project's internal resources are not sufficient to overcome the inefficiencies during a recession. This means that there is a *discrete* reduction in the expected life time of the project *even* when the value of Δ_{Φ_p} is arbitrarily small. In particular, it drops from $1/(1 - \rho)$ to $1/(1 - \rho + \rho(1 - \pi))$. Consequently, the owner of the project with $\phi_p = \chi$ and $\phi_c = \tilde{\phi}_{c,\text{no-bc}}$ would *strictly* prefer not to activate the project when $\Delta_{\Phi_p} > 0$. The NPV of activating also drops for projects with a lower value of ϕ_c and the cut-off point value of ϕ_c for projects with ϕ_p equal to χ in the presence of business cycles, $\tilde{\phi}_{c,\text{bc}}$, is given by

$$\tilde{\phi}_{c,\text{bc}}(\phi_p, \Phi_+) = \frac{(1 + \Delta_{\Phi_p})\phi_p - \mu}{1 - \beta\rho\pi} < \frac{\phi_p - \mu}{1 - \beta\rho} = \tilde{\phi}_{c,\text{no-bc}}. \quad (12)$$

The value of $\tilde{\phi}_{c,\text{bc}}(\phi_p, \Phi_-)$ would be negative for cyclical projects. Since startup costs are assumed to be positive, we do not have to consider this possibility. Unless stated otherwise, the variable $\tilde{\phi}_{c,\text{bc}}$ will refer to the formula given in equation (12), that is, to the cut-off level of cyclical projects for which the expected duration is shortened by business cycles.

For any positive value of Δ_{Φ_p} , no matter how small, $\tilde{\phi}_{c,\text{bc}}$ is below $\tilde{\phi}_{c,\text{no-bc}}$ with a discrete amount. A project with $\phi_p = \chi$ and $\phi_c = \tilde{\phi}_{c,\text{no-bc}}$ is faced with a discrete drop in the value of *activating* the project when Δ_{Φ_p} increases from 0 to a positive value, even an arbitrarily small number. The NPV of this project with $\phi_p = \chi$ and $\phi_c = \tilde{\phi}_{c,\text{no-bc}}$ does not drop, however, since the option of never activating the project remains available and the NPV of a project with $\phi_c = \tilde{\phi}_{c,\text{no-bc}}$ is by definition equal to the NPV of not activating. In contrast, the NPV of projects with $\phi_p = \chi$ and $\phi_c < \tilde{\phi}_{c,\text{no-bc}}$ will display a discrete reduction in value when Δ_{Φ_p} takes on a positive value.

The top panel of figure 2 plots the NPV of projects with $\phi_p = \chi$ as a function of ϕ_c , both when Δ_{Φ_p} is positive (and arbitrarily small) and when Δ_{Φ_p} is equal to zero. The bottom panel of this figure plots the change in the NPV value when business cycles of arbitrarily small value are introduced. Consider values for ϕ_c such that $\phi_c > \tilde{\phi}_{c,\text{bc}}$, i.e., ϕ_c is too high to make entry worthwhile given the reduction in expected duration brought about by the increase in Δ_{Φ_p} . The negative impact of business cycles is larger for smaller values of ϕ_c , since the projects with lower values for ϕ_c are more profitable, which means that the loss of not activating the project is larger.

Cyclical projects with low values for ϕ_c . Next, we consider cyclical projects with values of ϕ_c that are less than $\tilde{\phi}_{c,bc}$. These projects cannot overcome the inefficiencies in a recession, *but* the values of their startup costs are so low that it is still worth activating the project even though the expected duration of the project is now shortened. When $\Delta_{\Phi_p} > 0$, then there is no difference between the strict-inequality and the weak-inequality projects with $\phi_p = \chi$. But there is a difference when $\Delta_{\Phi_p} = 0$. We first consider the weak-inequality projects and then the strict-inequality projects.

Temporarily driven out of producing market output. Consider weak-inequality projects, with $\phi_p = \chi$ and $\tilde{\phi}_c \leq \tilde{\phi}_{c,bc}$. These projects can always overcome the inefficiency when $\Delta_{\Phi_p} = 0$. This means that the expected duration is equal to $1/(1-\rho)$. An arbitrarily small increase in Δ_{Φ_p} affects these projects in two ways. First, during recessions they only generate μ instead of the larger χ . Second, the project has to stop producing during a recession and the startup costs have to be paid again at the beginning of the boom. The NPV of these projects is higher in a world without business cycles than in a world with business cycles, as is documented in the top panel of figure 2. The magnitude of the drop in value is shown graphically in the bottom panel of the same figure.¹⁸

Temporarily driven into producing market output. Now consider strict-inequality projects with $\phi_p = \chi$ and $\tilde{\phi}_c \leq \tilde{\phi}_{c,bc}$. For these projects, the presence of business cycles is beneficial as long as their value of ϕ_c is low enough. The reason is that in a world without business cycles they always generate μ , whereas in a world with business cycles they generate $\chi > \mu$ in a boom *and* their startup costs are low enough to take advantage of this. The benefits are reduced by the fact that startup costs have to be paid. Consequently, the welfare gains are largest for the project with zero startup costs, as is shown in the bottom panel of figure 2.

It is important to realize that the welfare losses of the weak-inequality projects exceed the welfare gains of the strict-inequality projects. In terms of output, the losses and the gains exactly offset each other. But the shortening in the expected duration of a project means that the total amount of startup costs paid is higher when $\Delta_{\Phi_p} > 0$ than when $\Delta_{\Phi_p} = 0$. Thus, when $\phi_c \leq \phi_{c,bc}$, the loss of a weak-inequality project together with the gain of a strict-inequality project is a loss except when $\phi_c = 0$. This is documented in the lower panel of figure 2. When $\phi_c = 0$, then the gain of the weak-inequality project does exactly off-set the loss of the strict-inequality project, because it does not matter that the entry costs have to be paid more often.

¹⁸In the graph, the project with $\phi_c = 0$ has the smallest loss among the weak-inequality projects with $0 \leq \phi_c \leq \tilde{\phi}_{c,bc}$. But this depends on parameter values, and the project with $\phi_c = 0$ could also have the largest loss. What matters is how the NPV decreases with ϕ_c when there are and when there are no business cycles. On the one hand you only pay entry costs when you start in a boom which means that the NPV is less sensitive to ϕ_c in a world with business cycles. On the other hand you are expected to pay them more often at future dates in a world with business cycles. This would make the NPV more sensitive to ϕ_c in a world with business cycles. If $\rho = 0.5/(\beta(1 - 0.5\pi))$, then the effects exactly offset each other and the loss is the same for all weak inequality projects with $0 \leq \phi_c \leq \tilde{\phi}_{c,bc}$.

Magnitude of the impact of tiny business cycles on cyclical projects. We measure the impact of business cycles on individual projects as the permanent increase (or decrease) in per-period income that would make the owner of a project in a world *with* business cycles as well off as the owner of the same project in a world *without* business cycles. To standardize the measure we scale by ϕ_p . The formula is given in the following definition.

Definition 8 *The impact of business cycles on an individual project is given by*

$$L(\phi_c, \phi_p, \Delta_{\Phi_p}) = (1 - \beta) \left(\frac{N_{no-bc}(\phi_c, \phi_p, 0) - E[N_{bc}(\phi_c, \phi_p, 0, \Phi_{p,t})]}{\phi_p} \right),$$

where

$$E[N_{bc}(\phi_c, \phi_p, 0, \Phi_{p,t})] = \frac{(NPV_{bc}(\phi_c, \phi_p, 0, 1 + \Delta_{\Phi_p}) + NPV_{bc}(\phi_c, \phi_p, 0, 1 - \Delta_{\Phi_p}))}{2}.$$

The following proposition gives the results for projects at the boundary, i.e., when $\phi_p = \tilde{\phi}_{p,no-bc} = \chi$.

Proposition 9 *Suppose that (i) $\Delta_{\Phi_p} > 0$, (ii) assumptions 1 and 3 are satisfied, and (iii) the first part of 2 holds.¹⁹ Then the following properties hold:*

1.

$$\begin{aligned} \tilde{\phi}_{c,bc} &= \frac{\phi_p(1 + \Delta_{\Phi_p}) - \mu}{1 - \beta\rho\pi} \text{ if } \tilde{\phi}_{p,bc}(\Phi_+) \leq \phi_p < \tilde{\phi}_{p,bc}(\Phi_+), \\ \tilde{\phi}_{c,no-bc} &= \frac{\phi_p - \mu}{1 - \beta\rho} \text{ if } \phi_p \geq \tilde{\phi}_{p,no-bc}, \\ \tilde{\phi}_{c,bc} &< \tilde{\phi}_{c,no-bc} \text{ if } \tilde{\phi}_{p,no-bc} \leq \phi_p < \tilde{\phi}_{p,bc}(\Phi_+). \end{aligned}$$

2. *For any cyclical projects with $\phi_p > \tilde{\phi}_{p,no-bc}$ and for weak-inequality projects with $\phi_p = \tilde{\phi}_{p,no-bc}$, the change in welfare is given by*

$$L(\phi_c, \phi_p, \Delta_{\Phi_p}) = \begin{cases} \frac{1 - \Delta_{\Phi_p} - \mu/\phi_p}{2} + \frac{\phi_c}{\phi_p} \left(\frac{1 - \beta\rho\pi - 2(1 - \beta\rho)}{2} \right) > 0 & \text{if } 0 \leq \phi_c \leq \tilde{\phi}_{c,bc} \\ 1 - (1 - \beta\rho) \frac{\phi_c}{\phi_p} - \frac{\mu}{\phi_p} > 0 & \text{if } \tilde{\phi}_{c,bc} < \phi_c < \tilde{\phi}_{c,no-bc} \\ 0 & \text{if } \phi_c \geq \tilde{\phi}_{c,no-bc} \end{cases}$$

¹⁹If the second part of condition 2 holds, then a similar set of formulas are valid, but the role of the boom and the recession are switched.

For cyclical projects with $\phi_p < \tilde{\phi}_{p,no-bc}$ and for strict-inequality projects with $\phi_p = \tilde{\phi}_{p,no-bc}$, the change in welfare is given by

$$L(\phi_c, \phi_p, \Delta_{\Phi_p}) = \begin{cases} \frac{\mu/\phi_p - (1+\Delta_{\Phi_p})}{2} + \frac{\phi_c}{\phi_p} \left(\frac{1-\beta\rho\pi}{2} \right) < 0 & \text{if } 0 \leq \phi_c \leq \tilde{\phi}_{c,bc} \\ 0 & \text{if } \tilde{\phi}_{c,bc} < \phi_c < \tilde{\phi}_{c,no-bc} \\ 0 & \text{if } \phi_c \geq \tilde{\phi}_{c,no-bc} \end{cases}$$

The average of an affected weak-inequality and an affected strict-inequality project with $\phi_p = \tilde{\phi}_{p,no-bc}$ is given by²⁰

$$L(\phi_c, \phi_p, \Delta_{\Phi_p}) = \begin{cases} \frac{1}{2} \left(-\Delta_{\Phi_p} + \frac{\phi_c}{\phi_p} \beta\rho(1-\pi) \right) < 0 & \text{if } 0 < \phi_c < \frac{\phi_p \Delta_{\Phi_p}}{\beta\rho(1-\pi)} \\ \frac{1}{2} \left(-\Delta_{\Phi_p} + \frac{\phi_c}{\phi_p} \beta\rho(1-\pi) \right) = 0 & \text{if } \phi_c = \frac{\phi_p \Delta_{\Phi_p}}{\beta\rho(1-\pi)} \\ \frac{1}{2} \left(-\Delta_{\Phi_p} + \frac{\phi_c}{\phi_p} \beta\rho(1-\pi) \right) > 0 & \text{if } 0 < \phi_c \leq \tilde{\phi}_{c,bc} \\ 1 - (1-\beta\rho) \frac{\phi_c}{\phi_p} - \frac{\mu}{\phi_p} > 0 & \text{if } \tilde{\phi}_{c,bc} < \phi_c < \tilde{\phi}_{c,no-bc} \\ 0 & \text{if } \phi_c = \tilde{\phi}_{c,no-bc} \end{cases}$$

3.

$$\lim_{\Delta_{\Phi_p} \rightarrow 0} \tilde{\phi}_{c,bc} = \frac{\phi_p - \mu}{1 - \beta\rho\pi} < \tilde{\phi}_{c,no-bc} = \frac{\phi_p - \mu}{1 - \beta\rho}.$$

4. For weak-inequality projects with $\phi_p = \tilde{\phi}_{p,no-bc}$, the change in welfare of introducing arbitrarily small business cycles is given by

$$\lim_{\Delta_{\Phi_p} \rightarrow 0} L(\phi_c, \phi_p, \Delta_{\Phi_p}) = \begin{cases} \frac{1-\mu/\phi_p}{2} + \frac{\phi_c}{\phi_p} \left(\frac{1-\beta\rho\pi-2(1-\beta\rho)}{2} \right) > 0 & \text{if } 0 \leq \phi_c \leq \tilde{\phi}_{c,bc} \\ 1 - (1-\beta\rho) \frac{\phi_c}{\phi_p} - \frac{\mu}{\phi_p} > 0 & \text{if } \tilde{\phi}_{c,bc} < \phi_c < \tilde{\phi}_{c,no-bc} \\ 0 & \text{if } \phi_c \geq \tilde{\phi}_{c,no-bc} \end{cases}$$

and for strict-inequality projects with $\phi_p = \tilde{\phi}_{p,no-bc}$ by

$$\lim_{\Delta_{\Phi_p} \rightarrow 0} L(\phi_c, \phi_p, \Delta_{\Phi_p}) = \begin{cases} \frac{\mu/\phi_p - 1}{2} + \frac{\phi_c}{\phi_p} \left(\frac{1-\beta\rho\pi}{2} \right) < 0 & \text{if } 0 \leq \phi_c \leq \tilde{\phi}_{c,bc} \\ 0 & \text{if } \tilde{\phi}_{c,bc} < \phi_c < \tilde{\phi}_{c,no-bc} \\ 0 & \text{if } \phi_c \geq \tilde{\phi}_{c,no-bc} \end{cases}$$

²⁰If $\phi_c \leq \phi_{c,bc}$, this is the average of the weak-inequality and the strict-inequality project. If $\tilde{\phi}_{c,bc} < \phi_c \leq \tilde{\phi}_{c,no-bc}$, this is just equal to the effect of the weak-inequality project.

For projects affected by business cycles, the average impact for arbitrarily small changes is given by

$$\lim_{\Delta\Phi_p \rightarrow 0} L(\phi_c, \phi_p, \Delta\Phi_p) = \begin{cases} \frac{1}{2} \frac{\phi_c}{\phi_p} \beta \rho (1 - \pi) = 0 & \text{if } \phi_c = 0 \\ \frac{1}{2} \frac{\phi_c}{\phi_p} \beta \rho (1 - \pi) > 0 & \text{if } 0 < \phi_c \leq \tilde{\phi}_{c,bc} \\ 1 - (1 - \beta \rho) \frac{\phi_c}{\phi_p} - \frac{\mu}{\phi_p} > 0 & \text{if } \tilde{\phi}_{c,bc} < \phi_c < \tilde{\phi}_{c,no-bc} \\ 0 & \text{if } \phi_c = \tilde{\phi}_{c,no-bc} \end{cases}$$

The following observations can be made. First, parts 3 and 4 of this proposition make clear that the impact of business cycles of arbitrarily small magnitude is not arbitrarily small for the affected projects, i.e., the projects at the boundary. Second, business cycles are beneficial for some projects, because they allow them to overcome inefficiencies during some periods. Third, the combined loss for a strict-inequality and a weak-inequality project is equal to zero when $\phi_c = 0$, but the combined impact is a strictly positive loss when $\phi_c > 0$ as long as $\phi_c < \tilde{\phi}_{c,no-bc}$. The maximum combined loss, L^{\max} , is attained at $\phi_c = \tilde{\phi}_{c,bc}$ and is given by the following expression:

$$L^{\max} = \frac{1}{2} \frac{\phi_p - \mu}{\phi_p} \frac{\beta \rho (1 - \pi)}{1 - \beta \rho \pi}.$$

Table 1 reports the gains and losses for cyclical projects using the parameter values discussed at the end of section 2. There are non-trivial numbers. For some projects the impact is as high as 62.1%.

Table 2 is the analogue of table 1 for a typical value of $\Delta\Phi_p$. We set $\Phi_+ - 1$ (and thus $1 - \Phi_-$) equal to 0.007, which means that the standard deviation of $\Phi_{p,t}$ is equal to 0.007, a standard value used in the literature. The numbers are similar to those of table 1. That is, what matters for individual projects is in the first place the *presence* of business cycles and whether it affects their ability to satisfy the efficiency requirement. The magnitude of business cycles is of secondary importance.

3.3.2 Impact of arbitrarily small business cycles on timed-entry projects

Now consider projects at the other boundary, i.e., when $\phi_c = \tilde{\phi}_{c,no-bc}$. These projects have a value of ϕ_p such that inefficiencies are not an issue, but the value of ϕ_c is such that the owner is indifferent between activating and not activating the project when there are no business cycles. We will see that there is a fundamental difference between these boundary projects and the cyclical projects discussed in the previous subsection.

If $\phi_c = \tilde{\phi}_{c,no-bc}$, then the owner of this project is (by definition) indifferent between activating and not activating in a world without business cycles, i.e., when $\Phi_{p,t} = 1 \forall t$. The owner of this project would *strictly* prefer to activate the project when the economy is in a boom, i.e., when $\Phi_{p,t} > 1$ even when at some point in the future the economy gets into a recession, i.e., when at some point $\Phi_{p,t} < 1$. The reason is that

the startup costs are the same in both cases, but the NPV of the revenues are higher if the projects starts in a boom than if it starts in a world without business cycles. Thus $N_{bc}(\tilde{\phi}_{c,no-bc}, \phi_p, 0, 1 + \Delta_{\Phi_p}) > N_{no-bc}(\tilde{\phi}_{c,no-bc}, \phi_p, 0) = \mu/(1 - \beta)$ for $\Delta_{\Phi_p} > 0$. If the economy is in a recession, i.e., when $\Phi_{p,t} < 1$, then the owner would *strictly* prefer to wait. Interestingly, the value of the project exceeds $\mu/(1 - \beta)$ in a recession as well, since the project generates μ while the economy is in a recession and when the economy enters a boom it starts generating a sequence of revenues with an NPV that exceeds the NPV of generating μ each period.²¹

The magnitude of the positive impact of business cycles on projects with $\phi_c = \tilde{\phi}_{c,no-bc}$ is given by the following proposition.

Proposition 10 *Suppose that (i) $\phi_c = \tilde{\phi}_{c,no-bc}$, (ii) $\phi_p > \max \tilde{\phi}_{p,bc} = \max \left\{ \frac{\chi(\Phi_+)}{\Phi_+}, \frac{\chi(\Phi_-)}{\Phi_-} \right\}$,²² and (iii) the values of β , ρ , and π are in between 0 and 1. Then*

$$\begin{aligned} L(\phi_c, \phi_p, \Delta_{\Phi_p}) &= -\frac{(1 - \beta\rho)}{2(1 - \beta\rho\pi)}\Delta_{\Phi_p} \\ \lim_{\Delta_{\Phi_p} \rightarrow 0} L(\phi_c, \phi_p, \Delta_{\Phi_p}) &= 0. \end{aligned}$$

In contrast to the results for cyclical projects, the loss function for the timed-entry projects is not characterized by discontinuities and the impact of business cycles is equal to 0 for arbitrarily small values of Δ_{Φ_p} . Table 2 reports that the individual welfare consequences of business cycles for these projects remain small when a realistic value for Δ_{Φ_p} is considered, especially compared with the numbers for the cyclical projects.

4 Impact of regular aggregate fluctuations

In this section, we analyze the impact of business cycles when fluctuations are not arbitrarily small. The analysis here is actually very similar to the one in the previous section that focused on arbitrarily small fluctuations *except* that as fluctuations get larger they affect more projects and it is no longer true that only projects with values of ϕ_c or ϕ_p at cut-off levels are affected. We start with a graphical representation. Next, we discuss the quantitative consequences of business cycles for the cyclical and timed-entry projects. We will show that the results from the last section on the impact of business cycles on projects with values of ϕ_c or ϕ_p at cut-off levels provide a lower (upper) bound of the negative (positive) consequences of business cycles for projects with other values for ϕ_c or ϕ_p .

²¹Note that the NPV could never decrease below $\mu/(1 - \beta)$, since there is always the option to never activate the project.

²²The condition that $\phi_p > \max \tilde{\phi}_{p,bc}$ ensures that the project is not affected by the efficiency requirement, not when $\tilde{\phi}_{p,bc}$ is countercyclical and not when it is procyclical. Since timed-entry projects are not affected by inefficiencies, the condition in Assumption 2 is not needed and $\chi_t/\Phi_{p,t}$ could be constant.

For the analysis in this section, the distinction between strict-inequality and weak-inequality projects is no longer necessary.²³ From now on we assume that the efficiency requirement is given by equation (2), unless explicitly stated otherwise.

4.1 Graphical representation of regular business cycles.

In section 3.3, we identified four types of projects that are affected by business cycles of arbitrarily small magnitude. Those are three types of cyclical projects and timed-entry projects. When considering business cycles of non-trivial magnitude we can distinguish the exact same four groups, but there are some differences with the analysis of arbitrarily small business cycles. The main difference is that business cycles of non-trivial magnitude not only affect projects at the boundary, i.e., not only projects with $\phi_p = \tilde{\phi}_{p,\text{no-bc}}$ or $\phi_c = \tilde{\phi}_{c,\text{no-bc}}$.

This is documented in figure 3, which illustrates where the four different types of projects are located. The timed-entry projects have a value of ϕ_c just above or just below $\tilde{\phi}_{c,\text{no-bc}}$. Their value for ϕ_p is above $\tilde{\phi}_{p,\text{bc}}(\Phi_-)$ so that they can overcome inefficiencies even during downturns. Cyclical projects can have a value of ϕ_c above or below $\tilde{\phi}_{c,\text{bc}}$. Affected cyclical projects with a value of ϕ_c below $\tilde{\phi}_{c,\text{bc}}$ can have a value of ϕ_p below or above $\tilde{\phi}_{p,\text{no-bc}}$. In contrast, all affected cyclical projects with a value of ϕ_c above $\tilde{\phi}_{c,\text{bc}}$ have a value of ϕ_p above $\tilde{\phi}_{p,\text{no-bc}}$. The corresponding projects with a value of ϕ_p below $\tilde{\phi}_{p,\text{no-bc}}$ can overcome inefficiencies during a boom, but their entry costs are too high to benefit from this given that inefficiencies would force them to cease operations during a recession. In the remainder of this section, we discuss the quantitative impact of non-trivial business cycles.

4.2 Impact of regular business cycles on timed-entry projects

For values of Δ_{Φ_p} that are not arbitrarily small, the set of timed-entry projects is not limited to projects with $\phi_c = \tilde{\phi}_{c,\text{no-bc}}$, but includes the set $I_{\text{timed entry}}$ defined by

$$I_{\text{timed entry}} = \left\{ \phi_c, \phi_p : \begin{array}{l} \phi_p > \max \tilde{\phi}_{p,\text{bc}}(\Phi_p) \\ \tilde{\phi}_{c,\text{bc}}(\phi_p, \Phi_-) < \phi_c \leq \tilde{\phi}_{c,\text{bc}}(\phi_p, \Phi_+) \end{array} \right\}. \quad (13)$$

The duration of these projects are not shortened by business cycles. Consequently, the formula for $\tilde{\phi}_{c,\text{bc}}(\Phi_p)$ is not given by equation (12), but is given by a system of equations.²⁴ We have to distinguish between the projects in $I_{\text{timed entry}}$ with values of ϕ_c above $\tilde{\phi}_{c,\text{no-bc}}$ and those with values of ϕ_c below $\tilde{\phi}_{c,\text{no-bc}}$. We will now discuss these two groups in turn, each time keeping the value of ϕ_p fixed.

Consider projects with values of ϕ_c above $\tilde{\phi}_{c,\text{no-bc}}$ and below $\tilde{\phi}_{c,\text{bc}}(\Phi_+)$. These projects would never produce market output in a world without business cycles. Although $\phi_p > \mu$, their startup costs are too high to make entry profitable. In a world with business cycles,

²³Recall that this distinction was introduced to capture *both* the gains and the losses of business cycles in a framework with arbitrarily small fluctuations.

²⁴See appendix B.1.2 in Den Haan and Sedlacek (2009).

they would enter in a boom. Moreover, they would continue producing when the economy gets into a recession. The smaller the value of ϕ_c , the larger the gains from business cycles. In particular, projects with $\phi_c = \tilde{\phi}_{c,bc}(\Phi_+)$ do not benefit at all from business cycles and projects with $\phi_c = \tilde{\phi}_{c,no-bc}$ benefit the most. Thus, the welfare gains reported in Proposition 10 and tables 1 and 2 are an upper bound for the gains for projects with values of ϕ_c in between $\tilde{\phi}_{c,no-bc}$ and below $\tilde{\phi}_{c,bc}(\Phi_+)$.

Now consider projects with values of ϕ_c below $\tilde{\phi}_{c,no-bc}$ and above $\tilde{\phi}_{c,bc}(\Phi_-)$. In a world without business cycles, these projects always immediately enter. In a world with business cycles, they delay entry during a recession. The larger the value for ϕ_c , the more valuable the benefit to delay entry. So the largest gain is again achieved by projects with $\phi_c = \tilde{\phi}_{c,no-bc}$. Consequently, the welfare gains reported in Proposition 10 are an upper bound for the welfare gains achieved by *all* timed-entry projects.

Proposition 11 *If (i) $(\phi_c, \phi_p) \in I_{timed\ entry}$, (ii) the values of β , ρ , and π are in between 0 and 1, and (iii) $\Delta_{\Phi_p} > 0$, then*

$$L(\tilde{\phi}_{c,no-bc}, \phi_p, \Delta_{\Phi_p}) < L(\phi_c, \phi_p, \Delta_{\Phi_p}) \leq 0 \text{ for } \phi_c \neq \tilde{\phi}_{c,no-bc}.$$

As discussed in section 3.3.2, even these maximum gains are relatively small. Note that the gains are equal to zero at the boundaries of $I_{timed\ entry}$, i.e., when $\phi_c = \tilde{\phi}_{c,bc}(\phi_p, \Phi_-)$ or $\phi_c = \tilde{\phi}_{c,bc}(\phi_p, \Phi_+)$.

Three reasons why timed-entry projects are unlikely to be important. Table 2 documents that the welfare gains of business cycles for projects with a value of ϕ_c equal to $\tilde{\phi}_{c,no-bc}$ are small and these are an upper bound for the welfare gains of the other timed-entry projects, as documented in Proposition 11. This is not too surprising. These are marginal projects (in a true economic sense) and business cycles just make it possible to create a bit of value by alternative timing.

Another reason why timed-entry projects are unlikely to be important is that these projects are truly marginal from an economic point of view. The question arises how many of such projects exist. Why bother starting a business and producing market output when you only marginally improve upon your outside option? Cyclical projects are also marginal, but not from an economic point of view; if these projects overcome the inefficiency, then there are substantial rents to be earned.

The last reason why timed-entry projects are unlikely to be important is that the presence of such projects is not a robust outcome of models with inefficiencies. In particular, it seems reasonable that timed-entry projects are especially prone to inefficiencies given that these are truly marginal projects. For example, consider the following alternative formulation of the efficiency requirement:

$$\phi_p \Phi_{p,t} - r\phi_c \geq \chi_t, \tag{14}$$

where $r = 1 - \beta\rho$ is the interest rate. In Appendix A, this efficiency requirement is derived using a model with financial frictions. In this specification of the efficiency requirement,

the net per-period revenues matter, i.e., the revenues after amortization of the entry costs. With this alternative efficiency requirement, there would be no timed-entry projects at all. In contrast, the same three types of cyclical projects still exist. This case is discussed in more detail in Appendix.A

4.3 Impact of regular business cycles on cyclical projects

Let I_{cyclical} be the set of cyclical projects. Thus,²⁵

$$I_{\text{cyclical}} = \left\{ \phi_c, \phi_p : \begin{array}{l} \tilde{\phi}_{p,\text{bc}}(\Phi_+) \leq \phi_p < \tilde{\phi}_{p,\text{bc}}(\Phi_-) \\ \phi_c \leq \tilde{\phi}_{c,\text{no-bc}} \end{array} \right\}. \quad (15)$$

The welfare consequences of business cycles for cyclical projects with $\phi_p = \chi = \tilde{\phi}_{p,\text{no-bc}}$ are given in Proposition 9. How do these welfare consequences compare with the welfare consequences for the cyclical projects with $\phi_p \neq \tilde{\phi}_{p,\text{no-bc}}$? Keeping the value of ϕ_c fixed, then the welfare losses of projects with $\phi_p > \tilde{\phi}_{p,\text{no-bc}}$ are *bigger* than the losses of the weak-inequality projects with $\phi_p = \tilde{\phi}_{p,\text{no-bc}}$ and the welfare gains of projects with $\phi_p < \tilde{\phi}_{p,\text{no-bc}}$ are *smaller* than the gains of the strict-inequality projects with $\phi_p = \tilde{\phi}_{p,\text{no-bc}}$.

First, consider projects with $\phi_p < \tilde{\phi}_{p,\text{no-bc}}$ and $\phi_c \leq \tilde{\phi}_{c,\text{bc}}$. The entry costs of these projects are low enough so that entry is profitable even though they only survive until the next recession. In a world with business cycles, these projects generate output equal to $(1 + \Delta_{\Phi_p}) \phi_p > \mu$ during a boom and μ during a recession, whereas they always generate μ in a world without business cycles.²⁶ The larger the value of ϕ_p , the larger the gains (for equal values of ϕ_c). Thus, the welfare *gain* attained by strict-inequality projects with $\phi_p = \tilde{\phi}_{p,\text{no-bc}}$ is an *upper* bound for the gains achieved by cyclical projects.

Next, consider projects with $\phi_p \geq \tilde{\phi}_{p,\text{no-bc}}$. In this case, the welfare *loss* attained by weak-inequality projects with $\phi_p = \tilde{\phi}_{p,\text{no-bc}}$ are a *lower* bound for the losses suffered by cyclical projects. Business cycles permanently reduce market output of cyclical projects when ϕ_c is above $\tilde{\phi}_{c,\text{bc}}$ and temporarily (namely during recessions) when ϕ_c is below $\tilde{\phi}_{c,\text{bc}}$. Consequently, given the value for ϕ_c , the loss is larger when the value of ϕ_p is larger.

The following proposition summarizes these results.

Proposition 12 *Suppose that (i) $(\phi_c, \phi_p) \in I_{\text{cyclical}}$, (ii) assumptions 1, 2, and 3 are satisfied, and (iii) $\Delta_{\Phi_p} > 0$. Then*

$$L(\phi_c, \tilde{\phi}_{p,\text{no-bc}}, \Delta_{\Phi_p}) < L(\phi_c, \phi_p, \Delta_{\Phi_p}) \text{ for } \phi_p \neq \tilde{\phi}_{p,\text{no-bc}}.$$

²⁵Here we still adopt the assumption that $\tilde{\phi}_{p,\text{bc}}(\Phi_p)$ is countercyclical. If $\tilde{\phi}_{p,\text{bc}}(\Phi_p)$ would be procyclical, then the set is given by

$$I_c = \left\{ \phi_p : \tilde{\phi}_{p,\text{bc}}(\Phi_-) \leq \phi_p < \tilde{\phi}_{p,\text{bc}}(\Phi_+) \right\}.$$

²⁶The result that $(1 + \Delta_{\Phi_p}) \phi_p > \mu$ follows directly from Condition 1.

5 Necessary ingredients

In this subsection, we explain why both entry costs and inefficient operating decisions are needed for business cycles to be costly.

Why are entry costs essential? Suppose that entry costs are equal to zero for all projects. This would mean that the whole graph would collapse onto the x -axis in figure 3. If there are no entry costs, then projects cannot be permanently driven out of business. In our model, business cycles shorten the expected duration of the project. For some projects this makes the entry costs too high relative to the reduced expected revenue stream. This cannot matter when ϕ_c is equal to zero. If there are no entry costs, then business cycles allow some additional projects to produce during a boom and make production impossible for some projects during a recession. The gain for a project just below $\tilde{\phi}_{p,\text{no-bc}}$ and the loss for a project just above $\tilde{\phi}_{p,\text{no-bc}}$ roughly offset each other.²⁷ Given that we are in the lower tail of the distribution, it is possible that there are more projects with values of ϕ_p above $\tilde{\phi}_{p,\text{no-bc}}$ than below $\tilde{\phi}_{p,\text{no-bc}}$. Then business cycles could be costly even when entry costs are equal to zero. We prefer to build our argument, however, without relying on such distributional assumptions.

Why are inefficient operating decisions essential? There are two aspects of the inefficient operating decision that are important. First, the inefficiency makes it impossible to offset worsened conditions during a recession with improved conditions during a boom. That is, the inefficiency specified in equation (2) has to hold at each point in time, not just on average. This means that the reason for the efficiency requirement must be a private inefficiency. If there are no private inefficiencies, then there is no reason why the higher revenues in a boom could not offset the lower revenues in a recession. Second, in the presence of inefficient operating decisions, marginal projects have a positive surplus when defined relative to the true outside option. This means that it is costly from a social welfare point of view that business cycles prevent such projects from being created.

In an earlier version of this paper,²⁸ we describe in detail the model without the efficiency requirement. In that case, business cycles are beneficial for the following reason. Without the efficiency requirement, the entry and continuation decisions in a world with business cycles *could* be identical to those in a world without business cycles. If these decisions are the same, then the revenues would be more volatile in the world with business cycles, but—given our assumption of risk neutrality—the NPVs would be the same. Consequently, business cycles cannot make agents worse off. In fact, when the outside option μ is acyclical (or at least less cyclical than $\phi_p \Phi_{p,t}$), then the possible revenues, $\min \{ \mu, \phi_p \Phi_{p,t} \}$

²⁷As documented in proposition 9, the gain of the strict-inequality project and the loss of the weak-inequality project *exactly* offset each other for arbitrarily small fluctuations when $\phi_p = \tilde{\phi}_{p,\text{no-bc}}$. For larger fluctuations, there actually would be a net gain for the two projects with $\phi_p = \tilde{\phi}_{p,\text{no-bc}}$. But note that the output gained by a project with $\phi_p < \tilde{\phi}_{p,\text{no-bc}}$ is less than the output lost by a project with $\phi_p > \tilde{\phi}_{p,\text{no-bc}}$. So for all affected projects there still could be a net loss.

²⁸See Den Haan and Sedlacek (2009).

is a convex function of $\Phi_{p,t}$ and increased volatility in $\Phi_{p,t}$ would increase the expected value.

6 Approach to measure *overall* cost of business cycles

In this section, we outline the procedure used to obtain a quantitative estimate for the welfare costs of business cycles for the economy as a whole. In the first subsection, we discuss the key assumptions underlying our procedure. In the second subsection, we describe the basic idea. Details are given in appendix B.

6.1 Key elements underlying our approach

The reader may wonder whether a credible quantitative answer can be provided with the simple framework presented here. For example, different types of projects may face different types of inefficiencies, which would be associated with different values of χ_t . Moreover, even if all projects would face the same type of inefficiency, then the value of χ_t would still not have to be the same for all individual projects. For example, projects with a higher value of ϕ_p could have a higher value of χ_t .

Before explaining our strategy, we make explicit what we do *not* do. We do not focus on one particular type of inefficiency, calibrate the function $\chi(\Phi_{p,t})$, or possibly $\chi(\Phi_{p,t}, \phi_c, \phi_p)$, and finally calibrate the distribution of ϕ_c and ϕ_p . Just the calibration of the distribution of ϕ_p and ϕ_c would be very difficult given that what matters is the mass in a very specific, relatively small, part of the distribution. Moreover, by focusing on only one type of inefficiency, the results are likely to provide an incomplete picture of the impact of business cycles.

First element: Countercyclical efficiency requirement. Business cycles shorten the duration of projects when $\tilde{\phi}_{p,bc}(\Phi_{p,t})$ is countercyclical (and projects are eliminated during recessions), but also when $\tilde{\phi}_{p,bc}(\Phi_{p,t})$ is procyclical (and projects are eliminated during expansions). Business cycles are costly under both assumptions, but having a countercyclical number of projects does not seem very plausible. Therefore, to calibrate the model we assume that $\tilde{\phi}_{p,bc}(\Phi_{p,t})$ is countercyclical.

Second element: Assumption about importance of inefficient continuation. If the actual economy gets into a recession, then there are two reasons why aggregate output decreases. First, the output of existing projects decreases. Second, the number of projects decreases. For example, a recession goes together with job destruction. But even projects of existing workers could be eliminated. Thus, there is an intensive as well as an extensive margin to changes in output, just like there are two margins to changes in the total number of hours worked. We discuss below how to obtain estimates for the extensive margin of output changes. But this is not sufficient for our analysis. We have to identify which

part of the adjustment along the extensive margin is due to inefficiencies.²⁹ In terms of the terminology of the paper, we have to determine which of the observed changes in the number of projects are due to "cyclical projects". The numbers presented are based on the assumption that *all* observed changes along the extensive margin are due to inefficiencies.

There are, of course, other possible reasons why projects are terminated during downturns and restarted during expansions. One possible reason is the presence of timed-entry projects. Timed-entry projects continue operating during a downturn, but timed-entry projects that faced the exogenous separation shock during a recession are only restarted during the next expansion. Timed-entry projects could, thus, be responsible for some of the observed cyclical movements in the number of projects. In section 4.2, we gave reasons why these projects are unlikely to matter much quantitatively. A second possible reason for adjustment along the extensive margin not related to inefficiencies is that projects are terminated during a recession because the value of $\phi_p \Phi_-$ is too low relative to the value of the true outside option, μ . For example, during a recession, the value of market production could be too low compared to the value of leisure and home production. We doubt very much that most workers that become unemployed during an economic downturn prefer unemployment over being employed and we ignore this possibility. If the reader thinks otherwise and for example believes that say only 50% of all observed movements along the extensive margin is due to inefficiencies, then he/she should multiply the presented numbers by one half.

Third element: Lower tail of the distribution. For a given value of ϕ_c , we assume that

$$\int_{\tilde{\phi}_{p,\text{no-bc}}}^{\tilde{\phi}_{p,\text{bc}}(\Phi_-)} \phi_p f(\phi_p | \phi_c) d\phi_p \geq \int_{\tilde{\phi}_{p,\text{bc}}(\Phi_+)}^{\tilde{\phi}_{p,\text{no-bc}}} \phi_p f(\phi_p | \phi_c) d\phi_p, \quad (16)$$

where $f(\phi_p | \phi_c)$ is the density of ϕ_p conditional on ϕ_c . That is, the output produced by projects above $\tilde{\phi}_{p,\text{no-bc}}$ is higher than the output produced by projects below $\tilde{\phi}_{p,\text{no-bc}}$. A sufficient condition for this to be true is that $\partial f(\phi_p | \phi_c) / \partial \phi_p \geq 0$, which is likely to be true given that we are in the left tail of the distribution.

Fourth element: Cyclical changes in output along the extensive margin. The assumption that changes in output along the extensive margin are due to inefficiencies simplifies the analysis considerably. But we still have to take a stand on how to calculate which part of the observed cyclical changes in output is due to changes along the intensive margin (changes in output due to changes in productivity of existing projects) and which part is due to the extensive margin (changes in output due to change in the number of projects). We follow two approaches to estimate the magnitude of the latter, i.e., Y_{cyclical}/Y . The first (and simple) approach is discussed here. In appendix D, we discuss a more elaborate method that is based on a panel data set for German wages. With this procedure we obtain a quantitatively similar estimate.

²⁹Here inefficiencies include a wide range of possibilities including sticky wages, the inability to overcome moral hazard problems related to the financing of positive NPV projects, and the motivation of the participants in the project.

Using US total nonfarm employment from 1948Q1 to 2007Q4, we find measures for the volatility of detrended employment ranging from 0.0145 to 0.0388.³⁰ In our model, in which there are two regimes, a boom and a recession, the difference in employment levels between a recession and a boom would then range between 2.90% and 7.76%. The corresponding numbers for the cyclical change in output are 3.36% to 8.30%. The observed cyclical change in the number of employed workers is used as a proxy for the cyclical change in the *number* of projects. To determine the importance of the extensive margin for *output*, we need to know how productive the workers are that are associated with these changes in employment along the extensive margin.

Let N_{cyclical} stand for the number of projects that stop operating during a recession, let N stands for the time series average number of all projects, and let $\phi_{p,\text{ave}}$ stand for the average productivity of the N_{cyclical} cyclical projects. This means that

$$\frac{Y_{\text{cyclical}}}{Y} = \frac{N_{\text{cyclical}}}{N} \phi_{p,\text{ave}}.$$

Suppose that $\phi_{p,\text{ave}}$ is equal to 40%. This seems conservative.³¹ Combined with the observed range for N_{cyclical}/N , this implies a range for Y_{cyclical}/Y from 1.16% to 3.10%. This means that roughly 36% of the observed cyclical variation in GDP is assumed to be due to inefficiencies, i.e., to Y_{cyclical}/Y . This does not seem to be an excessive estimate for the role of inefficiencies. The higher this number, the higher the costs of business cycles.

Fifth element: Using what is known to extrapolate. With the assumptions made above, it is possible to get an estimate for the importance of cyclical projects with values of ϕ_c below $\tilde{\phi}_{c,\text{bc}}$, that is, values of ϕ_c that are low enough to make entry worthwhile in a boom even though the project has to be terminated during a recession. Here we discuss how to obtain an estimate for the importance of the cyclical projects with values of ϕ_c in between $\tilde{\phi}_{c,\text{bc}}$ and $\tilde{\phi}_{c,\text{no-bc}}$, that is values of ϕ_c that are such that entry is no longer attractive in a world with business cycles while entry is attractive in a world without business cycles. We will refer to the total output level of these projects as $Y_{\text{cyclical-PL}}$. Here PL stands for permanent loss, since in the world with business cycles their projects are never activated.

The difficulty with these projects is that they are never observed in the actual world, which does have business cycles. Still, several characteristics of this set of projects are known. First, the productivity levels of these cyclical projects are in the *upper* half of the range of values for ϕ_p associated with observed cyclical projects.³² This means that we only have to know the mass of projects in between $\tilde{\phi}_{c,\text{bc}}$ and $\tilde{\phi}_{c,\text{no-bc}}$ relative to the mass

³⁰The standard deviation is equal to 0.0145 (0.0220) when data are detrended using the Hodrick-Prescott filter with a smoothing coefficient equal to 1,600 (100,000). It is equal to 0.0388 when a linear trend is used. If we extend the sample up to 2010Q4 then the ranking of these three numbers remain the same and their magnitudes are equal to 0.0146, 0.0231, and 0.0487.

³¹According to Goldin and Katz (2008), those without a college degree earn wages that are roughly half of those with a college degree. Thus, if all non-cyclical workers are as productive as college graduates and all cyclical workers are as productive as those without a college degree then we still *underestimate* the productivity of the cyclical workers.

³²This is documented in figure 3. The observed cyclical projects are in two areas referred to as "gain"

below $\tilde{\phi}_{c,bc}$. Second, the values of $\tilde{\phi}_{c,bc}$ and $\tilde{\phi}_{c,no-bc}$ are known functions of parameters that can be calibrated. The key aspect of this part of our procedure is the assumption that conditional on ϕ_p , the mass in between $\tilde{\phi}_{c,bc}(\phi_p)$ and $\tilde{\phi}_{c,no-bc}(\phi_p)$ relative to the mass in between 0 and $\tilde{\phi}_{c,bc}(\phi_p)$, is equal to the length of the interval in between $\tilde{\phi}_{c,bc}(\phi_p)$ and $\tilde{\phi}_{c,no-bc}(\phi_p)$ relative to the length of the interval in between 0 and $\tilde{\phi}_{c,bc}(\phi_p)$.

6.2 Implementation: the bottom line

In appendix B, we give the exact formulas, but one can obtain a quite accurate estimate using the following simple procedure.

1. Using the formulas in the second part of Proposition 9, obtain a minimum and a maximum value of the impact of business cycles for projects with $0 \leq \phi_c \leq \tilde{\phi}_{c,bc}$. Take the average value.
2. These welfare consequences are expressed relative to the affected projects own productivity levels. Combining the estimate from the first step with the estimate for Y_{cyclical}/Y gives the total cost of business cycles due to cyclical projects with $0 \leq \phi_c \leq \tilde{\phi}_{c,bc}$.
3. Using the formulas in the second part of Proposition 9, obtain a minimum and a maximum value of the impact of business cycles for projects with $\tilde{\phi}_{c,bc} < \phi_c \leq \tilde{\phi}_{c,no-bc}$. Take the average value.
4. The magnitude of $(\tilde{\phi}_{c,no-bc} - \tilde{\phi}_{c,bc})$ relative to the magnitude of $(\tilde{\phi}_{c,bc} - 0)$ determines the magnitude of $Y_{\text{cyclical-PL}}/Y$ relative to the magnitude of Y_{cyclical}/Y . This makes it possible to calculate $Y_{\text{cyclical-PL}}/Y$.
5. Combining the estimate of step 3 with $Y_{\text{cyclical-PL}}/Y$ gives the total cost of business cycles due to cyclical projects with $\tilde{\phi}_{c,bc} < \phi_c \leq \tilde{\phi}_{c,no-bc}$.

For example, suppose that the average individual welfare costs are equal to 15.3% and 31.0% for cyclical projects with $0 \leq \phi_c \leq \tilde{\phi}_{c,bc}$ and $\tilde{\phi}_{c,bc} \leq \phi_c \leq \tilde{\phi}_{c,no-bc}$, respectively. These numbers are averaged across values of ϕ_c and when $0 \leq \phi_c \leq \tilde{\phi}_{c,bc}$ across the gains and losses of the affected projects.³³ Suppose that Y_{cyclical}/Y is equal to 3.1%, the upper end of our range of estimates. This means that the welfare costs for cyclical projects with $\phi_c \leq \tilde{\phi}_{c,bc}$ relative to *aggregate output* is equal to $15.3 \times 0.031 = 0.474\%$.

The magnitude of $(\tilde{\phi}_{c,no-bc} - \tilde{\phi}_{c,bc}) / (\tilde{\phi}_{c,bc} - 0)$ together with the value of Y_{cyclical}/Y imply that the output that could be produced by projects with $\phi_c > \tilde{\phi}_{c,bc}$ is equal to 5.3%

and "loss". The cyclical projects that are not observed are in the area referred to as "permanent loss", which is associated with the higher values of ϕ_p .

³³The example is based on the case with $\rho = 0.975$ and $\hat{\mu} = 0.2$ documented in table 2. For example, at $\phi_c = 0$ the average loss of a weak and strict-inequality project is equal to $(39.65 - 40.35)/2$. At $\phi_c = \tilde{\phi}_{c,bc}$ this average is equal to $(61.9547 + 0)/2$. The average of these two numbers is equal to 15.3.

of aggregate output. Combining this number with the individual welfare costs of 31.0% means that the welfare costs for cyclical projects with $\phi_c > \tilde{\phi}_{c,bc}$ is equal to $31.0 \times 0.053 = 1.643\%$. The total cost of business cycles would then be equal to $0.474\% + 1.643\% = 2.12\%$.

7 Quantitative impact of business cycles

The top panel of table 3 reports the welfare costs of business cycles. In Lucas (1987), the welfare costs of business cycles are estimated to be less than 0.1% when the coefficient of relative risk aversion is equal to 10.³⁴ In the basic version of our model, we find welfare costs ranging from 0.13% to 2.12% with risk-neutral agents. Roughly one fourth is due to cyclical projects with values of ϕ_c below $\tilde{\phi}_{c,bc}$ (that have to pay startup costs more often) and the remainder is due to cyclical projects with values of ϕ_c above $\tilde{\phi}_{c,bc}$ (that are permanently driven out of business by business cycles). The results are quite sensitive to ρ , the value of the survival rate of projects *in a world without business cycles*.³⁵ In our model, business cycles are costly because they shorten the duration of projects. Consequently, the higher the expected duration of projects in a world without fluctuations, the stronger this effect will be. The highest value for ρ considered in table 3 is 0.975, which corresponds to an expected duration of 10 years. If the value of ρ is increased from 0.975 to 0.99, then the welfare costs of business cycles increase from 2.12% to 3.77% (when $\mu = 0.2$).

Although these numbers are a magnitude larger than the ones reported in Lucas (1987), they are still relatively small. In the next section, we discuss modifications of the model in which the costs of business cycles are substantially larger.

8 Extensions with larger effects

In this section, we discuss two modifications of the model. Both substantially increase the costs of business cycles.

Non-zero lower bound for ϕ_c . The results presented above are based on the assumption that some projects can be created for free. This leads to a conservative estimate for the costs of business cycles.³⁶ Quantitatively, this matters a lot. Instead, suppose that the lowest possible value for ϕ_c is equal to 20% of the highest value of ϕ_c at which entry is still profitable when inefficiencies and business cycles do not matter, thus, $\min\{\phi_c\} = \tilde{\phi}_{c,no-bc}/5$. As documented in table 3, the modification increases the upper bound of our

³⁴The costs of business cycles according to the Lucas formula are equal to $0.5\sigma_c^2\gamma$, where σ_c is the standard deviation of the cyclical component of aggregate consumption and γ is the coefficient of relative risk aversion. Thus, if σ_c is equal to 0.013 and γ is equal to 10, then the costs of business cycles are equal to 0.084%.

³⁵Note that we cannot calibrate this parameter to observed separation rates, because in the real world economic downturns are important for separations.

³⁶The second part of proposition 9 shows that business cycles have a stronger negative impact on the combined well being of owners of strict-inequality and weak-inequality projects as ϕ_c increases.

estimates for the welfare costs of business cycles from 2.12% to 15.5%.³⁷

This modification has a stronger effect on the outcome for higher values of ρ . The reason is that the lower bound of ϕ_c depends on the value of ρ . As the value of ρ increases, the value of $(\tilde{\phi}_{c,\text{no-bc}} - \tilde{\phi}_{c,\text{bc}}) / (\tilde{\phi}_{c,\text{bc}} - \min\{\phi_c\})$ increases, which means that the number of projects that would be resurrected if business cycles are eliminated increases relative to the level of observed cyclical projects.

Entrepreneurs and inefficient entry decision. Our current setup differs from the usual setup in which an entrepreneur pays the entry costs and makes the entry decision, but the revenues are shared with others, like workers. This model is in many aspects similar to the one described above and the equations are the same or almost the same. For example, the cut-off levels for the entry costs in the world without and with business cycles are now given by

$$\tilde{\phi}_{c,\text{no-bc}} = \omega_e \frac{(\phi_p - \mu)}{1 - \beta\rho}, \quad (17)$$

$$\tilde{\phi}_{c,\text{bc}} = \omega_e \frac{\phi_p (1 + \Delta\Phi_p) - \mu}{1 - \beta\rho\pi} \quad (18)$$

where ω_e is the entrepreneur's share of the surplus. If ω_e is equal to 1, then there is no change in model.

One might think that changing ω_e is not important since it would simply scale the curves for $\tilde{\phi}_{c,\text{bc}}$ and $\tilde{\phi}_{c,\text{no-bc}}$ and the relative values of these two cut-off levels play a key role in our calibration. But lowering the value of ω_e has large implications for the numerical results. The reason is the following. A lower value of ω_e implies that the entry costs relative to ϕ_p are lower. When calculating the welfare costs of business cycles, we take into account that cyclical projects that are permanently driven out of business because of business cycles no longer pay entry costs. This positive aspect of ceasing to produce is smaller when the entry costs paid are smaller, that is, it is smaller when ω_e is smaller. Thus, welfare costs of business cycles are larger for smaller values of ω_e .

There is another reason why the welfare costs are larger when ω_e is smaller. When entry is efficient, i.e., when $\omega_e = 1$, then business cycles have a positive effect, because business cycles introduce the worthwhile option to postpone entry. As discussed above, this effect is quantitatively very small. As ω_e decreases, this small gain of business cycles can turn into a cost. The reason is the following. The option to wait still has positive value for the entrepreneur. The problem is that the entrepreneur does not take into account that by postponing entry he also postpones the worker getting his share of the revenues, which would exceed μ if entry is inefficient, i.e., when $\omega_e < 1$. The value of ω_e , thus, determines

³⁷In understanding these numbers it is important to keep in mind the following. When $(\tilde{\phi}_{c,\text{no-bc}} - \tilde{\phi}_{c,\text{bc}}) / (\tilde{\phi}_{c,\text{bc}} - \min\{\phi_c\})$ increases *and* the output produced by cyclical projects with $\phi_c \leq \tilde{\phi}_{c,\text{bc}}$ remains equal to the calibrated value (either 1.16% or 3.1%), then the output that can be produced by projects with $\tilde{\phi}_{c,\text{bc}} \leq \phi_c \leq \tilde{\phi}_{c,\text{no-bc}}$ increases.

whether business cycles are beneficial for timed-entry projects or not. Quantitatively, however, the impact of business cycles on timed-entry projects is always small.³⁸

What value for ω_e to use? This parameter captures the reward for the pure entrepreneurial activity of starting the project, thus, excludes revenues to other participants like workers and those that provide financing. Consequently, one would think that the value of ω_e is small. Here we set ω_e equal to 0.125. This is likely to be still high.³⁹ If this value is indeed too high, then our estimates are conservative, since the lower ω_e , the higher the costs of business cycles.

Table 3 reports the results for the model with entrepreneurs when $\min\{\phi_c\}$ is equal to 0 and when $\min\{\phi_c\}$ is equal to 20% of $\tilde{\phi}_{c,\text{no-bc}}$.⁴⁰ When the lower bound for ϕ_c is equal to zero, then the upper bound of our estimates increase from 2.12% to 3.95%. If we set the lower bound for ϕ_c equal to 20% of $\tilde{\phi}_{c,\text{no-bc}}$ then the welfare costs of business cycles are equal to 34.6%.

9 Related literature

Following the classic Lucas (1987) paper, there have been numerous attempts to develop models in which business cycles are costly.⁴¹ One strand of the literature considers preferences in which fluctuations are more harmful to the agent.⁴² But if agents are truly highly risk averse, then—as pointed out in Lucas (2003)—the question arises why high risk aversion does not show up in, for example, the diversification of individual portfolios, the level of insurance deductibles, or the wage premiums of jobs with high earnings risk. A second strand of the literature considers the possibility that risk is not spread evenly across agents. When idiosyncratic risk is persistent, then this line of research generates estimates for the cost of business cycles that are an order of magnitude larger than those found by Lucas.⁴³ The idea is that unemployment has very negative consequences for the individual and is a relatively rare event. Business cycles can be costly if risk sharing among agents is sufficiently limited, but the agents' degree of risk aversion is again important.

Lucas' calculations on the cost of business cycles are based on a comparison between two economies, one with and one without business cycles, that both have the same long-run growth path. But this is not necessarily the case. It could very well be that the presence of business cycles has long-term level or even growth effects. Empirical evidence for this view can be found in Ramey and Ramey (1995), Martin and Rogers (2000), Loayza, Ranciere, Serven, and Ventura (2007), Burnside and Tabova (2009), and Den Haan and Sedlacek

³⁸In this version of the paper, we ignore the timed-entry projects. In Den Haan and Sedlacek (2009), we explicitly take these projects into account, which makes the analysis much more cumbersome, but has virtually no effect on the results.

³⁹For example, Den Haan and Kaltenbrunner (2009) set ω_e equal to 0.0228 to match the observed employment volatility.

⁴⁰See Den Haan and Sedlacek (2009), for details on how welfare costs of business cycles are calculated when $\omega_e < 1$.

⁴¹See Lucas (2003) for a summary.

⁴²Examples of this line of research are Alvarez and Jermann (2005) and Tallarini (2000).

⁴³See Storesletten, Telmer, and Yaron (2001), Krebs (2007), Santis (2007), and Krusell, Mukoyama, Sahin, and Smith (2009).

(2009).⁴⁴ Although the relationship seems stronger for low and middle income countries, the link also exists for OECD countries.

Our paper is related to a set of papers that build theoretical models in which fluctuations have level or growth effects. As shown in Ramey and Ramey (1991), Jones, Manuelli, and Stacchetti (2000), Epaulard and Pommeret (2003), and Barlevy (2004), there is a relationship between growth and volatility in endogenous growth models. Barlevy (2004) points out that for a reduction in volatility to have a quantitatively important effect on output, it is important that the increase in investment induced by a reduction in volatility not only increases the growth rate of consumption, but also does *not* lead to an initial reduction in the level of consumption.⁴⁵ Barlevy (2004) accomplishes this by introducing diminishing returns to investment into an endogenous growth model. The nonlinearity makes it possible for a reduction in fluctuations to have a positive effect on the growth rate, even if average investment levels, and thus the initial consumption level, are not affected by fluctuations.

Such a nonlinearity is also important in papers that show that volatility has a negative effect on the average *level* of real activity. Gali, Gertler, and Lopez-Salido (2007) consider a simple New-Keynesian model in which the efficiency losses due to mispricing in a recession are not offset by the efficiency gains in a boom. Business cycles are then welfare reducing, although the effects turn out to be small. Jung and Kuester (2008) and Hairault, Langot, and Osotimehin (2010) show that the matching model contains a nonlinearity that causes volatility in job finding rates to reduce average unemployment. In particular, increases in job finding rates during booms have less of an impact than decreases in job finding rates during recessions, because the unemployment rate is smaller during booms than recessions.

How does our explanation compare with the models of these papers? Our main mechanism is *not* that expansions and contractions have asymmetric effects on inefficiencies or asymmetric effects on entry decisions. That is, if we would add a third state to our model in which Φ_p is equal to $\Phi_0 \equiv (\Phi_+ + \Phi_-)/2$, then there is no robust reason for asymmetric effects if Φ_p increases from Φ_0 to Φ_+ or decreases from Φ_0 to Φ_- .⁴⁶ The main reason why fluctuations are costly in our framework is that some projects are permanently driven out of business. The reason is that the inefficiency does not allow the worsening of the friction during a recession to be offset with a loosening during a boom. This loss does not introduce an asymmetry in generated business cycle fluctuations. Fluctuations are also costly for projects with an entry cost that is low enough to make entry worthwhile, even if the project is only viable during an expansion. For this type of project there is a negative *and* a positive effect in the sense that fluctuations end some projects during a recession

⁴⁴These papers establish an unconditional negative correlation between business cycles and real activity. There are also channels that generate a positive correlation. For example, Levchenko, Ranciere, and Thoenig (2009) show that financial liberalization leads to an increase in both volatility and expected economic growth.

⁴⁵Mertens (2008) shows how fluctuations can affect the average level of investment in a model without endogenous growth. In his model, agents have distorted beliefs and fluctuations worsen this distortion. Consequently, the risk premium is higher and the average capital stock is lower in a world with business cycles.

⁴⁶The model could generate such asymmetries, but the mass of projects would have to be distributed in a particular way.

but make additional projects viable during a boom. Even if the effects cancel out in terms of output, they do not in terms of welfare, since entry costs have to be paid more often in a world with fluctuations.

10 Concluding comments

According to our model, business cycles are costly for two types of reasons. First, since business cycles shorten the expected duration of projects and projects would pay startup costs less often in a world without business cycles. Second, business cycles make some projects completely impossible. In the simple version of our model, the first type of cost does not exceed 0.5% of aggregate output per period. In the experiments we considered, this type of cost is usually not much higher than this. But larger numbers are possible.⁴⁷ How large the second type of cost is depends on how many projects would come into existence if there are no business cycles. In the real world, we never see these potential projects, so it is not easy to obtain an estimate. The estimates we report for the simple model are based on what we feel is a conservative "extrapolation" of the environment we do observe.⁴⁸ This leads to non-trivial costs, but—combined with the first type of costs—our maximum cost estimate is "only" equal to 2.12% of aggregate output. However, we also show that one can quite easily get much larger costs. This occurs when one agent is responsible for paying the startup costs, but the revenues of the projects are shared with others and when there are not many projects with very low entry costs.

We end the paper with the following somewhat disturbing observation. Our numerical work focuses on the costs of business cycles. But our time-varying shock, $\Phi_{p,t}$, does not have to be an aggregate shock. It also could be a sectoral, geographical, and even an idiosyncratic shock. Consequently, if even the costs of business cycles are already non-trivial, then the costs of *all* fluctuations could very well be a staggering number.

A Theories consistent with our efficiency requirement

In the main text, we simply imposed the friction that firms can only operate if

$$\phi_p \Phi_{p,t} \geq \chi_t. \tag{19}$$

In this section, we show that that two very different theories generate such a requirement. The first is the contractual fragility framework of Ramey and Watson (1997) and the second is a framework in which the entrepreneur has to borrow to finance the investment made and borrowing is subject to a standard agency problem. Section A.1 discusses the

⁴⁷Suppose that (i) there is no mass below $\tilde{\phi}_{p,\text{no-bc}}$, that is, there are no projects that gain from the presence of business cycles, (ii) cyclical fluctuations in output are due solely to changes in the extensive margin, and (iii) entry costs are equal to zero. An upper bound for these types of costs would then be the observed difference in the level of output during a boom and a recession, which is roughly 4%. If entry costs are not zero, then these costs could be higher than 4%

⁴⁸In particular, we assume that the lower bound for startup costs is equal to zero and cyclical projects are only 40% as productive as other projects.

contractual fragility framework and section A.2 discusses the framework with the financial friction. The contractual fragility specification results in a condition that is exactly equal to the one given in equation (19). The financial friction leads to a slightly different specification in which one can expect the costs of business cycles to be even higher.

A.1 Contractual fragility of Ramey and Watson (1997)

The physical environment of the model described here is identical to the one described in the main text. The difference is that the efficiency requirement is not simply imposed, but is an implication of the model. In Ramey and Watson (1997), both participants have the option to cheat. For our purpose, it is sufficient if just the entrepreneur has such an option. The cheating option may be privately attractive, but is inefficient from the relationship's point of view. For example, the entrepreneur may deviate from the original business plan and choose one that is riskier, but gives him personally more prestige. It is also possible that he diverts funds to himself or acquaintances. The total current-period private benefits the entrepreneur can obtain by cheating are equal to χ_e and these consist of the actual funds the entrepreneur receives, $\chi_{e,\$}$, plus any non-pecuniary benefits, $\chi_e - \chi_{e,\$}$. The entrepreneur obtains pecuniary benefits by extracting a larger share of the resources than agreed upon, for example, by not paying out overtime or by not promoting workers. Increases in prestige or human capital and improvements of the entrepreneur's network are examples of non-pecuniary benefits. If the entrepreneur chooses the alternative business plan, then output is equal to

$$\phi_\chi \phi_p \Phi_{p,t}. \quad (20)$$

The alternative business plan causes the worker extra *disutility* equal to $\chi_w \geq 0$.⁴⁹ For the alternative choice to be inefficient, it must be the case that

$$(1 - \phi_\chi) \phi_p \Phi_{p,t} > (\chi_e - \chi_{e,\$}) - \chi_w. \quad (21)$$

That is, the loss in the project's revenues is larger than the net utility gain (when $\phi_\chi < 1$) or the gain in revenues is smaller than the net utility loss (when $\phi_\chi > 1$).

An entrepreneur is only willing to choose the original business plan if

$$\begin{aligned} \phi_p \Phi_{p,t} - w_t + \beta \mathbf{E}_t [\rho N_e(\phi_c, \phi_p, 1, \Phi_{p,t+1}) + (1 - \rho) N_e(\phi_c, \phi_p, 0, \Phi_{p,t+1})] \\ \geq \\ \chi_e + \beta \mathbf{E}_t [\rho N_e(\phi_c, \phi_p, 1, \Phi_{p,t+1}) + (1 - \rho) N_e(\phi_c, \phi_p, 0, \Phi_{p,t+1})], \end{aligned} \quad (22)$$

where w_t is the wage rate of the worker under the original business plan.⁵⁰ For simplicity, we assume that the entrepreneur's choice to cheat does not affect his continuation value.⁵¹ Equation (22) can, then, be written as

$$\phi_p \Phi_{p,t} - w_t \geq \chi_e. \quad (23)$$

⁴⁹The alternative business plan may not only require more effort from the worker, but may also force him to move to a different plant and may even result in dismissal.

⁵⁰Under the alternative business plan, the worker receives $\phi_\chi \phi_p \Phi_{p,t} - \chi_{e,\$}$.

⁵¹In the unfair world we live in, it is probably not an unrealistic assumption that the employer can impair his workers' well being without this having much effect on the options available to him in the next period. The case when the entrepreneur's continuation value is affected is discussed at the end of this section.

The current-period benefits of the worker when he is not employed are equal to μ . A worker is only willing to participate in a project if

$$\begin{aligned} w_t + \beta \mathbf{E}_t [\rho N_w(\phi_c, \phi_p, \Phi_{p,t+1}) + (1 - \rho)U_w(\phi_c, \phi_p, \Phi_{p,t+1})] \\ \geq \\ \mu + \beta \mathbf{E}_t [U_w(\phi_c, \phi_p, \Phi_{p,t+1})], \end{aligned} \quad (24)$$

where w_t is the wage rate of the worker, $N_w(\phi_c, \phi_p, \Phi_{p,t+1})$ is the discounted value of current and future benefits that accrue to the worker when he starts next period in a relationship, $U_w(\phi_c, \phi_p, \Phi_{p,t+1})$ the discounted value of current and future benefits that accrue to the worker when he starts next period not being in a relationship. Since the matching probability is equal to 1, it does not matter whether you leave period t in a relationship or not; in period $t + 1$ the worker still has the freedom to choose what is best. Consequently, $N_w(\phi_c, \phi_p, \Phi_{p,t+1}) = U_w(\phi_c, \phi_p, \Phi_{p,t+1})$ and the condition given in equation (24) can simply be written as

$$w_t \geq \mu. \quad (25)$$

A necessary and sufficient condition to satisfy both the participation condition of the worker and the no-cheating condition of the entrepreneur is given by⁵²

$$\phi_p \Phi_{p,t} \geq \chi_e + \mu. \quad (26)$$

If we let $\chi = \chi_e + \mu$, then we get exactly the condition in equation (2) used in the main text to model the friction.⁵³

In the remainder of this section, we provide some more intuition on why contractual fragility makes production impossible when $\phi_p \Phi_{p,t}$ is less than χ . Consider the case when

$$\mu < \phi_p \Phi_{p,t} < \chi_e + \mu. \quad (27)$$

If an existing project does not operate when $\phi_p \Phi_{p,t} > \mu$, then this is not efficient. However, for these values of $\phi_p \Phi_{p,t}$ it is not possible to both pay the entrepreneur enough so that he will not choose the alternative business plan and pay the worker enough so that his wage exceeds μ . In this case, the project does not operate. The idea of the contractual fragility of Ramey and Watson (1997) is that no credible contracts can be written that will prevent the entrepreneur from cheating and implementing the inefficient alternative business plan. The entrepreneur may promise that he will pay the worker a wage above μ and that he will not implement the alternative business plan, but if $\phi_p \Phi_{p,t} < \chi_e + \mu$, then the entrepreneur cannot both pay the worker more than μ and satisfy his own incentive compatibility condition. The worker knows that the entrepreneur will face this dilemma and chooses not to work for this entrepreneur.

The beauty of the contractual fragility framework of Ramey and Watson (1997) is that it allows for the possibility that projects are not activated even though everybody would

⁵²Since the entrepreneur can never earn any benefits outside of a relationship, there is no participation constraint for the entrepreneur.

⁵³For simplicity, we assumed that χ_e and μ are not time-varying, but the analysis here allows for these values to be time-varying.

be better off if the project is activated. The reason is that it is not possible to write contracts that prevent cheating.

The framework described here results in a restriction that is identical to the one used in the main text. There are, of course, more general specifications. For example, the values of χ_e and χ_w could depend on ϕ_c . A higher entry cost means larger investments in the firm and possibly more options for the entrepreneur to extract resources from the firm. The financial friction also predicts that χ depends positively on ϕ_c and we will argue that the costs of business cycles are higher when there is such positive dependence. But if the entrepreneur cannot continue operating his project after having exploited the worker and has to pay part of ϕ_c again, then this would dampen and possibly even overturn this effect.

A.2 Financial friction

There are many different models with financial frictions. In this section, we develop a model in which the friction is such that firms have to be sufficiently productive to make operating the project possible. The condition on firm productivity has a somewhat different form than the ad hoc efficiency requirement imposed in the main text. But this difference actually implies larger costs of business cycles.

We assume that the entrepreneur does not have any net worth and has to borrow to finance the entry costs. Also, we assume that the entrepreneur simply rolls over this debt every period until he is hit by the exogenous destruction shock and he stops producing. Let the interest rate be equal to r , which includes a premium for the fact that producers default on the debt obligation when they stop producing. For simplicity, we assume that all firms face the same interest rate, for example, because lenders cannot distinguish between different types of borrowers.⁵⁴ A standard financial friction is a limit on the amount that can be collateralized. In particular, assume that the entrepreneur can extract χ_e when he defaults on his loan. Examples of assets of the firm that cannot be collateralized are human capital, the value of the good will created, and the value of the networks built up.

The entrepreneur would not default on his loan if

$$\begin{aligned} \phi_p \Phi_{p,t} - w_t - r\phi_c + \beta \mathbf{E}_t [\rho N_e(\phi_c, \phi_p, 1, \Phi_{p,t+1}) + (1 - \rho) N_e(\phi_c, \phi_p, 0, \Phi_{p,t+1})] & \quad (28) \\ & \geq \\ \chi_e + \beta \mathbf{E}_t [N_e(\phi_c, \phi_p, 0, \Phi_{p,t+1})] & \end{aligned}$$

If the entrepreneur borrows the funds to finance the entry costs, then the value of the project's revenues that accrue to the entrepreneur before or after entry costs have been paid are identical, that is, $N_e(\phi_c, \phi_p, 1, \Phi_{p,t+1}) = N_e(\phi_c, \phi_p, 0, \Phi_{p,t+1})$ and we can write the last equation as

$$\phi_p \Phi_{p,t} - w_t - r\phi_c \geq \chi_e. \quad (29)$$

⁵⁴To ensure that even the amount borrowed does not reveal the type of the borrower, one could assume that one can scale each project. An entrepreneur with a high value for ϕ_c can then invest in say half a project and, thus, hide that he has a project with a high value for ϕ_c .

Since $w_t \geq \mu$, it must be true that

$$\phi_p \Phi_{p,t} \geq \chi_e + \mu + r\phi_c \quad (30)$$

and if we let $\chi = \chi_e + \mu$, then we get

$$\phi_p \Phi_{p,t} \geq \chi + r\phi_c. \quad (31)$$

This condition differs from the one we used in the main text, because it implies an upward sloping cut-off curve in the (ϕ_c, ϕ_p) space, whereas the condition that $\phi_p \Phi_{p,t} \geq \chi$ implies a vertical cut-off for the production decision.

Disappearance of timed-entry projects. Although the efficiency requirement looks somewhat different than the one used in the main text, the main implications are the same. There are two differences, both will make the costs of business cycles stronger. The first is that there are no longer timed-entry projects. The reason is the following. The entry condition when there are no business cycles and no frictions is given by

$$N_{\text{no-bc}}(\phi_c, \phi_p, 1) - \phi_c \geq \mu + \beta N_{\text{no-bc}}(\phi_c, \phi_p, 0), \quad (32)$$

which implies that

$$\phi_c \leq \tilde{\phi}_{c,\text{no-bc}}(\phi_p) = \frac{\phi_p - \mu}{1 - \beta\rho}. \quad (33)$$

From equation (31) we get for the case of no business cycles that

$$\phi_c \leq \hat{\phi}_{c,\text{no-bc}}(\phi_p) = \frac{\phi_p - \chi}{r}. \quad (34)$$

The value of r is given by $\beta\rho/(1 - \beta\rho)$.⁵⁵ If we strengthen assumption 1 slightly by assuming that $\mu < \beta\rho\chi$, then

$$\phi_c \leq \beta\rho \frac{\phi_p - \chi}{1 - \beta\rho} = \hat{\phi}_{c,\text{no-bc}}(\phi_p) < \tilde{\phi}_{c,\text{no-bc}}(\phi_p) = \frac{\phi_p - \mu}{1 - \beta\rho}. \quad (35)$$

This means that the entry condition given in equation (33) is not binding. As long as aggregate fluctuations are not too large this remains true for projects that continue until they are hit by the exogenous destruction shock. This means that timed-entry projects have disappeared out of the model. This case is illustrated in figure 4. Whereas in figure 3 the cut-off level of the inefficiency condition intersects with the entry condition, in figure 4 the cut-off level condition of the inefficiency condition makes the entry condition redundant.

Quantitatively, we always found the results for timed-entry projects to be small. So the disappearance of the timed-entry projects cannot matter much for the results. What matters is that the cyclical projects are still there. To this question we turn next.

⁵⁵The interest rate is higher than the discount rate, since the entrepreneur stops paying interest when he is hit by an exogenous destruction shock.

Cyclical projects and larger welfare costs of business cycles. Figure 4 documents that the same three groups of cyclical projects are present in this version of the model. In the previous subsection, we showed that the condition that the entry costs should be less than the NPV of the revenue stream is no longer binding if the efficiency condition is given by equation (31). That is, $\tilde{\phi}_{c,\text{no-bc}}(\phi_p) < \tilde{\phi}_{c,\text{no-bc}}(\phi_p)$. This inequality remains true in the presence of business cycles if business cycles do not shorten the project's life expectancy. If projects can only satisfy the efficiency requirement in a boom, then the life expectancy is shorter. This results in a downward shift of $\tilde{\phi}_c$ and for some projects $\tilde{\phi}_{c,\text{bc}}(\phi_p, \Phi_+) < \tilde{\phi}_{c,\text{no-bc}}(\phi_p, \Phi_+)$. This means that there are again projects that do not enter in a boom even though they satisfy the efficiency requirement.

Although the same three types of cyclical projects exist, the welfare costs of business cycles could very well be higher in this version of the model. Consider the cyclical projects that are permanently eliminated by business cycles. As in the model discussed in the main text, these are the projects with the higher values for ϕ_c . But in this version of the model, they have higher levels of output, whereas in the model discussed in the main text they had the same level of output as one group of cyclical projects. If this is the correct model, then our calibration procedure may have underestimated the output potential of the projects that are permanently eliminated because of business cycles.

The analysis so far assumed that the interest rate was not affected by business cycles. But given that some firms only survive until the next recession one would expect the interest rate to be higher in a world with business cycles. For now, assume that all firms have the same interest rate. Then the increase in the interest rate would be relatively small. Nevertheless, it would mean that business cycles do not just put a mean-preserving spread on the efficiency condition given in equation (31), they actually make it more difficult to satisfy it on average.

The situation would be worse if lenders can identify firms that will only survive until the next recession. Then these firms will have to face a further downward shift in the efficiency condition, increasing the number of firms that are permanently eliminated.

B Implementation: detailed description

Our strategy to determine the net welfare losses for all cyclical projects is divided in two parts. In the first part, we determine the net loss for cyclical projects with values of ϕ_c below $\tilde{\phi}_{c,\text{bc}}$. In the second part, we determine the loss for cyclical projects with values of ϕ_c above $\tilde{\phi}_{c,\text{bc}}$. The joint density of ϕ_c and ϕ_p is denoted by $f(\phi_c, \phi_p)$.

1. To calculate the net loss for cyclical projects with $0 < \phi_c \leq \tilde{\phi}_{c,\text{bc}}$, we do the following:
 - (a) *Average of ϕ_c given ϕ_p .* The question arises what to use for the distribution of ϕ_c and ϕ_p conditional on the project being a cyclical project with $\phi_c \leq \tilde{\phi}_{c,\text{bc}}$. In this step, we deal with the relevant aspects of the distribution of ϕ_c . We assume that the lower bound for ϕ_c is given by 0. It may not be realistic to assume that there are projects with zero startup costs, but we obtain a conservative

estimate by assuming that the lower bound is equal to 0. The reason is that the net welfare costs are lower if more projects have low values of ϕ_c . The upper bound for ϕ_c in this group of projects is given by $\tilde{\phi}_{c,bc}$ which is given by

$$\tilde{\phi}_{c,bc} = \frac{\phi_p (1 + \Delta_{\Phi_p}) - \mu}{1 - \beta\rho\pi}.$$

Finally, we assume that the average value of ϕ_c , conditional on ϕ_p , for projects with a value of ϕ_c in between 0 and $\tilde{\phi}_{c,bc}$ is equal to the average of the two endpoints, i.e., we assume it to be equal to $(\tilde{\phi}_{c,bc} + 0)/2$. Besides this conditional average, we do not need any further information about the distribution of ϕ_c .

(b) The net loss for cyclical projects with $\phi_c \leq \tilde{\phi}_{c,bc}$ is given by

$$\begin{aligned} & \int_{\tilde{\phi}_{p,bc}(\Phi_+)}^{\tilde{\phi}_{p,bc}(\Phi_-)} \int_0^{\tilde{\phi}_{c,bc}(\phi_p)} \frac{L(\phi_c, \phi_p, \Delta_{\Phi_p}) \phi_p}{Y} f(\phi_c, \phi_p) d\phi_c d\phi_p = \quad (36) \\ & \int_{\tilde{\phi}_{p,no-bc}}^{\tilde{\phi}_{p,bc}(\Phi_-)} \int_0^{\tilde{\phi}_{c,bc}(\phi_p)} \frac{(1 - \Delta_{\Phi_p}) \phi_p - \mu + \phi_c (1 - \beta\rho\pi - 2(1 - \beta\rho))}{2Y} f(\phi_c, \phi_p) d\phi_c d\phi_p \\ & + \int_{\tilde{\phi}_{p,bc}(\Phi_+)}^{\tilde{\phi}_{p,no-bc}} \int_0^{\tilde{\phi}_{c,bc}(\phi_p)} \frac{-(1 + \Delta_{\Phi_p}) \phi_p + \mu + \phi_c (1 - \beta\rho\pi)}{2Y} f(\phi_c, \phi_p) d\phi_c d\phi_p \leq \\ & \int_{\tilde{\phi}_{p,bc}(\Phi_+)}^{\tilde{\phi}_{p,bc}(\Phi_-)} \int_0^{\tilde{\phi}_{c,bc}(\phi_p)} \frac{-\Delta_{\Phi_p} \phi_p + \phi_c \beta\rho (1 - \pi)}{2Y} f(\phi_c, \phi_p) d\phi_c d\phi_p = \\ & \int_{\tilde{\phi}_{p,bc}(\Phi_+)}^{\tilde{\phi}_{p,bc}(\Phi_-)} \int_0^{\tilde{\phi}_{c,bc}(\phi_p)} \frac{-2\Delta_{\Phi_p} + \tilde{\phi}_{c,bc} \beta\rho (1 - \pi)}{4Y} \phi_p f(\phi_c, \phi_p) d\phi_c d\phi_p = \\ & \int_{\tilde{\phi}_{p,bc}(\Phi_+)}^{\tilde{\phi}_{p,bc}(\Phi_-)} \int_0^{\tilde{\phi}_{c,bc}(\phi_p)} \frac{-2\Delta_{\Phi_p} + \frac{(1 + \Delta_{\Phi_p})^{-\mu/\phi_p}}{1 - \beta\rho\pi} \beta\rho (1 - \pi)}{4Y} \phi_p f(\phi_c, \phi_p) d\phi_c d\phi_p = \\ & \frac{-2\Delta_{\Phi_p} + \frac{(1 + \Delta_{\Phi_p})^{-\tilde{\mu}}}{1 - \beta\rho\pi} \beta\rho (1 - \pi)}{4} \int_{\tilde{\phi}_{p,bc}(\Phi_+)}^{\tilde{\phi}_{p,bc}(\Phi_-)} \int_0^{\tilde{\phi}_{c,bc}(\phi_p)} \frac{\phi_p}{Y} f(\phi_c, \phi_p) d\phi_c d\phi_p = \\ & \frac{-2\Delta_{\Phi_p} + \frac{(1 + \Delta_{\Phi_p})^{-\tilde{\mu}}}{1 - \beta\rho\pi} \beta\rho (1 - \pi)}{4} \beta\rho (1 - \pi) \frac{Y_{\text{cyclical}}}{Y} \end{aligned}$$

In the first row, the value of $L(\cdot)$ is multiplied by ϕ_p since $L(\cdot)$ is scaled by ϕ_p . The expressions used for $L(\cdot)$ are given in proposition 9.⁵⁶ The inequality in the next line is based on the assumption that the mass of projects below $\tilde{\phi}_{p,no-bc}$ does not exceed the mass above $\tilde{\phi}_{p,no-bc}$. Given that we are in the left tail of the distribution, this is unlikely to be the case. In the following step, we use that the average value of ϕ_c , conditional on ϕ_p , is given by the midpoint of the interval as discussed in part a. The remaining steps are simple algebra,

⁵⁶See appendix C for a derivation.

where we define $\tilde{\mu}$ is the mean value of the outside option of the project as a fraction of ϕ_p across all cyclical projects with ϕ_c below $\tilde{\phi}_{c,bc}$.

The parameters Δ_{Φ_p} , β , ρ , $\tilde{\mu}$, and π can be relatively easily calibrated. See section 6.1 and appendix D for a discussion on how we calculate Y_{cyclical}/Y .

2. To calculate the net loss for cyclical projects with $\tilde{\phi}_{c,bc} < \phi_c \leq \tilde{\phi}_{c,no-bc}$ we do the following:

- (a) *Average of ϕ_c given ϕ_p .* As above, the average value of ϕ_c for projects with a value of ϕ_c in between $\tilde{\phi}_{c,bc}$ and $\tilde{\phi}_{c,no-bc}$ is assumed to be the average of the end points, that is, it is assumed to be equal to $(\tilde{\phi}_{c,bc} + \tilde{\phi}_{c,no-bc})/2$.
- (b) The welfare loss of cyclical projects with a value of ϕ_c above $\phi_{c,bc}$ is given by

$$\begin{aligned}
& \int_{\tilde{\phi}_{p,no-bc}}^{\tilde{\phi}_p(\Phi_-)} \int_{\tilde{\phi}_{c,bc}(\phi_p)}^{\tilde{\phi}_{c,no-bc}(\phi_p)} \frac{L(\phi_c, \phi_p, \Delta_{\Phi_p}) \phi_p}{Y} f(\phi_c, \phi_p) d\phi_c d\phi_p = \quad (37) \\
& \int_{\tilde{\phi}_{p,no-bc}}^{\tilde{\phi}_p(\Phi_-)} \int_{\tilde{\phi}_{c,bc}(\phi_p)}^{\tilde{\phi}_{c,no-bc}(\phi_p)} \frac{\phi_p - \mu - (1 - \beta\rho) \phi_c}{Y} f(\phi_c, \phi_p) d\phi_c d\phi_p = \\
& \int_{\tilde{\phi}_{p,no-bc}}^{\tilde{\phi}_p(\Phi_-)} \int_{\tilde{\phi}_{c,bc}(\phi_p)}^{\tilde{\phi}_{c,no-bc}(\phi_p)} \frac{\phi_p - \mu - (1 - \beta\rho) \frac{(\tilde{\phi}_{c,bc}(\phi_p) + \tilde{\phi}_{c,no-bc}(\phi_p))}{2}}{Y} f(\phi_c, \phi_p) d\phi_c d\phi_p = \\
& \int_{\tilde{\phi}_{p,no-bc}}^{\tilde{\phi}_p(\Phi_-)} \int_{\tilde{\phi}_{c,bc}(\phi_p)}^{\tilde{\phi}_{c,no-bc}(\phi_p)} \frac{\beta\rho(1-\pi)(\phi_p - \mu) - (1 - \beta\rho)\phi_p \Delta_{\Phi_p}}{2(1 - \beta\rho\pi) Y} f(\phi_c, \phi_p) d\phi_c d\phi_p = \\
& \frac{\beta\rho(1 - \pi)(1 - \tilde{\mu}) - (1 - \beta\rho) \Delta_{\Phi_p}}{2(1 - \beta\rho\pi) Y} \int_{\tilde{\phi}_{p,no-bc}}^{\tilde{\phi}_p(\Phi_-)} \int_{\tilde{\phi}_{c,bc}(\phi_p)}^{\tilde{\phi}_{c,no-bc}(\phi_p)} \phi_p f(\phi_c, \phi_p) d\phi_c d\phi_p = \\
& \frac{\beta\rho(1 - \pi)(1 - \tilde{\mu}) - (1 - \beta\rho) \Delta_{\Phi_p}}{2(1 - \beta\rho\pi)} \frac{Y_{\text{cyclical-PL}}}{Y},
\end{aligned}$$

- (c) *Output generated by cyclical projects with $\tilde{\phi}_{c,bc} < \phi_c \leq \tilde{\phi}_{c,no-bc}$.* If a project's value of ϕ_c is above $\tilde{\phi}_{c,bc}$, then it is too high to make entry profitable in a world with business cycles. Consequently, we do not observe these projects in the real world. These projects, however, have one characteristic in common with projects we do observe and that is their productivity level, ϕ_p . In particular, the productivity levels of these projects are in between $\tilde{\phi}_{p,no-bc}$ and $\tilde{\phi}_p(\Phi_-)$, which means that they are in the upper half of the productivity levels of the cyclical projects that produce Y_{cyclical} . Let $f(\phi_c|\phi_p)$ be the density of ϕ_c given ϕ_p . To calculate the value of $Y_{\text{cyclical-PL}}$ given the value of Y_{cyclical} , we would need to know the value of

$$\frac{\int_{\tilde{\phi}_{c,bc}(\phi_p)}^{\tilde{\phi}_{c,no-bc}(\phi_p)} f(\phi_c|\phi_p) d\phi_c}{\int_0^{\tilde{\phi}_{c,bc}(\phi_p)} f(\phi_c|\phi_p) d\phi_c},$$

that is, the mass of cyclical projects with a value of ϕ_c above $\tilde{\phi}_{c,bc}$ to the mass of cyclical projects with a value of ϕ_c below $\tilde{\phi}_{c,bc}$. We assume that a reasonable estimate for this ratio is to use the lengths of the intervals, thus

$$\frac{\int_{\tilde{\phi}_{c,bc}(\phi_p)}^{\tilde{\phi}_{c,no-bc}(\phi_p)} f(\phi_c|\phi_p)d\phi_c}{\int_0^{\tilde{\phi}_{c,bc}(\phi_p)} f(\phi_c|\phi_p)d\phi_c} \approx \frac{\tilde{\phi}_{c,no-bc}(\phi_p) - \tilde{\phi}_{c,bc}(\phi_p)}{\tilde{\phi}_{c,bc}(\phi_p) - 0}.$$

This implies that

$$\begin{aligned} & \frac{Y_{\text{cyclical-PL}}}{Y} & (38) \\ = & \int_{\tilde{\phi}_{p,no-bc}}^{\tilde{\phi}_{p,bc}(\Phi_-)} \int_{\tilde{\phi}_{c,bc}(\phi_p)}^{\tilde{\phi}_{c,no-bc}(\phi_p)} \frac{\phi_p}{Y} f(\phi_c, \phi_p) d\phi_c d\phi_p \\ = & \int_{\tilde{\phi}_{p,no-bc}}^{\tilde{\phi}_{p,bc}(\Phi_-)} \frac{\int_{\tilde{\phi}_{c,bc}(\phi_p)}^{\tilde{\phi}_{c,no-bc}(\phi_p)} f(\phi_c|\phi_p) d\phi_c}{\int_0^{\tilde{\phi}_{c,bc}(\phi_p)} f(\phi_c|\phi_p) d\phi_c} \frac{\phi_p \int_0^{\tilde{\phi}_{c,bc}(\phi_p)} f(\phi_c|\phi_p) d\phi_c}{Y} f(\phi_p) d\phi_p \\ \approx & \int_{\tilde{\phi}_{p,no-bc}}^{\tilde{\phi}_{p,bc}(\Phi_-)} \frac{\tilde{\phi}_{c,no-bc}(\phi_p) - \tilde{\phi}_{c,bc}(\phi_p)}{\tilde{\phi}_{c,bc}(\phi_p) - 0} \frac{\phi_p \int_0^{\tilde{\phi}_{c,bc}(\phi_p)} f(\phi_c|\phi_p) d\phi_c}{Y} f(\phi_p) d\phi_p \\ = & X \int_{\tilde{\phi}_{p,no-bc}}^{\tilde{\phi}_{p,bc}(\Phi_-)} \frac{\phi_p \int_0^{\tilde{\phi}_{c,bc}(\phi_p)} f(\phi_c|\phi_p) d\phi_c}{Y} f(\phi_p) d\phi_p \\ = & \frac{X}{Y} \int_{\tilde{\phi}_{p,no-bc}}^{\tilde{\phi}_{p,bc}(\Phi_-)} \int_0^{\tilde{\phi}_{c,bc}(\phi_p)} \phi_p f(\phi_c, \phi_p) d\phi_c d\phi_p \\ > & \frac{X}{2} \frac{Y_{\text{cyclical}}}{Y}, \end{aligned}$$

where

$$X = \frac{1 - \bar{\mu}}{1 + \Delta - \bar{\mu}} \frac{1 - \beta\rho\pi}{1 - \beta\rho} - 1.$$

The inequality follows from the fact that the output levels in $Y_{\text{cyclical-PL}}$ are all above $\tilde{\phi}_{p,no-bc}$ whereas the output levels in Y_{cyclical} are both above and below $\tilde{\phi}_{p,no-bc}$. That is, we assume that⁵⁷

$$\int_{\tilde{\phi}_{p,no-bc}}^{\tilde{\phi}_{p,bc}(\Phi_-)} \phi_p f(\phi_p|\phi_c) d\phi_p \geq \int_{\tilde{\phi}_{p,bc}(\Phi_+)}^{\tilde{\phi}_{p,no-bc}} \phi_p f(\phi_p|\phi_c) d\phi_p. \quad (39)$$

⁵⁷See section 6.1.

C Proofs

Part 1 of proposition 9. $\tilde{\phi}_{c,\text{no-bc}}(\phi_p)$ is defined as the value of ϕ_c at which the value of activating the project is equal to the value of never activating the project.⁵⁸ Thus,

$$N_{\text{no-bc}}(\tilde{\phi}_{c,\text{no-bc}}, \phi_p, 1) - \tilde{\phi}_{c,\text{no-bc}} = \frac{\mu}{1 - \beta}. \quad (40)$$

The value of an activated project is given by

$$N_{\text{no-bc}}(\phi_c, \phi_p, 1) = \phi_p + \beta\rho N_{\text{no-bc}}(\phi_c, \phi_p, 1) + \beta(1 - \rho) N_{\text{no-bc}}(\phi_c, \phi_p, 0). \quad (41)$$

If assumption 1 is satisfied, then activating a project is profitable if the efficiency condition is satisfied and the startup costs are low enough. Thus,

$$N_{\text{no-bc}}(\phi_c, \phi_p, 1) = N_{\text{no-bc}}(\phi_c, \phi_p, 0) + \phi_c \text{ when } \begin{cases} \phi_c \leq \tilde{\phi}_{c,\text{no-bc}} \text{ and} \\ \phi_p \geq \phi_{p,\text{no-bc}} \end{cases}. \quad (42)$$

Combining the last three equations gives

$$\tilde{\phi}_{c,\text{no-bc}}(\phi_p) = \frac{\phi_p - \mu}{1 - \beta\rho} \text{ if } \phi_p \geq \phi_{p,\text{no-bc}}. \quad (43)$$

Next, we calculate the value of $\tilde{\phi}_{c,\text{bc}}(\phi_p)$ for cyclical projects, that is, when $\tilde{\phi}_{p,\text{bc}}(\Phi_+) \leq \phi_p < \tilde{\phi}_{p,\text{bc}}(\Phi_-)$.⁵⁹ If the startup costs are low enough, then these projects are activated in a boom, but not in a recession.⁶⁰ Thus, if $\phi_c \leq \tilde{\phi}_{c,\text{bc}}(\phi_p)$, then

$$\begin{aligned} & N_{\text{bc}}(\phi_c, \phi_p, 0, 1 + \Delta_{\Phi_p}) \\ &= \\ & -\phi_c + \phi_p(1 + \Delta_{\Phi_p}) + \beta \begin{pmatrix} \pi & \rho & (N_{\text{bc}}(\phi_c, \phi_p, 0, 1 + \Delta_{\Phi_p}) + \phi_c) \\ +\pi & (1 - \rho) & N_{\text{bc}}(\phi_c, \phi_p, 0, 1 + \Delta_{\Phi_p}) \\ +(1 - \pi) & \rho & N_{\text{bc}}(\phi_c, \phi_p, 0, 1 - \Delta_{\Phi_p}) \\ +(1 - \pi) & (1 - \rho) & N_{\text{bc}}(\phi_c, \phi_p, 0, 1 - \Delta_{\Phi_p}) \end{pmatrix}. \end{aligned} \quad (44)$$

By definition, $\tilde{\phi}_{c,\text{bc}}(\phi_p)$ is the value of ϕ_c such that

$$N_{\text{bc}}(\tilde{\phi}_{c,\text{bc}}, \phi_p, 0, 1 + \Delta_{\Phi_p}) = \frac{\mu}{1 - \beta}. \quad (45)$$

In a boom this type of project could either produce or not produce. The NPV would be equal to $\mu/(1 - \beta)$ for both choices. In a recession this project cannot produce, so the

⁵⁸To economize on notation we typically write $\tilde{\phi}_{c,\text{no-bc}}$ instead of $\tilde{\phi}_{c,\text{no-bc}}(\phi_p)$.

⁵⁹The ordering is based on the first part of assumption 2. If the second part of this assumption would hold instead, then the formulas would be very similar, but the role of booms and expansions would be switched.

⁶⁰Assumption 1 ensures that the value of μ is low enough to ensure that entry is profitable as long as the efficiency condition is satisfied and the entry costs are low enough.

revenues are equal to μ until the economy gets out of a recession at which point the NPV by definition is equal to $\mu/(1-\beta)$. Consequently,

$$N_{bc}(\tilde{\phi}_{c,bc}, \phi_p, 0, 1 - \Delta_{\Phi_p}) = \frac{\mu}{1-\beta}. \quad (46)$$

Combining the last equations gives

$$\tilde{\phi}_{c,bc}(\phi_p) = \frac{\phi_p(1 + \Delta_{\Phi_p}) - \mu}{1 - \beta\rho\mu}. \quad (47)$$

Assumption 3 directly implies that

$$\tilde{\phi}_{c,bc}(\phi_p) < \tilde{\phi}_{c,no-bc}(\phi_p) \text{ if } \tilde{\phi}_{p,no-bc} \leq \phi_p < \tilde{\phi}_{p,bc}(\Phi_-). \quad (48)$$

Proof of part 2 of proposition 9. To calculate the formulas in this part of the proposition, we have to first calculate the relevant NPVs. First consider a world without business cycles. If the project cannot satisfy the efficiency condition or if the startup costs are too high, then the project is never activated and the revenues are μ each period. Thus

$$N_{no-bc}(\phi_c, \phi_p, 0) = \frac{\mu}{1-\beta} \text{ if } \phi_p < \tilde{\phi}_{p,no-bc} \text{ or } \phi_c > \tilde{\phi}_{c,no-bc}(\phi_p). \quad (49)$$

If assumption 1 is satisfied, then activating is profitable if the efficiency condition is satisfied and the entry costs are low enough. From equations 41 and 42, it follows that

$$N_{no-bc}(\phi_c, \phi_p, 0) = \frac{\phi_p - (1-\beta\rho)\phi_c}{1-\beta} \text{ if } \phi_p \geq \tilde{\phi}_{p,no-bc} = \chi \text{ and } \phi_c \leq \tilde{\phi}_{c,no-bc}(\phi_p). \quad (50)$$

Now consider the case without business cycles. Here we focus on cyclical projects. If the project can never satisfy the efficiency conditions or if the startup costs are too high, then the project will not operate and revenues are equal to μ . Thus,

$$\begin{aligned} & \text{if } \phi_p < \tilde{\phi}_{p,bc}(\Phi_+) \text{ or } \phi_c > \tilde{\phi}_{c,bc}(\phi_p), \text{ then} \\ E[N_{bc}(\phi_c, \phi_p, 0, \Phi_{p,t})] &= NPV_{bc}(\phi_c, \phi_p, 0, 1 + \Delta_{\Phi_p}) = NPV_{bc}(\phi_c, \phi_p, 0, 1 - \Delta_{\Phi_p}) \\ &= \frac{\mu}{1-\beta}. \end{aligned} \quad (51)$$

Now suppose that $\phi_p \geq \tilde{\phi}_{p,bc}(\Phi_+)$, $\phi_c \leq \tilde{\phi}_{c,bc}(\phi_p)$, and (since we focus on cyclical projects) $\phi_p < \tilde{\phi}_{p,bc}(\Phi_-)$. The value for

$$E[N_{bc}(\phi_c, \phi_p, 0, \Phi_{p,t})] = \frac{(N_{bc}(\phi_c, \phi_p, 0, 1 + \Delta_{\Phi_p}) + N_{bc}(\phi_c, \phi_p, 0, 1 - \Delta_{\Phi_p}))}{2}$$

is equal to

$$E[N_{bc}(\phi_c, \phi_p, 0, \Phi_{p,t})] = \frac{1}{2} \left(\frac{\phi_p(1 + \Delta_{\Phi_p}) + \mu - (1 - \beta\rho\pi)\phi_c}{1 - \beta} \right). \quad (52)$$

This formula follows from equation (44) and

$$N_{bc}(\phi_c, \phi_p, 0, 1 - \Delta_{\Phi_p}) = \mu + \beta \left(\begin{array}{c} \pi \\ + (1 - \pi) \end{array} \begin{array}{c} N_{bc}(\phi_c, \phi_p, 0, 1 - \Delta_{\Phi_p}) \\ N_{bc}(\phi_c, \phi_p, 0, 1 + \Delta_{\Phi_p}) \end{array} \right). \quad (53)$$

The formulas in the proposition follow directly from combining the formulas for the appropriate NPVs.

Proof of proposition 10. This proposition focuses on jobs that can always overcome the efficiency condition and have a value of ϕ_c that is exactly at the boundary. Thus,

$$\phi_c = \tilde{\phi}_{c,\text{no-bc}} \text{ and } \phi_p > \max \tilde{\phi}_{p,\text{bc}} = \max \left\{ \frac{\chi(\Phi_+)}{\Phi_+}, \frac{\chi(\Phi_-)}{\Phi_-} \right\}. \quad (54)$$

In a world without business cycles, the value of activating a project with $\phi_c = \tilde{\phi}_{c,\text{no-bc}}$ would be equal to the value of not activating. Thus,

$$N_{\text{no-bc}}(\phi_c, \phi_p, 0) = \frac{\mu}{1 - \beta} \text{ if } \phi_c = \tilde{\phi}_{c,\text{no-bc}} \text{ and } \phi_p > \max \tilde{\phi}_{p,\text{bc}} = \max \left\{ \frac{\chi(\Phi_+)}{\Phi_+}, \frac{\chi(\Phi_-)}{\Phi_-} \right\}. \quad (55)$$

In a world with business cycles, projects with $\phi_c = \tilde{\phi}_{c,\text{no-bc}}$ and $\phi_p > \max \tilde{\phi}_{p,\text{bc}}$ are timed-entry projects. Timed-entry projects are only activated during expansions, but an already activated project continues operating in a recession. The NPVs for timed-entry projects are given by the following equations:

$$N_{bc}(\phi_c, \phi_p, 0, 1 + \Delta_{\Phi_p}) = -\phi_c + \phi_p (1 + \Delta_{\Phi_p}) \quad (56)$$

$$+ \beta \left(\begin{array}{c} \pi \rho N_{bc}(\phi_c, \phi_p, 1, 1 + \Delta_{\Phi_p}) \\ + \pi (1 - \rho) N_{bc}(\phi_c, \phi_p, 0, 1 + \Delta_{\Phi_p}) \\ + (1 - \pi) \rho N_{bc}(\phi_c, \phi_p, 1, 1 - \Delta_{\Phi_p}) \\ + (1 - \pi) (1 - \rho) N_{bc}(\phi_c, \phi_p, 0, 1 - \Delta_{\Phi_p}) \end{array} \right),$$

$$N_{bc}(\phi_c, \phi_p, 1, 1 + \Delta_{\Phi_p}) = N_{bc}(\phi_c, \phi_p, 0, 1 + \Delta_{\Phi_p}) + \phi_c, \quad (57)$$

$$N_{bc}(\phi_c, \phi_p, 0, 1 - \Delta_{\Phi_p}) = \mu + \beta \left(\begin{array}{c} \pi N_{bc}(\phi_c, \phi_p, 0, 1 - \Delta_{\Phi_p}) \\ + (1 - \pi) N_{bc}(\phi_c, \phi_p, 0, 1 + \Delta_{\Phi_p}) \end{array} \right), \quad (58)$$

$$N_{bc}(\phi_c, \phi_p, 1, 1 - \Delta_{\Phi_p}) = \phi_p (1 - \Delta_{\Phi_p}) \quad (59)$$

$$+ \beta \left(\begin{array}{c} \pi \rho N_{bc}(\phi_c, \phi_p, 1, 1 - \Delta_{\Phi_p}) \\ + \pi (1 - \rho) N_{bc}(\phi_c, \phi_p, 0, 1 - \Delta_{\Phi_p}) \\ + (1 - \pi) \rho N_{bc}(\phi_c, \phi_p, 1, 1 + \Delta_{\Phi_p}) \\ + (1 - \pi) (1 - \rho) N_{bc}(\phi_c, \phi_p, 0, 1 + \Delta_{\Phi_p}) \end{array} \right).$$

Let

$$D(\phi_c, \phi_p) = N_{bc}(\phi_c, \phi_p, 0, 1 - \Delta_{\Phi_p}) - N_{bc}(\phi_c, \phi_p, 1, 1 - \Delta_{\Phi_p}) + \phi_c. \quad (60)$$

Then

$$\begin{aligned} E[N_{bc}(\phi_c, \phi_p, 0, \Phi_{p,t})] &= \frac{N_{bc}(\phi_c, \phi_p, 0, 1 + \Delta_{\Phi_p}) + N_{bc}(\phi_c, \phi_p, 0, 1 - \Delta_{\Phi_p})}{2} \\ &= \frac{-(1 - \beta\rho)\phi_c + \phi_p(1 + \Delta_{\Phi_p}) + \mu - \beta\rho(1 - \pi)D(\phi_c, \phi_p)}{2(1 - \beta)}. \end{aligned} \quad (61)$$

For these projects the cut-off level for ϕ_c is given by

$$\tilde{\phi}_{c,\text{no-bc}} = \frac{\phi_p - \mu}{1 - \beta\rho}. \quad (62)$$

Using this last expression we get that

$$\mathbb{E} \left[N_{\text{bc}}(\tilde{\phi}_{c,\text{no-bc}}, \phi_p, 0, \Phi_{p,t}) \right] = \frac{\phi_p \Delta_{\Phi_p} + 2\mu - \beta\rho(1 - \pi) D(\phi_c, \phi_p)}{2(1 - \beta)}. \quad (63)$$

The expression for D is calculated as follows. Working out the terms in the definition of D we get the following

$$\begin{aligned} D(\phi_c, \phi_p) &= \phi_c + N_{\text{bc}}(\phi_c, \phi_p, 0, 1 - \Delta_{\Phi_p}) - N_{\text{bc}}(\phi_c, \phi_p, 1, 1 - \Delta_{\Phi_p}) \quad (64) \\ &= \phi_c + \mu + \beta \left(\begin{array}{c} \pi N_{\text{bc}}(\phi_c, \phi_p, 0, 1 - \Delta_{\Phi_p}) \\ + (1 - \pi) N_{\text{bc}}(\phi_c, \phi_p, 0, 1 + \Delta_{\Phi_p}) \end{array} \right) \\ &\quad - \left(\phi_p (1 - \Delta_{\Phi_p}) + \beta \left(\begin{array}{c} \pi \rho N_{\text{bc}}(\phi_c, \phi_p, 1, 1 - \Delta_{\Phi_p}) \\ + \pi (1 - \rho) N_{\text{bc}}(\phi_c, \phi_p, 0, 1 - \Delta_{\Phi_p}) \\ + (1 - \pi) \rho N_{\text{bc}}(\phi_c, \phi_p, 1, 1 + \Delta_{\Phi_p}) \\ + (1 - \pi) (1 - \rho) N_{\text{bc}}(\phi_c, \phi_p, 0, 1 + \Delta_{\Phi_p}) \end{array} \right) \right) \\ &= \phi_c + \mu + \beta \left(\begin{array}{c} \pi N_{\text{bc}}(\phi_c, \phi_p, 0, 1 - \Delta_{\Phi_p}) \\ + (1 - \pi) N_{\text{bc}}(\phi_c, \phi_p, 0, 1 + \Delta_{\Phi_p}) \end{array} \right) \\ &\quad - \left(\phi_p (1 - \Delta_{\Phi_p}) + \beta \left(\begin{array}{c} \pi (N_{\text{bc}}(\phi_c, \phi_p, 0, 1 - \Delta_{\Phi_p}) + \rho(\phi_c - D)) \\ + (1 - \pi) N_{\text{bc}}(\phi_c, \phi_p, 0, 1 + \Delta_{\Phi_p}) + \rho\phi_c \end{array} \right) \right) \\ &= \phi_c (1 - \beta\rho) + \mu - \phi_p (1 - \Delta_{\Phi_p}) + \beta\pi\rho D. \quad (65) \end{aligned}$$

From this, we get

$$D(\phi_c, \phi_p) = \frac{\phi_c (1 - \beta\rho) + \mu - \phi_p (1 - \Delta_{\Phi_p})}{1 - \beta\pi\rho}. \quad (66)$$

Moreover,

$$D(\phi_c, \phi_p) = \frac{\phi_p \Delta_{\Phi_p}}{1 - \beta\pi\rho} \text{ if } \phi_c = \tilde{\phi}_{c,\text{no-bc}} = \frac{\phi_p - \mu}{1 - \beta\rho}. \quad (67)$$

If we combine this expression for $D(\phi_c, \phi_p)$ with the expression in equation (63) and the definition of the welfare loss, then we get that

$$\begin{aligned} L(\phi_c, \phi_p, \Delta_{\Phi_p}) &= (1 - \beta) \left(\frac{\mu}{1 - \beta} - \frac{\phi_p \Delta_{\Phi_p} + 2\mu - \beta\rho(1 - \pi) D(\phi_c, \phi_p)}{2(1 - \beta)} \right) \quad (68) \\ &= \frac{\phi_p \Delta_{\Phi_p} (1 - \beta\rho)}{2(1 - \beta\pi\rho)}. \end{aligned}$$

D Cyclical changes in output: Estimate of the extensive margin

In this appendix, we use a German panel data set on wage data to obtain a more direct estimate of that part of the cyclical change in output that is due to cyclical changes in the number of projects (the extensive margin). This data set does not exactly contain what we need. First, the data set gives information about employment positions, not about projects. That is, it is possible that new projects are created (eliminated) during a boom (recession) without additional workers being hired (fired). Second, the data set gives information about wages, whereas we would like to know the total value added created by the job, not just the wage component.

Nevertheless, we think that this data set provides direct evidence that cyclical changes in output along the extensive margin are nontrivial. We already know that the extensive margin is important for cyclical changes in total hours. But it is still possible that the workers being hired during a boom are not very productive, which would mean that these additional workers are not important in terms of explaining cyclical changes in *output*. The analysis here suggests that this is definitely not the case.

Data set used. We use the IAB monthly employment panel, a 2% representative subsample from the German social security and unemployment records. It is described in more detail in Jung and Kuhn (2009). The data set excludes self-employed and civil servants, but nevertheless covers 80% of the West German labor force.⁶¹

Constructing the estimate. As the economy gets out of a recession, the total wage sum earned increases. There are two reasons why this happens. First, there are workers that have found a job, *because* the recession has ended. In our model, these are cyclical workers.^{62,63} Second, workers whose employment status is not affected by the recession earn higher wages as economic conditions improve. We will refer to these workers as "non-cyclical" workers. The objective is to determine which part of the increase in total wages, observed as the economy gets out of a recession is due to cyclical workers gaining employment and which part is due to non-cyclical workers earning higher wages.

To answer this question we do the following. For each month, the workers are divided into two groups, group A and group B.

A. Workers in group A did *not* have a non-employment spell in the last 24 months. To

⁶¹In Den Haan and Sedlacek (2009), we document that aggregated wage data according to this panel data set follow true aggregate wages very closely.

⁶²According to definition 6, cyclical projects are all projects for which the stance of the business cycle determines whether they can satisfy the efficiency requirement. This includes projects with values of ϕ_c above $\tilde{\phi}_{c,bc}$ that will never operate in a world with business cycles. With cyclical workers we refer here to workers that belong to projects with values of ϕ_c below $\tilde{\phi}_{c,bc}$. Those with of $\phi_c > \tilde{\phi}_{c,bc}$ would never show up in our sample.

⁶³They also could be timed-entry workers, but these are likely to be not very important as explained in section 4.2.

do this, we first determine whether the worker was employed 24 months ago.⁶⁴ If the worker was employed T months ago *and* the total number of days the worker was "not employed" during the last T months was less than 30 days, then we include her/him.⁶⁵ Thus, a worker that experiences a job-to-job transition, but takes a "vacation" in between the two jobs is not excluded from this group of workers.

- B. Workers in group B are all other workers. These workers did experience a non-trivial period of non-employment during the last 24 months.

Clearly, all workers in group A are non-cyclical workers and all cyclical workers in our sample are in group B. The problem is that group B also contains non-cyclical workers. For example, new entrants to the labor force or workers who rejoin the labor force for personal reasons are also part of group B, but they are not cyclical workers. Below, we show how we deal with this aspect of the data set.

We find that on average the sum of wages earned by workers in group B is equal to 15% of the total wage sum. This is obviously a non-trivial number, but it is still possible that this 15% is mainly earned by new entrants and other non-cyclical workers and that the sum of wages earned by cyclical workers is small. To analyze the quantitative importance of cyclical workers we investigate how this fraction *changes* over the cycle. Such changes must by definition be due to cyclical workers. In particular, we look at the HP-filtered value of the ratio f_t , where

$$f_t = \frac{\text{sum of wages earned by workers in group B}}{\text{sum of all workers}}.$$

Figure 5 plots this series, together with the filtered unemployment rate. For convenience, we have added the means of the two series to the filtered data. As the economy moves from the trough of the recession to the peak of a boom, the fraction of wages earned by group B workers goes from roughly 13% to 17%. This is true for both the expansion of the eighties as the expansion of the late nineties. It is this *increase* from 13% to 17%—a non-trivial quantity—that we use as our measure of what cyclical workers can produce. It would not be right to use this difference between the observed values at the troughs and peaks as our estimate of Y_{cyclical}/Y . In our model, we only have two states, so it would be more prudent to use the difference between the average value of (detrended) f_t in a boom and the average value of (detrended) f_t in a recession. This would give an estimate for Y_{cyclical}/Y equal to roughly 2%.

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⁶⁴To be precise, we check whether he was employed in the reference week 24 months ago. In Den Haan and Sedlacek (2009), we document that the results are robust to using 36 instead of 24 months.

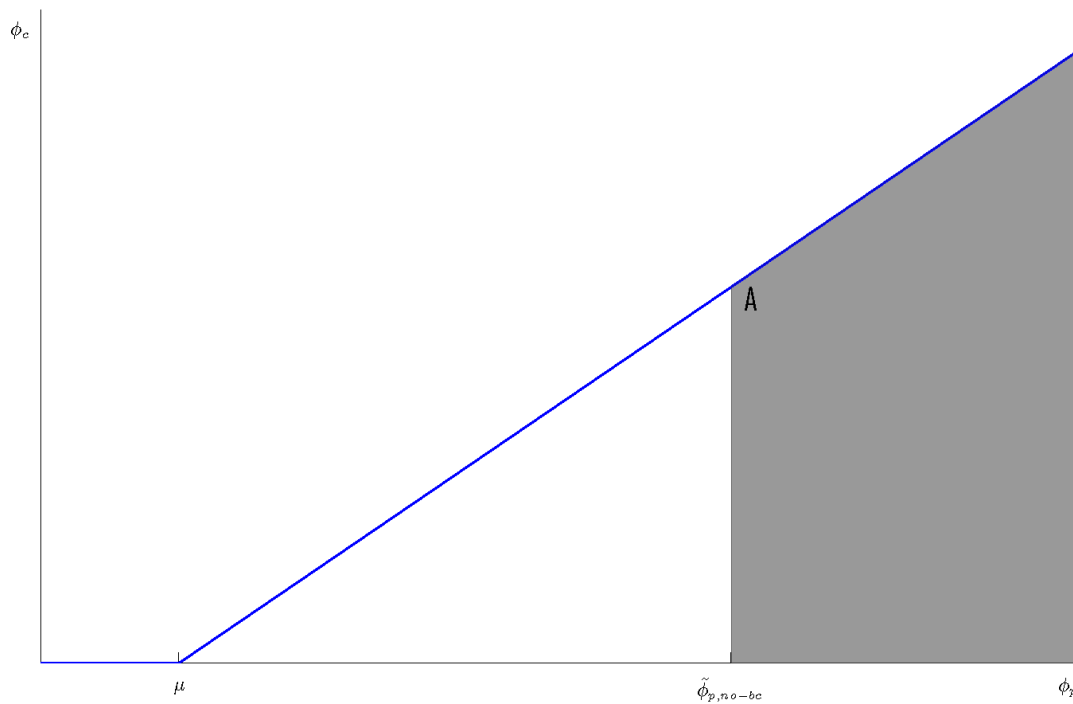
⁶⁵The data set keeps track of an experience variable that counts the total number of days a worker has worked. By looking at the increase in this variable, it is easy to check which fraction of a particular period a worker was actually working.

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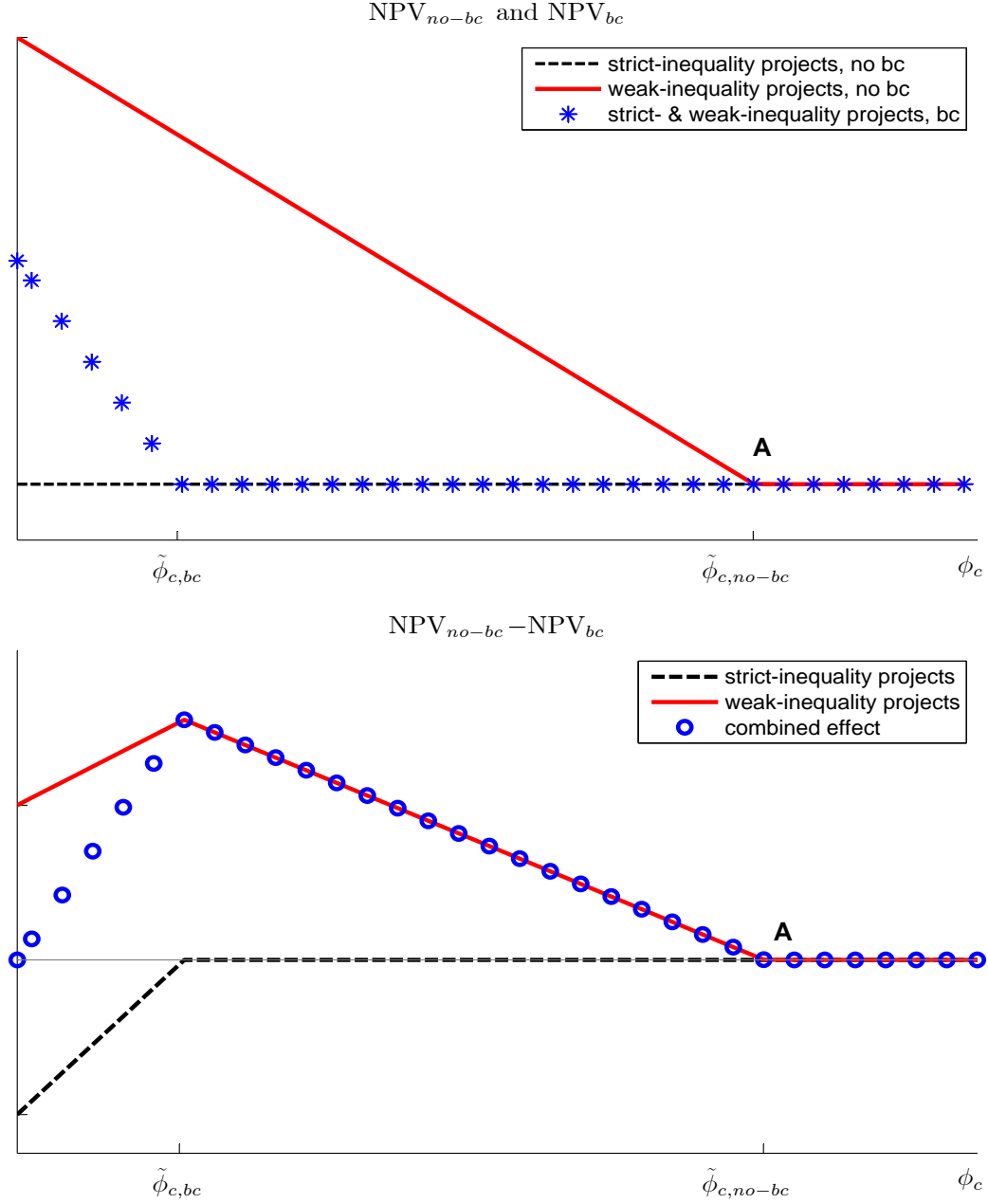
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Figure 1: Projects operating in a world without business cycles



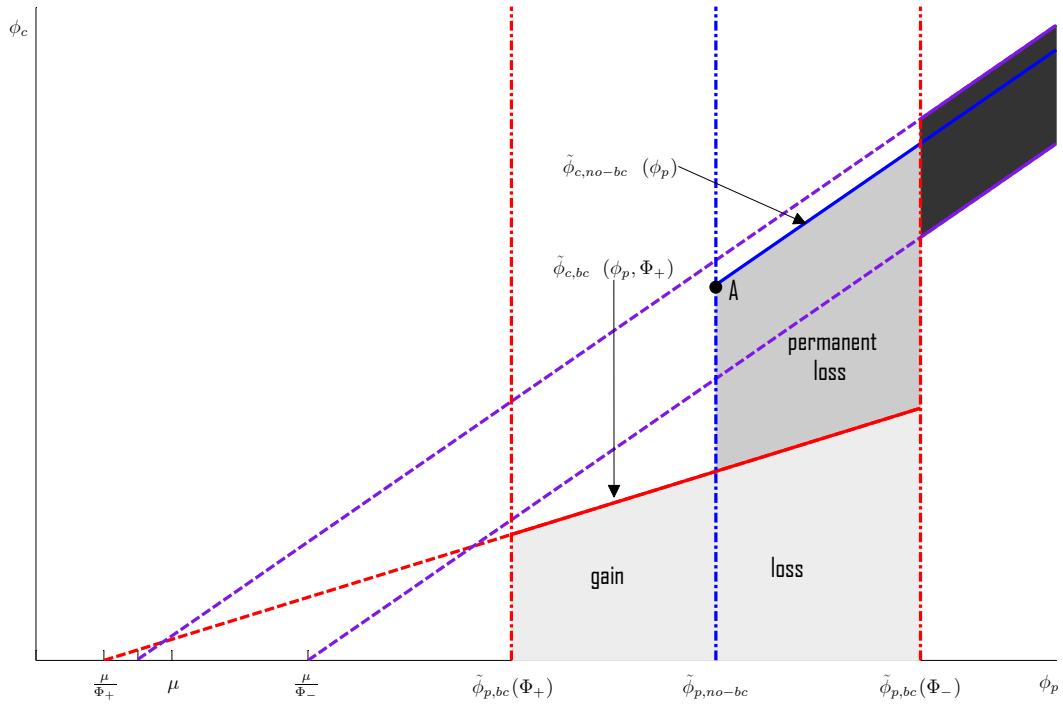
Notes: The shaded areas in this graph indicate the projects that (i) are productive enough to satisfy the efficiency requirement and (ii) have an entry cost that is low enough to make entry profitable. Point A corresponds to point A in the other figures.

Figure 2: Effect of arbitrarily small business cycles on boundary projects



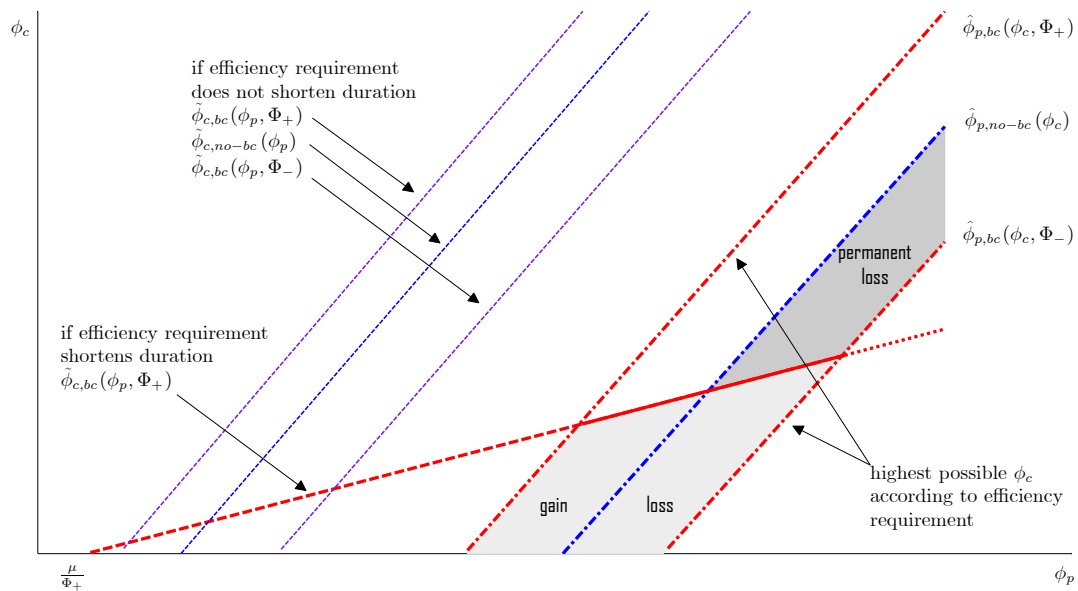
Notes: The top panel plots the NPV when there are no business cycles and when there are arbitrarily small business cycles for projects that are just affected by or are just not affected by inefficiencies. The bottom panel plots the difference between the two NPV values. Point A corresponds to point A in the other figures.

Figure 3: Projects affected by business cycles



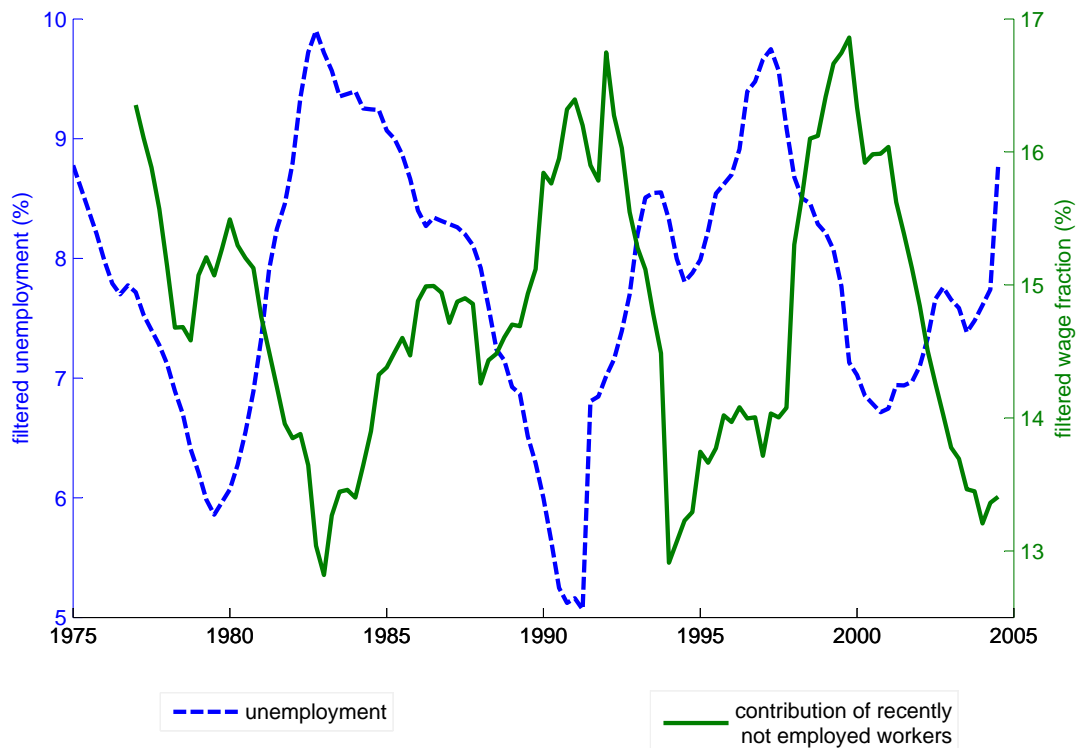
Notes: The shaded areas in this graph indicate the projects that are affected by business cycles. Light grey: Cyclical projects that operate during a boom and do not operate during a recession. Projects in the "gain" ("loss") area never (always) operate in a world without business cycles. Grey: Cyclical Projects that can overcome inefficiencies during a boom, but their entry costs are too high to make entry worthwhile given that inefficiencies will force exit during a recession. Black: Timed-entry projects. Point A corresponds to point A in the other figures.

Figure 4: Projects affected by business cycles in the presence of financial frictions



Notes: The shaded areas in this graph indicate the projects that are affected by business cycles. Light grey: Cyclical projects that operate during a boom and do not operate during a recession. Projects in the "gain" ("loss") area never (always) operate in a world without business cycles. Grey: Cyclical Projects that can overcome inefficiencies during a boom, but their entry costs are too high to make entry worthwhile given that inefficiencies will force exit during a recession. Point A corresponds to point A in the other figures.

Figure 5: Fraction of total wages earned by recently non-employed workers



Notes: This graph plots the German unemployment rate (left-side axis) together with the fraction of total wages earned by workers that recently (i.e., in the last 24 months) had a "non-employment spell" (right-side axis). Both series are quarterly averages of monthly series. The series are the HP-filtered series using a smoothing coefficient equal to 10^5 plus the mean.

Table 1: Impact of arbitrarily small business cycles for projects at the boundary

	$\rho = 0.875$	$\rho = 0.9167$	$\rho = 0.975$
cyclical projects; $\phi_p = \tilde{\phi}_{p,\text{no-bc}}$			
weak-inequality cyclical project with $\phi_c \leq \tilde{\phi}_{c,\text{bc}}$			
$\hat{\mu} = 0.2$	[35.8%,40.0%]	[40.0%,44.1%]	[40.0%,62.1%]
$\hat{\mu} = 0.43$	[25.5%,28.5%]	[28.5%,31.4%]	[28.5%,44.3%]
strict-inequality cyclical project with $\phi_c \leq \tilde{\phi}_{c,\text{bc}}$			
$\hat{\mu} = 0.2$	[-40.0%,0%]	[-40.0%,0%]	[-40.0%,0%]
$\hat{\mu} = 0.43$	[-28.5%,0%]	[-28.5%,0%]	[-28.5%,0%]
weak-inequality cyclical project with $\tilde{\phi}_{c,\text{bc}} \leq \phi_c \leq \tilde{\phi}_{c,\text{no-bc}}$			
$\hat{\mu} = 0.2$	[0%,35.8%]	[0%,44.1%]	[0%,62.1%]
$\hat{\mu} = 0.43$	[0%,25.5%]	[0%,31.4%]	[0%,44.3%]
timed-entry project; $\phi_c = \tilde{\phi}_{c,\text{no-bc}}$			
$\hat{\mu} = 0.2$	[0%,0%]	[0%,0%]	[0%,0%]
$\hat{\mu} = 0.43$	[0%,0%]	[0%,0%]	[0%,0%]

Notes: This table gives the welfare losses corresponding to a marginal increase in Δ_{Φ_p} starting at $\Delta_{\Phi_p} = 0$ for individuals with a project for which either $\phi_p = \tilde{\phi}_{p,\text{no-bc}}$ or $\phi_c = \tilde{\phi}_{c,\text{no-bc}}$. $\hat{\mu}$ is the value of μ relative to the value of ϕ_p , thus, $\mu = \hat{\mu}\phi_p$. Each cell contains the smallest and the largest outcome found across the possible values for ϕ_c .

Table 2: Impact of business cycles for projects at the boundary

	$\rho = 0.875$	$\rho = 0.9167$	$\rho = 0.975$
cyclical projects; $\phi_p = \tilde{\phi}_{p,\text{no-bc}}$			
weak-inequality project with $\phi_c \leq \tilde{\phi}_{c,\text{bc}}$			
$\hat{\mu} = 0.2$	[35.4%,39.7%]	[39.7%,43.8%]	[39.7%,62.0%]
$\hat{\mu} = 0.43$	[25.1%,28.2%]	[28.2%,31.1%]	[28.2%,43.9%]
strict-inequality project with $\phi_c \leq \tilde{\phi}_{c,\text{bc}}$			
$\hat{\mu} = 0.2$	[-40.4%,0%]	[-40.4%,0%]	[-40.4%,0%]
$\hat{\mu} = 0.43$	[-28.9%,0%]	[-28.9%,0%]	[-28.9%,0%]
weak-inequality project with $\tilde{\phi}_{c,\text{bc}} \leq \phi_c \leq \tilde{\phi}_{c,\text{no-bc}}$			
$\hat{\mu} = 0.2$	[0%,35.4%]	[0%,43.8%]	[0%,62.0%]
$\hat{\mu} = 0.43$	[0%,25.1%]	[0%,31.1%]	[0%,43.9%]
timed-entry project; $\phi_c = \tilde{\phi}_{c,\text{no-bc}}$			
$\hat{\mu} = 0.2$	[-0.19%,0%]	[-0.16%,0%]	[-0.08%,0%]
$\hat{\mu} = 0.43$	[-0.19%,0%]	[-0.16%,0%]	[-0.08%,0%]

Notes: This table gives the welfare losses corresponding to an increase in Δ_{Φ_p} from 0 to 0.007 for individuals with a project for which either $\phi_p = \tilde{\phi}_{p,\text{no-bc}}$ or $\phi_c = \tilde{\phi}_{c,\text{no-bc}}$. $\hat{\mu}$ is the value of μ relative to the value of ϕ_p , thus, $\mu = \hat{\mu}\phi_p$. Each cell contains the smallest and the largest outcome found across the possible values for ϕ_c .

Table 3: Welfare costs of business cycles

	$\rho = 0.875$	$\rho = 0.9167$	$\rho = 0.975$
Simple model			
no entrepreneurs; $\phi_{c,\min} = 0$; $\frac{Y_{\text{cyclical}}}{Y} = 1.16\%$			
$\tilde{\mu} = \bar{\mu} = 0.2$	0.18%	0.28%	0.79%
$\tilde{\mu} = \bar{\mu} = 0.43$	0.13%	0.20%	0.56%
no entrepreneurs; $\phi_{c,\min} = 0$; $\frac{Y_{\text{cyclical}}}{Y} = 3.1\%$			
$\tilde{\mu} = \bar{\mu} = 0.2$	0.49%	0.74%	2.12%
$\tilde{\mu} = \bar{\mu} = 0.43$	0.34%	0.52%	1.50%
Extended model			
no entrepreneurs; $\phi_{c,\min} = \tilde{\phi}_{c,\text{no-bc}}/5$; $\frac{Y_{\text{cyclical}}}{Y} = 3.1\%$			
$\tilde{\mu} = \bar{\mu} = 0.2$	0.71%	1.22%	15.5%
$\tilde{\mu} = \bar{\mu} = 0.43$	0.50%	0.86%	10.7%
with entrepreneurs; $\phi_{c,\min} = 0$; $\frac{Y_{\text{cyclical}}}{Y} = 3.1\%$			
$\tilde{\mu} = \bar{\mu} = 0.2$	0.91%	1.38%	3.95%
$\tilde{\mu} = \bar{\mu} = 0.43$	0.64%	0.98%	2.80%
with entrepreneurs; $\phi_{c,\min} = \tilde{\phi}_{c,\text{bc}}/5$; $\frac{Y_{\text{cyclical}}}{Y} = 3.1\%$			
$\tilde{\mu} = \bar{\mu} = 0.2$	1.41%	2.47%	34.6%
$\tilde{\mu} = \bar{\mu} = 0.43$	0.99%	1.74%	23.9%

Notes: The table reports the permanent percentage increase in GDP needed to make the welfare of agents living in the economy with business cycles equal to the welfare of agents living in the economy without business cycles. $\phi_{c,\min}$ is the lower bound of the distribution of ϕ_c . Y_{cyclical}/Y is the amount of output that is earned by cyclical projects in a boom relative to total output. $\tilde{\mu}$ and $\bar{\mu}$ are averages for the amount produced (in terms of leisure and/or home production) if the project does not operate as a fraction of market production for projects with values of ϕ_c below and above $\tilde{\phi}_{c,\text{bc}}$, respectively.