Discussion of 'Searching for Prosperity'
by Michael Kremer, Alexei Onatski, and James Stock

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October 2001

ABSTRACT

Kremer, Onatski, and Stock (KOS) criticize twin peaks dynamics in the evolution of cross-country income dynamics. They suggest instead convergence to a single peak at high incomes, with a prolonged transition when polarization and inequality increase. This article makes three points. First, the data are as consistent with a twin peaks characterization as they are for KOS's preferred description—in KOS's own analysis as well as across other studies. Second, the substantive economic message is identical in both twin peaks and KOS views: the global poor are substantial and will continue so—whether for centuries or for infinity is nitpicking. Finally, getting the empirics right matters greatly for theories of economic growth.

Keywords: convergence, discretization, distribution dynamics, growth, inequality, Markov chain, polarization, stochastic kernel, transition probability, twin peaks

JEL Classification: C23, D30, F43, O11, O57

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In their paper, Kremer, Onatski, and Stock (hereafter KOS) criticize the *twin peaks* characterization of cross-country incomes, previously developed in Bianchi (1997), Desdoigts (1999), Paap and van Dijk (1998), Quah (1993a,b), Quah (1996b), Quah (1997), and elsewhere. That characterization describes an emergent tendency for the cross-country distribution of per capita incomes to converge to a limit distribution having two clusters, one at the high end of the income distribution, another at the low. With high probability already-rich countries remain rich; currently-poor countries remain poor; middle-income countries become either rich or poor, depending. Over time these tendencies reinforce.

Such a picture should be contrasted with the alternative where all countries become eventually comparable to one another, where there is no long-run global divide across the deprived and the well off, where the poor converge to the rich.

This empirical work on economic growth—where growth and distribution are considered jointly and simultaneously—contrasts with that using cross-country growth regressions. The differences are sharp in the alternative economic issues at stake, in the alternative economic hypotheses examined, and in the alternative economic models motivated by the different empirics (see, e.g., Durlauf and Quah, 1999). It is this research on growth and distribution that KOS address.

1 General Background

As with all limiting or asymptotic statements, the right perspective on twin peaks is not literal—that the characterization takes literally an *infinite* number of steps to obtain, or that the two modes are identical twins—but instead that the characterization provides a useful approximation.[1] The issue is, Useful for what economic questions?

Quah (1996b, 1997) takes those questions to address the incipient tendency for the world income distribution to to cluster into and polarize across subgroups of rich and poor. Such dynamics suggest economic mechanisms for growth different from more standard ones such as: "Should a country accumulate more of this or that factor of production?"

Instead, the empirical patterns suggest two other key hypotheses. First, multiple steady states and thresholds matter, as documented empirically and theoretically in Azariadis and Drazen (1990), Durlauf and Johnson (1995), Galor and Zeira (1993), and Quah (1996a). Probalistically, countries above a certain threshold cluster around a high-income growth path; those below, a low-income growth path. Second, mechanisms of explicitly directed cross-country interaction—trade, exchange of ideas, technology transfer—are important drivers for economic growth (e.g., Coe and Helpman, 1995; Eaton and Kortum, 1999; Keller, 2001; Quah, 2001a). Getting to be in a high-performing group of countries aids one's own growth performance.

KOS challenge the twin peaks characterization, and suggest an alternative hypothesis to explain their preferred empirical cross-country pattern of growth. Their bottom-line empirical finding is that, given

[1] Asymptotic analysis in econometrics and statistics often falls victim to the same conceptual misunderstanding. As one example, take functional central limit theory in unit root asymptotics: This theory isn't useful just when a time series process has a root exactly equal to 1.0 and a researcher has an infinite, or even very large, number of observations. Indeed, even when neither of those conditions holds, such a theory provides a good approximation for answering many inferential questions of interest (see, e.g., Stock, 1991).

the data, many other limiting descriptions cannot be rejected either. Some of their point estimates continue to indicate twin-peakedness in the limit distribution. However, their preferred point estimate implies a single-peaked limit distribution, with most of its mass at the high end of the incomes range. However, that limit is reached only after centuries. During the prolonged transition, polarization and inequality rise.

In this discussion, I provide some further background and motivation to the KOS analysis, and criticize the way they present their empirical findings. At this level of analysis, there is little hard evidence against their preferred explanation for the cross-country dynamics, just as they present no hard evidence against the threshold models explaining twin peaks that they criticize. The data are consistent with a broad range of possibilities. Indeed, KOS's final policy prescription—that countries should learn from those who are successful—is very much in the spirit of some hypotheses previously advanced in twin peaks research (Quah, 1997).

In a Fukuyama-type "End of History" scenario, with capitalism and liberal democracy manifestly the regimes by which economic success is to be universally achieved, all countries eventually converge to the peak at high incomes (actually, then just the average income). Leader, already successful economies experience no loss from having others be similarly capitalistic liberal democracies. However, some successful policies might require the explicit cooperation of all involved parties—trade and technology transfer (more prosaic than exporting Big Fukuyama-type Ideas) are obvious examples. Then, there is an importing country and an exporting country; there is a country that receives new technology, there is one that transmits new technology. These cases, however, pose further questions: Why should the countries already rich *let* those emerging economies emulate their own hard-earned success? What do they get out of it? What are the economic forces and incentives that lead to particular patterns of polarization and inequality across the world?

2 KOS Contribution

KOS examine the statistical significance and robustness of the twin peaks description in three principal ways. First, they provide an explicit statistical test of restrictions on the functional form of the implied limit density: a "twin peaks" shape in that density can be specified as a collection of inequality restrictions. They implement this idea for a special case, that of tridiagonal transition probability matrices.

Second, KOS consider what the estimated transition probability matrices imply, not in ergodic limits but instead in real-time, long-horizon (but finite) dynamics. Third, KOS allow Markov transitions at five-year intervals, as well as annual frequencies. They find that then the higher-income peak becomes larger, and that projecting forwards the historical transition probability matrices, convergence to the limit can be slow. During the transition, measures of polarization and inequality can rise.

KOS's statistical analysis does not reject that the limit distribution is unimodal or uniform or has monotone density or any of a range of interesting possibilities. They find some point estimates for the limit distribution that, although imprecise, remain twin-peaked. Thus, although they don't describe their results quite this way, their statistical methods do not reject that the limit distribution is twin-peaked. Which characterization for the limit density one should favor remains an open question. My own bet currently rests with the twin-peaked point estimates as central tendencies, with positive probability that something else might transpire. KOS prefer what they call a "rosy long-run" forecast displaying a large high-income peak, obtained by taking transition steps at five-year rather than annual frequencies: Such a point estimate must be correspondingly noisy, with even fewer observations informing the estimate. I describe below why I don't believe such an estimate has greater legitimacy and confidence.

Next, KOS find that transitions are slow, so that from where the world was in the 1980s, the cross-section distribution could take centuries to converge to the ergodic limit. That the relevant time horizon extends to hundreds of years was also previously described in the transition-time analysis in Quah (1996a), using continuous stochastic kernels. But two things are important here: First, along the centuries-long transition period, inequality and polarization increase, implying long-lived twin peaks. Second, however long the cross-country distribution takes to converge to its ergodic limit (if single-peaked), emerging bimodality is already observable in actual point-in-time distributions from the 1980s (Bianchi, 1997; Quah, 1997). Both of these reinforce a twin peaks view of the world.

Finally, KOS use their findings to motivate an alternative description of policy and economic growth: In that view, economies search for unknown, successful policies—or as KOS put it, until other policies become too costly to experiment with further. Once located, such policies comprise an absorbing state, so that many or all economies, after a period of experimentation and subsequent successful discovery, converge to the same steady state. In KOS's analysis, such a dynamic process can be socially improved if economies learn from those already more successful or more advanced.

3 Method

All analyses of the dynamics of distributions have the same central underlying structure. The most transparent form—used by both KOS and Quah (1993a)—is as follows.

Divide up the space of possible income values into a collection of discrete cells: The cross-country income distribution is then a histogram defined over the chosen discretization. No country need belong forever in a given cell—it simply moves across cells as its income evolves.

Denote at time t, for integer $t \geq 0$, the histogram p_t, defined over the discretization just specified. Then describe the transition of individual countries across these cells by a *transition probability matrix* Q, that collects together entries $Q_{j,k}$ describing the probability of moving from the j-th cell to the k-th over a single time period, conditional on being in the k-th cell. Transition probabilities might, in general, also be time-varying, whereupon we would need to consider an entire sequence of matrices

$$\{Q(t) : t = 0, 1, 2, \ldots\}.$$

We hereafter specialize to the time-invariant case. By construction this admits the following:

$$p_{t+1} = Q' p_t \implies p_{t+s} = (Q^s)' p_t \quad \forall t, s \geq 0. \tag{1}$$

Under regularity conditions (1) implies

$$p_\infty \stackrel{\text{def}}{=} \lim_{s \to \infty} p_s = (Q^\infty)' p_0 \quad \forall p_0, \tag{2}$$

i.e., the limit of the iterative scheme (1) exists and is unique, and is therefore independent of initial condition p_0. When (2) holds, we can also write the final implication as:

$$p_\infty = Q' p_\infty. \tag{3}$$

The p_∞ defined in (2) and described in (3) admits at least two different interpretations. Equation (2) asserts p_∞ is the limit, as time unfolds sequentially and infinitely, of the real-time evolution (1). Alternatively, equation (3) says p_∞ is just another particular (atemporal) characteristic (functional) of matrix Q, namely one satisfying the static equation (3): no dynamics need be involved, so that it is then nonsense to say p_∞ depends on what happens in the infinite future.

Up until now, Q has been just a non-negative matrix whose rows sum to unity, satisfying regularity conditions to ensure (2). If, further, Q is diagonalizable, i.e., there exists a full-rank matrix V and a diagonal matrix Λ such that

$$Q = V \Lambda V^{-1}$$

(where, without loss of generality, Λ has its diagonal entries λ_j sorted by absolute value, largest to smallest), then:

1. the first eigenvalue λ_1 equals 1;

2. Q^∞ is rank 1;
3. p_∞ is the left eigenvector for Q corresponding to eigenvalue λ_1; and
4. the asymptotic speed of convergence to p_∞ is dictated by the absolute value of the second largest eigenvalue $|\lambda_2| < 1$; or, more precisely, convergence to the limit distribution occurs in geometric powers of λ_2.

From equation (3) and implication 3. above, given Q, the limit distribution p_∞ satisfies a *linear* equation in the space of probability vectors. However, individual elements in p_∞ depend *nonlinearly* on the entries in Q, and indeed in the general case have no closed-form analytic expression in individual entries of Q.

For notational economy, I hereafter drop the ∞ subscript in denoting the limit distribution when doing so is without ambiguity.

KOS consider when Q is tridiagonal, i.e., when all entries are zero except along the diagonal and the first super- and sub-diagonals. Then, by inspection, vector equation (3) becomes the following sequence of scalar relations:

$$p_1 = (1 - Q_{1,2})p_1 + Q_{2,1}p_2 \tag{4}$$

$$\text{for } j \geq 2 \quad p_j = Q_{j-1,j}p_{j-1} + Q_{j,j}p_j + Q_{j+1,j}p_{j+1}, \tag{5}$$

where, if the number of cells is finite, there is an endpoint equation for the last p_j that parallels equation (4). Iterating on the recurrence relation (5) from initial condition (4) gives:

$$\frac{p_j}{p_{j+1}} = \frac{Q_{j+1,j}}{Q_{j,j+1}}, \quad j = 1, 2, \ldots. \tag{6}$$

While equation (6) makes apparent how KOS's tridiagonal assumption has simplified the analysis, both equations (3) and (6) of course give entries of p that are *nonlinear* functions of individual entries in Q. The difference is that the relation in (6) has a closed-form representation, but not so in (3) in general. Nonlinearity, however, is common to both, just as when viewed as relations across variables in a space of distributions, both are at the same time also simply linear equations.

Within this structure, KOS carry out both asymptotic and Monte Carlo statistical tests on a range of hypotheses, including:

1. p_∞ equal to the observed p in 1992;
2. p_∞ is monotone increasing from the low income end;
3. p_∞ is flat.

KOS study the distribution sequence p_t generated from Q and p_0 by equation (1), and assess the implied dynamics of different indexes of polarization and inequality. They vary the assumed frequency of transitions from annual to multiple years.

4 Empirical Findings and Evaluation

KOS's principal empirical findings are threefold. First, with transitions assumed to occur annually, KOS estimate p_∞ to be twin-peaked, but cannot reject a null hypothesis that it is single-peaked at the high end of the incomes range. Do the data, therefore, prefer one peak to two? No: Although KOS do not explicitly say this, their statistical methods *guarantee* that they also cannot reject a twin-peaked null hypothesis—because that is what their point estimate turns out to be. Their methods do not allow discrimination across the two possibilities, or indeed any of a range of possible p_∞'s.

Reichlin (1999) has given an alternative depiction of the sensitivity of p_∞ estimates. Suppose that in Table 1 of KOS's paper, we alter only rows 1 and 5, changing Q_{21} to 0.04 from 0.03 and Q_{54} to 0.02 from 0.01. Then p_∞ changes from being bimodal to being exactly flat, i.e., p_∞ has 0.2 in each of its five entries. If we increase Q_{21} further to 0.05, then p_∞ becomes unimodal in the middle, i.e.,

$$p_\infty = \begin{pmatrix} 0.17 & 0.22 & 0.22 & 0.22 & 0.14 \end{pmatrix}.$$

It might therefore appear that p_∞ is highly sensitive to small changes in individual entries in Q—that, of course, is what KOS are documenting. However, what is a *small* change here? A change of Q_{21}

from 0.3 to 0.5 or of Q_{54} from 0.01 to 0.02 involves an absolutely small number. As is implicit in KOS's analysis, such changes would also be statistically insignificant. However, each is also a doubling of something very important—the fraction of nation-state economies transiting across the range of possible incomes. Viewed thus, this sensitivity becomes something informative, rather than something undesirable. Marginally different growth performances, here and there, can profoundly alter our views of inequality across the world.

The second principal finding is that with transitions assumed to occur at five-year periods, KOS estimate p_∞ to be single-peaked. This is their preferred characterization, off of which they subsequently discuss an economic model "consistent with the facts". However, contrary to practice in earlier parts of their paper, KOS here provide no statistical tests on characterizations for other than the point estimate. Do the data reject a twin-peaked p_∞, when transition frequencies are set to five-year periods? I suspect not.

Part of KOS's justification for preferring the five-year period can be described, using terminology different from theirs, as diagonal under-prediction in successive iterations of the annual periodicity model. This effect had early on been widely observed in applications of Markov chain methods in economics and sociology. However, that earlier literature led not to the use of multi-period transitions, but instead to mixture models, structured transition probabilities, and semi-Markov processes, all of which can replicate diagonal under-prediction. Examples of these include Blumen et al. (1955), Singer and Spilerman (1976a,b), and Spilerman (1972a,b). Which route should one take for studying cross-country distribution dynamics? KOS's argument seems, to me, not entirely definitive. Those arguments suggest semi-Markov models as much as they do 5-year transition Markov ones. In any case, until empirical research on growth data is undertaken using these ideas and the findings evaluated in comparison to KOS's proposal, no a priori argument can be compelling.

Third, KOS find that the generated sequence $p_t = (Q')^t p_0$ implies increasing polarization and inequality in the transition. This finding of course strengthens the conclusions that had come out of, among others, Quah (1996b, 1997), powerfully stated in more general form in Pritchett (1997).

5 Alternative Economic Models

KOS select two of their empirical findings, and suggest an economic model that fits them. The two empirical findings are, first, that the long-run distribution is single-peaked at the high-end of the incomes range; and, second, that polarization and inequality rise along the transition path.

KOS's model has economies searching for good policies. When an initially unsuccessful economy has not yet latched onto a policy that it can live with, economic performance is lackluster. However, when that economy has achieved a good enough policy—one where subsequent further experimentation is deeemd too costly—it stays with that policy. If all good-enough policies and economic structures are reasonably similar, then this mechanism produces in the long run a peak at the high end of the cross-country income distribution. Along the transition path, however, polarization and inequality increase—at least part of the time—as the newly-successful economies part company with the pool of still only experimenting ones.

The story is intriguing. However, a number of other growth models produce very similar dynamics. For a first example, reinterpret the model of firms in an industry from Aghion and Howitt (1998, Ch. 3) to be instead one of entire economies across the world (as in Howitt, 2000). Independent Poisson clocks determine a takeoff that then makes that economy the frontier or lead economy, leapfrogging all others originally ahead of it. The result from this dynamic process is a unique power-law limit distribution, that is single-peaked at the high or low end of the incomes range, depending on parameters. Since the limit distribution is unique, if the initial cross section is relatively more tightly compressed, then the transition path necessarily displays increasing inequality, and possibly increasing polarization as well. Details here—such as leapfrogging to exactly the frontier—should not, of course, be taken literally. Quite likely, jumping when indicated to do so by the Poisson mechanism to somewhere in the lead group rather than exactly to the frontier, or yet other reasonable changes in assumptions, would preserve the flavor of these results.

Lucas (2000) provides a model where countries learn from leader countries, where technology transfer is mechanistic. The time to when such transfer can occur is again determined by a Poisson counter; before then, countries simply languish at zero or low growth. Eventually, with probability one, all countries will have taken off, after which convergence occurs to a point mass limit distribution.

All three models—Aghion-Howitt's, KOS's, and Lucas's—display distribution dynamics where the cross section converges eventually to a single peak, while the transition exhibits a rich array of possibilities. Notable among those transition cross-country distributions is one with twin peaks, with the peak at a low income level representing that group of countries still awaiting a take-off or learning-success event.

To be clear, many growth models easily generate a single peak in the long-run cross-country distribution: Indeed, the simplest neoclassical growth model already does that. The subtlety here instead is in producing twin-peaked transitional dynamics that can show first increasing dispersion and then eventually convergence to a mass point in the cross-section distribution.

Solow (1997) provides an interesting contrast. He considers a learning and growth model where success feeds upon past success; failure feeds upon past failure. This model gives a semi-Markov persistence to the distribution dynamics, and the cross section distribution then displays a cluster of rich and a cluster of poor in the long-run limit, not just in the transition.

6 Other Related Literature

To add to the references already given in KOS and above, the ideas and findings in some other articles are relevant as well. This section collects together lessons from a range of diverse sources.

In addition to considering transitions over horizons other than just one year, Quah (1993a) also analyzed second-order Markov chains. He suggested that the twin-peaked characterization remained across a range of variations in the dynamic-distributional law of motion. Along similar lines, Quah (1993b) varied the definition of the transition matrix cells, so that they were not arbitrarily fixed a priori, but allowed to adapt as the cross-section distribution evolved.

Jones (1997) and Reichlin (1999) described how the limit distribution is sensitive to the necessarily arbitrary grouping from discretization. Indeed, a well-known property is that the Markov property itself can be distorted from inappropriate choice of discrete cells (e.g., Chung, 1960). Bulli (2001) considered optimal ways to discretize the income space while preserving the Markov property. She finds the twin peaks property to be dramatically strengthened at the optimal discretization.

Bianchi (1997) tested directly the nonparametric twin peaks characterization of the cross-section distribution, using Silverman-inspired bootstrap tests (Silverman, 1981, 1983). This procedure fails to exploit the dynamic information in transition probabilities. Nevertheless, Bianchi (1997) rejected unimodality in favor of a bimodal description. Paap and van Dijk (1998) used smooth parametrized mixture distributions to obtain results similar to those in Bianchi (1997). Quah (1997) examined visually the twin peaks hypothesis in nonparametric estimates of transition probabilities—so that no arbitrary discretization into cells was then necessary. That work claimed supporting evidence for emerging twin-peakedness. It circumvented the arbitrary-discretization difficulty, but provided no formal statistical tests.

Ultimately, there is a very simple and straightforward reason for going this continuous-kernel route and no longer studying empirical models with discrete cells arbitrarily defined by the researcher: Twin peaks in a discrete analysis is *always* an artifact of cell definition. Any mode in a histogram can be defined away by subdividing further the range of income values within the cells concerned. Without some natural definitions specific to a particular question, no researcher can be satisfied with a twin peaks description for a histogram version of the underlying distribution. Therefore, for the twin peaks question alone, although we have as yet no theory of inference for it, the continuum analysis in Bianchi (1997), Desdoigts (1999), Paap and van Dijk (1998), and Quah (1997) must be more informative than research with discretely-defined income cells.

This does not say we should ignore research such as in KOS or in Quah (1993a). It is simply to be clear on what one can and cannot learn from that research. Overly obsessing on the details will not be useful; more informative is combining the potential lessons there with those from other ways of examining the data.

7 Conclusions and Extensions

As the discussion in Section 5 suggests, the pattern of cross-country income distribution dynamics matters greatly—not just in the obvious sense of wellbeing and economic performance, but also for which growth models are useful for understanding the world's growth performance.

Should KOS's empirical findings change someone's view informed by, say, Quah (1996b) or Quah (1997) on world income distribution dynamics? As KOS themselves admit, the data and statistical models are such that for now we place too much weight on a researcher's prior beliefs. Having thought over KOS's arguments and evidence, I find that my posterior beliefs remain unshifted from the twin peaks prior I held before I encountered their work.

KOS have provided a substantive way forwards for thinking about the evidence more rigorously, and suggested an intriguing economic hypothesis for yet further work on the topic. But their empirical results alone don't convince, for reasons I have given above: First, on KOS's own grounds, their evidence and statistical results do not overwhelm sufficiently to change one's views. Second, the transition path analysis shows increasing inequality and polarization for hundreds of years, suggesting long-lived twin peaks, even when the limit distribution's point estimate is unimodal. Finally, discrete transition probabilities have not been the only way researchers have studied this twin peaks emergence, and the weight of evidence there, it seems to me, is on the side opposite to KOS's.

With all this discussion on subtleties, the key issue in this work might have been clouded over. It is useful therefore to repeat it: The second, low-income cluster in the twin peaks characterization is the global poor. Do KOS's empirical investigations suggest the global poor do not matter? No. While that second peak might not appear in KOS's preferred point estimate for the limit distribution, KOS cannot statistically reject its presence. Moreover, in KOS's own analysis, the prolonged transition over centuries to the long run carries exactly increased polarization and inequality, much as is suggested by the twin peaks dynamics that KOS criticize. Thus, that the global poor are substantial and will continue so, whether for centuries or for the infinite long run, is the common message, shared by both KOS's and twin peaks work.

Some technical extensions in this line of work are interesting. Although KOS do not do this, one can invert the mapping that takes a transition probability matrix to its limit density. Restrictions on the latter then imply, in turn, restrictions on the transition probability matrix directly. While conceptually straightforward, this is admittedly analytically very difficult, with the inverse mapping a set-valued correspondence, not just a function.

An obvious next step on the intellectual agenda is to obtain a theory for appropriate statistical inference in the smooth, nonparametric kernels case considered in Quah (1997), paralleling what KOS have done for the discrete case in Quah (1993a). But while it would close the logical gap in our scientific understanding, it is unclear what the substantive payoff will be for inference on economic growth. The cross-country incomes data are already stretched in statistical informativeness, even with relatively parametrized models. Completely nonparametric work, further adding in the near certainty of cross-country correlatedness, must push the informational content and statistical degrees of freedom in these cross-country incomes data to complete exhaustion.

This last observation, however, might usefully tell us where to go next. Both KOS's suggestions ("learning from each other's experience") and my discussion point to models of explicit cross-country interaction, not models of a representative economy from which the researcher then infers how the cross section must look. Economic and statistical dependence would then manifest both dynamically and cross-sectionally in parallel. Such work on cross-country interactions would usefully extend Coe and Helpman (1995), Helpman (1993), Keller (2001), Quah (1997), and Quah (2001a,b). An explicit model would permit yet tighter parametrization in these cross-country distributional dynamics. With luck, more can then be learnt from the data.

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