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Life in unequal growing economies

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Danny Quah*
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October 2008

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ABSTRACT

This paper characterizes the dynamic welfare implications of varying patterns of growth and inequality. In their impact on poverty, historically observed changes in aggregate growth overwhelm those in inequality. However, their welfare implications are more nuanced. For a reasonable range of utility functions the average person on earth is certainly better off living in the faster-growing more unequal economies, given historical patterns of growth and inequality. However, the relative benefits to that experience differs across countries, even within just the fast-growing ones. While the welfare impact of aggregate growth continues to be the most significant, that of inequality overwhelms that of business cycles.

Keywords: diffusion, distribution dynamics, income distribution, inequality, Markov, resolvent operator, transition density

JEL Classification: D31, D90, O10, O57

Communications to: D. Quah, Economics Department, LSE, Houghton Street, London WC2A 2AE

Tel: +44.20.7955.7535, Email: dq@econ.lse.ac.uk
(URL) http://econ.lse.ac.uk/staff/dquah/

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1 Introduction

That income inequality might go hand in hand with economic efficiency is a question long debated in economics, and at many different levels, ranging from frontier research to introductory university courses (e.g., Okun, 1975). In one traditional accounting, seeking egalitarianism dampens individual incentives, and thus lowers productivity and increases social inefficiency.

And while economic performance concerns much more than just economic growth, many studies on the relation between growth and inequality are significantly colored by the possibility that a tradeoff might exist between efficiency and equality.

Since at least Kuznets (1955) economists have sought theoretical and empirical basis to this hypothesized tradeoff in growth and inequality. Aghion, Caroli, and García-Peñalosa (1999) and Bénabou (1996) provide useful overviews. This large body of work contains at least two different and logically distinct strands of analysis.

One line of research seeks to establish whether inequality causes growth, or vice versa; and if so by how much (e.g., Banerjee and Duflo, 2003; Barro, 2000; Forbes, 2000; Galor and Zeira, 1993; Persson and Tabellini, 1994). A second strand takes no stance on causality but asks instead how the joint evolution of inequality and growth matters for yet other outcomes (e.g., Bourguignon, 2003; Córdoba and Verdier, 2005; Dollar and Kraay, 2002; Quah, 2003; Sala-i-Martin,

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2006; Schultz, 1998). The current paper falls within this second class of research. It asks what the observed dynamics in growth and inequality—whatever their underlying causes—imply for the welfare of people living in different unequal growing societies.

New in the recent literature, although not always explicitly acknowledged, is that we can now ask questions about growth and inequality globally—across over 120 countries and 6 billion people—not just within a single economy (Deininger and Squire, 1996; Milanovic, 2002, 2005). This means not just having more data to estimate yet more precisely the same given relation between inequality and growth within a hypothetical economy, say. Instead, global growth and inequality themselves can now be investigated directly.

Income inequality worldwide is an order of magnitude greater than within any single economy. In year 2000 the UK economy's income distribution had its 99th percentile earning 7 times median income. In contrast, even after adjusting for purchasing power parity the world's 99th percentile earned an amazing *56 times* world median income. In year 1993 the richest 1% of humanity earned as much income in total as the poorest 57% (Milanovic, 2002). Between 1988 and 1993 even as world per capita income increased 6%, average income in the bottom 5% fell by one quarter, while that in the top 5% rose by an eighth. In contrast, on average (i.e., using regression analysis) the typical economy has incomes in the poorest parts of its income distribution rise approximately one for one with its per capita income (Dollar and Kraay, 2002).

Not only do numbers like these indicate the overwhelming importance of inequality across economies, they suggest how what happens within a typical economy gives a misleading picture for what happens to the entire income distribution worldwide.

Bourguignon (2003), Dollar and Kraay (2002), Quah (2003), and Sala-i-Martin (2006) analyzed how, given historical patterns of changes in inequality, growth affects poverty. That research characterized in different ways the distribution dynamics in income implied by statis-

Table 1.1 Income and population in China and India, 1981–2004 (World Bank, 2008).

	GDP per capita (PPP 2000 Intl$)			Population (10^6)	
	1981	2004	Annual growth	1981	2004
China	792.8	5418.5	8.36%	993.9	1296.2
India	1228.8	2907.1	3.74%	702.8	1079.7

Table 1.2 Inequality in China and India, 1981–2004 [UNU (2007), PovcalNet July 2007]. UNU (2007) reports different estimates and so, in certain cases, a range of figures is available. By contrast, PovcalNet provides at most a single estimate and so, when available, the PovcalNet figures are always given after the last "/" in the Table.

	Gini coefficient (%)	
	1981	2004
China – Urban	15.0/16.1/16.7/18.5	32.9/34.0
China – Rural	23.9/23.9/25.1/25.0	33.4/34.0
China – All	29.1/	44.9/
India – Urban	34.1/33.3	34.7/37.6
India – Rural	30.1/30.1	26.3/30.5
India – All	31.4/	36.0/

tics on economic growth and income inequality.

Some flavor of those results is given in Tables 1.1–1.4. The first two tables summarize the dynamics of aggregate growth and inequality in China and India between 1981 and 2004. Those two economies held a combined population amounting to one third of the world. Tables 1.1 and 1.2 show how China was the faster-growing economy of the two at the same time that income inequality there rose significantly, while India grew slower but roughly maintained income inequality.[1] While there might be uncertainty over precise figures,

[1] PovcalNet provides no Gini coefficients for China and India,

Table 1.3 \$1-poverty in China and India, 1981–2004. Source: Chen and Ravallion (2007).

	$\underline{x}=1;\ HC_{\underline{x}}\ (N_{\underline{x}}, 10^6)$	
	1981	2004
China	63.8 (633.7)	9.9 (128.4)
India	51.8 (363.7)	34.3 (370.7)

Table 1.4 \$2-poverty in China and India, 1981–2004. Source: Chen and Ravallion (2007).

	$\underline{x}=2;\ HC_{\underline{x}}\ (N_{\underline{x}}, 10^6)$	
	1981	2004
China	88.1 (875.8)	34.9 (452.2)
India	88.9 (624.9)	80.4 (867.6)

the general trend is apparent.

If the world comprised just China and India, one would conclude (correctly) that fast-growing economies also become more unequal. But Tables 1.3 and 1.4 show a yet different implication of these statistics. An underlying income distribution implies both inequality measures like the Gini index and a so-called *poverty headcount*, the fraction of the population living below a given threshold, say, 1 Intl\$ a day. That fraction depends, of course, on the threshold income level taken to designate poverty, but is a number that can be readily calculated whatever that choice of threshold.

Table 1.3 shows the fraction of population living on less than 1 Intl\$ a day in the two economies in 1981 and 2004. Table 1.4 does the same for a threshold of 2 Intl\$ a day. In 1981, 634 million

except when divided into rural and urban. The Gini coefficients for all of China and India are taken therefore only from UNU (2007): for 1981 China averaging the 1980 and 1982 estimates; for 2004 China, using the unique 2003 estimate. For India the Povcal figures are for 1983 and 2004.5, in place of 1981 and 2004.

Chinese lived on less than 1 Intl$ a day; by 2004 the number living at those income levels had declined to between 128 million. In 1981 the number of $1-poor amounted to 64 percent of the total population; by 2004 the fraction was down to 9.9 percent. In other words, despite the increase in inequality noted earlier in Table 1.2, China's rapid growth lifted out of poverty hundreds of millions of its citizens. Table 1.4 shows these dynamics carry over to situations of less extreme ($2-)poverty; moreover, a lot of the Chinese population remained poor even in 2004.

Calculations like these show the consequences for poverty dynamics implied by varying patterns of growth and inequality. The current paper takes the natural next step and asks what such growth and inequality patterns signify instead for welfare more generally.

Section 2 presents the welfare calculations for growth and inequality, and discusses the conceptual and technical differences between them and similar ones undertaken elsewhere. Section 3 illustrates the relative importance of growth and inequality by numerical examples. Under reasonable calibrations, the welfare impact of inequality overturns the relatively sanguine impact of Tables 1.1–1.4. Moreover, in even the simplest cases such effects overwhelm, by orders of magnitude, the welfare costs of business cycles.

Section 4 then calibrates this welfare analysis on historical observations on growth and inequality worldwide. Section 5 concludes. Section 6 provides proofs for the analytical results.

The mathematics used in the analytical discussion—continuous-time stochastic processes, infinitesimal generators, resolvent operators, stochastic kernels, and so on—might be unfamiliar to some potential readers of this paper. Moreover, in the probability literature such presentation is relatively dispersed across texts such as Gihman and Skorohod (1975), Karlin and Taylor (1981), and Rogers and Williams (1987). Quah (2007) provides a self-contained, integrated summary.

2 Analytical framework

This section describes an analytical framework, slightly more general than that used later in this paper but illustrating all the key ideas and conceptually no more difficult.

Suppose an agent can live in economy A or economy B. Denote economy-wide characteristics—per capita income, income inequality, dynamic patterns of mobility, degree of urbanization, quality of the environment, and so on—by $\mathcal{Z}_A$ for A, and similarly $\mathcal{Z}_B$ for B.

Let the agent's consumption be C and suppose the agent's utility function U is defined over own consumption C and economy-wide characteristics $\mathcal{Z}$. That the latter appears in the agent's utility function does not, of course, require the agent to be public-spirited, egalitarian, or envious (nor does it rule these out). Understanding, say, the dynamics of the income distribution—part of $\mathcal{Z}$—matters to the agent simply because such information conditions her own individual likelihood of subsequent economic success.

If the agent's consumption is C_A when living in economy A but C_B when living in B, and the agent prefers B to A,

$$U(C_A, \mathcal{Z}_A) < U(C_B, \mathcal{Z}_B),$$

define the *welfare gain to* B *from* A to be that number Ψ such that

$$U(\,[1 + \Psi]\, C_A, \mathcal{Z}_A\,) = U(\,C_B, \mathcal{Z}_B\,). \tag{1}$$

The welfare gain Ψ is the fraction of consumption that just compensates the agent for having to live under $(C_A, \mathcal{Z}_A)$ instead of $(C_B, \mathcal{Z}_B)$.

Obviously, societies A and B need not be literally two distinct economies. Instead, A and B might refer to just a single nation operating under two alternative hypothesized regimes. When the problem is dynamic the pair $(C, \mathcal{Z})$ will represent timepaths or probability laws of motion. Even then, however, the analysis below will still construct the welfare gain as a scalar so that Ψ then bears interpretation as a compensating change in permanent or lifetime consumption.

Lucas (2003) has shown how such a framework, depending on choice of $\mathbb{Z}$, usefully organizes a broad range of macroeconomic research. A study of the welfare implications of business cycles and growth was famously performed in Lucas (1987), and in spirit is close to the analysis in this paper.

Here, the $\mathbb{Z}$ characteristics of interest are income inequality and aggregate economic growth. As already mentioned, this paper does not seek to disentangle the mechanism relating these two variables; it simply takes as given that they jointly evolve through time. Nonetheless, however, the relation between inequality and growth cannot be totally arbitrary. Being macroeconomic variables, inequality and growth have dynamics that result from aggregating individual income dynamics. Thus, by construction, the dynamics of individual circumstances need to be consistent with those of observable aggregates. In turn, agents' views about the evolution of their individual consumption and income are conditioned on the dynamics in those macroeconomic aggregates.

This paper constructs, for equation (1), $\mathbb{Z}$ probability laws of motion in income distributions and growth trends to evaluate the welfare consequences of living in different unequal growing economies. Such a stochastic dynamic description automatically embeds in a unified framework the answers to a range of questions: Are changes in inequality permanent or transitory? How much does mobility matter? Is duration or depth of poverty more important? Because the framework focuses on welfare dynamics, it flags how *current* inequality by itself need be neither informative nor important.

The current study shares two key features with work such as Blundell and Preston (1998), Heathcote, Storesletten, and Violante (2005), and Krueger and Perri (2004, 2006). First, it evaluates inequality through the latter's impact on individual agents' welfare: inequality is not, in itself, an object of direct interest. Second, it goes beyond point-in-time snapshots of disparities to look at inequality dynamics, or more generally the dynamics of the entire income distribution.

At the same time this study also differs from previous research in two significant ways. The current work concerns inequality set against long-run (average) growth trends, rather than stationary business cycle fluctuations or even permanent changes in the levels of income or consumption (as, for instance, in the random walk models used in Blundell and Preston (1998), Heathcote, Storesletten, and Violante (2005), or Krueger and Perri (2004, 2006)). Consequently, techniques and results differ. Because a case for using continuous-time stochastic process models here can be made (and will be in section 2.1) the analysis draws naturally on the theory of resolvent operators, rather than employing computational techniques more typically used in economics.[2] For comparison, however, Section 2.2 also provides more standard calculations for several simple discrete-time models.

Next, like all studies of cross-country and global inequality, this work uses data on *income* distributions. Extensive distributional or cross section data on consumption—the right variable for equation (1)—are available only for a small number of countries. A proper evaluation should then work out welfare consequences by first backing out the implied consumption dynamics—incorporating, say, smoothing and insurance arrangements—from varying patterns of income inequality and growth. Such an analysis is intricate, however, and might be more usefully done after some basic facts are first established for a wide range of countries, using the simplification employed here.[3] This paper, to focus on experiences across many different

[2] The mathematics of resolvent operators is available in probability texts such as Gihman and Skorohod (1975), Karlin and Taylor (1981), and Rogers and Williams (1987) but in a form dispersed relative to what's needed here. Quah (2007) contains a comparatively self-contained exposition, to aid reading the remainder of this paper.

[3] Heathcote, Storesletten, and Violante (2005) and Krueger and Perri (2004, 2006) carry out the more complicated analysis for US consumption and wage inequality dynamics.

economies, identifies income with consumption as a working assumption. However, since the technical machinery developed here works for arbitrary income processes, we can also easily investigate the effects of unobserved consumption dynamics diverging in specific ways from observed income timepaths.[4] Krueger and Perri (2004, 2006) and Slesnick (2001) note that for the US, between 1980 and 2003, consumption inequality rose much less than income inequality—this feature, by itself, is easily incorporated in the calculations that follow. I will return to this again in Section 3.

The work closest to the current one is that of Córdoba and Verdier (2005), who also use the same welfare framework (1). They consider consumption inequality in the US and, separately, the cross-country distribution of per capita incomes. But while their model contains a dynamic specification for consumption, they provide only steady-state welfare calculations, so that no statements are available on the effects of mobility. Moreover, their analysis uses only an autoregression in logs for consumption and thus (log) standard deviations to calibrate inequality, rather than—as given below—a number of alternative stochastic process characterizations for inequality dynamics and a range of different summaries of the cross-section distribution.

2.1 Continuous time stochastic process models

Bourguignon (1974) and Merton (1975) early on argued for the use of continuous-time stochastic process models in economic growth. Analytically, for one, characterizations of results are then convenient: suppose, for instance, the variables in a model are diffusion processes—almost all functions of diffusions remain diffusions while pretty much only linear transformations of linear sequences remain

[4] For instance, under certain assumptions the distribution dynamics of consumption can be deduced to comprise independent Brownian motions, regardless of the fine structure in the income distribution dynamics.

linear.

But the economic logic in continuous-time growth models is also compelling. First, relative to economic growth or income inequality over long stretches of time, the actions undertaken by or the incidents befalling economic agents every quarter (or even every year) occur with frequency high enough relative to the relevant horizon that they might as well be instantaneous. Those occurrences are then usefully modelled as perturbations on isolated epochs in a continuous timepath. Second, in studying economic growth a researcher might wish to analyze unanticipated take-offs or sudden disasters in consumption or income. Such events are not easily captured in discrete time structures but they are naturally modelled as discontinuities in a timepath traced in continuous time, i.e., as, perhaps, Poisson jumps or semi-Markov chain transitions overlaid on a continous-path, diffusion process.

Assume time is continuous and agents live for t in $[0, \infty)$. Write $C_j(t)$ for the consumption of individual j at time t. Suppose that the long-run conditional expectation

$$\xi = \lim_{t \to \infty} E\left[\log(C_j(t)/C_j(0)) \times t^{-1} \mid C_j(0)\right] \tag{2}$$

is invariant across j, i.e., that the economy has some constant long-run proportional growth rate in consumption, the same for everyone. For example, in balanced growth steady state for a Solow growth model this constant ξ would be the rate of technical progress. More generally, ξ is the aggregate growth rate of the economy.

Individual consumption fluctuations are described by j-indexed stochastic processes:

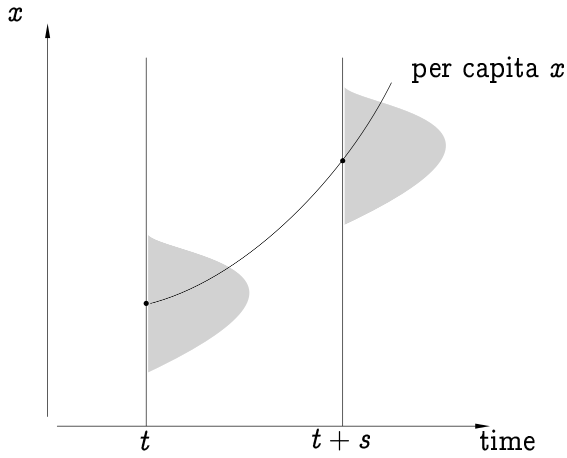
$$Z_j(t) \stackrel{\text{def}}{=} \log\left[C_j(t)e^{-\xi t}\right] \tag{3}$$

so that we can write

$$C_j(t) = e^{Z_j(t)}e^{\xi t}.$$

The collection Z describes consumption differences across people. Thus ξ represents aggregate growth and Z inequality. Strictly, the

Fig. 2.1 Growth and inequality Z fluctuations around a ξ growth path



collection Z of course characterizes everything about the cross-section distribution but referring to it as simply inequality denotes better the analysis's intent. If doing so misleads, at least it does so in an innocuous way. Fig. 2.1 illustrates the structure.

Equations (2)–(3) define Z by using ξ to transform C. They do not assume inequality and growth are functionally independent—both Z and ξ are induced from a single underlying mechanism for C. Moreover, while these equations restrict Z's trend, they do not require that Z be stationary with bounded variance. For instance, Z might be Brownian motion independent across j, and inequality would then be always increasing through time even as consumption obeys the trend growth in equation (2). However, if Z has bounded support then so too will C at any fixed time t.

Assume that the utility experienced by an agent in a growing but unequal society varies with that agent's consumption timepath, but not that of anyone else. Thus, an agent's utility does not depend on aggregate social indicators such as the shape, dispersion, or other characteristics of the economy-wide consumption distribution.[5]

For concreteness assume that the agent's utility, conditional on time-t information, is

$$W_j(t) = E_t \left[\int_t^{\infty} e^{-(s-t)\rho} U(C_j(s))\, ds \right], \qquad \rho > 0, \tag{4}$$

where the positive discount rate ρ will be required, in Theorem 2.1 later, to be sufficiently large. Say that the instantaneous utility function U shows constant relative risk aversion (CRRA) when

$$U(c) = \frac{c^{1-R} - 1}{1 - R}, \qquad R \geq 1. \tag{5}$$

In social welfare analysis of income inequality the coefficient R is known as the *inequality aversion parameter* (see, e.g., Cowell, 1995, Ch. 3).

How does welfare (4) vary with growth and inequality? Obtaining tractable answers to this question constitutes the remainder of this section. The first result, Theorem 2.1, provides a compact representation for W in the general case when inequality Z is Markov and time-homogeneous. Theorems 2.2 and 2.3 specialize to when Z is Brownian Motion and a (discrete) Markov chain respectively. The language is terse and relies on some familiarity with the mathematics in Quah (2007)—readers uninterested in the details can focus on just the final expressions in each case. Those concluding calculations will in turn be developed further in Section 3 to follow.

Theorem 2.1 *Let consumption* $\{ C(t) : t \geq 0 \}$*, obeying* (2)*, be Markov with time-homogeneous transitions; and lifetime utility* (4) *have* U

[5] This individualistic restriction is consistent with the *social welfare function* approach to income inequality—see, e.g., Cowell (1995, Ch. 3).

CRRA, as in (5). *Assume* $\rho > \max\{0, (1-R)\xi\}$. *Define*

$$\tilde{\rho} \stackrel{\text{def}}{=} \rho - (1-R)\xi;$$

$$\tilde{U}(z) \stackrel{\text{def}}{=} \begin{cases} (1-R)^{-1}e^{(1-R)z} & \text{for } R \neq 1; \\ z & \text{otherwise;} \end{cases}$$

and denote by $\mathbf{R}_\lambda$ *the resolvent for the inequality process* $Z(t) = C(t)e^{-\xi t}$. *Then conditional on* $C(0)$,

$$W(0) = (\mathbf{R}_{\tilde{\rho}}\tilde{U})(\log C(0)) + \begin{cases} -(1-R)^{-1}\rho^{-1} & \text{for } R \neq 1; \\ \xi\rho^{-2} & \text{otherwise.} \end{cases}$$

In words, up to an additive constant, lifetime utility equals the resolvent operator for $Z(t) = C(t)e^{-\xi t}$, growth-discounted consumption, evaluated at a modified instantaneous utility function $\tilde{U}$ and for the resolvent λ set to a suitably adjusted discount rate $\tilde{\rho}$. The resolvent expression $\mathbf{R}_{\tilde{\rho}}\tilde{U}$ can be either explicitly calculated or conveniently characterized for a range of inequality processes.[6]

Denote by B standard Brownian Motion, i.e., a continuous random function having independent increments and with $B(t) \sim \mathsf{N}(0, t)$ for all $t \in [0, \infty)$.

Theorem 2.2 *Assume the hypotheses of Theorem 2.1 and let inequality* Z *be Brownian Motion with variance parameter* $\sigma^2 > 0$, *i.e.,* $Z(t) = \sigma B(t)$. *Define the function* $r : \mathbb{R}^4 \to \mathbb{R}$ *by*

$$r(R, \rho, \xi, \sigma) = (1-R) \times \left[\rho - (1-R)\xi - \frac{1}{2}(1-R)^2\sigma^2\right]. \quad (6)$$

Provided $\tilde{\rho} > \frac{1}{2}(1-R)^2\sigma^2$ *then*

$$(\mathbf{R}_{\tilde{\rho}}\tilde{U})(z) = \begin{cases} r(R, \rho, \xi, \sigma)^{-1} \times e^{(1-R)z} & \text{for } R \neq 1; \\ \rho^{-1}z & \text{otherwise.} \end{cases}$$

[6] See, e.g., Rogers (1997).

Discontinuities in the consumption timepath, to capture sudden discrete upwards jumps or conversely economic disasters, can be modelled using Markov chains. The specialization of Theorem 2.1 to these can be stated as the following.

Theorem 2.3 *Assume the hypotheses of Theorem 2.1 and let inequality Z be a Markov chain whose values realize, with probability 1, on*

$$\overline{z}_0, \overline{z}_1, \overline{z}_2, \ldots, \overline{z}_{M-1}, \quad \overline{z}_m < \overline{z}_{m+1};$$

and with time-homogeneous transition probability matrices

$$P = \{\, P_t : t \geq 0 \,\}.$$

Let $\mathbf{G}$ be the infinitesimal generator of the semigroup P,

$$\mathbf{G} = \lim_{t \downarrow 0} \frac{P_t - I}{t},$$

and define the vector

$$\overline{U} = \begin{pmatrix} \tilde{U}(\overline{z}_0) & \tilde{U}(\overline{z}_1) & \cdots & \tilde{U}(\overline{z}_{M-1}) \end{pmatrix}^{\top}.$$

Then

$$(\mathbf{R}_{\tilde{\rho}}\tilde{U})(\overline{z}_m) = \left[(\tilde{\rho} - \mathbf{G})^{-1}\overline{U} \right](m), \quad m = 0, 1, \ldots, M-1.$$

Theorem 2.3 calculates utility for someone facing a continuous-time Markov chain in inequality using nothing more than matrix inversion.

While useful for modelling abrupt discontinuity, a Markov chain such as in Theorem 2.3, if its support is bounded $|\overline{z}_m| < \infty$ and if growth is positive $\xi > 0$, eventually has the consumption for all in the population almost surely permanently exceeding any fixed poverty threshold.

Combining Theorems 2.1–2.2 give that when Z is Brownian motion and $\log C(0) = Z(0) = z$ then

$$W(0) = \begin{cases} r(R, \rho, \xi, \sigma)^{-1} \times e^{(1-R)z} - (1-R)^{-1}\rho^{-1} & \text{for } R \neq 1; \\ \rho^{-1}z + \xi\rho^{-2} & \text{otherwise} \end{cases} \tag{7}$$

with multiplier r from (6).

[. . .]

2.2 Some discrete time examples

Discrete-time counterparts to the Section 2.1 calculations on welfare provide a direct check and further intuition to those continuous-time results.

Suppose time is discrete, $t = 0, 1, 2, \ldots$, and that preferences for agent j at time t are described by:

$$W_j(t) = E_t\left[\sum_{s=0}^{\infty} \delta^s U(C_j(s))\right], \qquad \delta \in (0, 1), \tag{8}$$

with instantaneous utility U again CRRA as in (5),

$$U(c) = \frac{c^{1-R} - 1}{1 - R}, \qquad R > 0.$$

Suppose consumption is:

$$C_j(t) = Z_j(t)\xi^t, \qquad \xi \geq 1, \tag{9}$$

for each j, i.e., a trending, lognormally-distributed first-order autoregression with the underlying growth rate ξ common across the population.

Let the cross-sectional differences $Z_j(t)$ satisfy:

$$Z_j(t) = \overline{z}_j \epsilon_j(t), \tag{10}$$

so that initial consumption conditional on the stochastic disturbance $\epsilon_j(0)$ is

$$C_j(0) = \overline{z}_j \epsilon_j(0).$$

Let ϵ be stationary and follow:

$$\log \epsilon_j(t) = -\frac{1}{2}(1+\alpha)^{-1}\sigma_j^2 + \alpha \log \epsilon_j(t-1) + \nu_j(t), \qquad (11)$$

with

$$|\alpha| < 1 \quad \text{and} \quad \nu_j(t) \sim \text{iid } \mathsf{N}(0, \sigma_j^2). \qquad (12)$$

Equations (11)–(12) imply ϵ_j is Markov. In the unconditional distribution

$$E \log \epsilon_j(t) = -\frac{1}{2}(1-\alpha^2)^{-1}\sigma_j^2$$

and

$$\text{Var} \log \epsilon_j(t) = (1-\alpha^2)^{-1}\sigma_j^2,$$

implying

$$\begin{aligned} E\epsilon_j(t) &= \exp\left[E \log \epsilon_j(t) + \frac{1}{2}\text{Var} \log \epsilon_j(t)\right] \\ &= \exp\left[-\frac{1}{2}(1-\alpha^2)^{-1}\sigma_j^2 + \frac{1}{2}(1-\alpha^2)^{-1}\sigma_j^2\right] = 1 \end{aligned}$$

and

$$\text{Var}\, \epsilon_j(t) = \exp\left[(1-\alpha^2)^{-1}\sigma_j^2\right] - 1,$$

increasing in σ_j^2.

Equation (9) then gives the unconditional expectation

$$EC_j(t) = \overline{z}_j \xi^t, \qquad t \geq 0,$$

i.e., $\overline{z}_j$ parametrizes the (unconditional) expected level of j's consumption profile and gives the initial value $EC_j(0) = \overline{z}_j$.

More generally, $\epsilon_j(t)$ is conditionally lognormal:

$$\log \epsilon_j(t_0+t) \mid \epsilon_j(t_0) \sim \mathsf{N}\left(\alpha^t \log \epsilon_j(t_0) - \frac{1-\alpha^t}{1-\alpha^2}\frac{\sigma_j^2}{2},\ \frac{1-\alpha^t}{1-\alpha^2}\sigma_j^2\right) \text{ for any } t>0. \quad (13)$$

The value to (8) can then be directly calculated:

Theorem 2.4 *Assume utility has the form* (5) *and that consumption evolves from the combination of growth and inequality given in* (9)–(12). *There are two cases:*

i. When $R=1$,

$$W_j(0;\epsilon_j(0),\xi,\overline{z}_j,\sigma_j^2,\alpha) = (1-\delta\alpha)^{-1}\log \epsilon_j(0) + \frac{\delta}{(1-\delta)^2}\log\xi + \frac{1}{1-\delta}\log\overline{z}_j - \frac{\delta}{(1-\delta)(1-\delta\alpha)(1+\alpha)} \times \frac{\sigma_j^2}{2}.$$

ii. Otherwise,

$$W_j(0;\epsilon_j(0),\xi,\overline{z}_j,\sigma_j^2,\alpha) = (1-R)^{-1}\left[-\frac{1}{1-\delta} + (\overline{z}_j)^{1-R}\times \left\{\epsilon_j(0)^{1-R} + e^{-(1-R)R(1-\alpha^2)^{-1}\sigma_j^2/2} \times \sum_{t=1}^{\infty}\delta^t\xi^{-(R-1)t}e^{D_1\alpha^t}e^{-D_2\alpha^{2t}}\right\}\right],$$

where

$$D_1 = (1-R)\left[\log\epsilon_j(0) + (1-\alpha^2)^{-1}\frac{\sigma_j^2}{2}\right] \text{ and}$$

$$D_2 = (1-R)^2(1-\alpha^2)^{-1}\frac{\sigma_j^2}{2} > 0.$$

2.3 Distributions and inequality

Stochastic process models of distribution dynamics describe income inequality as functions of distributional parameters. However, those parameters are not always the ones typically used for studying inequality.

The relation between distributional parameters and standard inequality indexes is not difficult to work out but is convenient, nonetheless, to have recorded in one place and in a unified notation. To that end, let $\mathcal{I}_G$ and $\mathcal{I}_\ell$ denote the *Gini coefficient* and the *log standard deviation*, respectively. These are not the only inequality indexes commonly used, but are the ones employed subsequently in this paper. These indexes *define* inequality and thus, by definition, increase (and intuitively so) with rising inequality. However, as we will see, since every inequality index folds an infinite-dimensional object—a distribution function—into a scalar, necessarily no one of them by itself can revealingly describe all conceivable changes in the underlying income distribution.[7]

Given a consumption or income distribution F call its mean or per capita value

$$\mathcal{E} \stackrel{\text{def}}{=} \int x \, d\mathsf{F}(x). \tag{14}$$

Record also the expectation of the logarithm,

$$\mathcal{E}_\ell \stackrel{\text{def}}{=} \int \log x \, d\mathsf{F}(x).$$

The Gini coefficient is

$$\mathcal{I}_G \stackrel{\text{def}}{=} \left[2^{-1}\mathcal{E}\right]^{-1} \int \left(\mathsf{F}(x) - \frac{1}{2}\right) x \, d\mathsf{F}(x) \tag{15}$$

(see, e.g., Cowell, 2000). The Gini index takes values in $[0,1]$. In (15) the index can be seen to be the correlation between (income

[7] This is true of *all* inequality indexes and is, of course, the subject of a massive literature; see, e.g., Cowell (1995, 2000).

or consumption) value and the cumulative distribution.[8] The log standard deviation is

$$\mathcal{I}_\ell = \left[\int (\log x - \mathcal{E}_\ell)^2 \, d\mathsf{F}(x) \right]^{1/2} . \tag{16}$$

When distribution F is lognormal,

$$d\mathsf{F} = (2\pi\theta_2)^{-1/2} \cdot x^{-1} \times \exp\left\{ -\frac{1}{2\theta_2}(\log x - \theta_1)^2 \right\} dx, \quad \theta_2 > 0, x > 0,$$

i.e., when the log of income has mean θ_1 and variance θ_2 then the inequality indexes $\mathcal{I}_G$ and $\mathcal{I}_\ell$ are particularly simple:

$$\mathcal{I}_G = 2 \times \mathsf{F}_{\mathsf{N}(0,1)}(\,{\theta_2}^{1/2}/\sqrt{2}\,) - 1$$

where $\mathsf{F}_{\mathsf{N}(0,1)}$ is the standard normal CDF, and

$$\mathcal{I}_\ell = \sqrt{\theta_2}\,.$$

In particular, the model in Theorem 2.4 has inequality, conditional on information at time $t = 0$, given by:

$$\mathcal{I}_G(t) = 2 \times \mathsf{F}_{\mathsf{N}(0,1)} \left(\left[\frac{1 - \alpha^t}{1 - \alpha^2}\, \sigma_j^2 \right]^{1/2} / \sqrt{2} \right) - 1, \tag{17}$$

independent of $(\xi, \overline{z})$.

3 Calibration

The results in Section 2, while explicit and available in closed form, can provide better intuitive understanding through numerical illustration. Providing such examples is the purpose of this section.

[8] The better-known, alternative description is that the Gini index is twice the area between the 45-degree line and the Lorenz curve; again see Cowell (1995).

Table 3.1 Growth matters. $\mathcal{I}_G = 0.32,\ \alpha = 0,\ \delta = 0.98$: For small rises in underlying growth rates, people will sacrifice relatively large cuts in permanent consumption levels

$\bar{\xi} = 1.02$		Compensating Ψ (%)			
ξ	R	1	2	5	10
1.005		107	58	26	14
1.01		62	32	14	7
1.03		−38	−19	−8	−4
1.04		−61	−32	−14	−8
1.05		−76	−42	−19	−10
1.06		−85	−49	−22	−12
1.07		−90	−54	−25	−14

3.1 Lognormal autoregression

First take the discrete-time lognormal autoregression in Theorem 2.4. Tables 3.1–3.5 show the workings of three factors of principal interest: growth ξ, inequality $\mathcal{I}_G$, and persistence α (i.e., the opposite of mobility). For this illustration all the tables take as base case per capita growth of 2% a year and inequality with Gini index set to 0.32. This last is the value in 1980 of the Gini index in both China and India and approximately that in the US (0.35).

Some features in these tables are intuitive and easy but they obviously do not constitute the principal message: In Table 3.1, uniformly across utility parametrizations R, growth lower than the base case $\xi = 1.02$ needs the compensating Ψ positive whereas growth higher needs the compensating Ψ negative; in Table 3.2 inequality higher than the base case $\mathcal{I}_G = 0.32$ requires compensating Ψ positive; and so on. Similarly, higher R means greater curvature in the utility function and so increases sensitivity to inequality but reduces that to growth.

It is of course not the signs on the entries in these Tables that are critical but their magnitudes. To understand those, begin with how

Table 3.2 Distribution matters but, when R is low, not as much. Base Var ϵ sets inequality $\mathcal{I}_G = 0.32$; growth $\xi = 1.02$; persistence $\alpha = 0$; and discount $\delta = 0.98$. To compensate for rises in inequality, permanent consumption levels have to increase some

	Compensating Ψ (%)					
R $\quad \mathcal{I}_G$	0.28	0.30	0.34	0.36	0.38	0.40
1	−4	−2	2	5	8	11
2	−8	−4	5	10	16	23
5	−19	−10	12	27	46	69
10	−34	−20	26	63	113	185

these numbers come about in a simple case. Theorem 2.4 for logarithmic utility ($R = 1$) says that the compensating Ψ in a neighborhood of the base case sets

$$\frac{\partial \log \overline{z}}{\partial \log \xi} = -\frac{\delta}{1-\delta}$$
$$\frac{\partial \log \overline{z}}{\partial\,[\,(1-\alpha^2)^{-1}\sigma^2\,]} = \frac{1}{2}\frac{1-\alpha}{1-\delta\alpha}.$$

Moreover, the steady-state ($t \to \infty$) version of equation (17) gives:

$$\frac{\partial \mathcal{I}_G}{\partial\,[\,(1-\alpha^2)^{-1}\sigma^2\,]} = \frac{1}{\sqrt{2}}\left[(1-\alpha^2)^{-1}\sigma^2\right]^{-1/2} \mathsf{f}_{\mathsf{N}(0,1)}\left(\left[\frac{\sigma^2}{1-\alpha^2}\right]^{1/2}/\sqrt{2}\right), \quad (18)$$

where $\mathsf{f}_{\mathsf{N}(0,1)}$ is the standard normal pdf. When $\delta = 0.98$ the growth effect $\partial \log \overline{z}/\partial \log \xi$ equals 50; when further $\alpha = 0$ the variance effect equals only 1/2 (this constitutes its upper bound in any case). Table 3.1 has in its column for $R = 1$ an approximately 50% compensating Ψ in the immediate neighborhood of the base case.

In general these multipliers, for logarithmic utility, are independent of where the agent is in the income distribution. For $R > 1$, take initial condition $\overline{z} = \log \epsilon = 1$, i.e., the agent is average when her utility is evaluated. The compensating Ψ in Table 3.1 falls rapidly as R increases: The greater the curvature in the utility function, the more steeply downweighted are consumption levels promised in the future from steady-state growth rates. The less Ψ then needs to compensate the agent.

Turning to Table 3.2 we first note that inverting equation (17) for the basecase $\mathcal{I}_G$ of 0.32 gives

$$\frac{\sigma^2}{1-\alpha^2} = 0.58.$$

Using this value on the right side of equation (18) gives 0.45 for that derivative. Then

$$\begin{aligned}\frac{\partial \log \overline{z}}{\partial \mathcal{I}_G} &= \frac{\partial \log \overline{z}}{\partial\,[\,(1-\alpha^2)^{-1}\sigma^2\,]} \times \left[\frac{\partial \mathcal{I}_G}{\partial\,[\,(1-\alpha^2)^{-1}\sigma^2\,]}\right]^{-1} \\ &\doteq \frac{1}{2} \times \frac{1}{0.45} \doteq 1.1\end{aligned}$$

Thus Table 3.2 has in its row for $R = 1$ a compensating Ψ that equals about 1.1 times the hypothesized change in the Gini index, an approximation that worsens the further $\mathcal{I}_G$ diverges from the basecase of 0.32: The compensating Ψ increases due to the curvature in the underlying relations. Looking down Table 3.2 Ψ becomes progressively larger: The more curved the utility function the more deeply inequality affects the individual. The sign of this relation is unsurprising; its magnitude, however, is.

Krueger and Perri (2004, 2006) have studied consumption and income inequality in the US from 1980 to 2003. They conclude that consumption inequality, as measured by the Gini index, rose from 0.23 to 0.27 whereas income inequality increased from 0.30 to 0.37. Obviously, consumption is smoothed relative to income. However, a change of 0.04 in the Gini index is nonetheless a large increase. Using models calibrated to the US economy and that look in greater detail at leisure and risk-sharing arrangements, Krueger and Perri (2004) conclude that the reduction in welfare due to consumption inequality amounts to at worst 6% in the level of permanent consumption. The multiplier $1.5 = 0.06/0.04$ is comparable to those for R between 1 and 2 in Table 3.2, although of course the model here is much simpler and strips out many interesting economic features.

In summary, two notable conclusions emerge. First, growth matters more than inequality in general. This is unsurprising and is much in line with the kinds of results uncovered in Tables 1.1–1.4 earlier. However, as utility become progessively more convex, this statement weakens correspondingly. Second, for typical values for the Gini index, the impact of even relatively small changes in inequality is appreciable. For instance, for $R = 5$, a worsening of inequality from a Gini index of 0.32 to just 0.34 requires a 12% compensating adjustment in the average level of consumption. One eighth of permanent consumption is extremely large: Lucas (1987) estimated the cost of business cycles to be less than only 0.1%, and that was with $R = 10$ (I will return to this below).

Next, put the magnitudes in these Tables in context by considering the example of China from Tables 1.1–1.4. Recall that there the Gini index increase from 0.29 to 0.45 and the per capita growth of 8.4% resulted, on net, in reducing the number in \$2-poverty by over 400 hundred million Chinese. Now, Tables 3.1 and 3.2 allow further conclusions. *(The following numbers need to be re-calibrated for a base Gini of 0.29 rather than 0.32.)* If the typical Chinese citizen discounted the future with discount factor $\delta = 0.98$ and had $R = 2$, and if before 1979 China's per capita growth had been 2%, then that citizen would have valued the 2004 unequal growing China at a 31% increase in lifetime consumption (54% from growth Table 3.1 and -23% from inequality Table 3.2). On the other hand, if his $R = 5$ then the result would have been equivalent to a net 44% *decline* in lifetime consumption.[9] How the Chinese historical timepath in growth and

[9] China's growth subsequently has, of course, been much higher, and therefore the net gain uniformly better than that discussed in

inequality is viewed depends, critically (if smoothly), on the shape of one's utility function, a message differing substantially from that of earlier work like that exemplified in Tables 1.1–1.4.

(The numbers in this paragraph need to be re-done using the new 2004 estimates.) Adding up the separate effects as just done is only for illustration; the actual change from historical experience of course combines these two effects simultaneously. Doing that calculation gives $\Psi = -58$ for $R = 1$, $\Psi = -22$ for $R = 2$, $\Psi = +25$ for $R = 5$, and finally $\Psi = +97$ for $R = 10$. The nonlinear interaction between growth and inequality gives numbers different from the additive discussion earlier, but not dramatically so.

Compared to the effects of business cycles estimated elsewhere, the inequality implications in Table 3.2 are large. For instance, Lucas (1987) estimates that had all US business cycle variability been eliminated the average citizen would benefit by only 0.1% of lifetime consumption. On the other hand that same representative agent would have benefited by 34% of lifetime consumption had the Gini index been reduced from only 0.32 to 0.28 (roughly speaking, from US inequality to Belgian, assuming growth did not change).

To understand this it is useful to return to the expression given in Theorem 2.4. When $\alpha = 0$ the conclusion there simplifies:

i. For $R = 1$,

$$W_j(0; \epsilon_j(0), \xi, \overline{z}_j, \sigma_j^2, 0) = \log \epsilon_j(0) + \frac{\delta}{(1-\delta)^2}\log\xi + \frac{1}{1-\delta}\log\overline{z}_j - \frac{\delta}{(1-\delta)^2} \times \frac{\sigma_j^2}{2}.$$

---

the text. More generally, had Chinese growth before 1979 been lower, or alternatively under-estimated after, or both then this welfare gain would be an under-estimate. It seems to me a not unreasonable lower bound for the changes occurring in that economy.

Table 3.3 Mobility matters. Initial $\epsilon(0)$ at 5th percentile of stationary distribution; base $\alpha = 0$; Var ϵ sets $\mathcal{I}_G = 0.32$. At a low initial level of consumption, to compensate for declines in mobility, permanent consumption levels have to rise (mostly). The relation is not monotone everywhere.

		Compensating Ψ (%)				
R	α	0.25	0.50	0.75	0.95	0.98
1		1	2	5	28	57
2		1	4	10	42	66
5		3	7	14	27	28
10		5	7	7	−8	−26

ii. For $R \neq 1$,

$$W_j(0; \epsilon_j(0), \xi, \overline{z}_j, \sigma_j^2, 0) = (1 - R)^{-1} \left[-\frac{1}{1-\delta} + (\overline{z}_j)^{1-R} \times \left\{ \epsilon_j(0)^{1-R} + e^{-(1-R)R\sigma_j^2/2} \times \frac{\delta \xi^{-(R-1)}}{1 - \delta \xi^{-(R-1)}} \right\} \right].$$

The experiment reported in Table 3.2 seeks that change in $\overline{z}$ that maintains a constant value in W as σ and thus $\mathcal{I}_G$ varies. In the expression for $R \neq 1$ such a calculation approximately (i.e., ignoring the product with $\epsilon_j(0)^{1-R}$) keeps invariant

$$(\overline{z})^{1-R} \times e^{-(1-R)R\sigma^2/2},$$

or

$$\Delta \log \overline{z} \doteq (R/2) \times \Delta \left(\sigma^2\right).$$

For US business cycles Lucas (1987) estimates σ to be 0.013. Then, even its doubling or quadrupling implies relatively small proportional change in lifetime consumption, $\Delta \log \overline{z}$. By contrast, for inequality, even a Gini coefficient of 0.32 already implies σ of 0.58, i.e., 45 times the corresponding number for business cycles!

Table 3.4 Mobility matters. Initial $\epsilon(0)$ at 50th percentile of stationary distribution; base $\alpha = 0$; Var ϵ sets $\mathcal{I}_G = 0.32$. At an average initial level of consumption, to compensate for declines in mobility, permanent consumption levels have to fall, although not by as much as in Table 3.5, where initial consumption is yet higher.

		Compensating Ψ (%)				
R	α	0.25	0.50	0.75	0.95	0.98
1		0	0	−1	−4	−8
2		0	−1	−2	−10	−17
5		−1	−3	−7	−28	−42
10		−2	−5	−11	−42	−63

Table 3.5 Mobility matters. Initial $\epsilon(0)$ at 95th percentile of stationary distribution; base $\alpha = 0$; Var ϵ sets $\mathcal{I}_G = 0.32$. At a high initial level of consumption, to compensate for declines in mobility, permanent consumption levels have to fall a lot.

		Compensating Ψ (%)				
R	α	0.25	0.50	0.75	0.95	0.98
1		0	−1	−4	−17	−28
2		−1	−2	−7	−26	−39
5		−2	−5	−12	−41	−59
10		−3	−6	−15	−50	−72

Finally, Tables 3.3–3.5 illustrate the impact of changes in mobility, holding constant the level of inequality. This is achieved by varying α but then adjusting σ so that $\mathcal{I}_G$ is fixed throughout. Intuition suggests, and indeed the Tables confirm, the initial position $\epsilon(0)$ matters greatly. If the agent is rich, high persistence enhances her welfare—her relatively high consumption can be maintained with high probability. The opposite holds, however, when the agent is initially poor—it is high mobility then that increases her well-being.

Tables 3.4 and 3.5 show that for initial conditions relatively high in the distribution (at 50% and 95% respectively in these two Tables), increases in α do indeed raise well-being. Compensating Ψ is negative throughout and increases in magnitude with α.

Table 3.3, on the other hand, shows positive Ψ, monotone increasing in α, except at high levels of R and α. When the agent is already poor, persistence is typically undesirable and has to be compensated for by increasing permanent consumption levels.

Why, however, is the relation not monotone at low initial levels of consumption? The reason is that in a dynamic stochastic model the autoregressive coefficient α regulates not just the expected timepath beginning from an initial condition, but also the growth rate of uncertainty conditional on current information (e.g., equation (13)). With inequality and the unconditional variance invariant (holding constant $(1-\alpha^2)^{-1}\sigma^2$), conditional uncertainty grows more slowly, the higher is persistence. Thus, other things equal, higher α or lower mobility *raises* well-being. When initial consumption is high, both the conditional uncertainty and expected timepath effects work in the same direction. Thus, the rich always prefer immobile societies.

The poor, on the other hand, dislike how high persistence keeps them poor but, conditional on that outcome, appreciate how it reduces uncertainty. Studying the proof of Theorem 2.4 (and in particular the expressions for D_1 and D_2), the threshold that defines those who are poor in this way is

$$\log \epsilon \leq \frac{1}{2}(1-\alpha^2)^{-1}\sigma^2.$$

For low ϵ people, whether the compensating Ψ is positive or negative when α increases will, in general, depend. The conditional uncertainty effect is significant only when people are sufficiently risk-averse, i.e., R is sufficiently high, as illustrated in Table 3.3 for α also sufficiently large.

These mobility effects are large. When, say, $R = 5$, persistence rising from 0 to 0.95 induces Ψ of -28% at the median, $+27\%$ at the 5th percentile, and -41% at the 95th percentile. At that curvature of instantaneous utility, a change of those magnitudes at the median or 5th percentile is equivalent to a change in the steady-state growth rate from 2% to 7% or a change in the Gini index from 0.32 to 0.36.

3.2 Markov chain

Next, turn to the continuous-time calculations. Calibrate the Markov chain in Theorem 2.3 to match global income distribution dynamics.

From the evidence in Björklund, Bratsberg, Eriksson, Jäntti, Naylor, Österbacka, Raaum, and Røed (2005) and Khor and Pencavel (2005),

[. . .]

3.3 Poisson overlays

A Poisson-related jump process can introduce dynamics corresponding to growth miracles and disasters.

[Calibration using Barro (2007), Hansen (2008) and Martin (2008)]

4 Empirics

This section provides some empirical context to the preceding analytical calculations.

Table 4.1 The importance of China. Source: Chen and Ravallion (2008) (PPP$ means constant 2005 international$).

	1981	1990	1999	2005
World GDP 10^{12} PPP$	26	35	45	56
per capita PPP$	5876	6704	7505	8662
World's $1-poor ($10^6$)	1904	1815	1695	1400
China's $1-poor ($10^6$)	835	683	447	208
Remainder (10^6)	1069	1132	1248	1192

An extreme form of utility U in the preceding sections might be one that penalizes extreme poverty but is then indifferent across incomes above a particular threshold. Then empirical evidence that bears on welfare analysis would include importantly only the evolution of extreme poverty.

Table 4.1 shows how world poverty—measured by the number of people living on less than PPP$1.25 a day—has changed in the last three decades.[10] In 1981 the world's population living in poverty numbered 1904 milion. By 2005 that number had fallen to 1400 million, a reduction of 504 million people (Chen and Ravallion, 2008).[11] Over the same period the population of China living in poverty fell from 835 to 208 million, a decline of 627 million. China singlehandedly lifted more people out of poverty than did the entire world.

Figs. 4.1–4.4 show four snapshots of the evolution of world poverty and economic growth, and provide a striking depiction of the signifi-

[10] I adopt here the practice, updated in Chen and Ravallion (2008), of using PPP$1.25 a day to define poverty. Numbers like these are subject to considerable debate and controversy. See, e.g., Asian Development Bank (2008); Bourguignon and Morrisson (2002); Chen and Ravallion (2004, 2008); Deaton (2005).

[11] These numbers use PPP indexes from the 2005 International Comparison Project (Asian Development Bank, 2008). In this time the world's population rose from 4.5 to 6.5 billion (World Bank, 2008).

Fig. 4.1 World Poverty 1981. The horizontal axis measures per capita income in thousands of PPP constant year-2005 US$; the vertical axis, millions of people with incomes less than PPP constant year-2005 US$1.25 a day. See text for abbreviations and further details.

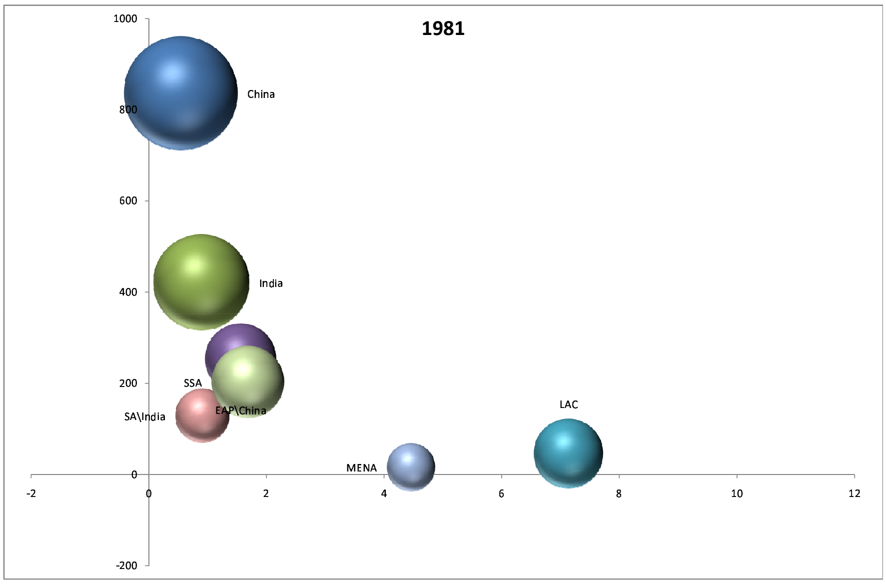


Fig. 4.2 World Poverty 1990. See notes for Fig. 4.1

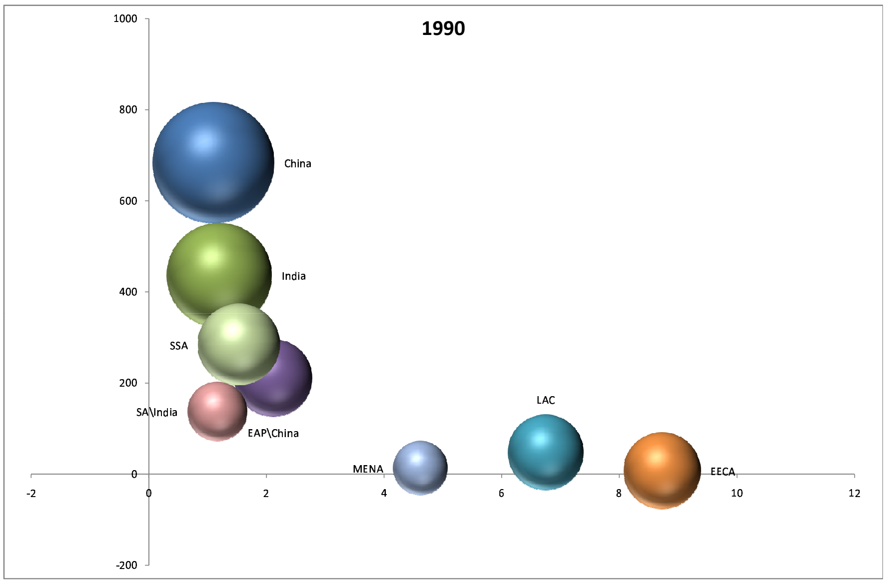


Fig. 4.3 World Poverty 1999. See notes for Fig. 4.1

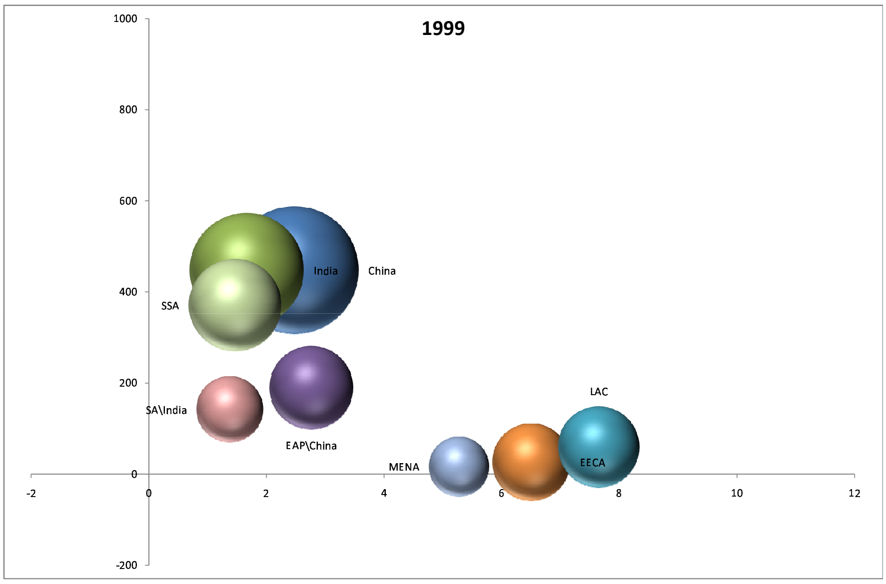
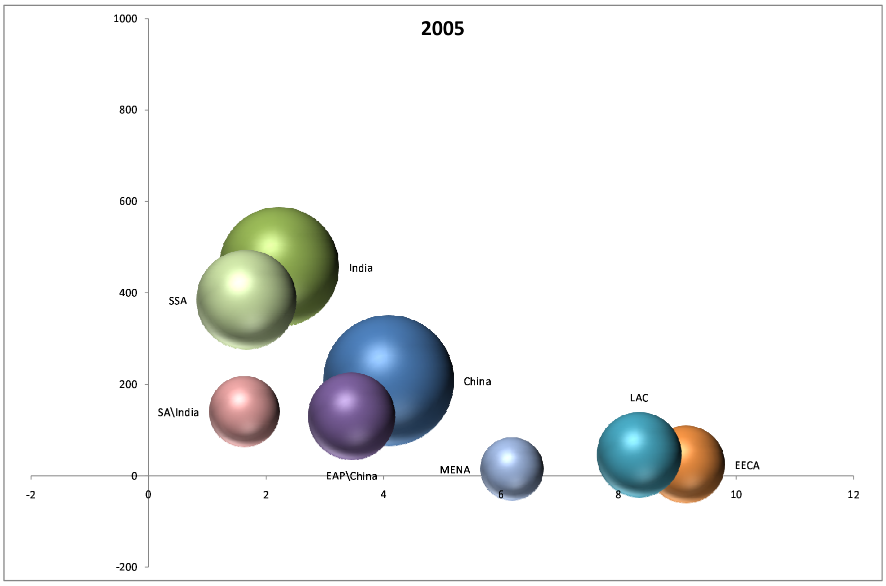


Fig. 4.4 World Poverty 2005. See notes for Fig. 4.1



cance of China in this history. An animation of the process appears in

`http://econ.lse.ac.uk/staff/dquah/p/2008.09-wpdyn-2005.gif`

and is more conveniently viewed online.

Each bubble in the Figures represents the state of a continental grouping or large economy. China and India, in particular, are explicitly given, as are East Asia and the Pacific (EAP) excluding China, and South Asia (SA) excluding India. Sub-Saharan Africa (SSA), Latin America and the Caribbean (LAC), and the Middle East and North Africa (MENA) also appear explicitly, with Eastern Europe and Central Asia (ECA) unavailable until 1990.

The horizontal axis in these Figures measures per capita GDP, in thousands of PPP constant year-2005 US$. The vertical axis measures in millions the number of people living on less than $1.25 a day, again in PPP constant-year 2005 US$. The location of each bubble is the state of the economy; the relative size of each bubble, the population. As population grows, so too does the bubble grow in size.

For 1981 China appears in the extreme upper left in Fig. 4.1: it is poor on average and holds many extremely poor people. Over time, the China bubble sinks and moves rightwards. Economic growth occurs and lifts hundreds of millions of Chinese out of extreme poverty. By 2005 China both holds fewer poor and is per capita richer than India and Sub-Saharan Africa.

Figs. 4.1–4.4 also show that the rest of East Asia and the Pacific region have in parallel with China also grown and successfully reduced poverty, although nowhere to the same magnitude as China alone.

By contrast the rest of the world has changed little in that same positive direction. The only other significant event is instead a negative one. Figs. 4.1–4.4 show Sub-Saharan Africa had per capita GDP decline and poverty nearly double between 1981 and 2005. By 2005 the population of all of Sub-Saharan Africa was only 58% that of

China but in absolute numbers Sub-Saharan Africa had 85% more people living in extreme poverty than had China.

However, the impact of China on world poverty is due not just to its having such a large population. Implicit in the Figures is also a comparison with India—the only other billion-people economy in the world. In this regard three features are particularly striking in the animation. First, compared to China over this sample period, India has both grown a lot slower and seen much less poverty reduction. Indeed, the number of people in India living on less than $1.25 a day has remained approximately constant despite economic growth. This rise in poverty is because of India's increased population and its relatively stable income distribution (Quah, 2003).

Second, while overall successful, China's poverty reduction has not been uniform throughout this time. Between 1987 and 1990 when economic growth slowed, poverty in China increased markedly as well.

Third, both from the 1987–1990 episode and by comparison to India, China's growth and size alone do not make automatic large-scale poverty reduction. Growth has to be sufficiently rapid to overturn the negative effects arising from increases in population and in inequality.

This last message, of course, carries also from the analytical and numerical calculations of the preceeding sections.

[...]

5 Conclusion

This paper has studied how varying patterns of growth and inequality affect the welfare of an economy's citizens. Previous work had shown how, for poverty, historical within- and cross-country patterns imply the effect of growth overwhelms that of inequality.

In this paper analytical calculations suggest that growth and inequality effects are more nuanced. For utility functions that are not unreasonable the welfare of the average person on earth is, certainly improved by the observed historical experiences of global growth and inequality. However, that experience is not uniform or unambiguous. It differs across even just the fast-growing economies.

[...]

6 Proofs

This appendix provides proofs of results in the body of the paper.

Proof of Theorem 2.1 *From* (3) *and* (5),

$$\begin{aligned} U(C(t)) = U(e^{Z(t)}e^{\xi t}) &= \frac{e^{(1-R)Z(t)}e^{(1-R)\xi t} - 1}{1-R} \\ \implies e^{-\rho t}U(C(t)) &= \frac{e^{(1-R)Z(t)}e^{-[\rho-(1-R)\xi]t} - e^{-\rho t}}{1-R} \end{aligned}$$

Using the definitions for $\tilde{\rho}$ and $\tilde{U}$, this gives

$$e^{-\rho t}U(C(t)) = e^{-\tilde{\rho}t}\tilde{U}(Z(t)) + \begin{cases} -(1-R)^{-1}e^{-\rho t} & \text{for } R \neq 1; \\ \xi t e^{-\rho t} & \text{otherwise.} \end{cases}$$

Then, conditional on $C(0) = e^z$,

$$\begin{aligned} W(0) &= E\left[\int_0^\infty e^{-\rho t}U(C(t))\,dt \mid C(0) = e^z\right] \\ &= (\mathbf{R}_{\tilde{\rho}}\tilde{U})(z) + \begin{cases} -\int_0^\infty (1-R)^{-1}e^{-\rho t}\,dt & \text{for } R \neq 1; \\ \int_0^\infty \xi t e^{-\rho t}\,dt & \text{otherwise.} \end{cases} \end{aligned}$$

The conclusion of the Theorem then follows from calculating, for $R \neq 1$,

$$\begin{aligned} \int_0^\infty -(1-R)^{-1}e^{-\rho t}\,dt &= (1-R)^{-1}\rho^{-1} \times \left(e^{-\rho t}\right)\Big|_0^\infty \\ &= -(1-R)^{-1}\rho^{-1}; \end{aligned}$$

and, for $R = 1$,

$$\int_0^\infty \xi e^{-\rho t} t\, dt = \xi \times (-\rho^{-2}) \times \left[(1+\rho t) e^{-\rho t} \right] \Big|_0^\infty = \xi \rho^{-2}.$$

Q.E.D.

Proof of Theorem 2.2 *The resolvent operator* $\mathbf{R}_\lambda$ *has kernel representation:*

$$(\mathbf{R}_\lambda \phi)(x') = \int_{-\infty}^{\infty} \phi(x)\, G_\lambda(x', x)\, dx,$$

where

$$G_\lambda(x', x)\, dx = \int_0^\infty e^{-\lambda t} \mathcal{M}_t(x', dx)\, dt,$$

with $\mathcal{M}_t$ *the time-t stochastic kernel (Quah, 2007, Section 3). Moreover, the resolvent kernel of standard Brownian motion is*

$$G_\lambda(x', x) = \sigma^{-1} \frac{1}{\sqrt{2\lambda}} e^{-|x-x'|\sqrt{2\lambda}/\sigma}. \tag{19}$$

(Quah (2007, Section 3), Karlin and Taylor (1981, p. 288)). Therefore,

$$(\mathbf{R}_{\widetilde{\rho}} \widetilde{U})(z^\dagger) = \int_{-\infty}^{\infty} \widetilde{U}(z) \times \sigma^{-1} \frac{1}{\sqrt{2\widetilde{\rho}}} \times e^{-|z-z^\dagger|\sqrt{2\widetilde{\rho}}/\sigma}\, dz.$$

For $R \neq 1$ *this becomes.*

$$(\mathbf{R}_{\widetilde{\rho}} \widetilde{U})(z^\dagger) = \frac{(2\widetilde{\rho})^{-1/2}\sigma^{-1}}{1-R} \int_{-\infty}^{\infty} e^{-(2\widetilde{\rho}/\sigma^2)^{1/2} \times |z-z^\dagger|} \times e^{(1-R)z}\, dz.$$

The integral on the right can be rewritten:

$$\int_{-\infty}^{z^\dagger} e^{-(2\widetilde{\rho}/\sigma^2)^{1/2}(z^\dagger - z)} \times e^{(1-R)z}\, dz$$
$$+ \int_{z^\dagger}^{\infty} e^{-(2\widetilde{\rho}/\sigma^2)^{1/2}(z - z^\dagger)} \times e^{(1-R)z}\, dz$$

$$= \frac{e^{-(2\widetilde{\rho}/\sigma^2)^{1/2} z^\dagger}}{(2\widetilde{\rho}/\sigma^2)^{1/2} + (1-R)} \times e^{[(2\widetilde{\rho}/\sigma^2)^{1/2} + (1-R)]z^\dagger} + \frac{e^{(2\widetilde{\rho}/\sigma^2)^{1/2} z^\dagger}}{(2\widetilde{\rho}/\sigma^2)^{1/2} - (1-R)} \times e^{-[(2\widetilde{\rho}/\sigma^2)^{1/2} - (1-R)]z^\dagger}$$

$$= e^{(1-R)z^\dagger} \times \frac{(2\widetilde{\rho}/\sigma^2)^{1/2} \times 2}{(2\widetilde{\rho}/\sigma^2) - (1-R)^2}$$

where convergence of the integral follows from $\widetilde{\rho} > \frac{1}{2}(1-R)^2\sigma^2$. *Then*

$$\begin{aligned}(\mathbf{R}_{\widetilde{\rho}}\widetilde{U})(z^\dagger) &= \frac{e^{(1-R)z^\dagger}}{1-R} \times \frac{2\sigma^{-2}}{(2\widetilde{\rho}/\sigma^2) - (1-R)^2} \\ &= \frac{e^{(1-R)z^\dagger}}{1-R} \times \frac{1}{\rho - (1-R)\xi - \frac{1}{2}(1-R)^2\sigma^2} \\ &= r(R,\rho,\xi,\sigma)^{-1} \times e^{(1-R)z^\dagger}.\end{aligned}$$

For $R = 1$ *analogous steps apply to*

$$(\mathbf{R}_{\widetilde{\rho}}\widetilde{U})(z^\dagger) = (2\widetilde{\rho})^{-1/2}\sigma^{-1} \int_0^\infty e^{-(2\widetilde{\rho}/\sigma^2)^{1/2} \times |z - z^\dagger|} \times z\, dz.$$

Again rewrite the integral on the right

$$\begin{aligned}&\int_{-\infty}^{z^\dagger} e^{-(2\widetilde{\rho}/\sigma^2)^{1/2}(z^\dagger - z)} \times z\, dz + \int_{z^\dagger}^{\infty} e^{-(2\widetilde{\rho}/\sigma^2)^{1/2}(z - z^\dagger)} \times z\, dz \\ &= \frac{e^{-(2\widetilde{\rho}/\sigma^2)^{1/2} z^\dagger}}{-(2\widetilde{\rho}/\sigma^2)} \left[(1 - (2\widetilde{\rho}/\sigma^2)^{1/2} z) e^{(2\widetilde{\rho}/\sigma^2)^{1/2} z} \right]\Big|_{-\infty}^{z^\dagger} \\ &\quad + \frac{e^{(2\widetilde{\rho}/\sigma^2)^{1/2} z^\dagger}}{-(2\widetilde{\rho}/\sigma^2)} \left[(1 + (2\widetilde{\rho}/\sigma^2)^{1/2} z) e^{-(2\widetilde{\rho}/\sigma^2)^{1/2} z} \right]\Big|_{z^\dagger}^{\infty} \\ &= \sqrt{2} \left[\sigma^2/\widetilde{\rho} \right]^{1/2} z^\dagger\end{aligned}$$

so that

$$(\mathbf{R}_{\widetilde{\rho}}\widetilde{U})(z^\dagger) = (2\widetilde{\rho})^{-1/2}\sigma^{-1}\sqrt{2} \left[\sigma^2/\widetilde{\rho} \right]^{1/2} z^\dagger = \rho^{-1} z^\dagger.$$

Q.E.D.

Of course equation (7) combining Theorems 2.1–2.2 can also be obtained directly using

$$E\left[e^{Z(t)} \mid Z(0)=z\right] = \exp\left(z + \sigma^2 t/2\right),$$

but such a direct approach would not generalize the way that using the resolvent operator allows.

Proof of Theorem 2.3 *From the definition*

$$\begin{aligned}(\mathbf{R}_{\tilde{\rho}}\tilde{U})(\overline{z}_m) &= \int_0^\infty e^{-\tilde{\rho}t} E\left[\tilde{U}(Z(t)) | Z(0) = \overline{z}_m\right] dt \\ &= \int_0^\infty e^{-\tilde{\rho}t}\left[\sum_{m=0}^{M-1} P_t(m,m')\tilde{U}(\overline{z}_{m'})\right] dt\end{aligned}$$

Collecting the right side across m gives the vector

$$(\mathbf{R}_{\tilde{\rho}}\tilde{U})(\overline{z}_m), \quad m = 0, 1, \ldots, M-1$$

as

$$\int_0^\infty e^{-\tilde{\rho}t}\left[P_t\overline{U}\right] dt = (\tilde{\rho} - \mathbf{G})^{-1}\overline{U},$$

thus establishing the Theorem. *Q.E.D.*

Proof of Theorem 2.4 *Performing the calculations directly for this case allows checking the results obtained more generally by resolvent operators. To that end use the conditional distributions in equation* (13) *to write:*

$$E_0 \log \epsilon_j(t) = \alpha^t \log \epsilon_j(0) - \frac{1-\alpha^t}{1-\alpha^2}\frac{\sigma_j^2}{2}.$$

Now take each of the cases in turn.

i. Suppose $R = 1$. When $t > 0$,

$$\begin{aligned} E_0 U(C_j(t)) &= E_0 \log C_j(t) \\ &= E_0 \left[\log \overline{z}_j + t \log \xi + \log \epsilon_j(t) \right] \\ &= \left[\log \overline{z}_j - (1-\alpha^2)^{-1} \sigma_j^2/2 \right] + (\log \xi) \times t \\ &\qquad + \left[\log \epsilon_j(0) + (1-\alpha^2)^{-1} \sigma_j^2/2 \right] \alpha^t, \end{aligned}$$

so that

$$\begin{aligned} & E_0 \sum_{t=0}^{\infty} \delta^t U(C_j(t)) \\ &\quad = U(C_j(0)) + E_0 \sum_{t=1}^{\infty} \delta^t U(C_j(t)) \\ &\quad = \log \overline{z}_j + \log \epsilon_j(0) + \left[\log \overline{z}_j - (1-\alpha^2)^{-1} \sigma_j^2/2 \right] \frac{\delta}{1-\delta} \\ &\qquad + \frac{\delta}{(1-\delta)^2} \log \xi + \left[\log \epsilon_j(0) + (1-\alpha^2)^{-1} \sigma_j^2/2 \right] \frac{\delta\alpha}{1-\delta\alpha}, \end{aligned}$$

using

$$\begin{aligned} \sum_{t=0}^{\infty} t\delta^t = \delta \sum_{t=1}^{\infty} t\delta^{t-1} &= \delta \frac{d}{d\delta} \sum_{t=1}^{\infty} \delta^t = \delta \frac{d}{d\delta} \frac{\delta}{1-\delta} \\ &= (1-\delta)^{-2} \delta. \end{aligned}$$

Collecting terms then gives

$$\begin{aligned} & E_0 \sum_{t=0}^{\infty} \delta^t U(C_j(t)) = (1-\delta\alpha)^{-1} \log \epsilon_j(0) \\ & + \frac{\delta}{(1-\delta)^2} \log \xi + \frac{1}{1-\delta} \log \overline{z}_j - \frac{\delta}{(1-\delta)(1-\delta\alpha)(1+\alpha)} \frac{\sigma_j^2}{2}. \end{aligned}$$

ii. Suppose $R \neq 1$. When $t > 0$,

$$\begin{aligned}
E_0 U(C_j(t)) &= (1-R)^{-1} E_0 \left[(\overline{z}_j)^{1-R} \xi^{-(R-1)t} \epsilon_j(t)^{1-R} - 1 \right] \\
&= (1-R)^{-1} \Bigg[(\overline{z}_j)^{1-R} \xi^{-(R-1)t} \times \\
&\quad \exp\Bigg([1-R] \left\{ \alpha^t \log \epsilon_j(0) - \frac{1-\alpha^t}{1-\alpha^2} \frac{\sigma_j^2}{2} \right\} \\
&\quad + \frac{1}{2} [1-R]^2 \frac{1-\alpha^{2t}}{1-\alpha^2} \sigma_j^2 \Bigg) \\
&\quad -1 \Bigg].
\end{aligned}$$

Collecting time-dependent terms in the exponent gives

$$\begin{aligned}
&[1-R] \left\{ \log \epsilon_j(0) + (1-\alpha^2)^{-1} \sigma_j^2 / 2 \right\} \alpha^t \\
&\quad - [1-R]^2 (1-\alpha^2)^{-1} (\sigma_j^2/2) \alpha^{2t},
\end{aligned}$$

while the time-invariant terms there become

$$\begin{aligned}
&- [1-R](1-\alpha^2)^{-1} \sigma_j^2/2 + [1-R]^2 (1-\alpha^2)^{-1} \sigma_j^2 / 2 \\
&\quad = -[1-R] R (1-\alpha^2)^{-1} \sigma_j^2 / 2.
\end{aligned}$$

For convenience, let

$$\begin{aligned}
D_1 &= [1-R] \left\{ \log \epsilon_j(0) + (1-\alpha^2)^{-1} \sigma_j^2 / 2 \right\} \\
D_2 &= [1-R]^2 (1-\alpha^2)^{-1} \sigma_j^2 / 2 > 0.
\end{aligned}$$

Then

$$\begin{aligned}
E_0 U(C_j(t)) = (1-R)^{-1} \Big[& (\overline{z}_j)^{1-R} e^{-[1-R]R(1-\alpha^2)^{-1}\sigma_j^2/2} \times \\
& \xi^{-(R-1)t} e^{D_1 \alpha^t} e^{-D_2 \alpha^{2t}} - 1 \Big]
\end{aligned}$$

so that

$$E_0 \sum_{t=0}^{\infty} \delta^t U(C_j(t))$$

$$= \frac{(\overline{z}_j)^{1-R} \epsilon_j(0)^{1-R} - 1}{1-R} + (1-R)^{-1} \times$$

$$\left[(\overline{z}_j)^{1-R} e^{-[1-R]R(1-\alpha^2)^{-1}\sigma_j^2/2} \sum_{t=1}^{\infty} \xi^{-(R-1)t} e^{D_1 \alpha^t} e^{-D_2 \alpha^{2t}} \right.$$

$$\left. - (1-\delta)^{-1}\delta \right]$$

$$= (1-R)^{-1} \left[-(1-\delta)^{-1} + (\overline{z}_j)^{1-R} \times \right.$$

$$\left\{ e^{-[1-R]R(1-\alpha^2)^{-1}\sigma_j^2/2} \sum_{t=1}^{\infty} \xi^{-(R-1)t} e^{D_1 \alpha^t} e^{-D_2 \alpha^{2t}} \right.$$

$$\left. \left. + \epsilon_j(0)^{1-R} \right\} \right].$$

Q.E.D.

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