

# A Macroeconomic Approach to Optimal Unemployment Insurance: Theory

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## Abstract

This paper develops a theory of optimal unemployment insurance (UI) in matching models. The optimal replacement rate is the conventional Baily-Chetty replacement rate, which solves the tradeoff between insurance and job-search incentives, plus a correction term, which is positive when an increase in UI pushes labor market tightness toward its efficient level. Labor market tightness is generally inefficient because in matching models, most wage mechanisms do not ensure efficiency. The effect of UI on tightness depends on the model: UI may raise tightness by alleviating a rat race for jobs or lower tightness by increasing wages through bargaining.

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Unemployment insurance (UI) is a key component of social insurance in modern welfare states. The microeconomic theory of optimal UI, developed by Baily [1978] and Chetty [2006], is well understood. It is an insurance-incentive tradeoff in the presence of moral hazard. UI helps workers smooth consumption when they are unemployed, but it also increases unemployment by discouraging job search. The Baily-Chetty formula resolves this tradeoff.

But the microeconomic theory only provides a partial description of the effects of UI on welfare because, while it accounts for workers' job-search behavior, it ignores firms' job-creation behavior. For instance, UI may exert upward pressure on wages by raising the outside option of unemployed workers, thereby discouraging job creation by firms. In that case, UI increases unemployment more than in the microeconomic theory. Alternatively, the labor market may operate like a rat race in which jobseekers queue for a fixed number of jobs offered by firms. A jobseeker who reduces her search effort is much less likely to find a job, but by moving down the queue, she improves the job prospects of the jobseekers who move ahead of her. In that case, although UI discourages job search, it has much less effect on unemployment than in the microeconomic theory. In these two examples, the microeconomic theory of optimal UI misses important channels through which UI affects unemployment and welfare.

In this paper we develop a macroeconomic theory of optimal UI that extends the microeconomic theory by accounting for firms' job-creation behavior. To that end we embed the Baily-Chetty model of UI into a matching model with generic production function and wage mechanism. The matching model is well suited for our purpose because it includes both workers searching for jobs and firms creating jobs.

The labor market tightness, defined as the ratio of aggregate vacancies to aggregate job-search effort, is central to our theory. Tightness is determined in equilibrium to equalize labor demand and labor supply. Tightness is important to workers because it influences their probability of finding a job; it is important to firms because it determines their recruiting costs. A higher tightness implies a higher job-finding rate per unit of effort, and it requires that a larger share of firms' workforce is allocated to recruiting instead of producing.

The microeconomic theory of UI is partial equilibrium in that it only considers the effect of UI on the labor supply, taking tightness as given. Our macroeconomic theory is general equilibrium in that it considers the effects of UI on the labor supply, labor demand, and tightness. As UI

generally affects tightness, our theory delivers results that are generally different from those of the microeconomic theory.

In matching models, the equilibrium is usually inefficient because most wage mechanisms do not ensure efficiency. The equilibrium is efficient if tightness maximizes welfare for a given UI; otherwise, it is inefficient. Tightness is inefficiently high if reducing tightness increases welfare; in that case, unemployment is inefficiently low and firms devote too much labor to recruiting workers. Tightness is inefficiently low if raising tightness increases welfare; in that case, unemployment is inefficiently high and too few jobseekers find a job.

When the equilibrium is efficient, the optimal UI replacement rate is given by the Baily-Chetty formula. Optimal UI follows the same principles as in the Baily-Chetty model with fixed tightness when the equilibrium is efficient because the marginal effect of UI on tightness has no first-order effect on social welfare.

When the equilibrium is inefficient, the replacement rate given by the Baily-Chetty formula is no longer optimal. This is our main result, and it has a simple intuition. Imagine for instance that tightness is inefficiently low. If UI raises tightness, UI is more desirable than the insurance-incentive tradeoff suggests, and the optimal replacement rate is higher than the Baily-Chetty replacement rate. Conversely, if UI lowers tightness, UI is less desirable than the insurance-incentive tradeoff suggests, and the optimal replacement rate is lower than the Baily-Chetty replacement rate.

Formally, we develop a formula that expresses the optimal replacement rate as the sum of the Baily-Chetty replacement rate plus a correction term. The correction term equals the effect of UI on tightness times the effect of tightness on welfare. The term is positive if UI brings tightness closer to its efficient level, and negative otherwise. Hence, the optimal replacement rate is above the Baily-Chetty replacement rate if and only if increasing UI brings tightness closer to its efficient level.

In matching models, increasing UI may raise or lower tightness. UI can lower tightness through a *job-creation mechanism*: when UI rises, the outside option of unemployed workers increases, wages rise through bargaining, firms create fewer jobs, and tightness therefore falls. UI can also raise tightness through a *rat-race mechanism*. Suppose to simplify that the number of jobs is fixed. In equilibrium, aggregate search effort times the job-finding rate per unit of

effort is equal to the number of jobs and thus fixed. By discouraging search, UI increases the job-finding rate and therefore tightness. When the number of jobs available is somewhat limited instead of completely fixed, the same logic applies and UI raises tightness. The overall impact of UI on tightness depends on which mechanism dominates. In the model of Pissarides [2000] with wage bargaining and linear production function, only the job-creation mechanism operates and UI lowers tightness. But in the job-rationing model of Michaillat [2012] with rigid wage and concave production function, only the rat-race mechanism operates and UI raises tightness.

To facilitate the application of the theory we express the optimal UI formula with estimable statistics, as in Chetty [2006]. Introducing estimable statistics allows us to develop two empirical criteria: one to evaluate whether tightness is inefficiently high or low, and another one to evaluate whether UI raises or lowers tightness. The first criterion is that tightness is inefficiently low if and only if the value of having a job relative to being unemployed is high compared to the share of labor devoted to recruiting. The second criterion is that UI raises tightness if and only if the microelasticity of unemployment with respect to UI is larger than the macroelasticity of unemployment with respect to UI. This criterion is simple to understand. The microelasticity measures the increase in unemployment caused by an increase in UI, accounting for the reduction in job search but keeping tightness constant. The macroelasticity, on the other hand, measures the increase in unemployment caused by an increase in UI, accounting both for the reduction in job search and the equilibrium response of tightness. Imagine for instance that an increase in UI raises tightness. In that case, the job-finding rate increases, which dampens the increase in unemployment caused by the reduction in job search, and which makes the macroelasticity smaller than the microelasticity.

The paper is organized as follows. Section I develops a generic model of UI. Section II expresses social welfare as a function of the generosity of UI and labor market tightness and computes the derivatives of the social welfare function with respect to these two variables. These derivatives are the building blocks of the optimal UI formula derived in Section III. Section IV shows that the formula continues to hold when workers can partially insure themselves against unemployment through home production, and when workers suffer a nonpecuniary cost from being unemployed. Section V studies the effect of UI on tightness in three specific models that illustrate the range of possibilities. Section VI concludes by discussing our companion paper

[Landais, Michaillat and Saez, 2015], which applies our theory to US data and explores how the generosity of UI should vary over the business cycle.

## I. A Generic Model

This section develops a generic model of UI. This model embeds the model of UI by Baily [1978] and Chetty [2006] into a matching model that uses the formalism from Michaillat and Saez [2015a]. The model is generic in that it can accommodate a broad range of labor demands, arising from diverse production functions and wage mechanisms. For simplicity we consider a static model; Landais, Michaillat and Saez [2015] present a dynamic extension of the model more adapted to quantitative analysis. Table 1 summarizes the notation.

### A. *The Labor Market*

There is a measure 1 of identical workers and a measure 1 of identical firms. Initially, all workers are unemployed and search for a job with effort  $e$ . Each firm posts  $v$  vacancies to recruit workers. The matching function  $m$  determines the number of worker-firm matches that are formed:  $l = m(e, v)$ , where  $l$  is the number of workers who find a job,  $e$  is aggregate job-search effort, and  $v$  is aggregate vacancies. The function  $m$  has constant returns to scale, is differentiable and increasing in both arguments, and satisfies  $m(e, v) \leq 1$ .

The labor market tightness  $\theta$  is defined by the ratio of aggregate vacancies to aggregate search effort:  $\theta = v/e$ . Since the matching function has constant returns to scale, the tightness determines the probabilities to find a job and fill a vacancy. A jobseeker finds a job at a rate  $f(\theta) = m(e, v)/e = m(1, \theta)$  per unit of search effort; hence, a jobseeker searching with effort  $e$  finds a job with probability  $e \cdot f(\theta)$ . A vacancy is filled with probability  $q(\theta) = m(e, v)/v = m(1/\theta, 1) = f(\theta)/\theta$ . The function  $f$  is increasing in  $\theta$  and the function  $q$  is decreasing in  $\theta$ . Accordingly, when the labor market is tighter, workers are more likely to find a job but vacancies are less likely to be filled. We denote by  $1 - \eta$  and  $-\eta$  the elasticities of  $f(\theta)$  and  $q(\theta)$ .

Table 1: Notation

	Description	Definition
<i>Panel A. The labor market</i>		
$e$	Job-search effort	
$v$	Number of vacancies	
$m(e, v)$	Matching function	
$\theta$	Labor market tightness	$\theta = v/e$
$f(\theta)$	Job-finding rate per unit of effort	$f(\theta) = m(1, \theta)$
$q(\theta)$	Vacancy-filling probability	$q(\theta) = f(\theta)/\theta$
$1 - \eta$	Elasticity of $f(\theta)$ wrt $\theta$	$1 - \eta = \theta \cdot f'(\theta)/f(\theta)$
$\rho$	Recruiting cost	
$\tau(\theta)$	Recruiter-producer ratio	$\tau(\theta) = \rho / (q(\theta) - \rho)$
<i>Panel B. Firms</i>		
$l$	Number of employees	
$n$	Number of producers	$n = l/(1 + \tau(\theta))$
$y(n)$	Production function	
$w$	Real wage	
$l^d(\theta, w)$	Labor demand	Equation (1)
<i>Panel C. The Unemployment Insurance Program</i>		
$c^e$	Consumption of employed workers	
$c^u$	Consumption of unemployed workers	
$\Delta c$	Consumption gain from work	$\Delta c = c^e - c^u$
$R$	Replacement rate	$R = 1 - \Delta c/w$
$\Delta U$	Utility gain from work	$\Delta U = U(c^e) - U(c^u)$
$\phi$	Harmonic mean of $U'(c^e)$ and $U'(c^u)$	Equation (12)
$SW(\theta, \Delta U)$	Social welfare function	Equation (4)
$\varepsilon^f$	Discouraged-worker elasticity	Equation (9)
$\varepsilon^m$	Microelasticity of unemployment wrt $\Delta U$	Equation (8)
$\varepsilon^M$	Macroelasticity of unemployment wrt $\Delta U$	Equation (20)
$\varepsilon_R^m$	Microelasticity of unemployment duration wrt $R$	Equation (25)
<i>Panel D. Workers</i>		
$U(c)$	Utility from consumption	
$\psi(e)$	Disutility from job-search effort	
$e^s(f, \Delta U)$	Effort supply	Equation (3)
$l^s(\theta, \Delta U)$	Labor supply	Equation (4)
$h$	Home production	
$\lambda(h)$	Disutility from home production	
$h^s(c^u)$	Home-production supply	Equation (27)
$c^h$	Consumption of unemployed workers with home production	$c^h = c^u + h^s(c^u)$
$\Delta U^h$	Utility gain from work with home production	$\Delta U^h = U(c^e) - U(c^h) + \lambda(h^s(c^u))$
$z$	Nonpecuniary cost of unemployment	

## B. Firms

The representative firm hires  $l$  workers, paid a real wage  $w$ , to produce a consumption good. The firm has two types of employees:  $n$  are producing output while  $l - n$  are recruiting employees by posting vacancies. The production function of the firm is  $y(n)$ . The function  $y$  is differentiable, increasing, and concave.

Posting a vacancy requires  $\rho \in (0, 1)$  recruiters. Since hiring  $l$  employees requires to post  $l/q(\theta)$  vacancies, the numbers of recruiters and producers in a firm with  $l$  employees are  $l \cdot \rho/q(\theta)$  and  $n = l \cdot (1 - \rho/q(\theta))$ . Accordingly, the firm's recruiter-producer ratio is  $\tau(\theta) = \rho/(q(\theta) - \rho)$ , and the numbers of employees and producers are related by  $l = (1 + \tau(\theta)) \cdot n$ . Since  $q(\theta) > \rho$  and  $q(\theta)$  is decreasing in  $\theta$ ,  $\tau(\theta)$  is positive and increasing in  $\theta$ .<sup>1</sup> When the labor market is tighter, the vacancy-filling rate is lower so firms must post more vacancies and allocate more workers to recruiting in order to hire a given number of employees; this is why the recruiter-producer ratio increases with tightness. The elasticity of  $\tau(\theta)$  is  $\eta \cdot (1 + \tau(\theta))$ .

The firm sells its output on a perfectly competitive market. Given  $\theta$  and  $w$ , the firm chooses  $l$  to maximize profits  $y(l/(1 + \tau(\theta))) - w \cdot l$ . The labor demand  $l^d$  gives the optimal number of workers employed by the firm. It is implicitly defined by the first-order condition of the maximization:

$$y' \left( \frac{l^d(\theta, w)}{1 + \tau(\theta)} \right) = (1 + \tau(\theta)) \cdot w. \quad (1)$$

Since  $y'(n)$  is decreasing in  $n$  and  $\tau(\theta)$  is increasing in  $\theta$ ,  $l^d(\theta, w)$  is decreasing in  $w$  and, for common specifications of the production function, in  $\theta$ .<sup>2</sup> For the profit-maximizing firm, producers should be hired to the point where their marginal product equals their marginal cost, which is the wage plus the recruiting cost  $\tau(\theta) \cdot w$ . This intuitively explains why the labor demand decreases with the wage and tightness.

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<sup>1</sup>The condition  $q(\theta) > \rho$  is necessary to have a positive number of producers. It limits tightness to a range  $[0, \theta^m)$  with  $\theta^m = q^{-1}(\rho)$ .

<sup>2</sup>The function  $l^d(\theta, w)$  is decreasing in  $\theta$  if and only if the elasticity of  $y'$  is in  $(-1, 0)$ . This condition is satisfied with the standard specification  $y(n) = n^\alpha$ ,  $\alpha \in (0, 1)$ .

### C. *The Unemployment Insurance Program*

The government's UI program provides employed workers with consumption  $c^e$  and unemployed workers with consumption  $c^u < c^e$ . UI benefits and taxes are not contingent on search effort because it is not observable. The generosity of UI is measured by the replacement rate

$$R \equiv 1 - \frac{\Delta c}{w},$$

where  $\Delta c \equiv c^e - c^u$  is the consumption gain from work. When a jobseeker finds work, she keeps a fraction  $\Delta c/w = 1 - R$  of the wage and gives up a fraction  $R$  because UI benefits are lost.<sup>3</sup>

The government must satisfy the budget constraint

$$y(n) = (1 - l) \cdot c^u + l \cdot c^e. \quad (2)$$

If firms' profits are equally distributed, the UI program can be implemented with a benefit  $b$  funded by a tax  $t$  on wages such that  $(1 - l) \cdot b = l \cdot t$ ,  $c^u = \text{profits} + b$ , and  $c^e = \text{profits} + w - t$ . If profits are unequally distributed, they can be taxed fully and rebated lump sum to implement the UI program.

### D. *Workers*

Workers cannot insure themselves against unemployment in any way, so they consume  $c^e$  if employed and  $c^u$  if unemployed. The utility from consumption is  $U(c)$ . The function  $U$  is differentiable, increasing, and concave. The disutility from job-search effort  $e$  is  $\psi(e)$ . The function  $\psi$  is differentiable, increasing, and convex.

Given  $\theta$ ,  $c^e$ , and  $c^u$ , the representative worker chooses  $e$  to maximize its expected utility

$$e \cdot f(\theta) \cdot U(c^e) + (1 - e \cdot f(\theta)) \cdot U(c^u) - \psi(e),$$

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<sup>3</sup>Consider a UI program that provides a benefit  $b$  funded by a tax  $t$  so that  $\Delta c = w - t - b$ . Our replacement rate is  $R = (t + b)/w$ . The conventional replacement rate is  $b/w$ . The conventional replacement rate ignores the tax  $t$  and is not the same as  $R$ , but it is approximately equal to  $R$  since  $t \ll b$  in practice (as unemployment is small relative to employment).



where  $e \cdot f(\theta)$  is the probability of finding a job and  $1 - e \cdot f(\theta)$  is the probability of remaining unemployed. The effort supply  $e^s$  gives the optimal job-search effort. It is implicitly defined by the first-order condition of the maximization:

$$\psi'(e^s(f(\theta), \Delta U)) = f(\theta) \cdot \Delta U, \quad (3)$$

where  $\Delta U \equiv U(c^e) - U(c^u)$  is the utility gain from work. When UI is less generous, the utility gain from work is higher. Since  $\psi'(e)$  is increasing in  $e$ ,  $e^s(f(\theta), \Delta U)$  is increasing in  $f(\theta)$  and  $\Delta U$ . Accordingly, jobseekers search more when the labor market is tighter and when UI is less generous. The utility-maximizing jobseeker should search to the point where the marginal disutility of search equals its marginal utility gain, which is the rate at which search leads to a job,  $f(\theta)$ , times the utility gain from having a job,  $\Delta U$ . A tighter labor market and a less generous UI lead to more search because they increase the marginal utility gain from search.

The labor supply  $l^s$  gives the number of workers who find a job when they search optimally. It is defined by

$$l^s(\theta, \Delta U) = e^s(f(\theta), \Delta U) \cdot f(\theta). \quad (4)$$

Since  $e^s(f(\theta), \Delta U)$  is increasing in  $f(\theta)$  and  $\Delta U$ , and  $f(\theta)$  is increasing in  $\theta$ ,  $l^s(\theta, \Delta U)$  is increasing in  $\theta$  and  $\Delta U$ . The labor supply is higher when the labor market is tighter because both the job-finding rate per unit of effort and the search effort are higher. The labor supply is higher when UI is less generous because search effort is higher.

### *E. The Wage Mechanism*

As in any matching model, we need to specify a wage mechanism. Common specifications of the wage mechanism include Nash bargaining or a fixed wage. Here we specify the most generic wage mechanism possible:

$$w = w(\theta, \Delta U).$$

In equilibrium the pair  $(\theta, \Delta U)$  determines all the other variables; therefore, the wage mechanism could be any function of any variable.

## II. Equilibrium and Social Welfare

In this section we characterize the equilibrium of the model and express the social welfare in an equilibrium as a function of the generosity of UI and labor market tightness in that equilibrium. We compute the derivatives of the social welfare function with respect to UI and tightness. These derivatives are the key building blocks of the optimal UI formula derived in Section III. To facilitate the application of the theory, we follow Chetty [2006] and express the derivatives with statistics that can be estimated empirically.

### A. Allocation and Equilibrium

Before introducing the equilibrium concept, we define a concept of allocation that helps separating issues of insurance and issues of efficiency when we analyze the social welfare function.

An *allocation* is a collection of quantities  $\{e, l, n, c^e, c^u, \theta, \Delta U\}$  that satisfies the following conditions: (i) effort is unobservable and determined by workers to maximize their utility:  $e = e^s(f(\theta), \Delta U)$ ; (ii) the recruiting cost imposes a wedge between the numbers of producers and employees:  $n = l/(1 + \tau(\theta))$ ; (iii) the matching function determines employment:  $l = l^s(\theta, \Delta U)$ ; (iv) the consumption levels  $c^e$  and  $c^u$  must satisfy the resource constraint, given by (2); and (v) the definition of  $\Delta U$  is satisfied:  $U(c^e) - U(c^u) = \Delta U$ . All the variables in the allocation can be expressed as a function of  $\Delta U$ , which describes the amount of insurance in the allocation, and  $\theta$ , which describes the amount of economic activity in the allocation.

Typically an allocation is a collection of quantities satisfying a resource constraint. Here we have additional constraints because of moral hazard, which links  $e$  to  $\Delta U$  and  $\theta$ , and because of the matching structure on the labor market, which links  $l$  and  $n$  to  $\theta$  and  $\Delta U$ . An allocation does not contain prices, but it is convenient to define a notional wage and a notional replacement rate that we use when we study the allocation:  $w \equiv y'(n)/(1 + \tau(\theta))$  and  $R \equiv 1 - (c^e - c^u)/[y'(n)/(1 + \tau(\theta))]$ . Of course, in an equilibrium, notional wage and replacement rate equal actual wage and replacement rate.

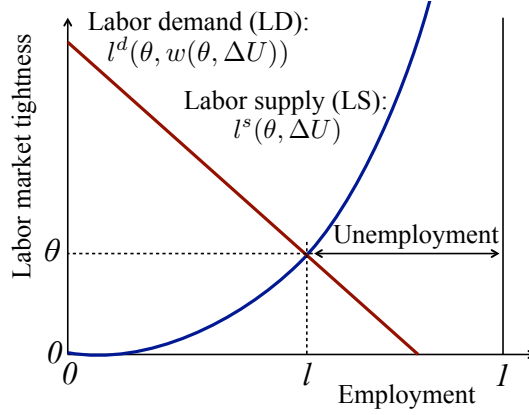


Figure 1: The Equilibrium

An *equilibrium* parameterized by  $\Delta U$  is a collection of variables  $\{e, l, n, c^e, c^u, \theta, w\}$  such that (i)  $\{e, l, n, c^e, c^u, \theta, \Delta U\}$  is an allocation; (ii)  $w$  is given by the wage mechanism:  $w = w(\theta, \Delta U)$ ; and (iii)  $\theta$  equalizes labor supply and labor demand:

$$l^s(\theta, \Delta U) = l^d(\theta, w(\theta, \Delta U)). \quad (5)$$

This equation defines  $\theta$  as an implicit function of  $\Delta U$ , denoted  $\theta(\Delta U)$ . This function describes the equilibrium level of tightness for a given  $\Delta U$ . All the variables in the equilibrium can be expressed as a function of  $\Delta U$  and  $\theta(\Delta U)$ .

The equilibrium is represented in a  $(l, \theta)$  plane as in Figure 1. The intersection of the labor supply and labor demand curves gives labor market tightness, employment, and unemployment. The labor supply curve shifts inward when UI increases. The labor demand curve responds to UI if the wage mechanism does.

### B. The Social Welfare Function

**DEFINITION 1.** The social welfare function  $SW$  is defined by

$$SW(\theta, \Delta U) = e^s(\theta, \Delta U) \cdot f(\theta) \cdot \Delta U + U(c^u(\theta, \Delta U)) - \psi(e^s(\theta, \Delta U)), \quad (6)$$

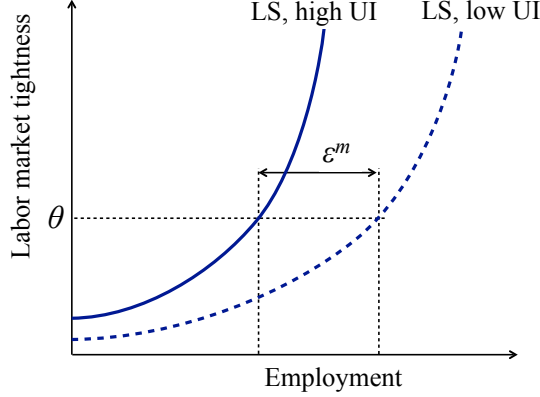


Figure 2: The Microelasticity of Unemployment With Respect to UI ( $\epsilon^m$ )

where  $c^u(\theta, \Delta U)$  is implicitly defined by

$$y\left(\frac{l^s(\theta, \Delta U)}{1 + \tau(\theta)}\right) = l^s(\theta, \Delta U) \cdot U^{-1}(U(c^u(\theta, \Delta U)) + \Delta U) + (1 - l^s(\theta, \Delta U)) \cdot c^u(\theta, \Delta U). \quad (7)$$

The social welfare function gives the social welfare in an allocation parameterized by  $\theta$  and  $\Delta U$ . The consumption level  $c^u(\theta, \Delta U)$  in (6) ensures that the government's budget constraint is satisfied in the allocation. The term  $U^{-1}(U(c^u(\theta, \Delta U)) + \Delta U)$  in (7) gives the consumption of employed workers when unemployed workers consume  $c^u(\theta, \Delta U)$  and the utility gain from work is  $\Delta U$ . The function  $SW$  plays a key role in the analysis because it can also be used to compute the social welfare in an equilibrium parameterized by  $\Delta U$ : in that equilibrium, the social welfare is  $SW(\theta(\Delta U), \Delta U)$ , where  $\theta(\Delta U)$  is the equilibrium level of tightness.

To facilitate the analysis of the social welfare function, we define two elasticities that measure the response of search effort to UI and labor market conditions.

**DEFINITION 2.** *The microelasticity of unemployment with respect to UI is*

$$\epsilon^m \equiv -\frac{\Delta U}{1-l} \cdot \frac{\partial(1-l^s)}{\partial \Delta U} \Big|_{\theta} = \frac{\Delta U}{1-l} \cdot \frac{\partial l^s}{\partial \Delta U} \Big|_{\theta}. \quad (8)$$

The microelasticity measures the percentage increase in unemployment when the utility gain from work decreases by 1 percent, taking into account jobseekers' reduction in search effort but ignoring the equilibrium adjustment of tightness. Because it keeps tightness constant, the micro-

lasticity measures a partial-equilibrium response of unemployment to UI. The ideal experiment to estimate the microelasticity is to offer higher or longer UI benefits to a randomly selected and small subset of jobseekers within a labor market and compare unemployment durations between treated and nontreated jobseekers. In Figure 2, an increase in UI reduces search effort, which shifts the labor supply curve in. The microelasticity measures this shift.

**DEFINITION 3.** *The discouraged-worker elasticity is*

$$\varepsilon^f = \frac{f(\theta)}{e} \cdot \frac{\partial e^s}{\partial f} \Big|_{\Delta U}. \quad (9)$$

The discouraged-worker elasticity measures the percentage decrease in search effort when the job-finding rate per unit of effort decreases by 1 percent, keeping UI constant. In our model, workers search less when the job-finding rate decreases so  $\varepsilon^f > 0$ . The discouraged-worker elasticity can be estimated by comparing the search effort of unemployed workers facing different local labor market conditions but receiving similar UI. Search effort can be measured directly using time-use surveys or indirectly using the number of job-application methods reported in household surveys.

Equipped with the elasticities  $\varepsilon^m$  and  $\varepsilon^f$ , we differentiate the social welfare function:

**LEMMA 1.** *The social welfare function admits the following partial derivatives:*

$$\frac{\partial SW}{\partial \theta} \Big|_{\Delta U} = \frac{l}{\theta} \cdot (1 - \eta) \cdot \phi \cdot w \cdot \left[ \frac{\Delta U}{\phi \cdot w} + R \cdot (1 + \varepsilon^f) - \frac{\eta}{1 - \eta} \cdot \tau(\theta) \right] \quad (10)$$

$$\frac{\partial SW}{\partial \Delta U} \Big|_{\theta} = (1 - l) \cdot \frac{\phi \cdot w}{\Delta U} \cdot \varepsilon^m \cdot \left[ R - \frac{l}{\varepsilon^m} \cdot \frac{\Delta U}{w} \cdot \left( \frac{1}{U'(c^e)} - \frac{1}{U'(c^u)} \right) \right], \quad (11)$$

where  $\phi$  is the harmonic mean of workers' marginal consumption utilities:

$$\frac{1}{\phi} = \frac{l}{U'(c^e)} + \frac{1 - l}{U'(c^u)}. \quad (12)$$

*Proof.* We first derive (10). Since workers choose effort to maximize expected utility, a standard application of the envelope theorem says that changes in the effort  $e^s(\theta, \Delta U)$  resulting from

changes in  $\theta$  have no impact on social welfare. The effect of  $\theta$  on welfare therefore is

$$\frac{\partial SW}{\partial \theta} = \frac{l}{\theta} \cdot (1 - \eta) \cdot \Delta U + U'(c^u) \cdot \frac{\partial c^u}{\partial \theta}. \quad (13)$$

After an increase in  $\theta$ , employment rises; the first left-hand-side term is the welfare gain following this employment gain. Higher employment is beneficial for welfare because it implies that more workers enjoy the high level of consumption  $c^e$  instead of the low level  $c^u$ . The first term is obtained by noting that  $1 - \eta = \theta \cdot f'(\theta)/f(\theta)$  so  $e \cdot f'(\theta) = (l/\theta) \cdot (1 - \eta)$ . This term accounts only for the change in employment resulting from a change in job-finding rate, not for that resulting from a change in effort. After a change in  $\theta$ , consumption must adjust to satisfy the government budget constraint, given by (7); the second left-hand-side term is the welfare change following this adjustment.

To compute the consumption change  $\partial c^u/\partial \theta$ , we implicitly differentiate  $c^u(\theta, \Delta U)$  with respect to  $\theta$  in (7). A few preliminary results are helpful. First, the definition of the notional wage imposes  $y'(n)/(1 + \tau(\theta)) = w$ . Second, since  $l^s(\theta, \Delta U) = e^s(f(\theta), \Delta U) \cdot f(\theta)$  and  $\varepsilon^f$  is the elasticity of  $e^s(f, \Delta U)$  with respect to  $f$  and  $1 - \eta$  is the elasticity of  $f(\theta)$  with respect to  $\theta$ , the elasticity of  $l^s$  with respect to  $\theta$  is

$$\frac{\theta}{l} \cdot \frac{\partial l^s}{\partial \theta} \Big|_{\Delta U} = (1 - \eta) \cdot (1 + \varepsilon^f). \quad (14)$$

Third, the definition of  $\Delta U$  implies that  $U^{-1}(U(c^u(\theta, \Delta U)) + \Delta U) - c^u = \Delta c$ . Fourth, the elasticity of  $1 + \tau(\theta)$  is  $\eta \cdot \tau(\theta)$  so the derivative of  $1/(1 + \tau(\theta))$  with respect to  $\theta$  is  $-\eta \cdot \tau(\theta)/[\theta \cdot (1 + \tau(\theta))]$ . Fifth, the derivative of  $c^e(c^u, \Delta U) = U^{-1}(U(c^u) + \Delta U)$  with respect to  $c^u$  is  $\partial c^e/\partial c^u = U'(c^u)/U'(c^e)$ . The implicit differentiation therefore yields

$$\frac{l}{\theta} \cdot (1 - \eta) \cdot (1 + \varepsilon^f) \cdot (w - \Delta c) - \frac{l}{\theta} \cdot \eta \cdot \tau(\theta) \cdot w = \left( \frac{l}{U'(c^e)} + \frac{1 - l}{U'(c^u)} \right) \cdot U'(c^u) \cdot \frac{\partial c^u}{\partial \theta}. \quad (15)$$

The first left-hand-side term is the budgetary gain from the new jobs created. Each new job increases government revenue by  $w - \Delta c$ . The new jobs result from a higher job-finding rate and a higher search effort. The term  $(1 + \varepsilon^f)$  captures the combination of the two forces. The

second left-hand-side term is the loss of resources due to a higher tightness, which forces firms to allocate more labor to recruiting and less to producing. The entire left-hand side is the change in resources available to fund the UI program after a change in tightness. This change dictates the consumption change  $\partial c^u / \partial \theta$ .

Finally to obtain (10), we substitute the value of  $\partial c^u / \partial \theta$  obtained from (15) into (13), and we introduce the variable  $\phi$  defined by (12).

We follow similar steps to derive (11). The effect of  $\Delta U$  on welfare is

$$\frac{\partial SW}{\partial \Delta U} = l + U'(c^u) \cdot \frac{\partial c^u}{\partial \Delta U}. \quad (16)$$

The first term on the left-hand side is the welfare gain enjoyed by employed workers after a reduction in UI contributions. After a change in  $\Delta U$ , consumption must adjust to satisfy the government budget constraint; the second term is the welfare change following this adjustment.

To compute the consumption change  $\partial c^u / \partial \Delta U$ , we implicitly differentiate  $c^u(\theta, \Delta U)$  with respect to  $\Delta U$  in (7). We need two preliminary results in addition to those above. First, the definition of the microelasticity implies that  $\partial l^s / \partial \Delta U = [(1 - l) / \Delta U] \cdot \varepsilon^m$ . Second, the derivative of  $c^e(c^u, \Delta U) = U^{-1}(U(c^u) + \Delta U)$  with respect to  $\Delta U$  is  $\partial c^e / \partial \Delta U = 1 / U'(c^e)$ . The implicit differentiation therefore yields

$$\frac{1-l}{\Delta U} \cdot \varepsilon^m \cdot (w - \Delta c) - \frac{l}{U'(c^e)} = \left( \frac{l}{U'(c^e)} + \frac{1-l}{U'(c^u)} \right) \cdot U'(c^u) \cdot \frac{\partial c^u}{\partial \Delta U}. \quad (17)$$

The first left-hand-side term is the budgetary gain from the new jobs created by reducing the generosity of UI. This is a behavioral effect, coming from the response of job search to UI. The second left-hand-side term is the budgetary loss coming from the reduction in the UI contributions paid by employed workers. This is a mechanical effect. These budgetary changes dictate the consumption change  $\partial c^u / \partial \Delta U$ .

Substituting the value of  $\partial c^u / \partial \Delta U$  obtained from (17) into (16) and introducing the variable  $\phi$  defined by (12), we obtain

$$\frac{\partial SW}{\partial \Delta U} = (1-l) \cdot \phi \cdot \left[ \frac{w}{\Delta U} \cdot \varepsilon^m \cdot R + \frac{l}{1-l} \cdot \left( \frac{1}{\phi} - \frac{1}{U'(c^e)} \right) \right]. \quad (18)$$

Equation (12) implies that  $1/\phi - 1/U'(c^e) = -(1-l) \cdot (1/U'(c^e) - 1/U'(c^u))$ . Substituting this into (18) yields (11).  $\square$

### C. A Condition for Efficiency

It is well understood that in matching models the equilibrium is generally inefficient because most wage mechanisms cannot ensure efficiency. This means that a small change in labor market tightness triggered by a small wage change generally has an effect on social welfare. If the increase enhances welfare, tightness is inefficiently low; in that case the wage is inefficiently high. If the increase reduces welfare, tightness is inefficiently high; in that case the wage is inefficiently low. Of course, if increasing tightness has no first-order effect on welfare, tightness is efficient. Proposition 1 formalizes this discussion.

**DEFINITION 4.** *The efficiency term is*

$$\frac{\Delta U}{\phi \cdot w} + R \cdot (1 + \varepsilon^f) - \frac{\eta}{1 - \eta} \cdot \tau(\theta). \quad (19)$$

**PROPOSITION 1.** *Consider an allocation parameterized by a utility gain from work  $\Delta U$  and a tightness  $\theta$ . A marginal increase in  $\theta$  raises social welfare when the efficiency term is positive; it has no first-order effect on social welfare when the efficiency term is zero; and it lowers social welfare when the efficiency term is negative.*

*Proof.* The result directly follows from (10) because social welfare in an allocation parameterized by  $\Delta U$  and  $\theta$  is given by the function  $SW(\theta, \Delta U)$ .  $\square$

Proposition 1 provides a condition for efficiency. It shows that an increase in tightness has a positive effect on welfare when the value of having a job relative to being unemployed ( $\Delta U$ ) is high compared to the share of labor devoted to recruiting ( $\tau$ ). Conversely, an increase in tightness has a negative effect on welfare when the value of having a job relative to being unemployed is low enough compared to the share of labor devoted to recruiting. Intuitively, the efficient level of unemployment is positive in matching models because some unemployment allows firms to devote fewer workers to recruiting and more to production, thus increasing output. While some unemployment is desirable, too much unemployment is costly because it makes too many workers



idle and unproductive and because unemployed workers are worse off than employed workers. Hence, the efficient levels of unemployment and tightness balance the amount of labor devoted to recruiting with the cost of being unemployed.

The efficiency condition in Proposition 1 is closely related to the famous Hosios [1990] condition. Equation (5) shows that by choosing the wage mechanism, one could manipulate the labor demand and select the equilibrium level of tightness. The higher the wage, the lower the labor demand, and the lower the resulting tightness. Hence, any allocation  $(\theta, \Delta U)$  could be implemented in equilibrium by choosing the appropriate wage mechanism. Accordingly, an interpretation of Proposition 1 is that it gives a condition that variables must satisfy to ensure that the wage mechanism is efficient—in the sense that the resulting equilibrium allocation maximizes welfare. This is exactly what the Hosios condition does. One difference, however, is that the Hosios condition involves a parameter of the model—it says that the wage is efficient if workers' bargaining power equals the elasticity of the matching function with respect to unemployment—whereas our condition involves estimable statistics. Another difference is that the Hosios condition only applies to models with risk-neutral workers whereas our condition applies to models with risk-averse workers and imperfect insurance.

### III. The Optimal Unemployment Insurance Formula

The government chooses UI to maximize social welfare subject to the equilibrium relationship between tightness and UI. Formally, the government chooses  $\Delta U$  to maximize  $SW(\theta(\Delta U), \Delta U)$ , where  $SW(\theta, \Delta U)$  is defined by (6) and  $\theta(\Delta U)$  is implicitly defined by (5). This section derives a formula characterizing the optimal replacement rate of the UI program. The formula is expressed with estimable statistics.

To obtain the formula, we need an elasticity measuring the response of unemployment to UI in equilibrium:

**DEFINITION 5.** *The macroelasticity of unemployment with respect to UI is*

$$\varepsilon^M \equiv -\frac{\Delta U}{1-l} \cdot \frac{d(1-l)}{d\Delta U} = \frac{\Delta U}{1-l} \cdot \frac{dl}{d\Delta U}. \quad (20)$$

The macroelasticity measures the percentage increase in unemployment when the utility gain from work decreases by 1 percent, taking into account jobseekers' reduction in search effort and the general-equilibrium adjustment of tightness. Because it accounts for the effect of UI on tightness, the macroelasticity measures the general-equilibrium response of unemployment to UI. The macroelasticity accounts in particular for the response of the wage mechanism to UI because this response conditions the adjustment of tightness. Estimating the macroelasticity is inherently more difficult than estimating the microelasticity because it necessitates exogenous variation in UI benefits across comparable labor markets instead of exogenous variations in UI benefits across comparable individuals within a labor market. The ideal experiment to estimate the macroelasticity is to offer higher benefits to all individuals in a randomly selected subset of labor markets and compare unemployment rates between treated and nontreated labor markets.

The optimal level of UI will depend on the response of tightness to UI. The macroelasticity matters because the wedge between macroelasticity and microelasticity determines this response:

**DEFINITION 6.** *The elasticity wedge is  $1 - \epsilon^M / \epsilon^m$ .*

**PROPOSITION 2.** *The elasticity wedge measures the equilibrium response of tightness to UI:*

$$\frac{\Delta U}{\theta} \cdot \frac{d\theta}{d\Delta U} = -\frac{1-l}{l} \cdot \frac{1}{1-\eta} \cdot \frac{\epsilon^m}{1+\epsilon^f} \cdot \left(1 - \frac{\epsilon^M}{\epsilon^m}\right). \quad (21)$$

*The elasticity wedge is positive if tightness increases with the generosity of UI, negative if tightness decreases with the generosity of UI, and zero if tightness does not respond to UI.*

*Proof.* Since  $l = l^s(\theta, \Delta U)$ , we have:

$$\epsilon^M = \frac{\Delta U}{1-l} \cdot \frac{dl}{d\Delta U} = \left( \frac{\Delta U}{1-l} \cdot \frac{\partial l^s}{\partial \Delta U} \right) + \left( \frac{\theta}{1-l} \cdot \frac{\partial l^s}{\partial \theta} \right) \cdot \left( \frac{\Delta U}{\theta} \cdot \frac{d\theta}{d\Delta U} \right).$$

Using (8) and (14), we obtain

$$\epsilon^M = \epsilon^m + \frac{l}{1-l} \cdot (1-\eta) \cdot (1+\epsilon^f) \cdot \frac{\Delta U}{\theta} \cdot \frac{d\theta}{d\Delta U}. \quad (22)$$

Dividing this equation by  $\epsilon^m$  and rearranging the terms yields (21). □

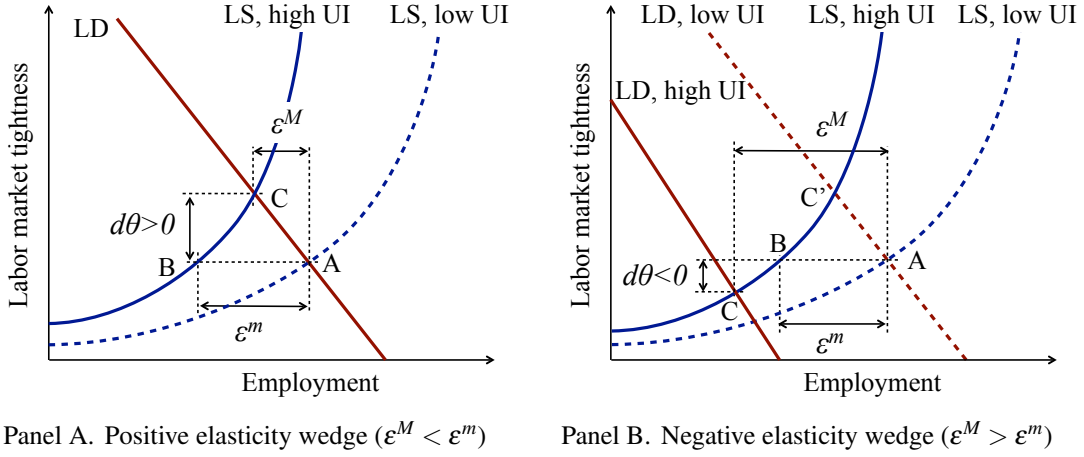


Figure 3: The Sign of the Elasticity Wedge ( $1 - \epsilon^M/\epsilon^m$ ) Gives the Effect of UI on Tightness

Notes: Panel A considers a downward-sloping labor demand curve that does not respond to UI. Panel B considers a downward-sloping labor demand curve that shifts inward when UI increases.

The proposition shows that a wedge appears between microelasticity and macroelasticity when UI affects tightness, and that this wedge has the same sign as the effect of UI on tightness. Figure 3 illustrates this result. The horizontal distance A–B measures the microelasticity and the horizontal distance A–C measures the macroelasticity. In Panel A, the labor demand curve is downward sloping, and it does not shift with a change in UI. After a reduction in UI, the labor supply curve shifts outward (A–B) and tightness increases along the new labor supply curve (B–C). Since tightness rises after the increase in UI, the macroelasticity is smaller than the microelasticity. In Panel B, the labor demand also shifts inward with an increase in UI. Tightness falls along the new supply curve after the labor demand shift (C'–C). In equilibrium, tightness can rise or fall depending on the size of the labor demand shift. In Panel B tightness falls so the macroelasticity is larger than the microelasticity. In Section V, we will consider specific models to describe the mechanisms through which UI affects tightness.

Having analyzed the effect of UI on tightness, we are equipped to derive the optimal UI formula:

**PROPOSITION 3.** *The optimal replacement rate satisfies*

$$R = \frac{l}{\epsilon^m} \frac{\Delta U}{w} \left[ \frac{1}{U'(c^e)} - \frac{1}{U'(c^u)} \right] + \left[ 1 - \frac{\epsilon^M}{\epsilon^m} \right] \frac{1}{1 + \epsilon^f} \left[ \frac{\Delta U}{w\phi} + (1 + \epsilon^f) R - \frac{\eta}{1 - \eta} \tau(\theta) \right]. \quad (23)$$

The first term in the right-hand side is the Baily-Chetty replacement rate, and the second term is the correction term.

*Proof.* The first-order condition of the government's problem is  $0 = dSW/d\Delta U$ , where the total derivative of social welfare with respect to  $\Delta U$  is  $dSW/d\Delta U = \partial SW/\partial \Delta U + (\partial SW/\partial \theta) \cdot (d\theta/d\Delta U)$ . The equation  $0 = \partial SW/\partial \Delta U$  is the Baily-Chetty formula. The term  $\partial SW/\partial \theta$  is proportional to the efficiency term (Proposition 1). The term  $d\theta/d\Delta U$  is proportional to the elasticity wedge (Proposition 2). Hence, the optimal UI formula is the Baily-Chetty formula plus a correction term proportional to the efficiency term times the elasticity wedge.

More precisely, we combine equation (21) with the derivatives in Lemma 1 to write the first-order condition  $0 = \partial SW/\partial \Delta U + (\partial SW/\partial \theta) \cdot (d\theta/d\Delta U)$ . We divide the resulting equation by  $(1-l) \cdot \phi \cdot w \cdot \varepsilon^m/\Delta U$  to obtain (23).  $\square$

**COROLLARY 1.** *If labor market tightness is efficient, the optimal replacement rate satisfies the Baily-Chetty formula:*

$$R = \frac{l}{\varepsilon^m} \cdot \frac{\Delta U}{w} \cdot \left[ \frac{1}{U'(c^e)} - \frac{1}{U'(c^u)} \right]. \quad (24)$$

*Proof.* Equation (24) obtains from Propositions 1 and 3.  $\square$

Formula (23) shows that the optimal UI replacement rate is the Baily-Chetty replacement rate plus a correction term. The Baily-Chetty replacement rate solves the tradeoff between the need for insurance, measured by  $1/U'(c^e) - 1/U'(c^u)$ , and the need for incentives to search, measured by  $\varepsilon^m$ , exactly as in the work of Baily [1978] and Chetty [2006]. The correction term is the product of the effect of UI on tightness, measured by the elasticity wedge, and the effect of tightness on welfare, measured by the efficiency term. Hence, the correction term is positive if and only if increasing UI pushes tightness toward its efficient level.

The structure of the formula—a standard term from public economics plus a correction term that is positive when the policy brings tightness toward its efficient level—is the same as the structure of the formula for optimal government purchases derived by Michaillat and Saez [2015b] in a matching model of the macroeconomy. More generally, the additive structure of the formula—a standard term plus a correction term—is similar to the structure of many optimal taxation formu-

has obtained in the presence of externalities. The reason is that tightness acts as a price influencing welfare when the labor market is inefficient, and the response of tightness to UI is akin to a pecuniary externality.

As in many optimal tax formulas, the right-hand-side of (23) is endogenous to UI. Even though the formula only characterizes optimal UI implicitly, it is useful because it transparently shows the economic forces at play, and it gives general conditions for the optimal replacement rate to be above or below the Baily-Chetty replacement rate.

There are two situations when the correction term is zero and the optimal replacement rate is given by the Baily-Chetty formula. The first situation is when UI has no effect on tightness such that the elasticity wedge is zero. In that case, our model is isomorphic to the Baily-Chetty model, in which tightness is fixed, so the optimal UI is the same. The second situation is when tightness is efficient such that the efficiency term is zero. This is the situation described by Corollary 1. In that case, the marginal effect of UI on tightness has no first-order effect on welfare; hence, optimal UI is governed by the same principles as in the Baily-Chetty model.

It may not be immediately apparent that (24) is equivalent to the traditional Baily-Chetty formula, but this becomes clear once we introduce the microelasticity of unemployment duration with respect to the replacement rate. This elasticity is denoted  $\varepsilon_R^m$  and defined by

$$\varepsilon_R^m = - \frac{\partial \ln(e^s \cdot f(\theta))}{\partial \ln(R)} \Big|_{\theta, c^e}. \quad (25)$$

It can be estimated by measuring the change in the average unemployment duration,  $1/(e^s \cdot f(\theta))$ , generated by a change in unemployment benefits,  $c^u$ , keeping constant tightness,  $\theta$ , and the consumption of employed workers,  $c^e$ . The microelasticity  $\varepsilon^m$  in our formulas is more convenient than  $\varepsilon_R^m$  to manipulate in the theoretical work; on the other hand, the elasticity  $\varepsilon_R^m$  is more commonly estimated in the empirical literature. The elasticities  $\varepsilon_R^m$  to  $\varepsilon^m$  are closely related. Consider a change  $dR$  keeping  $c^e$  and  $\theta$  constant. Since  $1 - l^s = s/(s + e^s \cdot f(\theta))$ ,  $d \ln(1 - l^s) = -l \cdot d \ln(e^s \cdot f(\theta))$ . As  $\Delta c = (1 - R) \cdot w$ , we have  $c^u = c^e - (1 - R) \cdot w$  and the change  $dR$  implies a consumption change  $dc^u = w \cdot dR$ , which implies a change  $d\Delta U = -U'(c^u) \cdot dc^u = -U'(c^u) \cdot w \cdot dR$ . Therefore,  $\varepsilon^m = -\Delta U \cdot d \ln(1 - l^s) / d\Delta U$  and  $\varepsilon_R^m = R \cdot d \ln(e^s \cdot f(\theta)) / dR$  are related by  $\varepsilon^m = \varepsilon_R^m \cdot l \cdot \Delta U / (U'(c^u) \cdot w \cdot R)$ . Using this relationship, we rewrite (24) as the traditional

Table 2: Optimal UI Replacement Rate Compared to Baily-Chetty Replacement Rate

	Elasticity wedge $< 0$	Elasticity wedge $= 0$	Elasticity wedge $> 0$
Efficiency term $> 0$	lower	same	higher
Efficiency term $= 0$	same	same	same
Efficiency term $< 0$	higher	same	lower

*Notes:* The UI replacement rate is  $R = 1 - (c^e - c^u)/w$ . The Baily-Chetty replacement rate is given by (24). The efficiency term is  $\Delta U / (\phi \cdot w) + (1 + \varepsilon^f) \cdot R - [\eta / (1 - \eta)] \cdot \tau(\theta)$ . The elasticity wedge is  $1 - \varepsilon^M / \varepsilon^m$ . Compared to the Baily-Chetty replacement rate, the optimal replacement rate is higher if the correction term in (23) is positive, equal if the correction term is zero, and lower if the correction term is negative.

Baily-Chetty formula:

$$\varepsilon_R^m = \left[ \frac{U'(c^u)}{U'(c^e)} - 1 \right]. \quad (26)$$

A weakness of the conventional expression of the Baily-Chetty formula is that the elasticity  $\varepsilon_R^m$  cannot be stable with  $R$  (it has to be zero when  $R = 0$ ). In contrast, the elasticity  $\varepsilon^m$  that we use in our expression (equation (24)) can potentially be stable with  $R$ .

When the correction term is nonzero and the optimal replacement rate departs from the Baily-Chetty replacement rate. The main implication of our formula is that increasing UI above the Baily-Chetty replacement rate is desirable if and only if increasing UI pushes tightness toward its efficient level. UI brings tightness closer to its efficient level either if tightness is inefficiently low and UI raises tightness or if tightness is inefficiently high and UI lowers tightness. In terms of the estimable statistics, UI brings tightness closer to its efficient level if the efficiency term and elasticity wedge are both positive or both negative. Table 2 summarizes all the possibilities.

An important implication of our formula is that even if UI is provided by private insurers, the public provision of UI is justified. Indeed, small private insurers do not internalize the effect of UI on tightness and offer insurance at the Baily-Chetty replacement rate. It is therefore optimal for the government to correct privately provided UI by a quantity equal to the correction term, which may be positive or negative.

Our formula reveals some interesting special cases. A first case is when UI has no adverse effect on unemployment ( $\varepsilon^M = 0$ ). Maybe surprisingly, full insurance is undesirable in that case. It is true that UI redistributes consumption from employed to unemployed workers without

destroying jobs, but because  $\varepsilon^M = 0$  while  $\varepsilon^m > 0$ , the logic of Figure 3 implies that UI raises tightness and forces firms to devote more workers to recruiting, thus reducing output available to consumption. Hence, the optimal replacement rate is below 1. In fact if tightness is efficient, UI is given by the Baily-Chetty formula and the magnitude of  $\varepsilon^M$  is irrelevant. To see this formally, set  $\varepsilon^M = 0$  and  $R = 1$  (which implies  $\Delta U = 0$  and  $c^e = c^u$ ) in formula (23). The resulting equation is  $1 = 1 - \eta \cdot \tau(\theta) / [(1 + \varepsilon^f) \cdot (1 - \eta)]$ , which never holds. Hence,  $R = 1$  is never optimal. Moreover, since the right-hand side is always smaller than the left-hand side, the effect of  $R$  on welfare at  $R = 1$  is negative and the optimal  $R$  is strictly below 1.

A second case is when workers are risk-neutral ( $U(c) = c$ ). Although there is no need for insurance, some UI remains desirable if increasing UI brings tightness closer to its efficient level. Since  $U'(c^e) = U'(c^u)$ , the Baily-Chetty replacement rate is 0 and the optimal replacement rate equals the correction term. Hence, the optimal replacement rate is positive when UI brings tightness toward its efficient level.

The third case is when UI has no adverse effect on search ( $\varepsilon^m = 0$ ). Although there is no need to provide search incentives, full insurance remains undesirable if the macroelasticity is strictly positive and tightness is inefficiently low. The Baily-Chetty replacement rate is 1. The optimal replacement rate is therefore below 1 only if the correction term is negative. If  $\varepsilon^M > 0$ , the logic of Figure 3 implies that UI lowers tightness. Hence, the optimal replacement rate is below 1 when tightness is inefficiently low. Formally, multiply (23) by  $\varepsilon^m$ , and set  $\varepsilon^m = 0$  and  $R = 1$  (which implies  $\Delta U = 0$  and  $c^e = c^u$ ). The resulting equation is  $0 = 0 - \varepsilon^M \cdot [1 + \varepsilon^f - \eta \cdot \tau(\theta) / (1 - \eta)]$ . When tightness is inefficiently low,  $1 + \varepsilon^f - \eta \cdot \tau(\theta) / (1 - \eta) > 0$  so the right-hand side is smaller than the left-hand side and the optimal  $R$  is strictly below 1.

To conclude, we discuss the empirical implementation of the formula. The formula is expressed with some observable variables ( $R$ ,  $l$ ,  $w$ ) and three types of statistics. The first type of statistics ( $\varepsilon^m$ ,  $\Delta U$ ,  $U'(c)$ ,  $\phi$ ) are involved in the Baily-Chetty formula. Starting with Gruber [1997], numerous studies have estimated these statistics (see Chetty and Finkelstein [2013] for a survey). The second type of statistics ( $\eta$  and  $\tau(\theta)$ ) are standard statistics from the matching literature. Many studies measure  $\eta$  (see Petrongolo and Pissarides [2001] for a survey). A small number of studies, such as Villena Roldan [2010], estimate  $\tau(\theta)$  or related quantities. The third type of statistics ( $\varepsilon^M / \varepsilon^m$  and  $\varepsilon^f$ ) are new to our formula. A growing number of papers, includ-

ing Lalive, Landais and Zweimüller [2015], Marinescu [2014], and Johnston and Mas [2015], attempt to measure  $\varepsilon^M/\varepsilon^m$ , and recent papers, such as Shimer [2004], Mukoyama, Patterson and Sahin [2014], DeLoach and Kurt [2013], and Gomme and Lkhagvasuren [2015], aim to estimate  $\varepsilon^f$ . In our companion paper [Landais, Michaillat and Saez, 2015], we use existing work and new empirical evidence to implement the formula.

## IV. Robustness of the Formula

This section shows that the optimal UI formula derived in the previous section is robust. It continues to hold when workers can partially insure themselves against unemployment through home production, and when workers suffer a nonpecuniary cost from being unemployed.

### *A. The Formula With Partial Self-Insurance Against Unemployment*

Section III assumes that workers cannot insure themselves against unemployment. In reality workers are able to partially insure themselves against unemployment using saving, spousal income, and home production [Aguiar and Hurst, 2005; Gruber, 1997]. Here we assume that workers partially insure themselves against unemployment with home production. Home production is a convenient representation of all the means of self-insurance available to workers.

The UI program provides employed workers with consumption  $c^e$  and unemployed workers with consumption  $c^u$ . In addition to consuming  $c^u$ , unemployed workers consume an amount  $h$  produced at home at a utility cost  $\lambda(h)$ . The function  $\lambda$  is differentiable, increasing, convex, and  $\lambda(0) = 0$ . Unemployed workers choose  $h$  to maximize their utility,  $U(c^u + h) - \lambda(h)$ .

The home-production supply  $h^s$  gives the optimal level of home production. It is implicitly defined by the first-order condition of the maximization:

$$\lambda'(h^s(c^u)) = U'(c^u + h^s(c^u)). \quad (27)$$

Since  $\lambda'(h)$  is increasing in  $h$  and  $U'(c)$  is decreasing in  $c$ ,  $h^s(c^u)$  is decreasing in  $c^u$ : unemployed workers produce less at home when UI benefits are more generous. To maximize utility, unemployed workers should produce to the point where the marginal disutility of home produc-



tion equals its marginal utility gain; higher UI benefits lead to less home production because they reduce the marginal utility gain from home production. In the presence of home production, the consumption of unemployed workers is  $c^h = c^u + h^s(c^u)$  and the utility gain from work is  $\Delta U^h = U(c^e) - U(c^h) + \lambda(h^s(c^u))$ .

When workers have partial access to self-insurance, the optimal UI formula becomes

$$R = \frac{l}{\varepsilon^m} \frac{\Delta U^h}{w} \left[ \frac{1}{U'(c^e)} - \frac{1}{U'(c^h)} \right] + \left[ 1 - \frac{\varepsilon^M}{\varepsilon^m} \right] \frac{1}{1 + \varepsilon^f} \left[ \frac{\Delta U^h}{w\phi} + \left( 1 + \varepsilon^f \right) R - \frac{\eta}{1 - \eta} \tau(\theta) \right],$$

where  $\phi$  is redefined using  $c^h$  instead of  $c^u$  and the elasticities  $\varepsilon^m$ ,  $\varepsilon^M$ , and  $\varepsilon^f$  are redefined using  $\Delta U^h$  instead of  $\Delta U$ . Hence, formula (23) carries over once the utility gain from work is adjusted from  $\Delta U$  to  $\Delta U^h$  and the marginal utility of unemployed workers from  $U'(c^u)$  to  $U'(c^h)$ . This new formula is obtained exactly like formula (23).<sup>4</sup> In a similar fashion, Chetty [2006] shows that the analysis of Baily [1978] can be generalized to account for partial self-insurance.

The availability of self-insurance affects both the Baily-Chetty replacement rate and the efficiency term in the formula. In the Baily-Chetty replacement rate, the need for insurance becomes  $1/U'(c^e) - 1/U'(c^h) < 1/U'(c^e) - 1/U'(c^u)$ . The need for publicly provided unemployment insurance is lower because unemployed workers are able to insure themselves partially against unemployment. Hence, the Baily-Chetty replacement rate is lower. In the efficiency term, the utility gain from work becomes  $\Delta U^h = \min_h \{U(c^e) - U(c^u + h) + \lambda(h)\} < U(c^e) - U(c^u) = \Delta U$ . Since the utility gain from work is lower, the efficiency term is lower, which implies that the efficient tightness is lower. The logic of Table 2 implies that the optimal replacement rate is higher

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<sup>4</sup>Repeating the derivation of (23) is simple except for three steps. First, the social welfare function admits a slightly different expression:

$$SW(\theta, \Delta U^h) = e^s(\theta, \Delta U^h) \cdot f(\theta) \cdot \Delta U^h + U(c^u(\theta, \Delta U^h) + h^s(c^u(\theta, \Delta U^h))) - \lambda(h^s(c^u(\theta, \Delta U^h))) - \psi(e^s(\theta, \Delta U^h)).$$

But, because unemployed workers choose home production to maximize their utility, the envelope theorem says that changes in home production  $h^s(c^u(\theta, \Delta U^h))$  resulting from changes in  $\theta$  and  $\Delta U^h$  have no impact on social welfare. Therefore, (13) and (16) remain valid once  $\Delta U$  and  $U'(c^u)$  are replaced by  $\Delta U^h$  and  $U'(c^h)$ . Second, since  $\Delta U^h = U(c^e) - U(c^u + h^s(c^u)) + \lambda(h^s(c^u))$ , the consumption of employed workers is given by

$$c^e(\theta, \Delta U^h) = U^{-1} \left( U(c^u(\theta, \Delta U^h) + h^s(c^u(\theta, \Delta U^h))) - \lambda(h^s(c^u(\theta, \Delta U^h))) + \Delta U^h \right).$$

But, because unemployed workers choose home production to maximize  $U(c^u + h) - \lambda(h)$ , changes in  $h^s(c^u(\theta, \Delta U^h))$  resulting from changes in  $\theta$  and  $\Delta U^h$  have no impact on  $c^e(\theta, \Delta U^h)$ . Therefore, (15) and (17) remain valid once  $\Delta U$  and  $U'(c^u)$  are replaced by  $\Delta U^h$  and  $U'(c^h)$ .

if the elasticity wedge is negative but lower if the elasticity wedge is positive. Overall, when self-insurance is available, the optimal replacement rate is unambiguously lower if the elasticity wedge is positive, but it may be higher or lower if the elasticity wedge is negative.

### *B. The Formula With a Nonpecuniary Cost of Unemployment*

Section III assumes that the well-being of unemployed and employed workers differs only because unemployed workers have consume less. But unemployment seems to have large detrimental effects on mental and physical health that to go well beyond what lower consumption would induce.<sup>5</sup> Some of the early studies on unemployment and health suffered from two issues. First, they were not able to separate between causality (unemployment causes low health) and selection (people who have low health become unemployed). But recent studies, such as Burgard, Brand and House [2007] and Sullivan and von Wachter [2009] for the United States, are able to identify the causal effect from unemployment to low health. Second, they were not able to control for the loss of income associated with unemployment and thus separate between the pecuniary and nonpecuniary cost of unemployment. But recent studies, such as Winkelmann and Winkelmann [1998], Di Tella, MacCulloch and Oswald [2003], and Blanchflower and Oswald [2004], find that unemployed workers report much lower well-being than employed workers even after controlling for household income and many other personal characteristics. This lower well-being seems to stem from higher anxiety, lower self-esteem, and lower life satisfaction [Darity and Goldsmith, 1996; Krueger and Mueller, 2011; Theodossiou, 1998].

Here we assume that unemployed workers have utility  $U(c^u) - z$ , where the parameter  $z$  captures the nonpecuniary cost of unemployment. Given the large literature in medicine, psychology, sociology, and economics documenting large nonpecuniary costs of unemployment, it is likely that  $z > 0$ . In theory, however, it is possible that  $z < 0$ ; in that case, workers enjoy nonpecuniary benefits from unemployment—for instance, additional time for leisure. In the presence of a nonpecuniary cost of unemployment, the utility gain from work is  $\Delta U^z = U(c^e) - U(c^u) + z$ .

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<sup>5</sup>The deleterious effects of unemployment on mental and physical health are documented by a large literature. See Dooley, Fielding and Levi [1996], Hawton and Platt [2000], and Frey and Stutzer [2002] for surveys and Murphy and Athanasou [1999] and McKee-Ryan et al. [2005] for meta-analyses.

Table 3: The Three Specific Models

	Model		
	Standard	Fixed-wage	Job-rationing
Production function	linear	linear	concave
Wage mechanism	bargaining	fixed	fixed
Elasticity wedge $1 - \varepsilon^M / \varepsilon^m$	negative	zero	positive
Reference	Pissarides [2000]	Hall [2005]	Michaillat [2012]

When unemployed workers suffer a nonpecuniary cost, the optimal UI formula becomes

$$R = \frac{l}{\varepsilon^m} \frac{\Delta U^z}{w} \left[ \frac{1}{U'(c^e)} - \frac{1}{U'(c^u)} \right] + \left[ 1 - \frac{\varepsilon^M}{\varepsilon^m} \right] \frac{1}{1 + \varepsilon^f} \left[ \frac{\Delta U^z}{w\phi} + \left( 1 + \varepsilon^f \right) R - \frac{\eta}{1 - \eta} \tau(\theta) \right],$$

where the elasticities  $\varepsilon^m$ ,  $\varepsilon^M$ , and  $\varepsilon^f$  are redefined using  $\Delta U^z$  instead of  $\Delta U$ . Hence, formula (23) carries over once the utility gain from work is adjusted from  $\Delta U$  to  $\Delta U^z$ . This new formula is obtained exactly like formula (23).

In the formula the nonpecuniary cost from unemployment affects the efficiency term but not the Baily-Chetty replacement rate. (In the Baily-Chetty replacement rate the  $\Delta U^z$  in the numerator cancels out with the  $\Delta U^z$  in the numerator of  $\varepsilon^m$ .) Hence, as already noted by Chetty [2006], the Baily-Chetty replacement rate is independent of  $z$  and thus of the overall level of well-being of unemployment workers. Furthermore, since  $\Delta U^z = \Delta U + z$ , the efficiency term is higher when  $z > 0$ . The logic of Table 2 therefore implies that when  $z > 0$ , the optimal replacement rate is higher if the elasticity wedge is positive but lower if the elasticity wedge is negative.

## V. The Elasticity Wedge in Three Specific Models

Section III shows that the optimal generosity of UI depends critically on the wedge between the macroelasticity and microelasticity of unemployment with respect to UI, but it remains vague on the economic mechanisms that create this wedge. To describe possible mechanisms, we now consider three specific models that differ by their wage mechanism and production function (see Table 3). The models illustrate the job-creation mechanism, which creates a negative elasticity wedge, and the rat-race mechanism, which creates a positive elasticity wedge.

### A. *The Standard Model of Pissarides [2000]*

The production function is linear:  $y(n) = n$ . When they are matched, a worker and a firm bargain over wages. The worker's bargaining power is  $\beta \in (0, 1)$ .<sup>6</sup> The outcome of the bargaining is that the surplus from the match is shared, with the worker keeping a fraction  $\beta$  of the surplus.<sup>7</sup>

We begin by determining the bargained wage. The worker's surplus from a match is  $\Delta U$ . The firm's surplus from a match is  $1 - w$  because once a worker is recruited, she produces 1 unit of good and receives a wage  $w$ . The total surplus from the match is  $1 - w + \Delta U$ . Worker and firm split this total surplus so  $\Delta U = \beta \cdot (1 - w + \Delta U)$  and  $1 - w = (1 - \beta) \cdot (1 - w + \Delta U)$ . Accordingly, the wage satisfies

$$w = 1 - \frac{1 - \beta}{\beta} \cdot \Delta U.$$

Increasing UI lowers  $\Delta U$  and thus raises wages. Intuitively, after an increase in UI the outside option of jobseekers increases and they are able to bargain higher wages.

We combine the wage equation with (1) and  $y'(n) = 1$  to obtain the labor demand:

$$\frac{\tau(\theta)}{1 + \tau(\theta)} = \frac{1 - \beta}{\beta} \cdot \Delta U. \quad (28)$$

This equation defines a perfectly elastic labor demand curve in a  $(l, \theta)$  plane, as depicted in Panel A of Figure 4. Since  $\tau(\theta)$  increases with  $\theta$  and  $\Delta U$  decreases with UI, (28) implies that the labor demand shifts downward when UI increases. Intuitively, when UI increases, wages rise through bargaining so it becomes less profitable for firms to hire workers.

Having obtained the labor demand, we can describe the elasticity wedge:

**PROPOSITION 4.** *In the standard model the elasticity wedge is negative:*

$$1 - \frac{\varepsilon^M}{\varepsilon^m} = -\frac{l}{1 - l} \cdot \frac{1 - \eta}{\eta} \cdot \frac{1 + \varepsilon^f}{\varepsilon^m} < 0.$$

<sup>6</sup>To obtain a positive wage, we impose that  $\beta/(1 - \beta) > \Delta U$ .

<sup>7</sup>In a seminal paper, Diamond [1982] also assumes a surplus-sharing solution to the bargaining problem. If workers and firms are risk neutral, the surplus-sharing solution coincides with the generalized Nash solution. Under risk aversion, the two solutions generally differ. We use the surplus-sharing solution for its simplicity.

*Proof.* We differentiate (28) with respect to  $\Delta U$ . Since the elasticities of  $\tau(\theta)$  and  $1 + \tau(\theta)$  with respect to  $\theta$  are  $\eta \cdot (1 + \tau(\theta))$  and  $\eta \cdot \tau(\theta)$ , we obtain  $(\Delta U / \theta) \cdot (d\theta / d\Delta U) = 1 / \eta$ . Then, using (21), we obtain the expression for the elasticity wedge.  $\square$

Panel A of Figure 4 illustrates the proposition. After an increase in UI, jobseekers search less, shifting the labor supply curve inward by a distance A–B. In addition, wages increase through bargaining, shifting the labor demand curve downward and further reducing employment by the horizontal distance B–C. The total reduction in employment is given by the horizontal distance A–C. Since A–B is smaller than A–C, the microelasticity is smaller than the macroelasticity and the elasticity wedge is negative.

The mechanism that causes the increase A–B in unemployment is the moral-hazard mechanism: an increase in UI leads to a reduction in search effort—a source of moral hazard because it is unobservable—and thus an increase in unemployment. The mechanism that causes the increase B–C in unemployment is the job-creation mechanism: an increase in UI leads to an increase in wages and thus a reduction in job creation. In sum, the standard model captures two mechanisms through which UI affects unemployment: the moral-hazard mechanism is at the origin of the microelasticity, and the job-creation mechanism explains why the macroelasticity is larger than the microelasticity.

Proposition 4 implies that in the standard model, an increase in UI reduces tightness. Thus, the optimal replacement rate is below the Baily-Chetty rate when tightness is inefficiently low and above it when tightness is inefficiently high. In the standard model, tightness is inefficiently low when the workers' bargaining power and thus wages are inefficiently high.

### *B. The Fixed-Wage Model of Hall [2005]*

The production function is linear:  $y(n) = n$ . The wage is fixed:  $w = \omega$ , where  $\omega \in (0, 1)$ . Unlike the bargained wage from the standard model, the fixed wage does not respond to UI. We combine the wage schedule with equation (1) to obtain the labor demand:

$$1 = \omega \cdot (1 + \tau(\theta)). \quad (29)$$

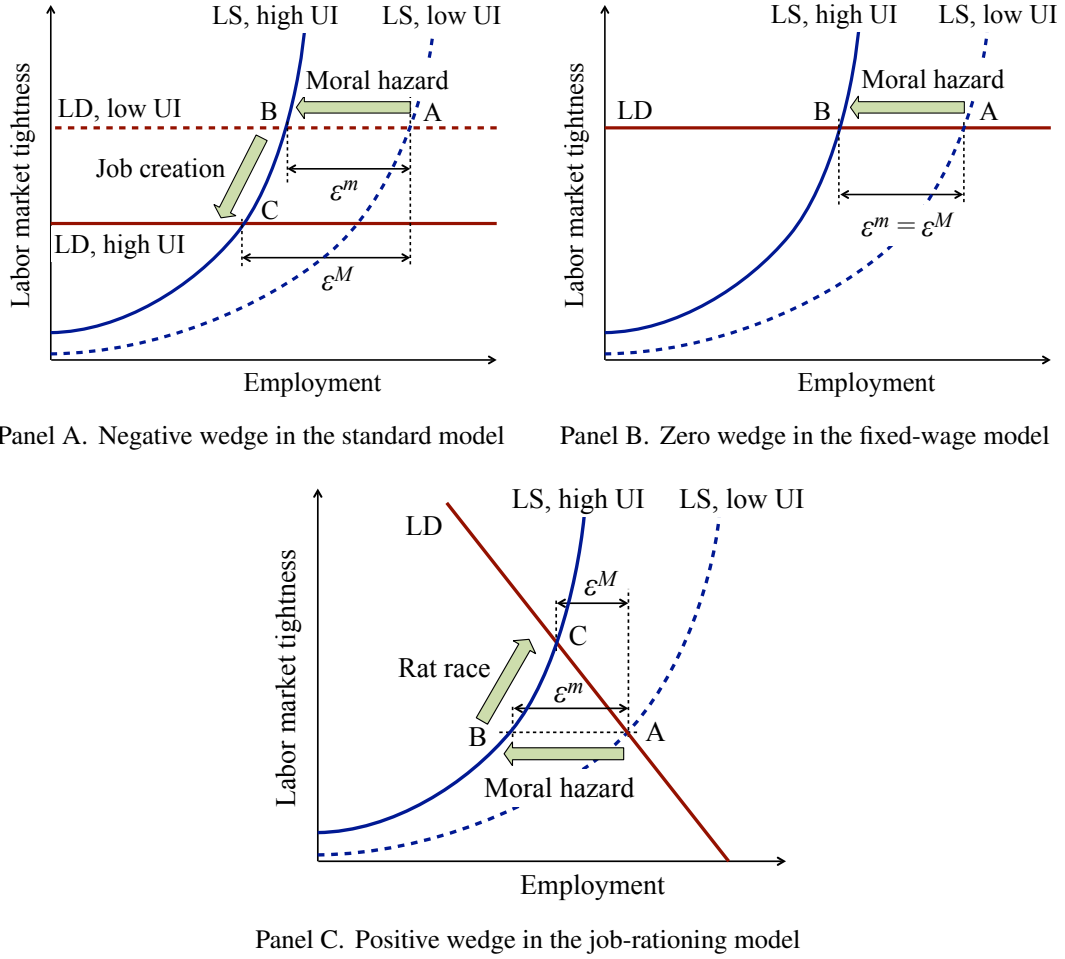


Figure 4: The Elasticity Wedge ( $1 - \epsilon^M / \epsilon^m$ ) in Specific Models

This equation defines a perfectly elastic labor demand in a  $(l, \theta)$  plane, as depicted in Panel B of Figure 4. The labor demand is unaffected by UI because the wage does not respond to UI.

Having obtained the labor demand, we can describe the elasticity wedge:

**PROPOSITION 5.** *In the fixed-wage model the elasticity wedge is 0.*

*Proof.* In equilibrium,  $\theta$  is determined by (29). This equation is independent of  $\Delta U$  so  $d\theta/\Delta U = 0$ . Using (21), we conclude that  $\epsilon^M = \epsilon^m$ .  $\square$

Panel B of Figure 4 illustrates the result. In the fixed-wage model UI affects employment only through the moral-hazard mechanism; the wage does not respond to UI so the job-creation mechanism is eliminated. As a result, the macroelasticity equals the microelasticity.

Since macroelasticity and microelasticity are equal, UI has no effect on tightness and optimal UI is given by the Baily-Chetty formula, even if tightness is inefficient. Basically, the fixed-wage model is isomorphic to the Baily-Chetty model of UI with fixed tightness.

### C. *The Job-Rationing Model of Michaillat [2012]*

The production function is concave:  $y(n) = n^\alpha$ , where  $\alpha \in (0, 1)$  parameterizes decreasing marginal returns to labor. The wage is fixed:  $w = \omega$ , where  $\omega \in (0, 1)$ . Unlike the bargained wage from the standard model, the fixed wage does not respond to UI.

We combine the wage schedule with equation (1) to obtain the labor demand:

$$l^d(\theta, \omega) = \left(\frac{\omega}{\alpha}\right)^{-\frac{1}{1-\alpha}} \cdot (1 + \tau(\theta))^{-\frac{\alpha}{1-\alpha}}. \quad (30)$$

Since  $\tau(\theta)$  is increasing in  $\theta$ , the labor demand is decreasing in  $\theta$ . Intuitively, when the labor market is tighter, hiring workers is less profitable as it requires a higher share of recruiters, and firms choose a lower level of employment. In the  $(l, \theta)$  plane of Figure 4, Panel C, the labor demand curve is downward sloping. Furthermore, the labor demand is unaffected by UI because the wage does not respond to UI.

The fact that the labor demand is downward sloping implies that jobs are rationed when the wage is high enough. Indeed, when  $\omega > \alpha$ , we have  $l^d(\theta = 0, \omega) < 1$ . In Figure 4, Panel C, the labor demand curve would cross the x-axis at  $l < 1$ . This means that jobs are rationed: firms would not hire all the workers even if workers searched infinitely hard and tightness were zero.

Having characterized the labor demand, we can describe the elasticity wedge:

**PROPOSITION 6.** *In the job-rationing model the elasticity wedge is positive:*

$$1 - \frac{\varepsilon^M}{\varepsilon^m} = \left(1 + \frac{\eta}{1-\eta} \cdot \frac{\alpha}{1-\alpha} \cdot \frac{1}{1+\varepsilon^f} \cdot \tau(\theta)\right)^{-1} > 0. \quad (31)$$

*Proof.* The elasticity of  $1 + \tau(\theta)$  with respect to  $\theta$  is  $\eta \cdot \tau(\theta)$ . From (30), we infer that the elasticity of  $l^d(\theta, a)$  with respect to  $\theta$  is  $-\eta \cdot \tau(\theta) \cdot \alpha/(1-\alpha)$ . By definition,  $\varepsilon^M$  is  $l/(1-l)$

times the elasticity of  $l$  with respect to  $\Delta U$ . Since  $l = l^d(\theta, a)$  in equilibrium, we infer that

$$\varepsilon^M = -\frac{l}{1-l} \cdot \eta \cdot \frac{\alpha}{1-\alpha} \cdot \tau(\theta) \cdot \frac{\Delta U}{\theta} \cdot \frac{d\theta}{d\Delta U}$$

We substitute the expression for  $(\Delta U/\theta) \cdot (d\theta/d\Delta U)$  from (21) into this equation and obtain

$$\varepsilon^M = \frac{\eta}{1-\eta} \cdot \frac{\alpha}{1-\alpha} \cdot \frac{1}{1+\varepsilon^f} \cdot \tau(\theta) \cdot (\varepsilon^m - \varepsilon^M).$$

Dividing this equation by  $\varepsilon^m$  and rearranging yields (31). □

Panel C of Figure 4 illustrates the proposition. After an increase in UI, jobseekers search less, shifting the labor supply curve inward by a distance A–B. Since the labor demand curve is downward sloping and does not respond to UI, the initial reduction in employment is attenuated by a horizontal distance B–C. The total reduction in employment is given by the horizontal distance A–C. Since A–B is larger than A–C, the microelasticity is larger than the macroelasticity and the elasticity wedge is positive.

The mechanism that causes the increase A–B in unemployment is the moral-hazard mechanism. The mechanism that causes the increase B–C in employment is the rat-race mechanism. This mechanism is absent from the standard and fixed-wage models. It operates as follows. The number of jobs available is somewhat limited because of decreasing marginal returns to labor. When a worker searches less, she reduces her probability of finding a job but mechanically increases others' probability of finding one of the few available jobs. Therefore, by discouraging job search, UI alleviates the rat-race for jobs and increases the job-finding rate per unit of effort and tightness. This increase in tightness leads to an increase in employment.<sup>8</sup> The rat-race mechanism explains why the macroelasticity is smaller than the microelasticity.

Proposition 6 implies that in the job-rationing model, an increase in UI raises tightness. Thus, the optimal replacement rate is above the Baily-Chetty rate when tightness is inefficiently low and

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<sup>8</sup>Consider an increase in UI and imagine that tightness,  $\theta$ , remains constant. Then the marginal recruiting cost,  $\tau(\theta)$ , is constant. As the wage,  $w$ , is constant, the marginal cost of labor,  $w \cdot (1 + \tau(\theta))$ , is also constant. Simultaneously, because jobseekers search less, firms employ fewer workers and the marginal product of labor is higher because of decreasing marginal returns to labor. Hence, firms face the same marginal cost but a higher marginal product of labor. This is suboptimal: firms could increase profits by posting more vacancies and hiring more workers. Consequently, the new equilibrium has a higher tightness.



below it when tightness is inefficiently high. In the job-rationing model, tightness is inefficiently low when the fixed wage is inefficiently high.

The rat-race mechanism appears in the job-rationing model because of decreasing marginal returns to labor, but alternative assumptions could give rise to it. Indeed, the mechanism operates as soon as the labor demand is downward sloping in a  $(l, \theta)$  plane, such that the number of jobs is limited for a given tightness. For instance, the presence of an aggregate demand on the product market as in the model from Michaillat and Saez [2015a] would give rise to the rat-race mechanism, even under constant marginal returns to labor.

## VI. Conclusion

This paper proposes a theory of optimal UI in matching models. The optimal UI replacement rate is the sum of the conventional Baily-Chetty replacement rate, which solves the tradeoff between insurance and job-search incentives, and a correction term, which is positive when an increase in UI pushes labor market tightness toward its efficient level. Hence, the optimal replacement rate is more generous than the Baily-Chetty replacement rate if tightness is inefficiently low and UI raises tightness, or if tightness is inefficiently high and UI lowers tightness.

In some countries, including the United States, the generosity of UI depends on the unemployment rate. Our formula provides guidance to link the generosity of UI to labor market conditions. It indicates that if UI has an influence on tightness and if tightness is inefficiently high or low, UI should be adjusted to bring tightness closer to its efficient level.

We propose empirical criteria to determine whether UI raises or lowers tightness and whether tightness is inefficiently high or low. Using these criteria, our companion paper [Landaïs, Michaillat and Saez, 2015] provides empirical evidence for the United States suggesting that UI raises tightness and that tightness is inefficiently low in slumps and inefficiently high in booms. Our theory combined with that evidence implies that the optimal replacement rate is above the Baily-Chetty rate in slumps but below it in booms. The companion paper also suggests that the departures from the Baily-Chetty formula are potentially large, especially in slumps.

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