# Gun Control and the Self-Defense Argument.[1]

Maitreesh Ghatak[2]

A key issue in the debate over gun control is how it will affect the relative incentives of criminals and law-abiding citizens to acquire guns. We propose a simple model of interaction of criminals and law-abiding citizens as a contest where the parties arm themselves in order to improve their chances of having an upper hand in an encounter. We study the effect of various gun control policies on crime and the total demand for guns taking into account the strategic interdependence between the demand for guns between criminals and law-abiding citizens.

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## I. Introduction

The debate over gun control often centers around how laws aimed at restricting the availability of guns will affect the relative incentives of criminals

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[2] Department of Economics, University of Chicago, 1126 E. 59th Street, Chicago IL 60637. Email: mghatak@midway.uchicago. On leave 2000-01 to: School of Social Science, Einstein Drive, Institute for Advanced Studies, Princeton, NJ 08540. Email : mghatak@ias.edu.

and law-abiding citizens to acquire guns. Supporters of gun control argue that in the absence of these laws, criminals will acquire guns too easily. Also, if citizens are armed, criminals may have an even greater incentive to acquire guns, leading to too many guns in circulation and a greater number of incidents of violent crime. In contrast, opponents of gun control voice the concern that these laws will undermine the ability of citizens to defend themselves or other citizens against criminals, and hence end up increasing crime and gun related violence. In this paper we formally analyze the effect of various types of gun control policies using a simple model that explicitly takes into account the self-defense argument against gun control.

In our model criminals and law-abiding citizens are engaged in a contest (e.g., Dixit, 1987) - the former wants to succeed in committing a crime and the latter to prevent the criminal from succeeding. Both parties choose the extent to which to arm themselves which depends on the cost of guns as well as the decisions of other individuals. In particular, a criminal's incentive to use a gun is likely to be affected by the likelihood that a potential victim will own a gun, and vice versa. We focus on the effects of two kinds of gun control policies on crime and the total demand for guns - those that affect the cost of guns facing all individuals, and those that selectively affect the cost of guns facing criminals or law abiding citizens.

We show that once the strategic interdependence between the demand for guns between criminals and law-abiding citizens is explicitly taken into account, some interesting conclusions emerge about the effect of various gun control policies. For example, measures to increase the cost of guns to criminals (a policy on which parties on both sides of the gun control debate seem to agree) will decrease crime but may increase the total demand for guns. On the contrary, measures to increase the general cost of guns will have no effect on crime, but will reduce the total demand for guns. Hence, according to this model, if the demand for guns of criminals and law abiding citizens for self defense were the only sources of demand for guns, the policy of a general tax on guns is preferable to those that aim at raising the relative cost of guns to criminals. A related interesting implication of our model is that the crime rate could be decreasing in the proportion of criminals in the population. The greater is the proportion of criminals in the population, the lower is the marginal return from having guns to criminals since there is a shortage of potential victims. Also, the higher is the marginal return from having guns to law abiding citizens since the threat of criminal attack goes up. This implies that the relative strength of law abiding citizens in terms of guns will go up, which will reduce the probability of a crime being successful. This can potentially outweigh the fact that there are more criminals in the population and lead to an overall decrease in the number of successful crimes.

It is hard to exaggerate the importance of the topic in the US context. About 30% of violent crimes involve the use of a firearm, and 86% of gun related crimes involve a handgun.[3] One-fifth of all gun owners and two-fifths of handgun owners cite self defense to be the most importance reason for owning guns (Cook, 1991). Estimates of the number of incidents in which guns are used for self defense purposes vary widely from 80,000 instances per year (Cook, 1991) to more than a million instances per year (Kleck and

[3] U.S. Department of Justice, 1995.

Gertz, 1995). Given that the number of violent crimes committed by guns is around 800,000 per year (Cook, 1991), these estimates suggest that for every 10 crimes committed with a gun the number of cases where a gun used in self defense prevents a crime ranges from 1 to 12. A recent study shows that victims who resist with a gun are less likely than other victims to lose their property in robberies, less likely to be injured by criminals, and less likely to be raped (Kleck and Gertz, 1995). Lott and Mustard (1997) have shown that increases in gun ownership caused by the passage of carrying concealed weapons legislation in ten states between 1985-91 in the US led to substantial reduction in violent crimes.[4] They argue that the law deterred criminals from committing crimes for the fear that potential victims could be armed.

Despite the importance of the topic in public policy debates, and a substantial and growing body of empirical work, very little theoretical work has been done in the economic literature on crime to consider individuals' incentives to acquire and use guns for crime and for self-defense in a framework that incorporates the strategic interdependence between these decisions. An exception is a recent short paper by Chaudhri and Geanakoplos (1998) which argues using a simple supply-demand diagram that the demand for guns is subject to externalities, and if taxing guns reduces the demand coming from criminals, it will also reduce demand for self-defense purposes.[5] As a result

[4]Several papers have studied the robustness of the Lott-Mustard results (e.g., Black and Nagin, 1998, Ludwig, 1998, and Duggan, 2001) with mixed results.

[5]Another paper that looks at a similar framework is by Donohue and Levitt (1998). It argues that easier availability of guns may reduce the predictability of fight outcomes and

such a policy can greatly reduce the total sale of guns. However, many argue (e.g., Bartley, 1999) that the demand for guns of criminals is likely to be less elastic compared to that of law abiding citizens, and in any case, criminals are less likely to be affected by changes in *laws* since they acquire their guns through illegal channels. As a result taxing guns may increase the rate of crime. The motivation for our paper comes from the fact that without explicitly modeling the interaction between criminals and law-abiding citizens taking into account the self-defense argument, and studying the properties of the equilibria of this game, it is hard to evaluate these alternative views emphasizing different *partial* effects of gun control policies.

In evaluating alternative policies we have to adopt some measure of social welfare. In other words, following Bartley (1999), we must address the question as to what is special about guns that society needs to regulate it. His argument is it is crimes committed with guns, and not guns *per se*. This assumes that the social welfare function puts more weight on the welfare of law abiding citizens than that of criminals, which is a reasonable position and is in fact implicitly assumed in the public debate on crime. If instead crime is viewed as a zero-sum game and society puts equal weight on the welfare of law abiding citizens and criminals, then the crime rate should not have any effect on social welfare. But even if one takes this view which ignores various costs associated with crime (e.g., injury) that makes crime a non-zero sum game, the resources spent by criminals to succeed in committing a crime, and by law abiding citizens to defend themselves represent a deadweight loss from

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hence lead to more violence through that channel. This is an interesting issue that is not captured in our model.

the social point of view. That is, in terms of our model, social welfare will negatively depend on the total amount of guns used by law abiding citizens and criminals. Gun control advocates typically point to another important social cost of easy availability of guns that has nothing to do with the crime rate. Guns involve some negative externalities due to possible accidental use or use by individuals who are unable to exercise their judgement (e.g., individuals with mental illnesses, those intoxicated with alcohol or drugs, and children). In addition, easy availability of guns may increase the likelihood that arguments between acquaintances will lead to fatal outcomes and that attempted suicides will successful. Hence, apart from the crime rate and the deadweight loss associated with resources spent to commit it and prevent it, one could argue that society also cares about the total amount of guns circulating in the population for the additional reason that they generate some negative externalities for society at large. In the light of this discussion, in this paper we will separately look at the effect of various gun control policies on both the crime rate and the total amount of guns that are demanded.[6]

[6]Those who invoke the Second Amendment to argue against gun control put a lot of weight on the liberty (or the participation constraints) of gun owners. According to this criterion this exercise is not meaningful since even if gun control reduces the amount of guns as well as crime, social welfare will decrease.

## II. The Model.

To focus on the self-defense argument against gun control, demand for guns in our model arises only from criminals and potential victims.[7] Criminals use it as a credible and efficient threat of violence to succeed in a crime such as robbery. Law abiding citizens use it as a credible and efficient threat of violence to resist a criminal. There are two types of economic activities, productive and criminal. The income from the productive activity is assumed to be constant, and is normalized to 1.[8] The criminal activity consists of trying to capture the income of an individual engaged in the productive activity. Individuals are drawn from a large population whose size is normalized to 1. We do not model the choice of individuals between engaging in the criminal or the productive activity. We simply assume that the proportion of criminals in the population, say $q$, is given. For notational simplicity, we will call a criminal a $C$ type player, or $C$, and a law-abiding citizen a $L$ type player, or $L$.

Criminals meet potential victims governed by a simple random matching process. Each player is randomly matched with another player. So $q$ is the probability that a given $L$ type player meets a $C$ type player, and $(1-q)$ is the probability that a given $C$ type player meets a $L$ type player. Also, $q^2$ is the probability that a given match has two $C$ type players, $(1-q)^2$ is the probability that it has two $L$ type players and $2q(1-q)$ is the probability that

[7] We do not explicitly deal with the purchase of guns for sports and hunting. The effects of various gun control policies on this source of demand is straightforward.

[8] We can easily extend the model to have the production function display diminishing returns with respect to the number of people engaging in the productive activity.

it has one $C$ type player and one $L$ type player. If two $C$ type players meet, both receive 0, and if two $L$ type players meet both receive 1. If one $C$ type and one $L$ type player meet, there is a certain probability that the former will be successful in robbing the latter. This probability is not a constant. $C$ type players can choose an appropriate criminal technology to improve their chances of committing a successful crime. Similarly, $L$ type players can choose an appropriate self defense technology to improve their chances of resisting a criminal. We assume that these technologies depend on their gun intensities, namely, the importance of guns relative to other potential instruments such as physical force, knives, etc. Let $x_C \in [\underline{x}, \overline{x}]$ denote the gun-intensity of the criminal technology to intimidate the potential victim, and let $x_L \in [\underline{x}, \overline{x}]$ denote the gun-intensity of the self defense technology a potential victim adopts to fend off a possible criminal attack where $1 \geq \bar{x} > \underline{x} > 0$. Let $z \equiv \frac{x_C}{x_L} \in \left[\frac{\underline{x}}{\overline{x}}, \frac{\overline{x}}{\underline{x}}\right]$ denote the relative strength of criminals with respect to law abiding citizens in terms of guns.

The interpretation of $x_i$ is that it is the resources spent by player $i$ to arm himself which could be in terms of money, effort or both. This could result in more guns, better quality guns, or the likelihood of obtaining a gun. In terms of the last interpretation, $x_i$ is the probability of player $i$ having a gun rather than another weapon (or a monotonically increasing function of this probability).[9]

[9]It has been widely observed that facing the same environment, not all citizens choose to arm themselves, and not all criminals use guns as their weapon of intimidation. An obvious explanation is that criminals and potential victims differ in terms of some private characteristics that govern the cost acquiring guns such as physical strength, how much

After a criminal and a law-abiding citizen are matched, let $P(x_C, x_L) \in [0, 1]$ denote the probability that an attempted crime is successful upon which $C$ receives 1 and $L$ receives 0. With probability $1 - P(x_C, x_L)$ the attempted crime is unsuccessful so that $C$ receives 0 and $L$ receives 1. We assume that the criminal receives the victim's entire income if successful and so $P(x_C, x_L)$ will be referred to as the expected return to a criminal from successfully committing a crime conditional on being matched with a victim.[10]

The function $P(x_C, x_L)$ giving the probability of a criminal succeeding in committing a crime is similar to the one used to depict the probability of winning in a contest (Dixit, 1987). We assume that the function $P(x_C, x_L)$ satisfies the following properties:

A1. $P_1(x_C, x_L) \geq 0$ and $P_{11}(x_C, x_L) \leq 0$ for all $x_C$ and $x_L$.

A2. $P_2(x_C, x_L) \leq 0$ and $P_{22}(x_C, x_L) \geq 0$ for all $x_C$ and $x_L$

A3. $P(x_C, x_L) = P(tx_C, tx_L)$ for any $t > 0$.

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they fear being attacked, contact with people who have access to guns, skill in using guns etc. Analytical simplicity is our only justification for using a model with a representative criminal and a representative law-abiding citizen, and favoring the interpretation of the demand for guns as some function of the probability of a player having a gun.

[10] We can easily allow the game to be non zero-sum by assuming that conditional on the criminal activity being successful criminals gain $A > 0$ and victims lose $-B < 0$ where $A \neq B$. This will allow for the possibility that the victim may lose more than what the criminal gains because of the psychological costs of being intimidated, or being hurt, and conversely, the criminal could gain less than what the victim loses because of the possibility of being hurt, or caught and punished by the police. However, it can be argued that $A$ and $B$ could themselves depend on $x_C$ and $x_L$. This makes the analysis much more complicated and is beyond the scope of this paper.

The first assumption implies that the more gun intensive the criminal technology, the more likely a criminal will succeed in committing a crime. As Cook (1991) puts it, the objective of a criminal is to gain the victim's compliance quickly, thereby preventing the victim from striking back, escaping or summoning help. In this regard guns are very effective since it gives the criminal the capacity to threaten bodily harm from a distance and forestall victim resistance. This assumption is based on the observation that a gun is more lethal, easier to conceal, and requires less effort or skill to use than other weapons and has a higher observed rate of success in commercial robberies compared to knives and other weapons (Cook, 1991).

The second assumption implies that the more gun intensive is the self defense technology, the less likely it is that the crime will be successful. This is one way of capturing the deterrence effect of gun ownership among law abiding citizens on crime, namely, the more they arm themselves, the less the crime is likely to be successful. The statistical record suggests that people who use guns to defend against robberies, assaults, and burglaries are generally more successful in foiling the crime and avoiding injury compared to people who resist using other weapons (Cook ,1991). Guns are more effective than other weapons or physical strength as a self defense measure since they are equalizers that tend to neutralize the natural advantage of people who select into criminal activity. We will call this the "weak deterrence effect".

We also assume that guns are subject to diminishing returns for both criminals and law abiding citizens, which ensures that an interior solution exists to the best response of each player, and it is unique.

Finally, we assume that $P(x_C, x_L)$ is homogeneous of degree zero. Hence we can express it in the following "ratio" form[11]:

$$P(x_C, x_L) = P(z, 1) \equiv p(z).$$

The following simple version of the logit functional form

$$P(x_C, x_L) = \frac{\alpha x_C}{\alpha x_C + \beta x_L}$$

where $\alpha > 0$ and $\beta > 0$, belongs to this class of functions.[12] The justification for this assumption is that in a contest it is the relative strength of the participants that matter in determining who will succeed, and not their absolute levels.

Notice that we do not explicitly model the interaction of the criminal and the target of the crime, in particular, how guns are actually used. All we assume is that upon being randomly matched with a law-abiding citizen (criminal), having a gun raises the criminal's (law-abiding citizen's) chances of successfully committing (preventing) the crime in an *ex ante* sense. If the criminal has a gun and the victim does not, it is likely that the former will able to scare the latter into submission. However, it is possible that the victim will be able to resist with physical force, or may be helped by others

[11]See Hirshleifer (1988) for a discussion of the two main functional forms used in the literature on conflict interactions, the "ratio" form (i.e., $p$ is an increasing and concave function of $\frac{x_C}{x_L}$) and the "difference" form (i.e., $p$ is an increasing and concave function of $x_C - x_L$). The latter form, a popular example of which is the logistic family of functions, $p = \frac{1}{1+e^{k(x_C - x_L)}}$, is not suitable for our setting because the only interesting equilibrium is always symmetric.

[12]The logit form is commonly used in the literature on contests, tournaments and patent races (see for example, Dixit, 1987, Rosen, 1986, and Loury, 1979).

(e.g., a police car may suddenly appear on the scene). On the other hand, if the victim has a gun and the criminal does not, it is possible that the victim will be able to scare off the criminal.[13] Even if both parties have guns, it is possible that whoever manages to draw it first will gain an upper hand, or it is possible that a shoot out may occur the outcome of which is unpredictable. There is evidence that for robbery victims resisting with a gun is not only effective when the criminal does not have a gun, but also when the criminal has a gun (Kleck and Gertz, 1995). In the absence of more evidence on this issue we prefer to remain agnostic, and simply assume that while the outcome of a conflict between two players is quite unpredictable *ex post*, the more likely one player has a gun and his opponent does not, the greater is the chance of success of the former in an *ex ante* sense.

The following relationship between the first and second derivatives of the functions $P(x_C, x_L)$ and $p(z)$ can be readily verified and will be useful for our subsequent analysis: $P_1 = p'(z)\frac{1}{x_L}, P_2 = -zp'(z)\frac{1}{x_L}, P_{11} = p''(z)\frac{1}{x_L^2}, P_{22} = zp'(z)\frac{1}{x_L^2}\left(2 + \frac{zp''(z)}{p'(z)}\right)$ and $P_{12} = P_{21} = -zp'(z)\frac{1}{x_L^2}\left(1 + \frac{zp''(z)}{p'(z)}\right)$. Let $\varepsilon \equiv \frac{-zp''(z)}{p'(z)}$ denote the elasticity of $p'(z)$. Clearly, for assumptions A1 and A2 to go through we need $p(z)$ to satisfy the conditions that $p'(z) \geq 0, p''(z) \leq 0$ and $2 \geq \varepsilon$. These are satisfied for the example $P(x_C, x_L) = \frac{\alpha x_C}{\alpha x_C + \beta x_L}$ which corresponds to $p(z) = \frac{\alpha z}{\alpha z + \beta}$, since $\varepsilon = 2p(z) \leq 2$.

The cross partial derivative of $P$ is important for the outcome of the game. When $P_{12} = 0$, an increase in the ownership of guns by law abiding citizens reduces the return to crime for criminals, but does not affect the

[13] According to Quigley (1990) most of the time people using guns to defend themselves merely have to show the gun and not use it.

marginal product of using guns for criminals. If $P_{12} < 0$ then an increase in the ownership of guns by law abiding citizens reduces the marginal return of using guns for criminals. This is an alternative, and stronger notion of the deterrence effect of gun ownership among law abiding citizens on crime: using guns in crime is more profitable if the victim is unarmed than otherwise. We can call this the "strong deterrence effect" - citizens having guns will not only reduce the level of crime, but also the marginal return of using guns by criminals since they now face victims who are potentially armed. Of course, there is no *a priori* reason for believing that this effect prevails in reality. One could as well make the opposite argument that the more the citizens are armed, the more criminals should arm themselves if they are to succeed in committing a crime. Indeed, Cook (1991) argues that the more vulnerable the victim, the lower is the marginal product of guns. This case, namely $P_{12} > 0$, captures an "arms race" effect.

The cost of acquiring guns is $\gamma(1 + t_C)x_C$ for a criminal and $\gamma(1 + t_L)x_L$ for a law abiding citizen where $\gamma > 0$ and $t_C \geq 0$ and $t_L \geq 0$.[14] It is helpful to think that the cost of guns to any player having a common component, $\gamma$, and a mark up representing a specific component, $t_i$ with $i = C, L$. Let

[14] For algebraic simplicity, we take the income that is subject to potential capture by criminals as exogenously given and independent of the cost of guns. One way to justify this is to interpret these costs as non-monetary effort costs of acquiring guns (which applies to non-tax gun control policies). Alternatively we can assume that both types of players have some initial endowment of $\omega$. In the first stage of the game they allocate it between purchasing guns and other consumption goods, and in the second stage they go out either to work or to commit crimes, and then finally they have another round of consumption.

$\tau \equiv \frac{1+t_L}{1+t_C}$ denote the ratio of the cost of guns to $L$ type players relative to $C$ type players.

If guns were completely unregulated, then the marginal cost of acquiring guns would be the same for all. Examples of measures to reduce the general availability of guns (i.e., raise $\gamma$) include taxes on guns and ammunition, waiting periods without background checks, gun bans of any kind, and restrictions on carrying of concealed weapons. In contrast measures to increase the cost of acquiring guns specifically to criminals (i.e., raise $t_C$) are background checks, waiting periods, restriction of the quantity of guns that can be purchased within a certain period, registration of guns, add on penalties for the commission of crime with a firearm, and the requirement that sales of guns to be made through a licensed gun dealer. Finally, laws that allow carrying concealed hand guns to law abiding citizens can be interpreted as a measure to reduce $t_L$. It is often argued that any gun control policy, even if it is "general" on paper, ends up affecting the cost of law abiding citizens relatively more than that of criminals since criminals obtain guns through illegal means. In terms of our framework this argument implies that all gun control policies lead to an increase in $\tau$.

## III. Equilibrium

The expected payoffs of a representative $C$ type player and a $L$ type player are, respectively,

$$
\begin{aligned}
\pi^C &= q.0 + (1-q)P(x_C, x_L) - \gamma(1+t_C)x_C \ = (1-q)P(x_C, x_L) - \gamma(1+t_C)x_C. \\
\pi^L &= (1-q).1 + q(1 - P(x_C, x_L)) - \gamma(1+t_L)x_L = 1 - qP(x_C, x_L) - \gamma(1+t_L)x_L \ .
\end{aligned}
$$

Notice that the utilitarian social welfare function is national income minus the cost of guns:

$$W = q\pi^C + (1-q)\pi^L = (1-q) - q\gamma(1+t_C)x_C \; - (1-q)\gamma(1+t_L)x_L \quad (1)$$

As a benchmark we note that :

> **Proposition 1**: *The joint welfare maximizing outcome is $x_C = \underline{x}$ and $x_L = \underline{x}$.*

This follows directly from maximizing (1) with respect to $x_C$ and $x_L$. This is a zero-sum game, and hence the net social marginal benefits of $x_C$ and $x_L$ are both zero. As a result, they would take their lowest possible values if social welfare is maximized.

We now study the Nash equilibrium of this game. By **A1** and **A2** the maximization problem facing each player is well-behaved. The first order conditions for interior solutions of $C$ type and $L$ type players are

$$\begin{aligned} (1-q)P_1(x_C, x_L) &= \gamma(1+t_C) \\ -qP_2(x_C, x_L) &= \gamma(1+t_L). \end{aligned}$$

The above equations can be solved to obtain the reaction functions, $x_C = R_1(x_L)$ and $x_L = R_2(x_C)$ which in turn can be simultaneously solved to find out the equilibrium values of $x_C$ and $x_L$. It is readily verified that the reaction functions of the two types of players have slopes in the opposite directions. In particular, $R_1'(x_L) = -\frac{P_{12}}{P_{11}}$ and $R_2'(x_C) = -\frac{P_{21}}{P_{22}}$. This is due to the zero-sum nature of the contests, i.e., in a given match between $C$ and $L$ type players, the expected payoff of a $C$ type player is $P$ and that of a $L$ type player is $-P$ (gross of the cost of guns). When $P_{12} < 0$, the reaction function of a $C$ type player is downward sloping. This is the strong deterrence effect at work - the marginal return from using a gun to a $C$ type player, $P_1$, decreases the more $L$ type players are armed. In contrast, the reaction function of a $L$ type player is upward sloping in this case. The marginal effect of $x_L$ on deterring crime is $-P_2 > 0$ and because $P_{12} < 0$ is equivalent to $-P_{12} > 0$, $-P_2$, *increases* with $x_C$. By an analogous argument, when $P_{12} > 0$ the reaction function of a $C$ type player is upward sloping, and that of a $L$ type player is downward sloping. In addition, note that since $-P_{11}P_{22} + P_{12}^2 > 0$ the *strategic stability* condition is satisfied irrespective of the sign of $P_{12}$. This ensures that the equilibrium is going to be stable, and the comparative static exercises will be meaningful.

Since the strategy spaces are non-empty compact convex subsets of an Euclidean space, and the payoff function for each player $i$ is continuous in $x_C$ and $x_L$, and quasi-concave in $x_i$ under our assumptions, an equilibrium in pure strategies exists. Since $P_1 = p'(z)\frac{1}{x_L}$ and $P_2 = -zp'(z)\frac{1}{x_L}$, we can rewrite the first-order conditions as:

$$x_C = \frac{1-q}{\gamma(1+t_C)} zp'(z) \tag{2}$$

$$x_L = \frac{q}{\gamma(1+t_L)} zp'(z). \tag{3}$$

Then these two conditions can be combined as

$$z^* = \tau \frac{1-q}{q} \tag{4}$$

where the superscript $(*)$ indicates that the equilibrium level of a variable is being considered. Since $z \in \left[\frac{\underline{x}}{\overline{x}}, \frac{\overline{x}}{\underline{x}}\right]$ so long as $\underline{x}$ is small enough, and $t_C, t_L$, and $q$ do not take extreme values, a unique interior equilibrium is guaranteed.

## IV. Comparative Statics

We want to solve for the equilibrium values of $x_C^*$ and $x_L^*$ in terms of $\gamma$, $t_C, t_L$ and $q$, and then examine the effect of various policy changes. From (4) we see that an increase in $\tau$ means guns are relatively more expensive to $L$ type players, which results in an increase in $z$, i.e., the relative strength of $C$ type players. Our main interest lies in the effect of various parameter changes on two variables: (i) the crime rate, which is the average number of encounters between $C$ and $L$ type players, times the probability that the crime is successful, i.e., $c^* = 2q(1-q)P(x_C^*, x_L^*)$; (ii) the total (or average) demand for guns, $x^* \equiv qx_C^* + (1-q)x_L^*$.

### IV.1 The Crime Rate

The comparative static analysis for the crime rate is straightforward. Since $c^* = 2q(1-q)p(z^*)$, and $p^*$ depends only on the relative cost of guns to $C$ type and $L$ type players, $\tau$, it is not affected by the general cost of guns, $\gamma$. Secondly, any policy that increases $t_C$ and/or decreases $t_L$ will reduce the crime rate. Third, $p^*$ is decreasing in $q$. That is, the greater the proportion of criminals in the population, the *lower* is probability that in a given encounter between a $C$ type and a $L$ type player a crime will be successful. This follows from the simple fact that the higher is $q$ the greater is the incentive for $L$ type players to arm themselves, and the lower is the incentive of $C$ type players to arm themselves. This reduces the ratio of demand for guns coming from $C$ type players to the demand coming from $L$ type players, $z^*$, which in turn decreases $p^*$. However, to look at the effect on the average crime rate, we also have to take into account the effect of an increase in $q$ on the probability that in a random match between two players, one is a $C$ type player and the other is a $L$ type player, i.e., $2q(1-q)$. It is straightforward to check that using (4):

$$\frac{\partial c^*}{\partial q} = 2((1-2q)p^* - z^* p'(z^*)).$$

Since $p$ is concave, $p^* > z^* p'(z^*)$ and so for low values of $q$, $\frac{\partial c^*}{\partial q} > 0$. On the other hand, so long as $q$ is close enough to $\frac{1}{2}$ (and indeed if it exceeds $\frac{1}{2}$) the equilibrium of level of crime would be decreasing in the number of criminals in the population. Hence we have the following result:

> Proposition 2*: An increase in the general cost of guns has no effect on the crime rate. An increase in the cost of guns faced by criminals relative to that of law abiding citizens will reduce the crime rate. If the proportion of criminals in the population is greater than some threshold level $\hat{q} \in (0, \frac{1}{2})$ then the crime rate is decreasing in the proportion of criminals.*

## IV.2 The Total Demand for Guns

Now let us turn to the effect of changes in $\gamma, t_C, t_L$ and $q$ on the total demand for guns. Using the first-order conditions (2) and (3) we have:

$$x^* = \frac{(1-q)^2 p'(z)}{\gamma(1+t_C)}(1+\tau).$$

It follows upon inspection that $\frac{\partial x^*}{\partial \gamma} < 0$. Straightforward algebra yields:

$$\frac{\partial x^*}{\partial t_C} = A_1\{\varepsilon - (1 + \frac{\tau}{1+\tau})\}, \frac{\partial x^*}{\partial t_L} = A_2\{\frac{\tau}{1+\tau} - \varepsilon\}, \text{ and } \frac{\partial x^*}{\partial q} = A_3(\varepsilon - 2q)$$

where $A_1 \equiv \frac{(1-q)^2\{(1+t_C)+(1+t_L)\}}{\gamma(1+t_C)^3}p'(z)$, $A_2 \equiv \frac{(1-q)^2\{(1+t_C)+(1+t_L)\}}{\gamma(1+t_C)^2\ (1+t_L)}p'(z)$ and $A_3 \equiv \frac{1}{\gamma}\ \ \frac{1}{1+t_C} + \frac{1+t_L}{(1+t_C)^2}\ \ \frac{(1-q)}{q}p'(z)$ are positive terms that do not affect the sign of the derivatives we are interested in.

From the first-order condition of the players we see that the *direct* effect (i.e., ignoring the strategic interaction in the demand for guns of the two types of players) of an increase in $t_C$ is to reduce the *level* of demand for guns for $C$ type players, and leave the *level* of demand of $L$ type players unaffected. If $P_{12} < 0$ (i.e., the "strong deterrence effect" is in operation), the reaction function of a $C$ type player is downward sloping and that of a $L$ type player is upward sloping. Hence the indirect effect will reduce the demand for guns of $L$ type players, which in turn will partly mitigate the extent to which the demand for guns by $C$ type players will fall. Since $P_{12} = P_{21} = -zp'(z)\frac{1}{x_L^2}(1-\varepsilon)$, when $P_{12} < 0$, $\varepsilon < 1$ and hence $\frac{\partial x^*}{\partial t_C} < 0$. However, if $P_{12} > 0$ (the "arms race" case) the opposite will happen. Now the direct effect of an increase in $t_C$ in terms of reduction in the demand for guns by $C$ type players will be met with an increase in the demand for guns by $L$ type players, which will partly mitigate the drop in demand from $C$ type players. If $\varepsilon$ is high, the indirect effects will be strong enough such that the net effect will be an increase in the total demand for guns.

If $t_L$ is increased, the direct effect is a decrease in $x_L$ and no effect on $x_C$. However, the indirect effect of this will be an increase in the demand for guns by $C$ type players if $P_{12} < 0$ (or $\varepsilon < 1$), which will in turn dampen the decrease in $x_L$. If $\varepsilon$ is small then the indirect effects will be strong enough in this case so as to lead to a net overall increase in the demand for guns. In contrast, if $P_{12} > 0$ (or, $\varepsilon > 1$) criminals will disarm themselves in response to the drop in demand by law abiding citizens, and the direct and indirect effects of an increase in $t_L$ will all tend to reduce overall demand.

The effect of an increase in $q$ is similar to that of a reduction in the relative price of guns faced by $L$ type players because it reduces the marginal benefit of criminals from having guns (they are less likely to match up with a $L$ type player). Hence the result is similar to that of an increase in $t_C$ : if $\varepsilon$ is low, the net result could be a reduction in the total demand for guns.

The above analysis can be summarized in the following result:

> Proposition 3*: An increase in the general cost of guns will reduce the total demand for guns. However, an increase in the cost of guns faced by criminals may increase the total demand for guns if* $P_{12} > 0$ *and conversely, an increase in the cost of guns faced by law abiding citizens may increase the total demand for guns if* $P_{12} < 0$.

For illustration, consider the following example of a logit function, $p(z) = \frac{\alpha z}{\alpha z+\beta}$ which corresponds to $P(x_C, x_L) = \frac{\alpha x_C}{\alpha x_C + \beta x_L}$. It is readily checked that $P_{12} = \alpha\beta\frac{\alpha x_C - \beta x_L}{(\alpha x_C + \beta x_L)^3}$, $P_{11} = -2\alpha\beta\frac{\alpha x_L}{(\alpha x_C + \beta x_L)^3}$ and $P_{22} = 2\alpha\beta\frac{\beta x_C}{(\alpha x_C + \beta x_L)^3}$. As noted earlier, $\varepsilon = 2p(z)$ in this case. Using the equilibrium value of $z$, we get $\varepsilon = 2\frac{\alpha(1+t_L)(1-q)}{\alpha(1+t_L)(1-q)+\beta(1+t_C)q}$. Notice that $\varepsilon$ is increasing in $\alpha$ and decreasing in $\beta$. Also, let $\eta^i$ denote the price elasticity of demand for guns of player $i$ $(i = C, L)$. It is straightforward to check that the reaction functions of $C$

and $L$ type players in this case are $x_C = \sqrt{\frac{\beta(1-q)}{\alpha\gamma(1+t_C)}}\sqrt{x_L} - \frac{\beta}{\alpha}x_L$ and $x_L = \sqrt{\frac{\alpha q}{\beta\gamma(1+t_L)}}\sqrt{x_C} - \frac{\alpha}{\beta}x_C$.

The best-response functions are depicted in Figures 1 and 2 corresponding to the cases $\alpha > \beta$ and $\alpha < \beta$. A key feature of these best response functions are that they change slope depending on the relative magnitude of $x_C$ and $x_L$. In particular, if $\alpha x_C > \beta x_L$ then $P_{12} > 0$ and the reaction function of $C$ is upward sloping and that of $L$ is downward sloping. If $\alpha x_C < \beta x_L$, then $P_{12} < 0$ and the reaction function of $C$ is downward sloping and that of $L$ is upward sloping. For $\alpha x_C = \beta x_L$, $P_{12} = 0$ and so the reaction functions have a slope of zero. In the figures, the line $P = \frac{1}{2}$ corresponds to the locus of points $\alpha x_C = \beta x_L$.

The explanation for this feature of reaction functions is as follows. If the probability of success of the crime was additively separable in $x_L$ and $x_C$, then from the point of view of a $C$ type player, the decision problem would be like a standard problem of profit maximization with respect to one input where output is subject to diminishing returns in the *absolute level* of the input. In contrast, we have a "rat-race" type situation where output is subject to diminishing returns in the *relative level* of the input with respect to the action chosen by his rival. If the rival starts of with an advantage (i.e., $\alpha x_C < \beta x_L$) then pulling up the ratio is an uphill task, and diminishing returns set in much faster. Any further increases in the rival's action in this region, accordingly leads to a reduction in the choice of the action level of the decisionmaker. In contrast, if a $C$ type player starts with a relative advantage, then pulling up the ratio is relatively easy and diminishing returns set in much more slowly. If the players are evenly matched (i.e., $P = \frac{1}{2}$ or, $\alpha x_C = \beta x_L$) the marginal product of $x_C$ is unaffected by (small) changes in $x_L$. Formally, that is why $P_{12} \gtreqless 0$ according as $\alpha x_C \gtreqless \beta x_L$.

Evaluated at the equilibrium point in a hypothetical initial situation of no gun control (i.e., $\tau = 1$) we find that

$$\begin{aligned} \eta^C &\equiv -\frac{\gamma(1+t_C)}{x_C} \frac{dx_C}{d\gamma(1+t_C)} = \frac{1}{2}\left(1+\frac{\beta}{\alpha}\right) \\ \eta^L &\equiv -\frac{\gamma(1+t_L)}{x_L} \frac{dx_L}{d\gamma(1+t_L)} = \frac{1}{2}\left(1+\frac{\alpha}{\beta}\right) . \end{aligned}$$

The higher is $\alpha$ relative to $\beta$, the greater is the elasticity of demand of law-abiding citizens, and the lower is the elasticity of demand of criminals, and conversely when $\beta$ is high relative to $\alpha$. This is what we would expect: if guns are more effective when used by criminals relative to law abiding citizens on the margin (in terms of increasing the probability of successfully committing a crime), $L$ type players start with a relative disadvantage (i.e., $\alpha x_C > \beta x_L$) and so a given change in the cost of guns would affect them much more sharply (i.e., $\varepsilon$ is large) than it would affect $C$ type players. As a result, an increase in $t_L$ would certainly lead to a drop in the demand for guns. But, since the demand for guns by citizens is elastic in this case, an increase in $t_C$ that has a direct effect of reducing $x_C$ will sharply increase the demand for guns by $L$ type players in turn. This indirect effect could be strong enough to lead to an increase in the total demand for guns. On the contrary, if $\alpha$ is small relative to $\beta$, then $\varepsilon$ is small and an increase in $t_C$ will reduce the total demand for guns since the negative response by criminals (whose demand is more elastic) will dominate. However, an increase in $t_L$ could increase the total demand for guns in this case.

When the demand for guns of criminals and law-abiding citizens are independent, raising the price faced by criminals only will naturally reduce their demand for guns and leave the demand of law-abiding citizens unchanged. As a result the total demand for guns will fall, and crime will decrease. This may be the reason why this is not a controversial issue in the debate over gun control. However as the above analysis suggests, this need not necessarily be the case if we recognize the interdependence in the demand for guns by criminals and law-abiding citizens.

## V. Concluding Remarks

The main contribution of the theoretical model described above is to show that the effects of various types of gun control policies on crime and on the total demand for guns are not obvious if one recognizes the interdependence between the demand for guns of criminals in order to commit crimes, and of law abiding citizens in order to defend themselves. We conclude that an increase in the general cost of guns will reduce the total demand for guns and will not affect crime. In contrast, while an increase in the relative cost of guns to criminals will reduce crime, its effect on the total demand for guns is ambiguous. We identify conditions under which this effect can be signed.

Our model is rather narrow in scope in many respects and clearly needs to be extended to answer many related questions of interest. We mention some possibilities below. While we explicitly model the strategic interdependence in the demand for guns by criminals and law abiding citizens, we ignore the issue of intra-group externalities. For example, an armed citizen confers a positive externality to other citizens by potentially deterring criminals.

Second, we do not deal with any dynamic issues that arise from the fact that guns are durable goods, and second hand markets for guns are well developed. A related point is that gun control affects only the sale of new guns, and not the existing stock of guns and this implies regulating the cost of ammunitions might be a better option than regulating guns. Third, the occupational choice between engaging in the criminal or the productive activity is given (as if individuals differ in terms of an extreme 'taste for crime' parameter) and not related to the returns to crime. In terms of our model, this would mean making the fraction of criminals in the population (i.e., $q$) depend on the returns to crime which depend positively on the probability of succeeding in a crime (i.e., $P$). Finally, we do not address some interesting issues arising from the fact that both criminals and law abiding citizens are likely to be heterogeneous in terms of their ability (or taste) for using guns, and the matching between these two types are unlikely to be random as assumed in the paper, but based on the potential vulnerability of a victim. This in turn will depend on the distribution of gun ownership in the population, which is related to the point on intra-group externalities noted above.

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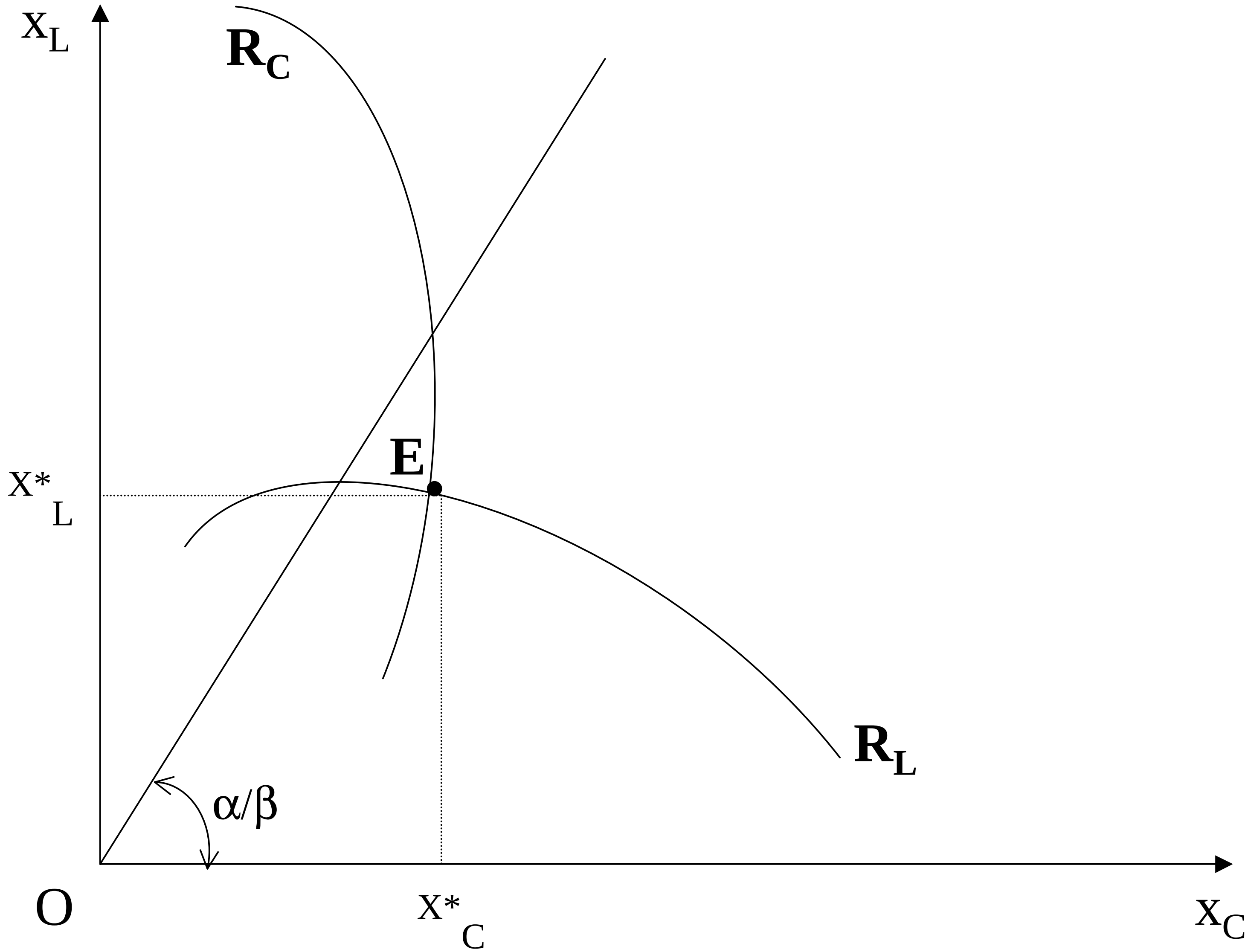


**Figure 1** : Equilibrium with $\alpha > \beta$

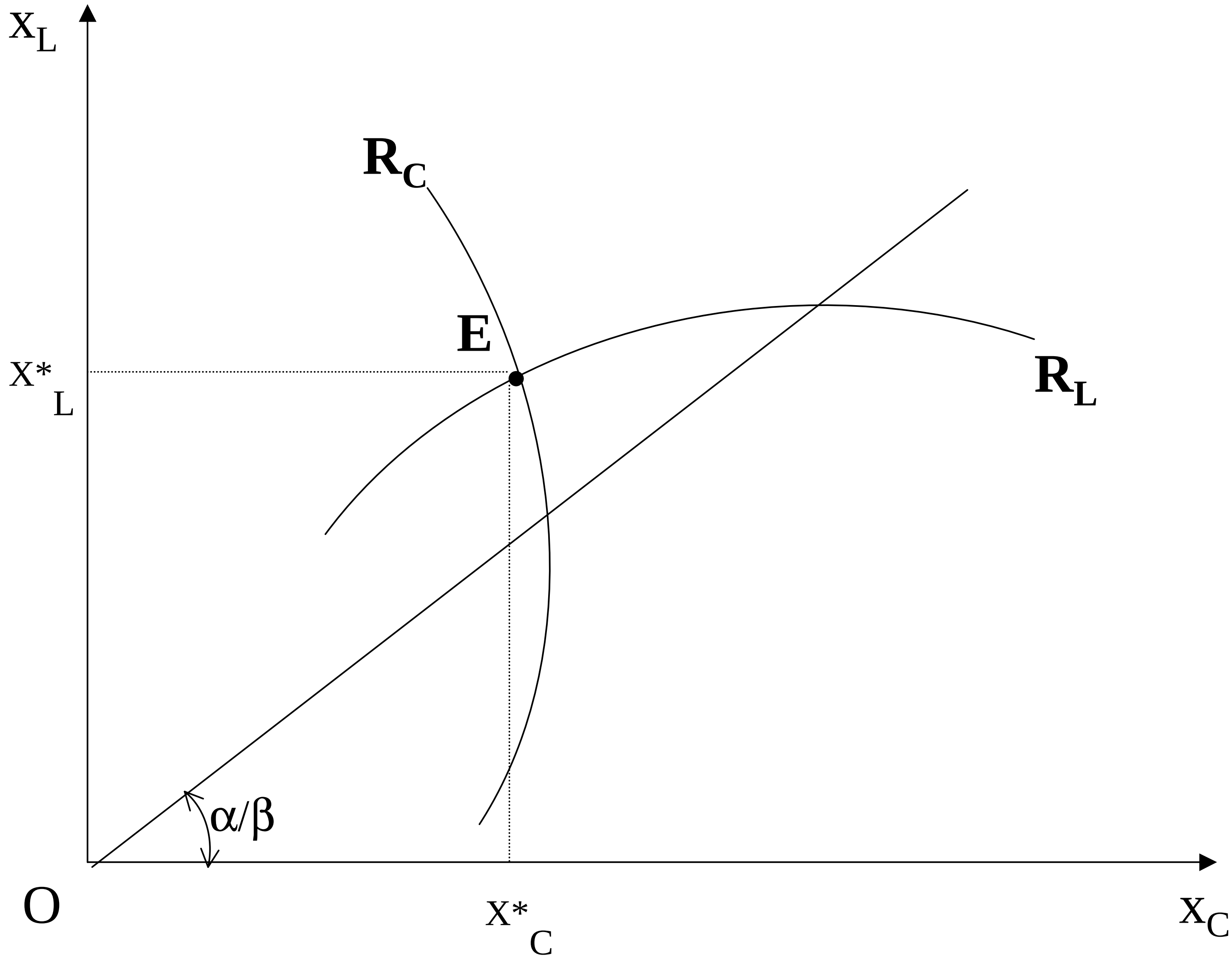


**Figure 2** : Equilibrium with $\alpha<\beta$