

Contractual Structure in Agriculture with Endogenous Matching*

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Abstract

We analyze optimal contractual forms and equilibrium matching in a double-sided moral hazard model of sharecropping similar to Eswaran and Kotwal (1985). We show that, with endogenous matching, the presence of moral hazard can reverse the matching pattern relative to the first best, and that even if sharecropping is optimal for an exogenously given pair of agent skills, it may not be observed in equilibrium with endogenous matching. The economy with endogenous matching features less sharecropping compared to an economy with agent skills drawn at random from the same distribution. This suggests that studies of agency costs in sharecropping may underestimate their extent if focusing only on the intensive margin and ignoring the extensive margin.

Keywords: tenancy; sharecropping; endogenous matching

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1 Introduction

The choice of contractual form in agriculture, in particular, the choice between wage, fixed rent, or sharecropping contracts has important implications for productive efficiency. Alternative theories of choice of contractual form generate sharply different predictions about the efficiency of sharecropping and the potential effects of agricultural reforms or policies such as tenancy or land reform.¹ For example, in models that involve moral hazard and wealth constraints (e.g., Mookherjee, 1997 or Banerjee, Gertler and Ghatak, 2002), a tenancy or land reform that gives a higher share or transfers ownership rights to tenants would unambiguously improve productive efficiency. On the other hand, Eswaran and Kotwal (1985) develop a model of contractual choice where sharecropping is a way of pooling non-contractible inputs of landowners and tenants, and in the context of this model, a tenancy reform would reduce rather than increase productive efficiency by limiting the efficient division of labour.

These alternative theories, however, take the pattern of matching between landlords and tenants (and land quality and tenant ability) as exogenously given. A more recent literature has begun to pay attention to the important issue of endogenous matching and its implications for the observed distribution of contractual forms and efficiency (e.g., Legros and Newman, 1996, 2002 and 2007; Akerberg and Botticini, 2002; Serfes, 2005; Dam and Perez-Castrillo, 2006; Alonso-Pauli and Perez-Castrillo, 2012).²

We contribute to the literature on contractual choice in agriculture by introducing endogenous matching in a double-sided moral hazard model similar to the classic Eswaran and Kotwal (1985) model. We extend and recast their analysis in an equilibrium matching setting and show that selection effects matter for contractual choices – for example, if sharecropping is optimal for an exogenously given pair of skill types it may not be observed in equilibrium with endogenous matching. We also compare the likelihood of observing sharecropping in our endogenous matching setting with that in an economy in which landlord and tenant skill levels are drawn at random from distribution with the same support and probabilities and show that sharecropping is less likely with endogenous matching.

We model the interplay of two economic forces in the context of organizing agricultural production, namely incentive provision under double-sided moral hazard and matching. The choice of tenancy organization form is influenced by these two important factors. On the one hand, it is shaped by incentive considerations since, more often than not in reality, the characteristics or actions of contract parties are unobservable and need to be revealed or implemented through constrained-optimal contract design. On the other hand, the organizational structure depends on the relative importance of the inputs that contracting parties bring in. By pooling various inputs organizations can exploit specialization and absolute advantage.³ In general, the gains from specialization have to be balanced against the agency costs stemming from incentive problems.

¹See Singh (1989) for a review of the literature on sharecropping tenancy and Banerjee (1999) for a review of the literature on land reform. More recent empirical contributions on the efficiency effects of tenancy reforms include Banerjee et al. (2002) and Bardhan and Mookherjee (2011).

²We discuss this literature and our contribution with respect to it below.

³As in Eswaran and Kotwal (1985), absolute advantage is key to specialization by task in sharecropping. Unlike in trade models, comparative advantage without absolute advantage (e.g., one party better at both tasks) does not lead to a partnership.

To fix ideas, suppose agricultural production depends on the labor effort and the ‘skill’ (ability, productivity) supplied by each of two parties, the landlord and the tenant. Individuals differ in their skill levels. As Eswaran and Kotwal (1985) put it, sharecropping can be viewed as a “partnership” in which each partner provides the unmarketed input in which he is better endowed or has an absolute advantage. For example, landowners may have an absolute advantage in managerial skills related to technology and market information while tenants may have an absolute advantage in labour supervision. Sharecropping pools ability but is subject to agency problems in effort provision. To give more incentives to one party means giving less incentives to the other. If ability and effort are complements in production, the higher the relative skill of one agent, the higher should be his output share. This means that he might be optimally matched with a low-skill agent. However, this undermines the gains from trade due to specialization, since a match with a high-ability person would produce more output. If the incentive costs outweigh the gains from specialization, sharecropping may not be observed in equilibrium and the high-ability agent may instead simply hire a low-ability agent from the market to work for him or produce on his own (corresponding to fixed-wage or fixed-rent contracts). That is, simply pooling ability is not a sufficient argument for sharecropping – sufficiently strong complementarities between the qualities of the supplied inputs (the agents’ skills) are also needed, otherwise with endogenous matching sharecropping may not be observed in equilibrium.

We show that the presence of double-sided moral hazard combined with weak complementarity in input quality in production leads to the possibility of agent skills being strategic substitutes in joint surplus and a reversal of the equilibrium matching pattern relative to the first best, from positive to negative assortative. In contrast, without incentive problems, even a small amount of complementarity between the quality of inputs implies positive assortative matching. This means that when individuals are heterogeneous in their skills, allowing for endogenous matching between landlords and tenants can affect the optimal choice of contractual form (sharecropping vs. sole production in fixed rent or wage contract) relative to the setting of Eswaran and Kotwal (1985) where isolated landlord-tenant pairs are considered. Specifically, there exist situations in which sharecropping is the optimal contractual form for exogenously given pairs of skill types, but with endogenous sorting the landlord or tenant performs both tasks in equilibrium. This has the implication that empirical studies on agency costs (e.g., see Chiappori and Salanie, 2003 for a review of the contracting literature, including sharecropping) may underestimate their full extent by focusing on the intensive margin and ignoring the extensive margin, that is, the fact that sharecropping may not arise at all and the consequent welfare loss in terms of efficient matching between landowners and tenants, as opposed to the loss of efficiency due to moral hazard when sharecropping is actually observed.⁴

In our theoretical analysis we use the general approach and results from Legros and Newman (2002), who study matching in transferable-utility economies (their companion paper, Legros and Newman, 2007 analyzes the non-transferable utility case) with or without market imperfections and provide weaker conditions for monotone matching than the standard sub- and super-modularity results from the earlier literature. Legros and Newman also give

⁴This fits Bastiat’s general dictum that an economist must take into account both what is seen and what is not seen (Bastiat, 1873).

examples of payoff functions in imperfect markets settings in which the equilibrium matching pattern is reversed (e.g., from positive to negative assortative) by changing a model parameter which could be interpreted as the “strength” of the market imperfection. In our application to contractual choice in agriculture we echo this by showing that the fact that a market imperfection exists, here arising endogenously from the moral hazard problem, can reverse the matching pattern relative to the first best by affecting the optimal effort choices made by the match parties.

The classical optimal sorting literature (e.g., Becker, 1993, Sattinger, 1975) has argued that the sub- or super-modularity of the payoff function is closely related to whether the agents’ characteristics over which matching is done (here, the skill levels) are substitutes or complements. With substitutes, negative assortative matching arises, while with complements the matching is positive assortative. As pointed out by Legros and Newman (2002), however, the link between the degree of substitutability of agents’ characteristics and the modularity of the payoff function may break down in more general environments featuring credit market imperfections (e.g., Legros and Newman, 1996; Sadoulet, 1998); restrictions on how output is shared (e.g., Farrell and Scotchmer, 1998); or particular production technologies (e.g., Kremer and Maskin, 1996). Our application to contract choice in agriculture with double-sided moral hazard fits into this list.

This paper is also related to the empirical literature on agricultural tenancy and the role of matching. Akerberg and Botticini (2002) pose the puzzle that output shares of tenants cultivating risky crops tend to be higher than those of tenants cultivating less risky crops which is contrary to the predictions of standard models of sharecropping under moral hazard in which greater output variance would lead to higher demand for insurance by the risk-averse tenant and, therefore, lower crop share (e.g., Stiglitz, 1974). Akerberg and Botticini argue that risk-neutral tenants are more likely to select riskier crops (and therefore, in equilibrium, their observed shares could be higher) and provide empirical evidence of matching between landowners and tenants consistent with this hypothesis. The argument has been formalized in Serfes (2005).

More generally, our work is related to previous theoretical literature on organizational choice with endogenous matching. Legros and Newman (1996) build a general equilibrium model with moral hazard, endogenous firm formation by agents heterogeneous in wealth, and a credit market with costly state verification problem. They show that, when borrowing is not costly, firms are efficiently organized and essentially consist of one type of agents but, when the information problem is sufficiently severe, firm organization and production efficiency depend on the distribution of wealth. Dam and Perez-Castrillo (2006) study a principal-agent model with risk-neutral principal and agents heterogeneous in initial wealth, moral hazard in effort choice, and limited liability. They analyze the simultaneous determination of stable pairwise matching and the contracts signed and show that the contracts in a stable outcome of their economy are more efficient than principal-agent contracts in which the principals obtain the full surplus.⁵ Alonso-Pauli and Perez-Castrillo (2012) study endogenous

⁵Besley and Ghatak (2005) consider a similar setting but allow agents to be intrinsically motivated and principals and agents to differ in terms of their ‘mission’ preferences which affect the level of agent motivation. They analyze the effect of endogenous matching on efficiency, and show that it may reduce incentive pay since agent and principal mission preferences are better aligned.

matching between heterogeneous managers and shareholders who can adopt an incentive-based contract or a “code of best practice” in a setting with moral hazard. They show that firms with the best projects tend to adopt a code when managers are not too heterogeneous while, when projects are similar, the best managers tend to be hired via incentive contracts. They also show the possibility of reversal in the optimal contract compared to the case of an exogenous agent pair. Our paper focuses exclusively on incentive problems with double-sided moral hazard and two-sided matching. In our model there are no limited-liability constraints and utility is fully transferable. Comparative advantage and not wealth is thus the key source of heterogeneity that we focus on.⁶

The rest of the paper proceeds as follows. Section 2 presents the model and characterizes the equilibrium matching and joint surplus maximizing organization forms that will be observed in the first best and under double-sided moral hazard allowing for a varying degree of complementarity in agents’ characteristics. We demonstrate the possibility for matching pattern reversal between the first best and moral hazard. Several empirically testable implications of the model are discussed in Section 3. The last section makes some concluding observations.

2 Model

Output depends on the effort levels, $e_1 \geq 0$ and $e_2 \geq 0$, supplied in two distinct tasks. Following Eswaran and Kotwal (1985) we call them ‘management’ and ‘effective labour effort’ (which may include supervision of wage labour) or more simply, ‘task 1’ and ‘task 2’. Production also requires one unit of land. There are two types of economic agents: those who are better at task 1 and those who are better at task 2. This will be formalized below. Suppose landlords are good at supplying managerial inputs (task 1) while tenants (who are assumed to be landless) are good at labour supervision (task 2). Each landlord has a single unit of land. A landowner (task 1 specialist) could work on both tasks on his own although he is not very good at task 2. Alternatively, he can rent out the land to a tenant who can work on both tasks in exchange of fixed rent. Finally, the landowner can enter a contract with a task 2 specialist in a sharecropping arrangement. This allows gains from specialization to be realized but will be subject to incentive problems when efforts are unobservable.

The quality (ability, skill) of a task 1 specialist (the landlord) is $\theta_1 \geq 1$ and the quality of task 2 specialist (the tenant) is $\theta_2 \geq 1$. Both θ_1 and θ_2 take values on the set $\Theta \subseteq [1, \theta^{\max}]$ where $\theta^{\max} > 1$ can be arbitrarily large but finite. The skill levels, θ_1 and θ_2 are observable and contractible. In contrast, the effort levels e_1 and e_2 may or may not be contractible. All agents are risk-neutral and there are no limited liability or wealth constraints.⁷ This means that, if it is efficient for the task 2 specialist to become the full residual claimant (e.g., through a fixed-rent contract), then there is nothing preventing that contractual form being chosen, in contrast to many models of tenancy where a rent-extraction vs. incentives

⁶A related recent paper by Dam (2010) analyzes a two-sided matching model involving entrepreneurs who vary in wealth and financial intermediaries who vary in monitoring ability and shows that, with endogenous matching, more efficient monitors lend to borrowers with lower wealth.

⁷We briefly discuss the case of wealth constraints at the end of Section 2.3.1 below.

trade-off exists due to a limited liability constraint, and therefore, even if fixed-rent contracts are efficient (e.g., when both parties are risk-neutral) they will not be chosen.⁸

Each agent has an additively separable payoff in their consumption, c and effort in each task, $u = c - \gamma(e_1) - \gamma(e_2)$. The disutility cost of providing effort is assumed quadratic, $\gamma(e_1) = \frac{1}{2}e_1^2$ and $\gamma(e_2) = \frac{1}{2}e_2^2$. The reservation payoff of each agent is set to 0. If a task 1 specialist of skill θ_1 and a task 2 specialist of skill θ_2 engage in joint production, output is:

$$q^J(e_1, e_2) = \alpha\theta_1\theta_2 + \theta_1e_1 + \theta_2e_2 + \varepsilon \quad (1)$$

where ε is a random variable with $E(\varepsilon) = 0$ and finite variance and α is a parameter capturing the extent of skill complementarity in production.

Assume that if a task i ($i = 1, 2$) specialist of type θ_i undertakes production on her own she can perform task k ($k = 1, 2$ with $k \neq i$) just as well as task- k specialist of the lowest possible type, $\theta_k = 1$. For example, a landlord (task 1 specialist) can produce on his own but will be only as productive as the lowest-skill tenant at task 2. This puts a natural lower bound on θ – otherwise there is no advantage in producing jointly. Given this, if a task i specialist of skill θ_i undertakes sole production (that is, she works on both tasks either using own or rented land), output is:

$$q^S(e_i, e_k) = \alpha\theta_i + \theta_i e_i + e_k + \varepsilon. \quad (2)$$

Using (1) and (2), we can write the production function, (1) for any organization type between agents of skills θ_i and θ_k in Θ as:

$$q(e_i, e_k) = \alpha\theta_i z_k + \theta_i e_i + z_k e_k + \varepsilon \quad (3)$$

where $z_k = \theta_k$ if production is done jointly and $z_k = 1$ if the task i specialist is sole producer, for $i, k = 1, 2$ with $i \neq k$.

Assume $\alpha \geq 0$.⁹ If $\alpha = 0$ agents' abilities are substitutes in production, while their complementarity increases in α for $\alpha > 0$. Suppose that each of θ_1 and θ_2 can take just two values, θ_a and θ_b in Θ with $\theta_a \geq \theta_b$ (high skill and low skill). From now on, subscripts 1 and 2 on θ pertain to the skill level of the task 1 or 2 specialist, while subscripts a and b on θ denote the numerical value of someone's skill level, i.e., $\theta_i \in \{\theta_a, \theta_b\}$ for $i = 1, 2$.

We assume transferable utility, i.e., agents can make any side payments or transfers among each other. Therefore, there are no restrictions on the level of wage or fixed rent coming from wealth or liquidity constraints, unlike in models with limited liability. Below, we use the qualifier “optimal” to refer to organization/contract forms that maximize expected joint payoff (total surplus). To determine the equilibrium matching pattern in our economy, we follow Legros and Newman (2002), henceforth LN(2002) and use the core as our equilibrium concept.

There are continuum of measure one of task-1 specialists and continuum of measure one task-2 specialists. Therefore, since land is essential in production, and since there are measure one land units in total, following LN(2002), we define an equilibrium in our model economy as

⁸See for example, Mookherjee (1997) and Banerjee, Gertler, and Ghatak (2002).

⁹We rule out the case $\alpha < 0$ in which higher partner ability reduces the joint payoff. In this case, if α is sufficiently negative, sole production always dominates joint production.

consisting of a pairwise matching correspondence between landlords and tenants and payoff allocation for each pair (see LN(2002), p.929 for details). Since there is transferable utility, we only keep track of the total expected payoff in each pair which, as well as aggregate surplus, will be maximized in equilibrium – see LN(2002), Proposition 1. Assume for now (this will be relaxed in Section 2.3.2) that there is an equal measure, $1/2$ of each skill level, θ_a and θ_b for both the landlords and tenants.

Given our assumption of two possible ability levels for each specialist, $\theta_a \geq \theta_b$, let $J(i, i)$, $i = a, b$ denote the expected total surplus when a landlord and a tenant both with skill θ_i produce jointly; let $J(a, b)$ denote the expected total surplus when a landlord and tenant of different skill levels produce jointly; and let $S(i)$, $i = a, b$ denote the expected total surplus when an agent of skill θ_i engages in sole production (with own or rented land). We can thus define the possible values for expected total surplus in any landlord-tenant pair as follows:

$$\begin{aligned}\pi(a, a) &\equiv \max\{J(a, a), S(a)\} \\ \pi(a, b) &\equiv \max\{J(a, b), S(a)\} \\ \pi(b, b) &\equiv \max\{J(b, b), S(b)\}.\end{aligned}\tag{4}$$

In writing the expression for $\pi(a, b)$ in (4) we used the fact that q^J is symmetric in θ_1 and θ_2 (see (1)) and that $S(a) \geq S(b)$, i.e., sole production by the low-skill type in a (θ_a, θ_b) pair is never optimal – a low-skill landlord can always rent his land to the high-skill tenant while a high-skill landlord can produce alone (the low-skill tenant he is matched to can be thought of sitting idle). To see this, note that

$$S(i) \equiv \max_{e_i, e_k} E(q^S(e_i, e_k)) - \gamma(e_i) - \gamma(e_k)$$

which, given the assumed functional forms for output $q^S(., .)$ (see (2)) and effort costs $\gamma(., .)$, yields optimal effort choices $e_i^S = \theta_i$ and $e_k^S = 1$, and so the maximized expected joint surplus with sole production is

$$S(a) = \alpha\sqrt{a} + \frac{1}{2}(a + 1) \text{ and } S(b) = \alpha\sqrt{b} + \frac{1}{2}(b + 1)\tag{5}$$

where we call $a \equiv (\theta_a)^2$ and $b \equiv (\theta_b)^2$ to simplify the notation. Clearly, $S(a) \geq S(b)$ whenever $a \geq b$.

The expected total surplus under joint-production, $J(a, a)$, $J(a, b)$ and $J(b, b)$ will be determined below since its form depends on the contractibility of effort.

2.1 First Best

As a benchmark, we first characterize the first best, that is, the case when both efforts e_1 and e_2 are contractible. Under joint production and no incentive problems, e_1 and e_2 are chosen by the agents to maximize total expected surplus, i.e., we have,

$$J(i, j) \equiv \max_{e_1, e_2} E(q^J(e_1, e_2)) - \gamma(e_1) - \gamma(e_2)$$

which, given our assumed functional forms for output $q^J(., .)$ and effort costs $\gamma(., .)$, yields first-best optimal efforts $e_1^J = \theta_1$ and $e_2^J = \theta_2$ where θ_i , $i = 1, 2$ denotes the skill level of the

type- i specialist in the pair. Plugging in, and using the superscript FB to denote the first best, we obtain:

$$\begin{aligned} J^{FB}(a, a) &= \alpha a + a, \quad J^{FB}(b, b) = \alpha b + b \\ \text{and } J^{FB}(a, b) &= \alpha\sqrt{ab} + \frac{1}{2}(a + b). \end{aligned}$$

where $a \equiv (\theta_a)^2$ and $b \equiv (\theta_b)^2$ as before. We have the following straightforward result:

Lemma 1: *In the first best, joint production is the joint-surplus maximizing organization form for any agent pair with skills $\theta_1, \theta_2 \in \{\theta_a, \theta_b\}$. That is, $\pi^{FB}(i, j) = J^{FB}(i, j)$ for $(i, j) \in \{(a, a), (a, b), (b, b)\}$.*

Proof: We simply check all three cases. $J^{FB}(a, a) \geq S(a)$ is equivalent to $\alpha a + a \geq \alpha\sqrt{a} + \frac{1}{2}(a + 1)$ which is true since $a \geq 1$. Similarly for b . We also have $J^{FB}(a, b) = \alpha\sqrt{ab} + \frac{1}{2}(a + b) \geq \alpha\sqrt{a} + \frac{1}{2}(a + 1) = S(a)$ since $b \geq 1$. ■

Given Lemma 1, we next characterize the equilibrium matching pattern in the first best.

Proposition 1 (First best)

Suppose agent efforts are contractible. Then, for any $\alpha \geq 0$, in equilibrium there is joint production with positive assortative matching of the ‘segregation’ type, (θ_a, θ_a) and (θ_b, θ_b) .

Proof: We already showed in Lemma 1 that $\pi^{FB}(i, j) = J^{FB}(i, j)$ for $(i, j) \in \{(a, a), (a, b), (b, b)\}$. Following LN (2002), define the *surplus function*, σ^{FB} as:

$$\begin{aligned} \sigma^{FB}(a, b) &\equiv \max\{0, \pi^{FB}(a, b) - \frac{1}{2}(\pi^{FB}(a, a) + \pi^{FB}(b, b))\} \\ &= \max\{0, J^{FB}(a, b) - \frac{1}{2}(J^{FB}(a, a) + J^{FB}(b, b))\} \\ &= \frac{1}{2} \max\{0, 2\alpha\sqrt{ab} + a + b - \alpha a - a - \alpha b - b\} \\ &= \frac{1}{2} \max\{0, -\alpha(\sqrt{a} - \sqrt{b})^2\} \\ &= 0 \quad \text{for all } a, b \text{ and } \alpha \geq 0. \end{aligned}$$

The proposition statement then follows directly from Proposition 4 in LN(2002), p.932. ■

As pointed out in LN(2002), the surplus function σ^{FB} being identically zero is a much weaker condition for segregation than strict super-modularity of the joint payoff function, π^{FB} (note, super-modularity does not hold here for $\alpha = 0$). The intuition for the Proposition 1 result is similar to that in Becker’s model of the marriage market – when agents’ types are complements in the joint payoff, positive assortative matching (henceforth, PAM) results (e.g., Becker, 1993, ch. 4). Clearly, this would also hold when there are more than two skill levels (assuming measure consistency) – if there were any heterogeneous pairs, a profitable deviation to segregation PAM supported by appropriate transfers would be possible.

2.2 Double-sided moral hazard

Now suppose e_1 and e_2 are non-contractible, that is, joint production has the advantage of exploiting gains from specialization (the fact that both agents can use their skills θ_1 and θ_2 strictly larger than 1 in production) but this has to be traded-off against the agency costs that arise from the double-sided moral hazard problem. Consistent with the tenancy literature, we restrict attention to linear contracts of the form (s, R) where $s \in [0, 1]$ represents the output share of the task 2 specialist (the ‘tenant’) and R represents a transfer from the task 1 specialist (the ‘landlord’) to the tenant which could be negative (a ‘rent’) or positive (a ‘wage’).¹⁰ Using the superscript MH to denote this setting with moral hazard, given (s, R) the agents’ optimal effort choices are:

$$\begin{aligned} e_1^{MH} &= \arg \max_{e_1} \left((1-s)(\alpha\theta_1\theta_2 + \theta_1e_1 + \theta_2e_2) - R - \frac{e_1^2}{2} \right) = (1-s)\theta_1 \\ e_2^{MH} &= \arg \max_{e_2} \left(s(\alpha\theta_1\theta_2 + \theta_1e_1 + \theta_2e_2) + R - \frac{e_2^2}{2} \right) = s\theta_2. \end{aligned}$$

Note that these are (weakly) lower than the first-best levels. Expected indirect total surplus when agents with skill levels θ_1 and θ_2 produce jointly (sharecrop) using contract (s, R) is then

$$\begin{aligned} J^{MH}(s) &= \alpha\theta_1\theta_2 + \theta_1e_1^{MH} + \theta_2e_2^{MH} - \frac{(e_1^{MH})^2}{2} - \frac{(e_2^{MH})^2}{2} = \\ &= \alpha\theta_1\theta_2 + \theta_1^2 \left((1-s) - \frac{1}{2}(1-s)^2 \right) + \theta_2^2 \left(s - \frac{1}{2}s^2 \right). \end{aligned} \quad (6)$$

The optimal sharing rule, s^* (the tenant’s crop share) is chosen to maximize indirect joint surplus (6), which incorporates the parties’ incentive compatible efforts, e_i^{MH} .¹¹ It is:

$$s^* = \frac{\theta_2^2}{\theta_1^2 + \theta_2^2}. \quad (7)$$

Intuitively, the optimal tenant share s^* is higher the greater is the tenant’s ability relative to the landlord’s ability. This follows from the fact that ability and effort are complements in the production function, and so the higher is ability the greater is effort for the same share. The marginal gain from raising the share to elicit greater effort is therefore increasing in agent’s ability. Since our assumptions on the range of θ_i imply $s^* \in (0, 1)$, we interpret the optimal contract in joint production under moral hazard as a sharecropping agreement.

Substituting s^* in (6) and simplifying, we obtain the following expression for the maximized value of joint surplus under sharecropping by agents with skills θ_1 and θ_2 ,

$$J^{MH}(\theta_1, \theta_2) = \alpha\theta_1\theta_2 + \frac{1}{2} \left\{ \theta_1^2 + \theta_2^2 - \frac{\theta_1^2\theta_2^2}{\theta_1^2 + \theta_2^2} \right\}. \quad (8)$$

¹⁰See Bhattacharya and Lafontaine (1995) for sufficient conditions under which the optimal sharing rule under double-sided moral hazard can be represented by a linear contract without loss of generality. These conditions are satisfied in our setting. Additionally, we assume that there is no budget-breaker and the first-best cannot be achieved as in Holmstrom (1982).

¹¹The same optimal sharing rule obtains if we maximize one party’s payoff subject to a participation constraint by the other.

Note that, because of the incentive problem, the modularity of indirect joint surplus under moral hazard, J^{MH} cannot be inferred directly from the substitutability or separability of agent characteristics in the production function $q^J(.,.)$ as we did in the first-best (or, as in Becker, 1973; Sattinger, 1975). Now it also depends on the endogenous choices by the parties and the sharing rule. The optimal share of each party is function of the magnitude of her ability relative to the other party's ability. The higher is θ_1 , the share of the task 1 specialist goes up while the share of the task 2 specialist goes down, and as a result, e_1 goes up and e_2 goes down. The positive effect of an increase in θ_2 on the share s^* and e_2 (relative to e_1) is therefore lower the higher is θ_1 . In other words, given that ability and effort are complements in production, a high-quality agent's skills are best utilized if he also has strong incentives, but the higher the quality of the other agent, this will not be the case. This is the intuition for the fact that the skills of the agents engaged in sharecropping under moral hazard can be substitutes in joint surplus – see (9).

Finally, if a task $i = 1, 2$ specialist performs both tasks (sole production), since there are no incentive problems, total surplus is $S(i)$ as obtained in (5) in the previous section.

2.3 Matching and organization forms under moral hazard

Denoting, as before $a = (\theta_a)^2$ and $b = (\theta_b)^2$ with $a \geq 1$ and $b \geq 1$, for any $\theta_a, \theta_b \in \Theta$ with $\theta_a \geq \theta_b$, and calling $\pi^{MH}(i, j) \equiv \max\{S(i), J^{MH}(i, j)\}$ for $(i, j) \in \{(a, a), (a, b), (b, b)\}$, using (5) and (8), the expressions for indirect expected total surplus under moral hazard are:

$$\begin{aligned}\pi^{MH}(a, a) &= \frac{1}{2} \max\{2\alpha\sqrt{a} + a + 1, 2\alpha a + \frac{3}{2}a\} \\ \pi^{MH}(a, b) &= \frac{1}{2} \max\{2\alpha\sqrt{a} + a + 1, 2\alpha\sqrt{ab} + a + b - \frac{ab}{a+b}\} \\ \pi^{MH}(b, b) &= \frac{1}{2} \max\{2\alpha\sqrt{b} + b + 1, 2\alpha b + \frac{3}{2}b\}.\end{aligned}\tag{9}$$

The equilibrium organization forms and matching pattern thus depend on the values of a, b and α . Below, we first analyze the case $\alpha = 0$ in which we show that the equilibrium displays negative assortative matching (henceforth, NAM) consisting of heterogeneous agent pairs a, b for any $\theta_a, \theta_b \in \Theta$ with $\theta_a \geq \theta_b$. We also characterize the joint-surplus maximizing organization form depending on the values of a and b . For most of the analysis we assume that the two skill levels differ, i.e., $\theta_a > \theta_b$, and so $a > b$. We analyze the special case $\theta_a = \theta_b$ ($a = b$) separately.

2.3.1 Moral hazard and negative assortative matching

Assume $\alpha = 0$ in this section. Therefore we have, from (9):

$$\begin{aligned}\pi^{MH}(a, a) &= \frac{1}{2} \max\{a + 1, \frac{3}{2}a\}, \quad \pi^{MH}(a, b) = \frac{1}{2} \{a + 1, a + b - \frac{ab}{a+b}\} \\ \text{and } \pi^{MH}(b, b) &= \frac{1}{2} \max\{b + 1, \frac{3}{2}b\}.\end{aligned}\tag{10}$$

We have the following result.

Proposition 2 (NAM under moral hazard)

Let $\alpha = 0$. For any $a > b \geq 1$,

$$\pi^{MH}(a, b) > \frac{1}{2}[\pi^{MH}(a, a) + \pi^{MH}(b, b)], \quad (*)$$

that is, the indirect expected joint payoff function π^{MH} is strictly sub-modular and the equilibrium matching is negative assortative (NAM).

Proof: There are four cases to consider:

Case (a): $b < a < 2$. Then, from (10), $\pi^{MH}(a, a) = \frac{a+1}{2}$ and $\pi^{MH}(b, b) = \frac{b+1}{2}$ since $a, b < 2$. Also, $\pi^{MH}(a, b) = \frac{a+1}{2}$ since $a + 1 > a + b - \frac{ab}{a+b}$ is equivalent to $a > b^2 - b$. The latter is true since $b^2 - b < b$ for $b < 2$ and so $a > b > b^2 - b$. Thus, in Case (a) sole production is optimal in all skill type pairs. Inequality (*) holds since it is equivalent to $a > b$.

Case (b): $b < 2 \leq a$. Then, from (10), using similar arguments as in Case (a), we have: $\pi^{MH}(a, a) = \frac{3}{4}a$, $\pi^{MH}(b, b) = \frac{b+1}{2}$ and $\pi^{MH}(a, b) = \frac{a+1}{2}$. Thus, (*) is equivalent to $a + 1 > \frac{3}{4}a + \frac{b+1}{2}$ or $\frac{a}{4} + \frac{1}{2} > \frac{b}{2}$, which is true since $a \geq 2$ and $b < 2$.

Case (c): $2 \leq b < a$ and $a \leq b^2 - b$. Then, from (10), $\pi^{MH}(a, a) = \frac{3}{4}a$, $\pi^{MH}(b, b) = \frac{3}{4}b$ and $\pi^{MH}(a, b) = \frac{1}{2}(a + b - \frac{ab}{a+b})$. In this case for a, b joint production is surplus-maximizing for all skill type pairs. Inequality (*) is now equivalent to $a + b - \frac{ab}{a+b} > \frac{3}{4}(a + b)$ which, after simplifying, reduces to $(a - b)^2 > 0$.

Case (d): $2 \leq b < a$ and $a > b^2 - b$. Then, from (10), $\pi^{MH}(a, a) = \frac{3}{4}a$, $\pi^{MH}(b, b) = \frac{3}{4}b$ and $\pi^{MH}(a, b) = \frac{a+1}{2} > \frac{1}{2}(a + b - \frac{ab}{a+b})$ (see Case (a) for the last inequality). Thus,

$$\pi^{MH}(a, b) > \frac{1}{2}(a + b - \frac{ab}{a+b}) > \frac{3}{8}a + \frac{3}{8}b = \frac{1}{2}(\pi^{MH}(a, a) + \pi^{MH}(b, b))$$

using the same argument as in Case (c) to show the second inequality.

Overall, we showed that inequality (*) holds for all $a > b \geq 1$. It corresponds to *Condition H* in LN(2002), which implies perfectly heterogeneous matching in equilibrium (see LN, 2002, p. 933). Since there are only two possible skill levels in our model, inequality (*) also implies that the indirect joint payoff function π^{MH} is strictly sub-modular, i.e., since π^{MH} is also symmetric, by Proposition 7 in LN(2002), the equilibrium sorting pattern is NAM. ■

Comparing with the result from Proposition 1, Proposition 2 implies that the presence of agency costs can reverse the equilibrium matching pattern relative to the first best.¹² Specifically, for $\alpha = 0$ we showed that PAM obtains in the first best but NAM obtains under moral hazard – the presence of moral hazard not only implies the usual loss of surplus from agency costs but also the opposite sorting pattern. Suppose we have sharecropping in heterogeneous pairs (θ_a, θ_b) in equilibrium under moral hazard (see Corollary A below for conditions) and would like to assess the welfare loss relative to the first-best outcome.

¹²See also Legros and Newman (1996) and Alonso-Pauli and Perez-Castrillo (2012) for related results and also Kaya and Vereshchagina (2010) who study matching with double-sided moral hazard and show that, depending on whether types and efforts are substitutes or complements, positive or negative assortative matching results.

If we took the (θ_a, θ_b) pairs as exogenously given, in isolation, then resolving the moral hazard problem would indicate a smaller increase in welfare compared to the gains that would obtain when the parties are allowed to endogenously re-match in PAM pattern, as they optimally would in the first best (Proposition 1). Conversely, if we started from the first-best benchmark, the welfare loss due to moral hazard may be overstated if, keeping the matching fixed, we were to compute the agency cost due to moral hazard. Therefore, agency costs are higher under exogenous matching compared to under endogenous matching.

Corollary A to Proposition 2 (optimal organization forms with moral hazard and NAM)

(a) *If $2 \leq b < a$ and $a \leq b^2 - b$, then the optimal organizational form is sharecropping in heterogeneous pairs, (θ_a, θ_b) . For all other a, b with $a > b \geq 1$ the optimal organization form is sole production by the high-skill agents, θ_a with own land (“fixed wage”) or with rented land (“fixed rent”).*

(b) *If $b = a = \theta^2$, then for $b < 2$ sole production by all agents is optimal and for $b \geq 2$ sharecropping by all in (θ, θ) pairs is optimal.*

Proof: the results follow directly from the discussion of the four cases listed in the proof of Proposition 2. ■

Proposition 2 and its Corollary A imply that, with double-sided moral hazard and endogenous matching, when joint surplus is sub-modular, sharecropping will be observed in equilibrium only if the skill levels of both parties are: (i) sufficiently large – so that gains from specialization are large, and (ii) relatively similar – since, due to NAM, with skill heterogeneity sole production is more likely to dominate joint production. Specifically, Corollary A, part (a) implies that a necessary condition to observe sharecropping in equilibrium is that the lower skill level θ_b ($=\sqrt{b}$) be larger or equal to $\sqrt{2}$. Also, note we can re-write the condition $a \leq b^2 - b$ as $a - b \leq b^2 - 2b$. Since $b^2 - 2b$ is increasing in b for $b \geq 1$, the smaller is b (θ_b), the smaller the (squared) skill difference $a - b$ must be for sharecropping to be optimally chosen in equilibrium.

Discussion and comparison with Eswaran and Kotwal (1985)

Eswaran and Kotwal (1985) study the choice of organization form in agriculture with double-sided moral hazard by taking pairs of agents with certain skill levels in isolation. We extend their analysis by recasting the basic model in a matching setting and study the choice of optimal organization form and equilibrium sorting pattern with endogenous matching between many agents.

We first derive conditions in our setting analogous to those in EK (1985), for two exogenously given agents (agent 1, a landlord and agent 2, a tenant) with skill levels θ_1 and θ_2 in Θ , respectively. Maintain the assumption $\alpha = 0$ for this sub-section. As seen before, there are three possible organization forms: (i) the agents produce jointly subject to the double-sided moral hazard problem (‘sharecropping’); (ii) Agent 1, the landlord, is sole producer (paying unskilled workers fixed wage); and (iii) Agent 2, the tenant, is sole producer paying the landlord a fixed rent. Since output is increasing in the skill level, in forms (ii) and (iii) it is optimal that the higher-skill agent be the sole producer.

The next Proposition re-states Eswaran and Kotwal’s (1985) results in our context:

Proposition 3

An exogenously given pair of agents with skills $\theta_1, \theta_2 \in \Theta$ with $\theta_1 \geq \theta_2$ would optimally use sharecropping if and only if $\theta_2 \geq \sqrt{2}$ and $\theta_1^2 \leq \theta_2^4 - \theta_2^2$. Otherwise, sole production by the high-skill type, θ_1 (a fixed rent or fixed wage arrangement) maximizes expected joint surplus. If the agents' skill levels are equal, $\theta_1 = \theta_2 = \theta$, then sharecropping is joint-surplus maximizing if and only if $\theta \geq \sqrt{2}$ and sole production is joint-surplus maximizing otherwise.

Proof: (see Appendix)

To illustrate the importance of endogenous sorting, suppose we took an isolated single pair of agents of the same skill, $\theta_1 = \theta_2 = \theta$. By Proposition 3, sharecropping would be optimally chosen if and only if

$$\theta \geq \sqrt{2} \quad (11)$$

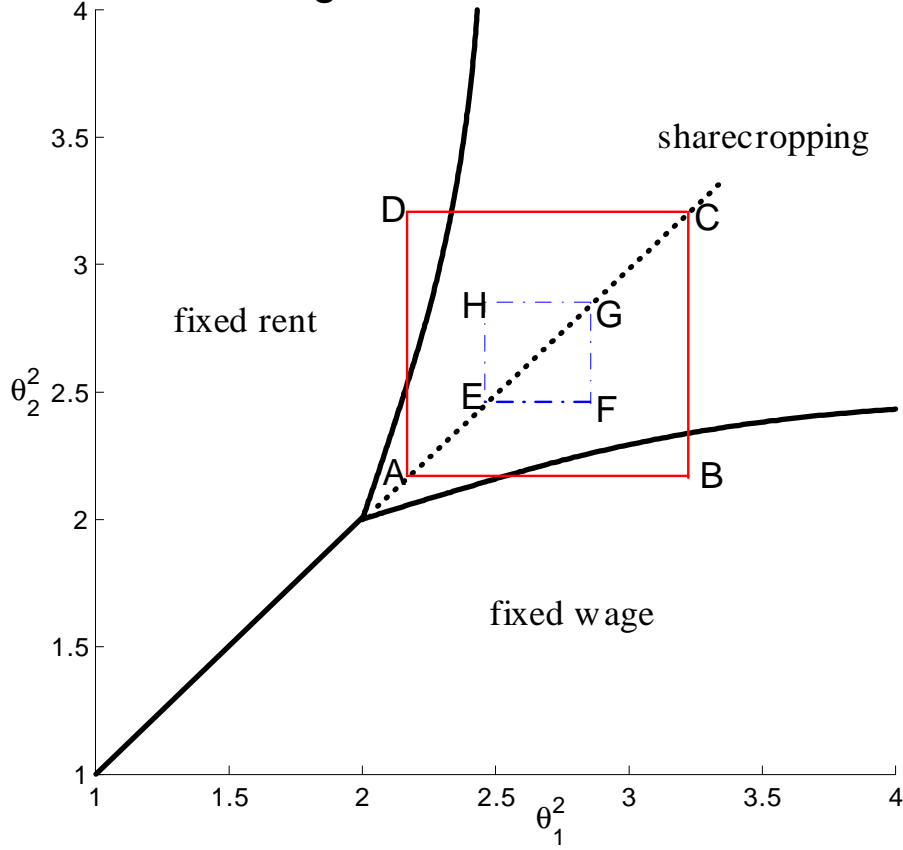
Note that condition (11) is stronger than the corresponding condition for the first best ($\theta \geq 1$) derived in Lemma 1, which is intuitive given the presence of agency costs.

If there are two skill levels θ_a and θ_b in Θ that each satisfy (11) and if two (θ_a, θ_a) and (θ_b, θ_b) homogeneous pairs were given exogenously, in isolation, we would conclude that sharecropping would be the only contractual form observed in the economy. For example, if we think of these agent pairs as living in two separate villages, we would expect to observe only sharecropping and no fixed wage or rent in each location. By taking landlord-tenant pairs in isolation and inferring the contractual form they would choose we, however, ignore the possibility that such pair of agents may never match in equilibrium with endogenous sorting. To see this, suppose there are a couple of tenants and a couple of landlords from each skill level θ_a and θ_b – that is, four agents in total. If they can match with any partner they want to, sharecropping may no longer be joint-surplus maximizing even if it were optimal when the pairs (θ_a, θ_a) and (θ_b, θ_b) are studied in isolation. Indeed, the sub-modularity of indirect joint surplus under moral hazard shown in Proposition 2 implies that if sharecropping is optimal agent pairs must take the heterogeneous form (θ_a, θ_b) . Therefore, with endogenous matching, it is no longer the case that sharecropping is joint-surplus maximizing for any θ_a and θ_b satisfying (11). Instead, Proposition 2 and Corollary A imply that, for any $\theta_a > \theta_b \geq \sqrt{2}$, if the high-skilled type is sufficiently productive so that $\theta_a^2 > \theta_b^4 - \theta_b^2$ (this is equivalent to $a > b^2 - b$), then sole production by the high-skill agents achieves higher expected joint surplus than sharecropping. For instance, $\theta_a^2 > \theta_b^4 - \theta_b^2$ always holds when $\theta_b = \sqrt{2}$. Hence, with endogenous matching, we would not observe sharecropping in our economy with double-sided moral hazard populated with two agents with skill $\theta_b = \sqrt{2}$ and two with skill any $\theta_a > \sqrt{2}$.

Graphically, for any landlord and tenant skill levels θ_1 and θ_2 in Θ , the (θ_1^2, θ_2^2) parameter space can be divided into three areas depicted on Figure 1.

- A1. ‘fixed wage’ area corresponding to the set $\{(\theta_1, \theta_2) : \theta_1 \geq \theta_2 \text{ and } \theta_1^2 > \theta_2^4 - \theta_2^2\}$
- A2. ‘fixed rent’ area corresponding to the set $\{(\theta_1, \theta_2) : \theta_2 \geq \theta_1 \text{ and } \theta_2^2 > \theta_1^4 - \theta_1^2\}$
- A3. ‘sharecropping’ area corresponding to the set $\{(\theta_1, \theta_2) : \theta_1 \geq \theta_2 \text{ and } \theta_1^2 \leq \theta_2^4 - \theta_2^2\} \cup \{(\theta_1, \theta_2) : \theta_2 \geq \theta_1 \text{ and } \theta_2^2 \leq \theta_1^4 - \theta_1^2\}$

Fig. 1 - Contract Forms



Looking at Figure 1, sharecropping is optimal (maximizes expected joint surplus) only when both agents have relatively high (here, above $\sqrt{2}$) and not too dissimilar skill levels θ_1, θ_2 . This is similar to Eswaran and Kotwal (1985). However, Eswaran and Kotwal focus on the contract that maximizes total surplus for any given pair of skill levels, i.e., they characterize the “contractual frontier” for all possible agent pairs but are silent on which matches and what organization forms are expected to arise in equilibrium. We extend and complement their analysis by embedding the double-moral hazard setting in a matching model and show that sharecropping may no longer be optimal for the same parameter values. For instance, on Figure 1 suppose we take two agents in isolation at either point A (both low-skill) or at point C (both high-skill). Since (11) is satisfied at both points A and C, taking each of these pairs in isolation, we would conclude that sharecropping will be observed in both. If we consider, instead, an economy with all four agents together and allowed to match endogenously, the joint expected surplus from two heterogeneous sharecropping pairs, corresponding to points B and D is higher than that from the two homogeneous pairs at points A and C (see Proposition 2). However, for the skill levels corresponding to points B and D, sharecropping in heterogeneous pair is dominated by other contractual forms (fixed wage or fixed rent) in which the high-skilled agent is a sole producer. That is, with endogenous matching no sharecropping would be observed in this economy.

Contrast the above discussion with another scenario in which we start with two isolated pairs at points E and G on Figure 1 (i.e., when the difference between the higher and lower skill levels is smaller than that in A and C). With exogenously given homogeneous pairs at E and G, by (11), we have sharecropping as joint-surplus maximizing. Unlike in the ABCD example, however, with all four agents together free to match with anyone they like, Proposition 2 and Corollary A imply that we would still observe sharecropping in equilibrium but in heterogeneous pairs corresponding to the points H and F. This has implications about the optimal crop share (we come back to this in Section 3).

Expected output

Using our previous results – expression (6) and the discussion right after, it is easy to compute expected total output in equilibrium with double-sided moral hazard depending on the values for the agent skill levels, the parameters a and b (we still hold $\alpha = 0$). Under sole production by skill type $k = a, b$, it is easy to verify that expected total output is $q_S^{MH}(k, k) = k + 1$; in a homogeneous sharecropping pair with skills k , it is $q_J^{MH}(k, k) = k$; and in a heterogeneous sharecropping pair, expected total output is $q_J^{MH}(a, b) = \frac{a^2 + b^2}{a + b}$.

Look back at Figure 1, points ABCD (the case $2 \leq b < a$ and $a > b^2 - b$, calling b the squared skill level at A and a that at C). With exogenously given agent pairs at points A and C (homogeneous sharecropping pairs), total expected output is $a + b$. With endogenous matching, by Proposition 2 and Corollary A, sole production by the high-skill types (at points B and D) is optimal, yielding total expected output $2a + 2$ – strictly higher. Note that the difference in expected output between the exogenous and endogenous cases is increasing in the (squared) skill difference $a - b$.

Consider now Figure 1, points EFGH (the case $2 \leq b < a$ and $a \leq b^2 - b$). With isolated pairs at points E and G (homogeneous sharecropping pairs), total expected output is $a + b$. With endogenous matching, by Proposition 2 and Corollary A, sharecropping in heterogeneous pairs at points F and H is optimal, yielding total expected output $2(\frac{a^2 + b^2}{a + b})$. The difference in expected output between the two cases is $2(\frac{a^2 + b^2}{a + b}) - (a + b)$ which equals $\frac{(a - b)^2}{a + b}$. Thus, total expected output is once again higher under endogenous matching and the output difference is increasing in $a - b$.

Wealth constraints

In the above analysis it was assumed that all three organization forms (sharecropping, fixed wage, fixed rent) are feasible for all types of agents. In particular, if sole production by high-skill tenants (“fixed rent” contract) occurs in equilibrium (see Corollary A, part (a)), it was assumed that these tenants have sufficient wealth to pay the rent. However, if they do not have the resources to rent the land (e.g., suppose they are poor and paying the rent upfront is required), then this organization form is infeasible and the next best form has to be chosen, which is either sharecropping or sole production by the low-skill landlord. Notice that this introduces an asymmetry in the joint payoff $\pi^{MH}(a, b)$ since a high-skill landlord can still engage in sole production. The equilibrium outcome under endogenous matching can be thus affected by the presence of wealth constraints. The full analysis of the possibility of wealth constraints remains outside the scope of our paper but we address it here by the following example.

Example A – Wealth constraints

Suppose the model parameters are as in Case (a) in the proof of Proposition 2, i.e., $b < a < 2$. If wealth constraints are not present, we know from Proposition 2 and Corollary A that sole production (fixed rent or wage) by the high-skill landlords or tenants is joint-surplus maximizing. Suppose, however, wealth constraints prevent high-skill tenants from sole production (fixed-rent contract). High-skill landlords can still produce on their own (fixed wage). Thus, the form of $\pi^{MH}(a, b)$ now depends on whether the high-skill party is a landlord or a tenant. We have $\pi^{MH}(a, b) = \pi_L^{MH}(a, b) \equiv \frac{1}{2} \max\{a + 1, a + b - \frac{ab}{a+b}\}$ if the high-skill agent is landlord (as before), but $\pi^{MH}(a, b) = \pi_T^{MH}(a, b) \equiv \frac{1}{2}(a + b - \frac{ab}{a+b})$ – with wealth constraints the only option for a high-skill tenant is to sharecrop with a low-skill landlord. Hence, in Case (a) in Proposition 2, for $b < a < 2$, if

$$\pi_L^{MH}(a, b) + \pi_T^{MH}(a, b) > \pi^{MH}(a, a) + \pi^{MH}(b, b), \quad (12)$$

which is equivalent to $a + 1 + a + b - \frac{ab}{a+b} > a + 1 + b + 1$, i.e., to $a^2 > a + b$, then there is sole production by high-skill landlords and sharecropping between low-skill landlords and high-skill tenants in equilibrium (instead of sole production by high-skill types in the case without wealth constraints). For instance, this holds for $a = 1.9$ and $b = 1.1$. If instead, $b < a < 2$ but $a^2 < a + b$ (e.g., $a = 1.2$ and $b = 1.1$), then (12) does not hold and NAM no longer obtains in equilibrium for this case.

In contrast, take Case (c) in Proposition 2 ($2 \leq b < a$ and $a \leq b^2 - b$). Now, the presence of wealth constraints does not affect our previous results since the equilibrium consists of sharecropping in heterogeneous pairs which is still feasible.

2.3.2 Comparison with a randomly drawn skill levels economy

In this sub-section we still maintain the assumption $\alpha = 0$ (thus, Proposition 2 applies) but allow the measure of agents to differ by skill level, as well as by task-specialization, i.e., tenants vs. landlords. We compare the fraction of sharecropping units in our economy (hereafter, “the GK economy”) with that which would obtain in an economy similar to Eswaran and Kotwal (1985), in which agents of different skill levels are drawn at random from skill distributions with the same support and probability (hereafter, “the EK economy”).

Formally, let $\lambda_k \in (0, 1)$ denote the fraction of low-skill agents (skill level θ_b) of type $k = 1, 2$ (landlords and tenants respectively). As before, call $a \equiv \theta_a^2$ and $b \equiv \theta_b^2$ to simplify the notation, with $a > b$ and $a, b \in [1, (\theta_{\max})^2]$. The case $a = b$ is analyzed separately at the end of this section. We study the equilibrium matching pattern depending on the values of a and b (the support of the skill distribution) and the agent distribution by type and skill levels, the values λ_k . There are four relevant cases for a and b , as listed in the proof of Proposition 2:

- case (a): $b < a < 2$
- case (b): $b < 2 \leq a$
- case (c): $2 \leq b < a$ and $a \leq b^2 - b$
- case (d): $2 \leq b < a$ and $a > b^2 - b$

The case $\lambda_1 = \lambda_2 = 1/2$

Begin with the case of equal measure of each of the two ability levels for both tenants and landlords ($\lambda_1 = \lambda_2 = 1/2$). This is the parametrization used in Section 2.2 and Figure 1. Proposition 2 implies that the equilibrium matching pattern is always NAM no matter which organization form is optimally chosen. Thus, we have that in cases (a), (b) or (d) for a, b (note case (d) corresponds to example ABCD on Figure 1), then $\pi^{MH}(a, b) = S(a)$ – sole production by the high-skilled is optimal. Thus, in equilibrium we have high-skill landlords producing alone and high-skill tenants producing alone renting land from the low-skill landlords. There is no sharecropping. In case (c) for a, b (which corresponds to example EFGH on Figure 1) we have $\pi^{MH}(a, b) = J^{MH}(a, b)$ – sharecropping in heterogeneous pairs obtains in equilibrium due to NAM and the fact that $S(a)$ is dominated. All agents are involved in sharecropping.

Compare these results with randomly drawing a landlord and a tenant from the corresponding skill distributions (the same a, b, λ_1 and λ_2) and looking at them in isolation as in Eswaran-Kotwal (1985). Sharecropping would be chosen if we draw skill values θ_1 and θ_2 that are both in the “triangular” region in the middle of Figure 1. Thus, in case (a) for a, b there is no sharecropping in both the GK economy (our model with endogenous matching) and the EK economy (the setting with isolated pairs). In case (b) the probability of sharecropping in the EK economy is $1/4$ (the probability of drawing two θ_a skill levels), while, as explained above, in the GK economy the fraction of sharecropping contracts in equilibrium is zero. In case (c) the fraction of sharecropping in both the EK and GK economies equals 1. Finally, in case (d) the probability of sharecropping in the EK economy is $1/2$ (the probability of drawing two θ_a skills or two θ_b skills), whereas in the GK economy the fraction of sharecropping pairs is zero.

We next look at the general case $\lambda_1, \lambda_2 \in (0, 1)$ and obtain the following result.

Proposition 4

In an economy in which a pair of types is drawn at random (the “EK economy”), there is a (weakly) higher likelihood of sharecropping than in an economy in which agents have skill levels whose measures and supports coincide with the probability distribution over skills in the EK economy and in which agents can match endogenously (the “GK economy”). This holds for any skill levels $\theta_a, \theta_b \in \Theta$ with $\theta_a \geq \theta_b$ and any probability/measure distribution $\lambda_1, \lambda_2 \in (0, 1)$ over the two skill levels where $\lambda_i, i = 1, 2$ denotes the probability/measure of low-skill task i specialists.

Proof: (see Appendix)

Proposition 4 shows that in an economy in which agents with different abilities are drawn at random there is a higher likelihood of sharecropping being the optimally chosen (surplus-maximizing) contractual form than in an economy with the same distribution of abilities but in which there is endogenous matching. This is a consequence of the NAM result shown earlier. Proposition 4 holds for any distribution of skill levels over agent types which further emphasizes the general message that selection effects are crucial to understand the nature of equilibrium contracts.

2.3.3 Moral hazard and positive assortative matching

In Proposition 2 we showed that for $\alpha = 0$, for any $a > b \geq 1$ the inequality $\pi^{MH}(a, b) > \frac{1}{2}[\pi^{MH}(a, a) + \pi^{MH}(b, b)]$ holds, and hence there is negative assortative matching in equilibrium. By continuity, the same result holds for sufficiently small and positive α . In this section we show that for α sufficiently large there can be positive assortative matching (PAM) in equilibrium under moral hazard. Since for $\alpha \neq 0$ the algebra in computing and comparing $\pi^{MH}(i, j)$ for $i, j \in \{a, b\}$ becomes intractable, we only provide sufficient conditions for PAM.

Proposition 5 (PAM under moral hazard – sufficient conditions)

Suppose the following sufficient conditions hold: (i) $\alpha \geq 1/2$; (ii) $a > b \geq 2$; and (iii) $a \leq b^2 - b$. Then

$$\pi^{MH}(a, b) < \frac{1}{2}[\pi^{MH}(a, a) + \pi^{MH}(b, b)] \quad (**)$$

which implies that the equilibrium matching pattern in the economy is positive assortative of the segregation type.

Given $\alpha \geq 1/2$, weaker but ‘messier’ forms of the sufficient conditions (ii) and (iii) in the Proposition statement can be derived using (15) and (16) from the proof of Proposition 5 in the Appendix. Specifically, for $\alpha \geq 1/2$, the left-hand side of (15), $(\frac{k}{2} - 1) + 2\alpha(k - \sqrt{k}) \geq \frac{3}{2}k - \sqrt{k} - 1$ which is non-negative for $\sqrt{k} \geq \frac{1+\sqrt{7}}{3} \approx 1.22$, i.e., $b \geq (1.22)^2$ is a weaker version of condition (ii). Similarly, for $\alpha \geq 1/2$, (16) implies that $b^2 \geq (a + b)[1 - \sqrt{a}(\sqrt{b} - 1)]$ is a weaker version of condition (iii).

Suppose the sufficient conditions (i)–(iii) in Proposition 5 are satisfied. Then, the optimal organization form is sharecropping in homogeneous pairs, (θ_a, θ_a) and (θ_b, θ_b) . Now the logic from the ABCD example depicted in Figure 1 goes the other way. In particular, if we started with two heterogeneous agent pairs at points B and D in isolation, we would conclude that no sharecropping would be observed. However, with endogenous matching allowed, sharecropping in homogeneous pairs corresponding to points A and C is the equilibrium outcome.

3 Discussion and Implications

In this section we discuss some applications and potentially testable implications of the theoretical analysis in Section 2.

1. Share equality and/or uniformity

Our results imply that, with double-sided moral hazard and endogenous matching, the optimal shares observed in sharecropping would typically be equal to or close to $1/2$. In the PAM case supposing there are multiple agents of each skill level and the conditions given in Proposition 5 hold, then with endogenous matching we would observe only sharecropping pairs between agents of equal skills and hence optimal share s^* from (7) equal to $1/2$ in

each.¹³ No other sharing rules would be observed. In addition, as long as the lowest skill level satisfies (11), all agents would use sharecropping and all will use a share of one-half – a uniform sharing rule across all production units independent of skill levels. Therefore, in the PAM case, our model gives an alternative explanation for why the observed share in most studies of sharecropping is clustered around 50:50.¹⁴

In the NAM case (see Proposition 2 and discussion afterwards), (7) implies that the optimal shares in a (θ_a, θ_b) sharecropping pair are $\frac{\theta_a^2}{\theta_a^2 + \theta_b^2}$ and $\frac{\theta_b^2}{\theta_a^2 + \theta_b^2}$. From the discussion after Figure 1, the optimal share when sharecropping is observed in equilibrium would be close to an equal split. The intuition is the same as in Eswaran and Kotwal (1985) – if the skill difference between agents is too high, any gains from specialization are smaller and cannot offset the agency costs. This is an attractive feature of theirs and our model given the predominance of such shares in data. What is new here, is that accounting for endogenous matching makes the dispersion in skill levels for which sharecropping is optimal narrower than if agent pairs were taken in isolation (e.g., see Figure 1, points E and G vs. points A and C). More specifically, an implication of our model is that the lower the low-skill value θ_b , the lower is the maximum possible deviation between 1/2 and the optimal share with endogenous matching. Indeed, by Corollary A, given b , the maximum high-skill agent share that can be observed in equilibrium is $s_{\max} = \frac{b^2 - b}{b^2} = 1 - \frac{1}{b}$. It is obtained by setting $a = b^2 - b$. Obviously, for low values of b the value s_{\max} is close to 1/2 (e.g., if $\theta_b = \sqrt{2}$, sharecropping can only be observed for $\theta_a = \theta_b$ and then $s^* = s_{\max} = 1/2$).¹⁵ Only if the minimum skill level θ_b in the economy is large can shares substantially differ from 1/2 in equilibrium.

2. Within vs. across group heterogeneity

Suppose NAM obtains under double-sided moral hazard (e.g., the case $\alpha = 0$). This means that there is more heterogeneity *within* groups (sharecropping pairs), but less heterogeneity *across* pairs. In our simple two-skill (θ_a, θ_b) , four-agent example on Figure 1, the joint surplus in both pairs H and F is the same. While we do not determine the exact individual payoffs, we showed that the optimal output shares in a heterogeneous pair are unequal which would result in general in unequal surplus shares. If, instead, we had positive sorting as in Proposition 5, there is no intra-group inequality but there is inter-group inequality. The endogenous sorting pattern can therefore have implications about inequality in the economy as a whole.

3. Unobserved ability

Negative sorting implies that landlord-tenant pairs are heterogeneous in equilibrium (Proposition 2). If skill level is not directly measured but instead proxied (e.g., by wealth), such heterogeneity is often interpreted as suggestive of risk-sharing – the relatively less risk-averse (richer) party insures the more risk-averse (poorer) party. We provide an alternative explanation for heterogeneous matching which does not rely on risk aversion and insurance motives.

4. Variance of observed shares

¹³This result applies more generally, for more than two skill levels – PAM results in ‘more homogeneous’ matches and thus in optimal shares closer to 1/2 relative to NAM (see Ghatak and Karaivanov, 2011).

¹⁴See Young and Burke (2001) for a theory based on conformity to custom.

¹⁵Remember, we must have $b \geq 2$ in order to have sharecropping in equilibrium and so $s_{\max} \in [\frac{1}{2}, 1)$.

A prediction of our model is that, holding other things equal, the observed distribution of crop shares would have greater variance under negative sorting than under positive sorting.

5. Variance of output

As shown earlier, total output in a sharecropping pair of agents with skills θ_1 and θ_2 under double-sided moral hazard equals:

$$\alpha\theta_1\theta_2 + \frac{\theta_1^4 + \theta_2^4}{\theta_1^2 + \theta_2^2} = \alpha\theta_1\theta_2 + \theta_1^2 + \theta_2^2 - \frac{2\theta_1^2\theta_2^2}{(\theta_1^2 + \theta_2^2)}$$

Hence, if $\alpha = 0$ or positive but not too high, like we did with joint surplus π^{MH} (see Proposition 2), it can be shown that total output is sub-modular in θ_1 and θ_2 . This implies that the variance of output across production units will be lower with negative sorting compared to with positive sorting. This observation, together with point 4 above, could be potentially useful to infer the likely matching pattern in sharecropping data when skills are unobservable to the researcher.

6. Skewness of the income distribution

With positive sorting Kremer (1993) shows that the income distribution is skewed to the right. His model has no agency problems, just a production function with complementarity in skills, $y = A\theta_1\theta_2$ which leads to PAM and homogeneous groups (with $\theta_1 = \theta_2$) being formed. Kremer shows that a given skill gap $\theta_a - \theta_b > 0$ translates into an income gap proportional to $\theta_a^2 - \theta_b^2$. Hence, with positive sorting, the income distribution in Kremer's economy is more skewed than the skill distribution.

Similar logic applies in our setting under moral hazard. Suppose $\alpha = 0$ and look at sharecropping in an exogenously given (θ, θ) pair with $\theta \geq \sqrt{2}$. Total surplus is $\pi_{pos} \equiv \frac{3}{4}\theta^2$ – see (8), where θ is θ_a or θ_b . Assuming equal surplus sharing, each party earns income $\frac{3}{8}\theta^2$ which is proportional to the square of her skill level, as in Kremer's model. Calling $a = \theta_a^2$ and $b = \theta_b^2$, note that the income difference across agents of different skills would be $\frac{3}{8}(a - b)$ if we took the (θ_a, θ_a) and (θ_b, θ_b) pairs in isolation.

The income distribution is in fact more skewed under negative endogenous sorting for the same $\theta_a > \theta_b \geq \sqrt{2}$ and $\alpha = 0$. Suppose the conditions in Corollary A are satisfied so that heterogeneous sharecropping units are optimal (a and b are as in Case (c) in the proof of Proposition 2). From (8), each such unit has total surplus $\pi_{neg} \equiv \frac{1}{2}(a + b - \frac{ab}{a+b})$. The optimal share of the high-skill party is $s^* = \frac{a}{a+b}$. Thus, assuming no other transfers, she earns expected income $s^*\pi_{neg}$ while the low-skilled agent earns $(1 - s^*)\pi_{neg}$. Therefore, the difference between the incomes of the high- and low-skill agents is:

$$(2s^* - 1)\pi_{neg} = \frac{1}{2}(a - b)\left[1 - \frac{ab}{(a + b)^2}\right] > \frac{3}{8}(a - b)$$

since $\frac{ab}{(a+b)^2} < \frac{1}{4}$. Thus, the income distribution is more skewed under negative endogenous sorting compared to if agents were exogenously paired into homogeneous (θ_a, θ_a) and (θ_b, θ_b) units. Intuitively, the unequal share s^* further biases income in favor of the high-skill agents – incentive reasons dictate that they earn higher share of total surplus, which enhances the rewards to skill in addition to what comes solely from technology. The reversal in equilibrium matching from positive to negative in the presence of moral hazard thus may increase income

inequality above and beyond the purely technological complementarity reasons identified by Kremer.

4 Conclusions

We analyze contractual arrangements between risk-neutral landowners and tenants taking as benchmark the classical model of Eswaran and Kotwal (1985) in which sharecropping exploits absolute advantage in two separate tasks but is subject to double-sided moral hazard. We study the consequences of allowing endogenous matching of landowners and tenants. We show that the presence of moral hazard can reverse the equilibrium matching pattern relative to the first best. Even if sharecropping is optimal (joint-surplus maximizing) for an exogenously given agent pair, with endogenous matching sharecropping may not be observed in equilibrium. An important implication of these results is that, in the distribution of possible contractual forms, sharecropping is less likely to be observed in equilibrium if matching is endogenous. This involves an efficiency loss because some optimal sharecropping arrangements do not form due to agency costs which must be taken into account, in addition to the usual efficiency loss coming directly from agency costs in an exogenously given sharecropping contract.

In the working paper version (Ghatak and Karaivanov, 2011) we investigate conditions that lead to sub- or super-modularity of the indirect joint surplus function under double-sided moral hazard and endogenous matching under general forms for the production and effort cost functions. In the double-sided moral hazard setting, the connection between substitutability in skills and modularity of joint surplus is not clear ex-ante due to the endogenous choice of effort levels and optimal sharing rule. We demonstrate that the modularity of indirect joint surplus depends on the degree of substitutability in agents' skills but the degree of substitutability in agent efforts plays an additional important role.

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5 Appendix

Proof of Proposition 3

Call $m = \theta_1^2$ and $n = \theta_2^2$. Given (8), for any $\theta_1 \geq \theta_2$ joint production would be chosen if and only if it yields higher expected total surplus, i.e., if

$$\frac{1}{2}(m + n - \frac{mn}{m + n}) \geq \frac{1}{2}(m + 1). \quad (13)$$

Simplifying this, the condition for sharecropping to maximize expected joint surplus is:

$$m \leq n^2 - n. \quad (14)$$

By assumption, $n \leq m$. For (14) to hold, we need $\theta_2 = \sqrt{n} \geq \sqrt{2}$ (otherwise, $n^2 - n < n$) and $m \leq n^2 - n$ for any such $\sqrt{2} \leq \theta_2 \leq \theta_1$. Conversely, if $\theta_2 \geq \sqrt{2}$ ($n \geq 2$), then $n^2 - n \geq n$, and so, for any $m \geq n$ with $m \leq n^2 - n$ (such m exist for $n \geq 2$) inequality (13) holds, i.e., sharecropping with share s^* from (7) achieves higher expected joint surplus than sole production by the high-skill agent. If $\theta_2 = \sqrt{n} < \sqrt{2}$ (which implies $n^2 - n < n \leq m$), or if $\theta_2 \geq \sqrt{2}$ but $m > n^2 - n$, then by (13) sole production by the high-skill agent achieves higher expected total surplus.

For the case of equal skills, note that if $\theta_1 = \theta_2 = \theta$ (i.e., $m = n = \theta^2$) then (13) is equivalent to $m \geq 2$ or $\theta \geq \sqrt{2}$. ■

Proof of Proposition 4

We already showed the proposition result for the case $\lambda_1 = \lambda_2 = 1/2$ in the main text. We consider the remaining possibilities for $\lambda_1, \lambda_2 \in (0, 1)$ in turn. We first analyze the unequal skills case $\theta_a > \theta_b$ in points 1-4 below. The special case $\theta_a = \theta_b$ is considered separately in point 5.

1. $\lambda_1 = \lambda_2 = \lambda < 1/2$ – equal measure of each skill per agent type (landlord or tenant) but less low-skill than high-skill agents overall.

Proposition 2 still implies NAM in equilibrium in the GK economy. However, in this case there are not enough low-skill agents to match all agents in NAM pairs. Hence, the equilibrium sorting pattern is to match as many as possible agents in NAM pairs and the rest, $1 - 2\lambda$ in homogeneous (θ_a, θ_a) pairs. By Proposition 2, any other matching will be blocked. Existence of any (θ_b, θ_b) pairs in equilibrium or of more than measure $1 - 2\lambda$ of (θ_a, θ_a) pairs (which would imply the existence of (θ_b, θ_b) pairs), leads to a contradiction as two (θ_a, θ_b) pairs achieve higher joint surplus than two homogeneous pairs (θ_a, θ_a) and (θ_b, θ_b) . Therefore, we have the following results depending on the values of a and b and using analogous arguments as in the case $\lambda_1 = \lambda_2 = 1/2$.

In case (a) for a and b there is no sharecropping in both the EK and GK economies. In case (b) the probability of sharecropping in the EK economy is $(1 - \lambda)^2$ (the probability of drawing two θ_a skilled agents) while in the GK economy the fraction of sharecropping pairs is strictly lower, $(1 - 2\lambda)^2$ (since in case (b) sharecropping is joint surplus maximizing for (θ_a, θ_a) pairs, see Corollary A to Proposition 2). In case (c) the probability/fraction of sharecropping in both the EK and GK economies is equal to 1. In case (d) the probability of sharecropping in the EK economy is $\lambda^2 + (1 - \lambda)^2$ (see Proposition 3), while in the GK economy the fraction of sharecropping pairs is strictly lower, $(1 - 2\lambda)^2$, as in case (b).

2. $\lambda_1 = \lambda_2 = \lambda > 1/2$ – equal measure of each skill per agent type (landlord or tenant) but more low-skill than high-skill agents overall.

Proceed analogously to point 1 above. Now there are not enough θ_a agents to match everyone in NAM pairs. Hence, the equilibrium sorting is to match as many as possible agents in NAM pairs and the rest, $2\lambda - 1$ in homogeneous (θ_b, θ_b) pairs. In case (a) for a and b there is no sharecropping in both the EK and GK economies. In case (b) the probability of sharecropping in the EK economy is $(1 - \lambda)^2$ while in the GK economy the fraction of sharecropping in equilibrium is strictly lower, equal to zero (since sole production is joint surplus maximizing for (θ_b, θ_b) pairs, see Corollary A). In case (c) the probability/fraction of sharecropping in both EK and GK economies is 1. In case (d) the probability of sharecropping in the EK economy is $\lambda^2 + (1 - \lambda)^2$ while in the GK economy the corresponding fraction is lower, $(2\lambda - 1)^2$ by the same arguments as in point 1 above.

3. $\lambda_1 \neq \lambda_2$ with $\lambda_1 + \lambda_2 \leq 1$.

Now there are not enough agents of one of the low-skill types to match all agents in NAM pairs. Hence, the equilibrium sorting is to match as many as possible agents in NAM pairs and the rest, $1 - \lambda_1 - \lambda_2$ (could be zero) in homogeneous high-skill (θ_a, θ_a) pairs. Note that $\lambda_1 \leq 1 - \lambda_2$ and $\lambda_2 \leq 1 - \lambda_1$ so $\lambda_1 + \lambda_2$ heterogeneous pairs can always be formed. Proceeding as before, we have that, in case (a) for a and b , there is no sharecropping in both the EK and GK economies. In case (b), the probability of sharecropping in the EK

economy is $(1 - \lambda_1)(1 - \lambda_2)$ while in the GK economy the corresponding fraction is strictly lower, $(1 - \lambda_1 - \lambda_2)^2$. This is true since $(1 - \lambda_1)(1 - \lambda_2) > (1 - \lambda_1 - \lambda_2)^2$ is equivalent to $(\lambda_1 + \lambda_2)(1 - \lambda_1 - \lambda_2) + \lambda_1\lambda_2 > 0$. In case (c), the probability/fraction of sharecropping in both the EK and GK economies is 1. In case (d), by Proposition 3, the probability of sharecropping in the EK economy is $\lambda_1\lambda_2 + (1 - \lambda_1)(1 - \lambda_2)$ while in the GK economy the corresponding fraction is strictly lower, $(1 - \lambda_1 - \lambda_2)^2$.

4. $\lambda_1 \neq \lambda_2$ with $\lambda_1 + \lambda_2 > 1$.

This is analogous to point 3 above but now there are not enough agents of one of the high-skill types to match all agents in NAM pairs. Hence, the equilibrium sorting is to match as many as possible agents in NAM pairs and the rest, $\lambda_1 + \lambda_2 - 1$ in homogeneous low-skill (θ_b, θ_b) pairs. In case (a) for a and b , there is no sharecropping in both the EK and GK economies. In case (b) the probability of sharecropping in the EK economy is $(1 - \lambda_1)(1 - \lambda_2)$, while in the GK economy the corresponding fraction is 0 since by Proposition 2 sole production is optimal for (θ_b, θ_b) tenant-landlord pair. In case (c) the probability/fraction of sharecropping in both the EK and GK economies is 1. In case (d), the probability of sharecropping in the EK economy is $\lambda_1\lambda_2 + (1 - \lambda_1)(1 - \lambda_2)$ while in the GK economy the corresponding fraction is strictly lower, $(\lambda_1 + \lambda_2 - 1)^2$ (this is shown as in point 3 above).

5. *The special case $\theta_a = \theta_b = \theta \in \Theta$ and $\lambda_1, \lambda_2 \in (0, 1)$*

In this case $a = b = \theta^2$, the measure of both landlords and tenants is one and all agents have the same skill θ . We thus have $\pi^{MH}(\theta, \theta) = \max\{S(\theta), J^{MH}(\theta, \theta)\} = \frac{1}{2} \max\{b + 1, \frac{3}{2}b\}$. By Corollary A, part (b), for $b < 2$ there is sole production by all agents in equilibrium and for $b \geq 2$ there is sharecropping by all agents in (θ, θ) pairs in both the EK and GK economies. ■

Proof of Proposition 5

Consider the expressions for expected joint surplus under moral hazard, (9). Note that $J^{MH}(k, k) \geq S(k)$ for $k = a, b$ is equivalent to

$$\begin{aligned} 2\alpha k + \frac{3}{2}k &\geq 2\alpha\sqrt{k} + k + 1 \text{ or} \\ \left(\frac{k}{2} - 1\right) + 2\alpha(k - \sqrt{k}) &\geq 0. \end{aligned} \tag{15}$$

It is clear from (15) that $k \geq 2$ is a sufficient condition for $J^{MH}(k, k) \geq S(k)$ for any $\alpha \geq 0$. Thus, under condition (ii) in the proposition statement, sharecropping is optimally chosen in homogenous pairs, i.e., J^{MH} achieves the maxima in $\pi^{MH}(a, a)$ and $\pi^{MH}(b, b)$.

Next, $J^{MH}(a, b) \geq S(a)$ is equivalent to

$$\begin{aligned} 2\alpha\sqrt{ab} + b - \frac{ab}{a + b} &\geq 2\alpha\sqrt{a} + 1 \text{ or} \\ b^2 &\geq (a + b)[1 - 2\alpha\sqrt{a}(\sqrt{b} - 1)]. \end{aligned} \tag{16}$$

Condition (iii) in the Proposition statement implies that (16) holds for any $\alpha \geq 0$, i.e., $J^{MH}(a, b) \geq S(a)$ for any $a, b \geq 1$ satisfying $a \leq b^2 - b$.

The above results ensure that $J^{MH}(i, j) \geq S(i)$ for $(i, j) \in \{(a, a), (b, b), (a, b)\}$ – share-cropping is optimal for any skills pair. Thus, $\pi^{MH}(i, j) = J^{MH}(i, j)$ for all i, j and so to show $(**)$ we only need to ensure that

$$J^{MH}(a, b) - \frac{1}{2}[J^{MH}(a, a) + J^{MH}(b, b)] < 0$$

which, using (9) and simplifying, is equivalent to

$$\frac{1}{2}(\sqrt{a} - \sqrt{b})^2 \left(-\frac{\alpha}{2} + \frac{(\sqrt{a} + \sqrt{b})^2}{8(a+b)} \right) < 0.$$

Note first that $\frac{(\sqrt{a} + \sqrt{b})^2}{8(a+b)} < 1/4$ is equivalent to $(\sqrt{a} - \sqrt{b})^2 > 0$ which is always true for $a > b$. Thus, for any $\alpha \geq 1/2$ we have $\frac{(\sqrt{a} + \sqrt{b})^2}{8(a+b)} < 1/4 \leq \frac{\alpha}{2}$ i.e., $J^{MH}(a, b) - \frac{1}{2}[J^{MH}(a, a) + J^{MH}(b, b)] < 0$ for any $a, b \geq 1$ and α satisfying (i)–(iii).

The above results imply that the surplus function $\sigma^{MH}(a, b) \equiv \max\{0, \pi^{MH}(a, b) - \frac{1}{2}[\pi^{MH}(a, a) + \pi^{MH}(b, b)]\}$ equals zero for a, b, α satisfying (i)–(iii). Thus, as in Proposition 1, by Proposition 4 in LN(2002), we obtain PAM of the segregation type in equilibrium. ■