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Bidding behavior in a repeated procurement auction: A summary

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Abstract

This paper considers bidding behavior in a repeated procurement auction setting. We study highway procurement data for the state of California between December 1994 and October 1998. We consider a dynamic bidding model that takes into account the presence of intertemporal constraints such as capacity constraints. We estimate the model non-parametrically and assess the presence of dynamic constraints in bidding. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

This paper examines repeated procurement auctions for highway paving contracts. We study whether previously won uncompleted contracts affect the ability to win further contracts. Two distinct effects may arise: First, since the duration of highway paving contracts is a number of months, winning a large contract may commit some of the bidder's machines and paving

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resources for the duration of the contract. Although rental of additional equipment is available, this may increase total cost. Second, a learning effect may arise, since supplying services on a large contract may give a bidder the necessary expertise to conduct further services. The learning effect may lower the cost for future contracts.

To examine the presence of intertemporal constraints, in Section 2, we consider a dynamic bidding game. The model assumes that bidders learn their costs every period anew. Costs are drawn from a distribution that depends on the bidders state. We assume that the state is determined by the backlog from previously won contracts. The backlog measures the amount of uncompleted work from previously won contracts. We do not specify the nature in which the state affects the distribution of costs. Instead, we let the data decide whether there is any relationship and if so, how the state affects costs. In examining equilibria of the game, we restrict our attention to Markovian strategies that depend on payoff relevant variables. Specifically, we assume that bidding strategies are only a function of the level of current backlogs of firms in the industry.

Section 3 discusses the econometric model. Paarsch (1992), Laffont et al. (1995) and others develop an empirical approach to estimate private information in auction environments. We follow their approach and estimate private information based on the dynamic model. We do not impose ex-ante implications of the capacity constraint on the estimation. In contrast, we examine to what extent implications of capacity constraints are satisfied by the data.

The empirical specification follows a two-stage approach similar to Elyakime et al. (1994) and Guerre et al. (1998). In the first stage, the beliefs of bidders concerning bids of other bidders are estimated non-parametrically. Bidder asymmetry, contract characteristics and state variables are accounted for in the estimation. In the second stage, privately known costs are inferred using a structural relationship of the model. In our case, the structural relationship requires an expression for the expected sum of future profits. We calculate the expected future discounted payoff of bidders numerically based on estimates of bidders' beliefs.

Section 4 describes the data and the industry. We examine data on highway procurement contracts in California between December 1994 and October 1998. During this 4-year time period contracts with a total value of \$5.9 billion nominal dollar were awarded in California. For comparison, the annual value of construction of highways, streets and related work in the US is about \$35 billion per year.[1] Descriptive data analysis suggests the expected effect of backlog. Estimates of the probability of submitting a bid reveal that bidders which committed only a small fraction of their capacity are about twice as likely to submit a bid than bidders who committed a large fraction.

[1] 1992 Census of Construction Industries.

Highway construction data have been studied by a number of authors. Feinstein et al. (1985) and Porter and Zona (1993) study issues of bidder collusion. Bajari (1997) assesses the importance of bidder asymmetry.

Section 5 presents the estimation results of our model. In general, we find that the distribution of bids exhibits the expected properties of capacity constraints. To illustrate the estimates, we evaluate the primitives of the model at sample characteristics of bidder 1. The bid distribution of bidder one at low backlog values stochastically dominates (in the first order sense) the distribution at high backlog values. Moreover, increasing the backlog appears to monotonically decrease the probability of submitting a bid. Although there are some exceptions to this rule.

The equilibrium of the model implies a monotone relationship between costs and bids. As suggested by Elyakime et al. (1994), a test of the adequacy of the bidding model can be based on the monotonicity property. We find that there is indeed a monotone relationship between bids and costs, thus, the assumption of the imposed bidding model is confirmed.

Estimates of the expected future sum of profits are hardly affected by changes in the state. In particular, the effect of current backlog of the bidder on future profitability appears minor. This result may not be surprising since the duration of highway contracts is a number of months on average and effects on future profitability should be small.

Finally, Section 6 summarizes the main conclusions of the paper.

2. The bidding model

The bidding model has the following features: There are an infinite number of periods, $t \in \{0, 1, \dots, \infty\}$. In every period the buyer offers a single contract for sale. There are two types of bidders: regular and fringe bidders. Fringe bidders have a short life and exit in the period they entered.[2] Regular bidders stay in the game forever. The set of regular bidders is denoted by $\{1, \dots, n_{\mathrm{r}}\}$ and the set of fringe bidders is denoted by $\{n_{\mathrm{r}} + 1, \dots, n\}$.

Each period bidder i learns her cost for the contract. The cost is privately known and denoted by c_i^t. The priors of all bidders about the cost of a regular bidder c_i^t are identical and represented by the distribution function $F(c \mid s_i^t, s_0^t)$ where s_i^t is a variable that summarizes the available capacity of bidder i at time period t and s_0^t denotes contract characteristics. We assume that contract characteristics are drawn identically and independently from the distribution of

[2] In the data we observe a number of bidders that submit a bid only once, or a small number of times. On the other hand, we observe a number of bidders that submit bids frequently. To account for this difference, we classify the largest 30 firms in $ value won as regular bidders and the remaining firms as fringe bidders.

contracts. The distribution of costs has a continuous density function $f(c \mid s_i^t, s_0^t)$ and support $[0, C]$. Similarly, the cost of a fringe bidder is drawn from a continuous distribution function $F(c \mid s_0^t)$ with associated density function $f(c \mid s_0^t)$ and support $[0, C_F]$. The bidders may submit a bid for each contract which is the price at which they are willing to provide the service. The bidder with the lowest bid wins the contract and receives his bid. All agents are risk neutral. Ties are resolved by the flip of a coin. The buyer imposes a fixed (non-random) reserve price R, meaning that bids above R are rejected. We assume that $R = C_F$.

The state variable includes the backlog stemming from past won contracts and bidder specific characteristics. The evolution of the state depends on the current backlog and the size of the project won this period. We introduce a common discount factor parameter, $\beta \in (0, 1)$, that measures firms' patience with regard to future profits.

We consider subgame perfect equilibria and restrict attention to Markovian strategies. A strategy for bidder i is a function of the state vector s and bidder i's cost at time period t. The strategy can be written as $b(c_i^t, s_i, s_{-i}, s_0)$ where s_{-i} denotes the vector of the states of the remaining regular bidders $(s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_n)$.[3]

The expected future payoff for regular bidder i can be written as

$$V_i(s_i, s_{-i}, s_0) = \int \max_b \{[b - c]\mathrm{Prob}(i \text{ wins} \mid b, s) + \beta \sum_{j=1}^{n} \mathrm{Prob}(j \text{ wins} \mid b, s) E_{s'} V_i(s' \mid s, j \text{ wins})\} f(c \mid s_i, s_0)\, \mathrm{d}c,$$

where the expectation operator, $E_{s'}$, accounts for uncertainty with respect to contract characteristics. The payoff equals the ex ante expected current period return plus the sum of discounted future returns. For a fringe bidder the ex ante payoff equals to

$$\int \max_b \{[b - c]\mathrm{Prob}(i \text{ wins} \mid b, s)\} f(c \mid s_0)\, \mathrm{d}c.$$

In subsequent sections we use the following notation: We denote the distribution function of equilibrium bids of bidder i by $G(b \mid s_i, s_{-i}, s_0)$ and the associated density function by $g(b \mid s_i, s_{-i}, s_0)$. The probability that a bid b wins the contract for bidder i can be written as $\prod_{j \neq i}[1 - G(b \mid s_j, s_{-j}, s_0)]$.

3. Econometric model

In this section, we describe our econometric model. First, we argue about the choice of state variables. Second, we describe the estimation of the bid distribution function, G, which represents bidders' beliefs. Finally, we explain how we use G to infer costs and the distribution function of costs, F.

[3] In order to simplify the notation, from now on we drop superscript t.

3.1. State variables

Bidders select their bids after observing a number of variables: These include the bidders' state variables, contract characteristics and private information. We assume that capacities and contract characteristics are observable to all bidders and the econometrician prior to the bidding. Private information is not observable to bidders.

Non-parametric methods impose limitations on the dimensionality of estimators.[4] We make a number of choices concerning the variables of interest which are in part aimed at reducing the dimension of the state space.

First, we project the logarithm of bids on a vector of contract characteristics and we estimate the density and distribution of the resulting residuals, b.

Second, we estimate the bid distribution function using the following five variables: $(a_i, a_{-i}, r, x_i, W_i)$. The variable a_i denotes the backlog of bidder i, i.e., the work amount measured in dollars that is left from previously won projects. The backlog variable is constructed in the following way:[5] For every contract previously won, we calculate the amount that is left to do by taking the initial size of the contract and multiplying it by the fraction of time that is left until the completion time. For contracts that finished prior to the end of the sample period, we use the actual completion date, while for contracts that did not finish by the end of the sample period, we use the planned completion date. Based on this calculation, we determine at any given point in time the total amount of work measured in dollars that is left to do. Finally, we standardize the variable, by applying the normal distribution to every observation. The parameters of the normal distribution are the bidder specific mean (calcultated using daily obvservations) and the bidder specific variance. The resulting backlog variable is a number between zero and one. Due to the standardization, the backlog variable is comparable across bidders.

The variables r and x_i are indices that capture the bidders' capacity and location characteristics: $r(L_t, L_1, \dots, L_n)$, measures the number of bidders that are located in close proximity to the contract location, L_t; $x_i(\mathrm{MB}_i, L_i, L_t, E_t)$ is constructed as a linear projection of the decision to submit a bid on the space of the maximum capacity of the firm, MB_i, its location, L_i, with respect to the contract location, L_t, and the engineer's estimate, E_t.

The variable W_i measures the number of contracts won by bidder i prior to the sample period. It is determined based on data on contract performance between December 1994 and April 1996 which is prior to the sample period.

[4] The limitations for fixed sample sizes are discussed in Silverman (1986) and Simonoff (1996).

[5] We experimented with different specifications of the backlog effects. In particular, we also used a variable that measures the fraction of the maximum capacity at any point in time. The estimation results were very similar.

The theoretical model implies that the state of the remaining bidders have a symmetric effect on bidder i's bid distribution and value function. I.e., bidder i's bid distribution function remains unchanged if the state of bidders j and l are exchanged. We maintain the exchangeability property with respect to the backlog variable, a_{-i}. Exchangeability implies that the exact order of the components of this vector does not matter and we can select a particular ordering. We order elements according to their size. We report estimates using the largest difference between individual order statistics, $a_{(-i)}$ and $a'_{(-i)}$, $i = 1, \dots, n-1$. We use this measure, i.e. $d(a_{-i}, a'_{-i}) = \sup_{j \neq i}\{|a_{(j)} - a'_{(j)}|\}$, to asses whether two vectors a_{-i} and a'_{-i} are close.

3.2. Bidder's beliefs

We estimate the density and distribution function of bids using Kernel estimators. Bids by bidder i are observed only if $b \leq R$. To account for the truncation, we estimate the conditional bid density $g(b \mid b \leq R;\, a_i, a_{-i}, r, x_i, W_i)$, and the truncation probability $\Gamma(b \leq R \mid a_i, a_{-i}, r, x_i, W_i)$. Thus the density function of bids is given by

$$g(b \mid a_i, a_{-i}, r, x_i, W_i) = g(b \mid b \leq R;\, a_i, a_{-i}, r, x_i, W_i) \cdot \Gamma(b \leq R \mid a_i, a_{-i}, r, x_i, W_i).$$

The theoretical model assumes that bidders observe a reserve price. Thus, bids are drawn from a distribution with bounded support. This poses a problem in the estimation, since Kernel estimators perform poorly close to the boundary. To address this boundary problem, we adopt the mirror image method proposed by Schuster (1985). The method involves creating additional data points by adding a mirror image of the data on the other side of the bound. Schuster establishes asymptotic properties of this method.

The data do not include the reserve price, or upper bound of the support of bids. We estimate the upper bound, or reserve price, using the maximum of the winning bids.[6] $\bar{b} = \max_t b_1^t$, where b_1^t denotes the winning bid on contract t.

3.3. Inference of costs

Optimal bids are chosen based on the privately known cost, the beliefs about other bidder's bids and the effect of all these factors on future payoffs. The necessary first-order condition for equilibrium bids summarizes this relationship. The equation can be rearranged to express the cost as an explicit function

[6] Winning bids are selected because we are certain that these bids are below the reserve price. Non-winning bids may have been inadvertently above the reserve price.

of the equilibrium bids, the beliefs about other bids and the value function. This approach has been proposed by Elyakime et al. (1994) in the context of a static auction models.

Let $\phi(.)$ denote the unobserved cost associated with a bid by bidder i. It is a function of the bid, b and the state, s. Let $\tau(b \mid s) = g(b \mid s)/(1 - G(b \mid s))$ denote the hazard function of bids. The first-order condition for optimal bids yields the following equation for privately known costs, ϕ:

$$\phi(b, s_i, s_{-i}, s_0)$$
$$= b - \frac{1 - \beta \sum_{j \neq i} \tau(b \mid s_j, s_{-j}, s_0) E_{s'}[V_i(s' \mid s, i \text{ wins}) - V_i(s' \mid s, j \text{ wins})]}{\sum_{j \neq i} \tau(b \mid s_j, s_{-j}, s_0)}.$$

The function $\phi(b, s_i, s_{-i}, s_0)$ provides an explicit expression of privately known costs that involves the hazard function of bids, τ, and the value function, V_i. It states that the cost equals the bid minus a mark-down. The mark-down accounts for the level of competition and the incremental effect on the future discounted profit if the firm wins the contract instead of another firm. In order to infer costs, we have to find estimators for the hazard and distribution of bids and the value function. As explained below, an estimator of the hazard and distribution function can be directly obtained from the data. On the other hand, we can express the value function as a recursive expression of the distribution of equilibrium bids. Thus, once estimates of bid distributions are obtained, we can calculate the value function numerically.[7]

There are a number of methods to estimate the model. We chose a non-parametric approach since we do not have any prior knowledge about the shape of bidder's beliefs. Before explaining the estimators, we briefly summarize in the next section the variables used in the estimation. Then, we describe our modeling choices.

4. The data and industry

In this section, we present our data and describe the awarding process for contracts. In addition, we report summary statistics and descriptive analysis of the data.

Our data consist of all California Department of Transportation (Caltrans) contract awards for highway and street construction made between December 1994 and October 1998.[8] Information on bids is available from from May 1,

[7] Judd (1998) discusses numerical approximation methods.

[8] We obtained our data from the Bid Results files available at the Bid Summary FTP Site of the California Department of Transportation Office Engineer internet site: http://tresc.dot.ca.gov/office/engineer.

1996 through October 30, 1998. During the latter period, Caltrans advertised 2083 projects from which 1850 were finally awarded and 233 cancelled or postponed.

The bid data contain the following information on every project awarded: Bid opening date; Contract number; Location; Number of Working Days and the Engineers' Estimate. Additionally, the data provides the Name, the Address, the Amount of the Bid and the Rank of the Bid for each of the bidding firms. In order to obtain a measure of past performance, we complement the bid data with the Caltrans Contract Awards database. This source contains information on contracts awarded between December 1994 to October 1998. It provides the dollar amount of the contract, the name and phone number of the contractor, the location and the completion time of the work.

Contracts are awarded by the California Department of Transportation subject to Federal Acquisition Regulations[9] and, therefore, is very similar to other states' procedures.[10] The process can be described in three steps: First, the Caltrans' Headquarters Office Engineer announces a project that is going to be let and the invitation to submit bids starts. This period is called the Advertising period and ranges between 4 and 10 weeks, depending on the size or complexity of the project. Occasionally, the Advertising period will be reduced to expedite project scheduling. Second, potential bidders may collect bid proposals that explain the plans and specifications of the work required.[11] Based on the proposal bidders may submit a sealed bid. For each bid, Caltrans checks that the bidding firm is among the firms that are qualified to do business with Caltrans[12]. Third, on the letting day, the bids are unsealed and ranked. The project is awarded to the lowest bidder provided that the required responsibility criteria are fullfilled[13]. After each letting, a list of all bids and their rankings is announced and made accessible to the public.

Between May 1st of 1996 and October 30th of 1998, the Caltrans awarded 1850 contracts. The total value of the contracts was $3834.35 million. A total of 9679 bids were received for these projects. According to Table 1, on average

[9] See the Project Submission and Estimate Guide at the Caltrans Office Engineer site, the Federal Acquisition Regulations and the Transport Acquisition Manual at the Department of Transportation Site.

[10] See Porter and Zona (1993) for a detailed explanation of New York State Department of Transportation, for instance.

[11] Project's characteristics, terms and identification number.

[12] Prior to the bidding, potential bidders have to qualify for contractual work for the Department of Transportation and are required deposit a predetermined amount of funds that have to be available. Violation of the contract specifications and delivery scheduling can cause the rejection of a submitted bid.

[13] The winning bid is compared to the Caltrans engineers' estimate. The bid is accepted if all computations and cost imputations are considered correct. The winning firm is awarded the project no more than 30 days after the letting date.

Table 1
Descriptive statistics of selected variables

	Number of observations	Mean	Standard deviation	Minimum	Maximum
Number of bids per contract	1850	4.96	2.15	1.00	9.00
Backlog[a]	55 770	0.32	0.29	0.00	1.00
Distance to the project	55 770	226	11 071	0	722 335
Estimate[b]	1850	13.39	1.35	9.47	18.31
(Ranked 1[c] – Estimate)/Estimate	1850	– 0.03	0.23	– 0.79	1.34
(Ranked 2[c] – Ranked 1)/Ranked 1	1800	0.10	0.18	0.00	5.08

[a] Backlog measures the amount of uncompleted work from previously won contracts.
[b] Logarithm of the estimate.
[c] Ranked 1 and Ranked 2 are the winning bid and the bid ranked in second position, respectively.

Table 2
Probit estimates of the bid submission decision[a]

Number of observations	68913	68913	68913
χ^2	3017.24	2952.74	699.81
Degrees of freedom	9	7	4
Pseudo R^2	0.124	0.121	0.025
Log likelihood	– 10977.93	– 11013.33	– 12213.92
Variable			
Reserve price	– 0.1913	– 0.1966	– 0.2088
	(0.042)	(0.042)	(0.040)
Estimate	0.3819	0.3864	0.3726
	(0.040)	(0.040)	(0.038)
Working days	– 0.1631	– 0.1649	– 0.1256
	(0.011)	(0.011)	(0.010)
Nbid-fringe	– 0.0514	– 0.0504	– 0.0465
	(0.017)	(0.017)	(0.016)
Distance	– 0.1698	– 0.1694	
	(0.013)	(0.013)	
Number of wins	0.0148	0.0148	
	(0.001)	(0.001)	
Number of wins in the region	0.1806	0.1793	
	(0.005)	(0.005)	
Backlog	– 0.0824		
	(0.009)		
Backlog within the region	0.0448		
	(0.014)		
Constant	– 3.3298	– 3.3065	– 3.2593
	(0.114)	(0.114)	(1.021)

[a] Reserve price, estimate, working days are in logarithm. The numbers in parentheses are robust standard deviations.

there were 5.23 bidders per contract, ranging from 1 to 24 bidders across projects. A total of 50 projects received one bid, 191 contracts received two, 288 contracts received three bidders and so on.

Table 2 reports the probit estimates of the decision to submit a bid. Explanatory variables include contract specific characteristics such as the estimate and the number of working days, and, bidder specific characteristics such as the bidder's maximum capacity, the distance of the bidder's closest plant to the project location. On average, a constrained bidder is 50% less likely to submit a bid than an unconstrained bidder. Distance to the project decreases the probability to submit a bid at an increasing rate. As the size, or estimate, of the project increases the probability of submitting a bid intially decreases and then increases at an increasing rate. The number of working days of the project have a negative effect.

5. Estimation results

This section describes the estimation results of the bidding model. First, we report estimates of the bid distribution. Second, we describe estimates of the value function. Third, we consider the condition of identification. Finally, we report cost estimates for one bidder.

The first problem when estimating the model is to choose a suitable bandwidth. On the one hand, the bandwidth must be large enough to produce a smooth curve, while on the other hand being small enough not too produce to much bias. In the calculations, we select the bandwidth using the ocular method.

Fig. 1 plots the estimates of the distribution of bids. Estimates are reported for backlog equalling 0.1, 0.05 and 0.9. Other state variables are fixed at their sample average values. From the figure, we see that an average regular bidder submits a bid on between 2 and 5% of all contracts. The participation in an auction is more likely with low backlog levels.

The stochastic dominance property of the distribution functions for different backlog levels in Fig. 1 is evident. Fig. 1 illustrates this property for fixed levels of the backlog variable.

Fig. 2 plots the estimates of the probability of submitting a bid as a function of the backlog and holding other variables constant at their sample average. The figure suggests that the probability of submitting a bid declines monotonically in backlog: When backlog equals 0, the probability equals 6%; it falls to 5% as backlog increases to 0.5; and when backlog exceeds 0.7 the probability of submitting a bid declines to 2%. Although the relationship is monotone at sample average values, there are instances where the relationship is not monotone.

Fig. 3 depicts estimates of the value function evaluated as a function of the backlog and holding other variables constant at the sample average state

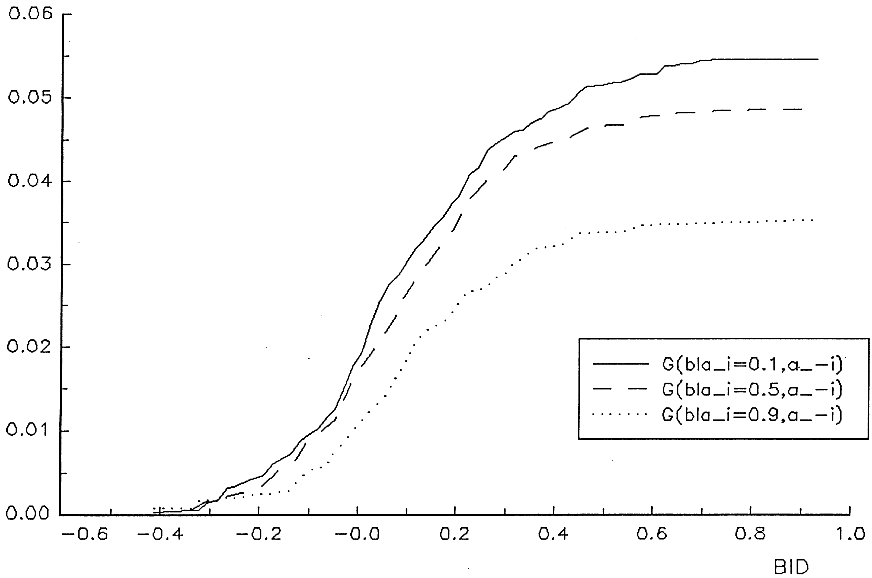

Fig. 1. Bid distribution at average values.

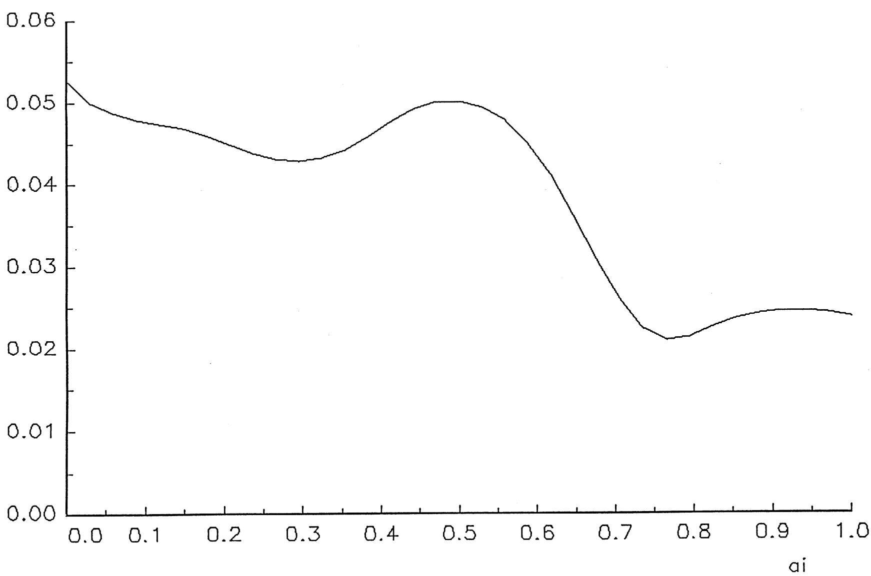

Fig. 2. Probability of submitting a bid as a function of a_i.

variables for bidder 1. The plus signs indicate confidence intervals at the approximation points of the value function. The confidence intervals account for the approximation error due to the numeric calculation of the value function. The value function is calculated assuming a 0.95 annual discount factor which corresponds to a 5% interest rate.

Fig. 3 indicates that the value function is almost flat over the range of the backlog variable. Thus, there is little effect of winning a large contract today on the future discounted profitability of the firm. A reason may be that with a discount factor of about 0.95, the duration of highway paving contracts is too short to have an effect on the long run profitability of the firm. In the most extreme case, in which a contract commits all resources for one year, we would expect at most an effect of 5% on the value function. Thus, the magnitude of this effect may be too small to affect future profits.

Inference of unknown costs from observed bids is based on a monotone relationship between the inverse bidding function and the cost. Next, we examine whether this assumption is satisfied in the data. Fig. 4 depicts the relationship between bids and estimated costs for bidder 1. For comparison the figure also plots the 45° line. Fig. 4 shows that there is a monotone relationship. Thus, for bidder 1, the inference is valid.

Based on Fig. 4, it is difficult to assess whether the difference between bids and costs is decreasing or increasing as bids increase. An examination of the relative

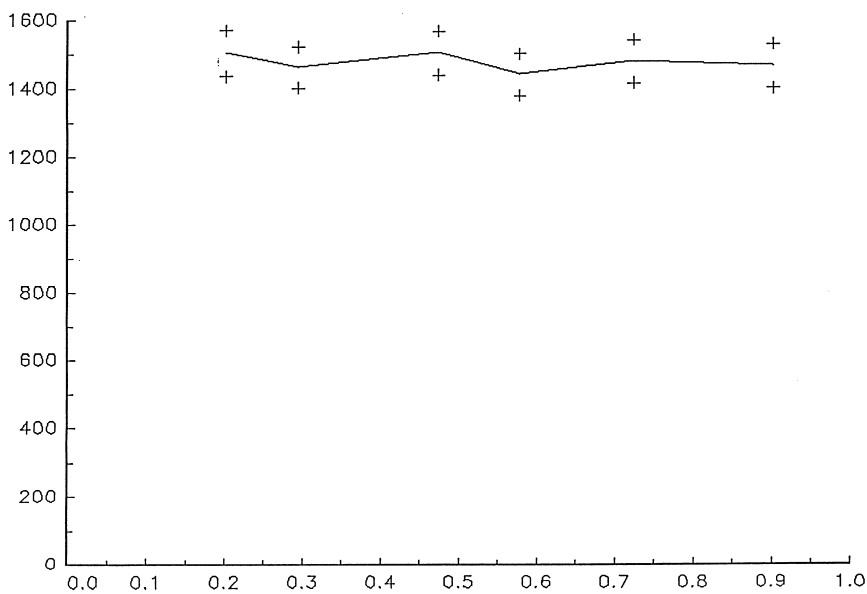

Fig. 3. Value function for bidder 1.

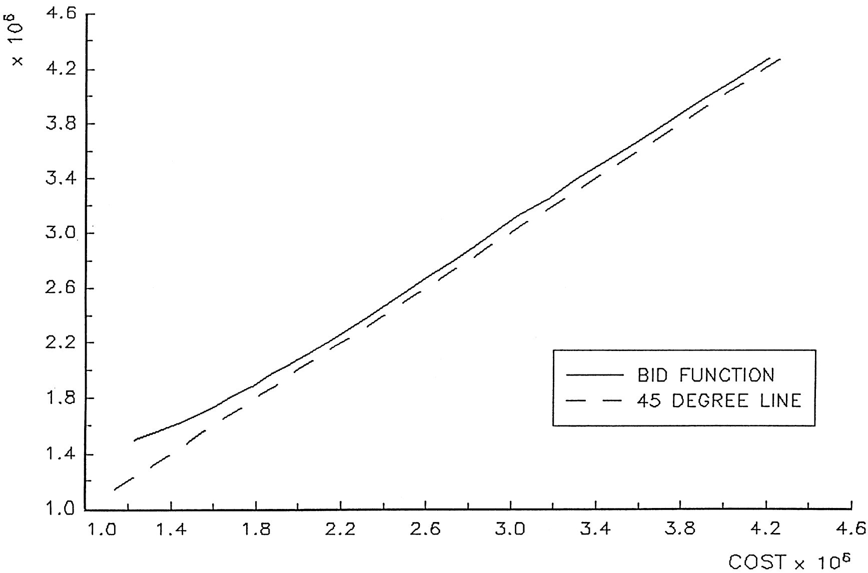

Fig. 4. Bid function at average values for bidder 1.

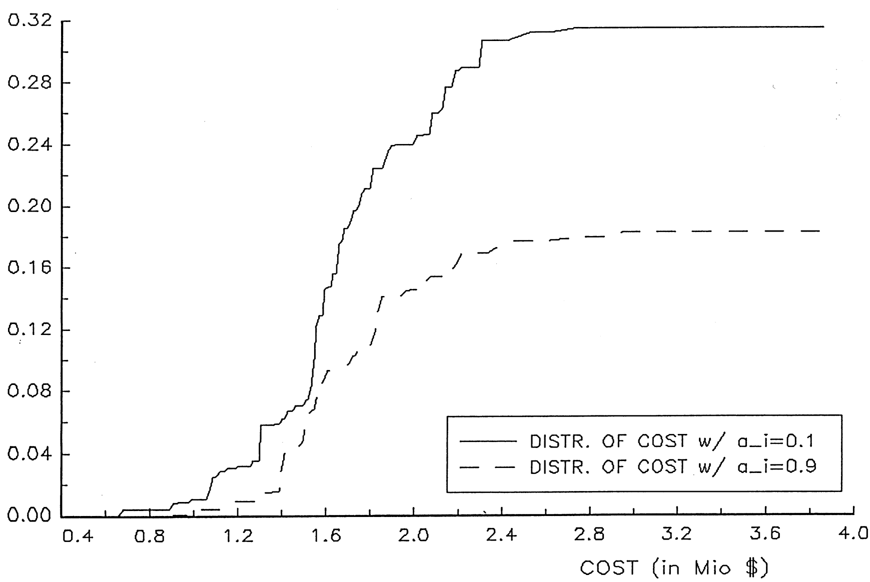

Fig. 5. Distribution of costs at average values for bidder 1.

mark-up of bidder 1 which is defined as the bid minus the cost divided by the bid reveals that the mark-up is declining in costs.

Fig. 5 illustrates the distribution of costs for bidder 1. Estimates are reported for backlog equaling 0.1 and 0.9. Other variables are fixed at sample average values. The cost distribution exhibits the stochastic dominance property. The average cost is lower with a low than with a high level of backlog. This suggests that capacity constraint play an important role in highway procurement contracts.

6. Conclusion

This paper presents a structural model of intertemporal effects in bidding. We estimate the model and find that the data confirm the intuition: Bidding behavior is affected by capacity constraints. Bidders with a high backlog have on average a higher cost than bidders with a low backlog.

Further research will quantify the inefficiency of the first price auction rule induced by the asymmetry stemming from backlog. In addition, we intend to evaluate alternative auction rules, such as second price auctions and/or different bid opening time profiles in terms of efficiency and cost for the department of transportation.

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