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A Study of Collusion in First-Price Auctions

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This paper examines the bidding for school milk contracts in Florida and Texas during the 1980s. In both states firms were convicted of bid-rigging. The data and legal evidence suggest that the cartels in the two states allocate contracts in different ways: One cartel divides the market among members, while the other cartel also uses side payments to compensate members for refraining from bidding. We show that both forms of cartel agreements are almost optimal, provided the number of contracts is sufficiently large.

In the auction the cartel bidder may face competition from non-cartel bidders. The presence of an optimal cartel induces an asymmetry in the auction. The selected cartel bidder is bidding as a representative of a group and has on average a lower cost than a non-cartel bidder. The data support the predicted equilibrium bidding behaviour in asymmetric auctions in accordance with optimal cartels.

1. INTRODUCTION

Bid rigging is a pervasive problem. Recent indictments in the U.S. have been brought against firms in the construction industry, real estate, antiques, utility procurement, and milk industry. This study considers the school milk market as a case study of collusive behaviour in auctions. Between May 1988 and September 1993, the Antitrust Division of the U.S. Department of Justice filed 106 cases in 14 states involving bid rigging in procurement auctions for the supply of milk to schools. Forty-five corporations and 49 individuals have been convicted for a total of $46 million in fines and an average of 180 days in prison for 26 persons.[1]

This paper examines the Florida and Texas school milk cartels from 1980 to 1991 and studies the organization of cartel agreements and the implications of the presence of a cartel on the bidding behaviour. There are several advantages in focusing on this narrow market. First, in both states, firms were convicted for bid-rigging and the operations and bidding practices of the cartels are well documented. Second, the data set collected by the Antitrust Division of the Department of Justice is quite detailed and permits the study of strategic behaviour of the firms involved. Finally, the school milk market is split into many contracts offered for sale annually. As a result, the interactions of firms occur on a frequent and a regular basis.

The empirical analysis suggests the following: The cartels in the two states allocate contracts in different ways. One cartel coordinates the behaviour of members by dividing the market among cartel members, while the other cartel also uses side payments to compensate cartel members for refraining from bidding or for submitting phony bids. The

1. Report on the Criminal Cases filed by the Antitrust Division of the U.S. Department of Justice involving the Milk Industry, 1993, in reply to the Freedom of Information Act request to the Antitrust Division of the Department of Justice from July 23, 1993, reference number AT 93-225.

two schemes can be modelled as collusive behaviour with and without side payments. It is shown that, if the number of contracts is sufficiently large, then both forms of cartel agreements almost maximize expected cartel payoffs. The bidding data are studied to assess the optimality of cartels. If cartel firms and non-cartel firms are identical, and the cartel selects the lowest cost firm among cartel members, then there will be an asymmetry between the selected cartel bidder and a non-cartel firm. The cartel bidder will have a lower cost on average. In addition, there may exist *ex ante* asymmetries between cartel and non-cartel firms. The bidding evidence in Florida and Texas is in accordance with asymmetric bidding.

In contrast to the empirical literature this paper does not seek to prove collusion, but rather presumes that the accusations of bid rigging were correct. The presumption is supported by results of statistical tests aimed at detecting the presence of collusion.[2] In addition, in Florida and also in Texas all accused firms pleaded guilty. The paper examines the behaviour of firms who were later convicted of collusion.

The analysis is theoretical and empirical, and focuses on two basic issues of bidder collusion: First, how does a cartel organize the collusive scheme? Second, how does the presence of a collusive scheme affect the bidding behaviour? The theoretical section examines the effectiveness of cartel schemes in the school milk market and provides implications on the cartel bidding behaviour. The empirical section assesses the relevance of the implications and provides a description of the collusive practices in Florida and Texas. Occasionally, the description of the Florida and Texas cartel may overburden the reader with institutional details.

In Section 3, we consider a theoretical model of cartel behaviour in a First Price procurement auction in which a buyer offers multiple contracts for sale. It is assumed that the bidders' (sellers') costs are privately known. Two forms of cartel agreements are distinguished: A strong cartel is one that can make side payments, while a weak cartel cannot.[3] A cartel is efficient if the sole cartel bidder is the lowest cost cartel firm. We construct a mechanism, which we call Ranking Mechanism, and show that it converges to the optimal outcome, as the number of contracts increases. The Ranking Mechanism can be implemented by a simple vote among cartel members prior to the auction in which bidders rank contracts according to their preferences. The feasibility of side payments affects the outcome of the collusive arrangement. The Ranking Mechanism induces a smaller variance in market shares than the efficient mechanism with side payments. Without side payments a cartel has to give every member a sufficiently large share of contracts to keep members satisfied with the agreements. A cartel with side payments need not, since it can compensate its members with side payments instead.

Section 5 describes the allocation of contracts to members in Florida and Texas. The data reveal that Florida market shares of cartel firms fluctuate substantially, while Texas market shares are almost constant over time. The conjecture that the Florida cartel uses side payments is substantiated by legal evidence. In Texas there is no legal evidence of side payments. Both cartels may be (almost) optimal, since the market is split into many contracts. In Texas more than a hundred contracts are awarded every year.

2. Hewitt, McClave and Sibley (1993) perform statistical tests to detect the presence of a cartel in the Texas school milk data. They examine the determinants of bids and document evidence for collusive behaviour in Texas. In Pesendorfer (1995) tests for the presence of a cartel are formulated based on the bid submission decision. The test results also confirm the presence of collusion in both, Florida and Texas.

3. This distinction is familiar to readers of the literature on collusion in Bertrand or Cournot games. For example Cramton and Palfrey (1990) examine cartel behaviour in Cournot/Bertrand games with side payments, while Roberts (1985) considers cartel behaviour without side payments.

In the buyer's auction the cartel bidder may face competition from non-cartel bidders. The bidding data are studied to assess the optimality of cartels. As described above, if cartel firms and non-cartel firms are identical and the cartel selects the lowest-cost firm, then there will be an induced asymmetry between the selected cartel bidder and a non-cartel firm. In addition, there may exist *ex ante* asymmetries. We assume that non-cartel firms are identical and examine equilibrium bidding behaviour in single object First-Price asymmetric auctions. The bidding equilibrium has two testable implications, which have originally been shown by Maskin and Riley (1993) in the context of two-bidder asymmetric auctions. First, the equilibrium strategy of the selected cartel bidder is to shade its bid up by more than do non-cartel bidders. Second, the *ex ante* bid distribution of the cartel is first-order stochastically dominated by the *ex ante* bid distribution of non-cartel bidders. The implications rely on the assumption that the cartel is efficient. Alternatively, if the cartel cannot select the lowest-cost firm, then the asymmetry need not arise and the predictions need not hold.

Section 6 presents the evidence on the presence of asymmetries between cartel and non-cartel bids. We report two results: First, an examination of the determinants of bids reveals that cartel and non-cartel bidding rules differ significantly. Second, the empirical distribution of cartel bids is first-order stochastically dominated by the empirical distribution of non-cartel bids. These two findings confirm the predictions of the bidding model with cost asymmetries between the selected cartel bidder and non-cartel firms. The results are reinforced by the fact that the evidence in Florida and Texas is similar, which is in accordance with the predicted outcome for optimal cartels.

The paper is organized as follows. The next section discusses the related literature. Section 3 considers a theoretical model of cartel behaviour and gives testable predictions. Section 4 describes the school milk market and the data and presents the features of the market that may encourage collusion. Section 5 examines the allocation of market shares among cartel firms. Section 6 characterizes the bidding behaviour of cartel and non-cartel firms and examines the presence of asymmetries. Section 7 gives conclusions.

2. RELATED LITERATURE

The empirical literature focuses on tests to detect the presence of collusion. Porter and Zona (1993, 1997), Zona (1986) and Baldwin, Marshall and Richard (1997) propose tests to detect collusion in auction markets. Porter and Zona (1993) devise a test based on the rank distribution of cartel and non-cartel bids. They apply the test to Long Island highway construction data and reject the hypothesis that there was no collusion. Baldwin, Marshall and Richard (1997) study forest timber sales and estimate structural models of bidder collusion and competitive behaviour. They compare the performance of these models and find that the collusive model performs better than the competitive model. Porter and Zona (1997) test for the presence of collusion in the Ohio school milk market and estimate its cost. Ways of calculating damages in bid-rigging cases are also proposed in Howard and Kasermann (1989) and Nelson (1993). McMillan (1991) describes the negotiation among bidders for Japanese public works contracts and estimates the cost of this collusive scheme to Japan's taxpayers.

The Texas school milk data have already been examined by Hewitt, McClave and Sibley (1993). They document that the supplier does not change from year to year on many occasions. They perform a number of tests to explain the high incumbency factor. Their first hypothesis is that incumbent firms have lower delivery costs due to their plant location, then second, that the Board of Education favours certain firms. Third, that firms collude. They conduct a regression based analysis in which they compare the bidding behaviour of firms in incumbent school districts and non-incumbent school districts. Hewitt, McClave and Sibley reject the first two hypotheses and conclude that the high incumbency factor constitutes evidence of collusion. They also provide loss estimates for the Board of Education.

The theoretical literature focuses on single object auctions. The efficiency properties of cartel agreements are studied by Graham and Marshall (1987), Mailath and Zemsky (1991) and McAfee and McMillan (1992). Graham and Marshall show that with side payments a cartel can achieve efficiency. McAfee and McMillan show that without side payments, the best a cartel can do is to randomly select a cartel member (or to let cartel members submit identical bids). Their argument relies on the assumption that there is only one object for sale. We shall explain in the next section why this scheme is not optimal for multiple objects.

3. THEORY

This section examines a theoretical model of cartel behaviour and describes implications for the school milk market. In Section 3.1 optimal cartel agreements for all-inclusive cartels with and without side payments are characterized. It is established that without side payments, cartel agreements achieve almost the maximum expected payoff per contract, if the number of contracts offered for sale is sufficiently large. In Section 3.2 the assumption that the cartel includes all bidders is relaxed and implications of the bidding equilibrium in the case of one large bidder (the cartel) and a number of identical small (non-cartel) bidders are given.

The game considered is static and there will be an individual incentive to cheat on the agreement. Compliance with the cartel mechanism can come from an infinitely repeated version of the game: Cooperative payoffs will be sustainable in equilibrium if the discount factor is sufficiently large. A possible enforcement strategy is a grim trigger strategy in which the cartel threatens deviant behaviour with competitive bidding behaviour in the future. However, we do not model dynamics explicitly and do assume that the cartel agreement is binding.

The underlying model of this section is a First-Price procurement auction in which a buyer simultaneously offers m contracts for sale. The set of possible bidders (sellers) is denoted by $N = \{1, \ldots, n\}$. Bidder i's costs for the contracts are known privately and given by $c_i = (c_i^1, \ldots, c_i^m)$, where c_i^j denotes i's cost for contract j. The priors of the other bidders and the buyer about c_i^j are identical and represented by the distribution function F with continuous density f and support $[0, C]$. So bidder i's costs $c_i = (c_i^1, \ldots, c_i^m)$, are m independent draws from the distribution function F. The bidders submit a bid for each contract which is a price at which they are willing to provide the service. The bidder with the lowest bid wins the contract and receives his bid. All agents are risk neutral. Ties are resolved by the flip of a coin. The buyer imposes a fixed (non random) reserve price R, with $R \leqq C$, meaning that bids above R are rejected.

3.1. *Cartel agreements*

The problem facing a ring of colluding bidders is how to select a sole bidder for any given contract. Rather than explicitly modelling a particular selection process, we study direct revelation mechanisms in which the sole cartel bidder is determined as a function of bidders' reported costs, $\hat{c} = (\hat{c}_1, \ldots, \hat{c}_n)$. It is assumed that all bidders are colluding. The cartel members choose a direct mechanism to maximize *ex ante* expected payoffs to cartel members. Two forms of cartels are considered. A strong cartel can make side payments and a weak cartel cannot. A cartel mechanism is efficient if it designates the member with the lowest cost to submit the lowest cartel bid.

We consider first a cartel in which cartel members are unable to make side payments to each other. This restriction is left unexplained, but one reason might be that side payments increase the risk of prosecution by antitrust authorities.

A (direct) cartel mechanism is a tuple $((q_i^j)_{j=1}^m)_{i=1}^n$, where $q_i^j: [0, C]^{n \cdot m} \to [0, 1]$ is the probability that cartel member i is the sole bidder on contract j in the buyers auction. Each bidder sends a (possibly untrue) report $\hat{c}_i$ of his cost to the mechanism. All bidders make their reports simultaneously. By the revelation principle the restriction to direct revelation mechanisms is without loss of generality: any Nash equilibrium outcome of any game will also be a Nash equilibrium outcome of some direct revelation game in which the bidders report their costs truthfully.

The gains for bidder i of being designated the sole cartel bidder on contract j are $R - c_i^j$. Let $U_i(c_i, \hat{c}_i)$ denote the expected payoff to cartel member i with costs c_i and reports $\hat{c}_i$. It is given by

$$U_i(c_i, \hat{c}_i) = \sum_{j=1}^m E_{-i}[R - c_i^j] q_i^j(\hat{c}_i, c_{-i}),$$

where c_{-i} is the vector of realized costs by all other bidders $(c_1, \ldots, c_{i-1}, c_{i+1}, \ldots, c_n)$ and $E_{-i}[\cdot]$ is the expectation operator with respect to the distribution of c_{-i}. In order for truth telling to be a Nash equilibrium, the following incentive constraint must be satisfied,

$$U_i(c_i, c_i) \geqq U_i(c_i, \hat{c}_i) \qquad \forall \hat{c}_i \in [0, C]^m. \tag{IC}$$

The following remark establishes an implication of the incentive constraint. Incentive compatible mechanisms cannot achieve efficiency in the absence of side payments. All proofs are given in the Appendix.

Remark 1. *For a finite number of contracts m, any incentive compatible mechanism without side payments is not efficient.*

The reason for the efficiency loss is the adverse selection problem. In order to induce truthful reports, the cartel mechanism has to give each member a sufficiently large winning probability independent of the reports. This restriction makes it impossible for weak cartels to achieve efficiency.[4]

Next, it is shown that with more than one contract, weak cartels can do better. McAfee (1992) obtains a bound for the efficiency loss in a related game. He considers a game of dissolving a partnership in which two agents alternate in picking objects. He shows that the total efficiency loss in this game is bounded regardless of the number of objects to be allocated. The game considered by McAfee induces an asymmetry between agents. The second mover does worse. In the context of collusion, asymmetric agreements appear difficult to implement and we restrict attention to symmetric agreements.

Consider the following symmetric mechanism, which we call the Ranking Mechanism: Each cartel member announces a ranking of the contracts according to his costs. The member who ranks a contract highest (assigns the highest preference) will be the sole

4. Specifically, the incentive constraint implies that the expected winning probability is constant for costs on rays through the point $(R, R, \ldots, R)$ in $\mathfrak{R}^m$. An extreme case occurs if there is only a single contract. The incentive constraint for a single contract implies that a weak cartel has to give the contract to each member with a constant probability.

cartel bidder for that contract. In the case of several cartel members ranking a contract at the same position, each of these members will be the sole bidder with equal probability (by virtue of a coin toss).

The Ranking Mechanism satisfies the incentive constraint. To see this, consider a particular bidder and suppose that everybody else submits truthful reports. Notice that the bidder may increase the winning probability of a contract by announcing a high rather than a low position in the ranking. Thus, payoff maximization is achieved by assigning the highest valued contract to the highest available position in the ranking which yields truthful cost reports.[5]

Theorem 1 shows that as the number of contracts increases, the *ex ante* expected payoff under the Ranking Mechanism converges to the maximum *ex ante* expected payoff per contract. With many contracts weak cartels are almost efficient.

Theorem 1. Suppose $R=C$. *As* $m\rightarrow\infty$ *the ex ante expected payoff per contract under the Ranking Mechanism converges to the maximum ex ante expected payoffs per contract.*

The announced position in the ranking determines a position in the joint distribution of costs. An estimator of the cost of a bidder is obtained by evaluating the inverse of the joint distribution at the point corresponding to the position in the ranking. As the number of contracts increases, the ranking determines the position in the joint distribution more precisely and the estimator becomes more accurate. With many contracts, the Ranking Mechanism awards contracts to the bidder with almost the lowest cost and, thus, performs almost as well as an efficient mechanism.

The assumption of symmetric cost distribution functions is not important in Theorem 1. An asymmetry may arise across contracts and/or bidders. With asymmetric distribution functions a cost estimator of a contract can also be determined from the announced position in the joint distribution. The accuracy of the cost estimator improves, as the number of contracts increases, which is a general property of order statistics. Under asymmetry it is no longer optimal to treat all bidders (or contracts) as symmetric. Instead, bidders with lower inferred costs are favoured. Theorem 1 assumes that costs are independently distributed. With costs correlated across contracts the result need not hold. Consider the extreme case of perfect correlation, which means that a bidder's cost realizations are the same across contracts. In this case, the allocation problem reduces to a problem with a single contract and it is optimal to randomize across bidders, which is inefficient.

As we describe in more detail in Section 5, two features of the Texas evidence suggests similarities to a cartel scheme that does not use side payments. First, a substantial fraction of cartel contracts stay with the owner from the preceding year. Second, every year a small fraction of contracts change ownership among cartel firms with trade occurring almost on a contract for contract basis leaving overall market shares roughly constant. Both features do not require side payments. In addition, the trading of contracts among Texas cartel members suggests attempts to improve efficiency. Unfortunately, we do not know the rules of the Texas trading game and it is difficult to infer the magnitude of

5. Truthful reports are the unique equilibrium in the Ranking game with two contracts and two bidders. However, when the number of bidders exceeds the number of contracts it need not be the only equilibrium. An example occurs with four bidders and two contracts: Two bidders announce a preference for the first contract and two bidders announce a preference for the second contract. This strategy profile constitutes an equilibrium provided the difference between cost realizations is small. The reason is that a deviation reduces the probability of winning from $\frac{1}{2}$ to $\frac{1}{3}$. For small differences between cost realizations, a deviation will, therefore, result in a loss. Although, there may exist other equilibria, the subsequent analysis considers only the equilibrium involving truthful reports.

efficiency improvements from observed outcomes of trade. However, Theorem 1 indicates that an almost efficient allocation of contracts is achievable with many contracts. In Texas there are 136 contracts awarded every year which may enable the cartel to be almost efficient.

Graham and Marshall (1987), Mailath and Zemsky (1991) and McAfee and McMillan (1992) consider a cartel agreement in which side payments among its members are allowed. We refer to these as strong cartels. They establish that a strong cartel can achieve efficiency. McAfee and McMillan give a mechanism which can be implemented as a first price auction held among cartel members for the right to bid in the buyer's auction. The highest bid of this auction is equally split among the losers. Clearly this result also applies to multiple contracts. For example, the cartel could use the same mechanism for each contract separately.

These results and Remark 1 imply that a strong cartel can solve the asymmetric information problem more efficiently than a weak cartel. However, if there are many contracts offered, the difference in performance between weak and strong cartels is small, by Theorem 1. When deciding whether to use side payments or not, a cartel has to weigh the additional efficiency gain with the negative effect of side payments. Side payments increase the risk of prosecution by antitrust authorities. This increase in the probability of being detected reduces the expected collusive gains and may outweigh efficiency gains. With many contracts it can be optimal for a cartel not to use side payments.

A difference in the behaviour between a weak and a strong cartel is manifested in the market share of each member. The following result establishes that the efficient mechanism with side payments induces a larger variance in the number of contracts per member than the Ranking Mechanism. Although we did not characterize optimal mechanisms for weak cartels in general, we use the Ranking Mechanism to establish Remark 2, since it is optimal in the limit, as the number of contracts increases.

Remark 2. Let y_m *be the number of contracts received by bidder j under the Ranking Mechanism with m contracts and let* z_m *be the number of contracts received by bidder j under the efficient mechanism with side payments with mcontracts. The following property holds*:

$$\lim_{m\to\infty} \frac{\text{Var}\,(z_m)}{\text{Var}\,(y_m)} \geqq 2 - \frac{1}{n}.$$

The result characterizes a property of the ratio of variances. In the limit, the variance of the efficient mechanism with side payments is at least $(2-1/n)$ times larger than the variance of the Ranking Mechanism. The intuition for Remark 2 is that a weak cartel has to give each member a sufficiently large share of contracts independent of the realization of costs in order to keep members satisfied with the agreement. Strong cartels need not, since they can give side payments instead.

3.2. *Cartel bidding behaviour*

This section considers the situation when the cartel bidder faces competition from non-cartel firms and explains properties of the bidding equilibrium. Although the results in Section 3.1 can be extended to the case where not all bidders are colluding,[6] in this section

6. Specifically, in Theorem 1 the payoff of the selected cartel bidder which equals the reserve price is replaced with the expected payoff of the cartel bidder in the procurement auction. With sufficiently many contracts a weak cartel selects the member with almost the lowest cost among the group of cartel members, which has implications on the cost distribution of the cartel bidder.

we do not require that the cartel bidder is the lowest cost bidder among the group of cartel bidders, but use a weaker condition that also includes weak cartels.

Let 1 denote the cartel bidder with prior distribution F_1 and let $i = 2, \ldots, n$ be identical non-cartel (small) bidders with prior distribution $F_i = F$. Let $H_i(c) = f_i(c)/(1 - F_i(c))$ denote the hazard rate. We assume that $H_1(c) > H_i(c)$ for all c and $i = 2, \ldots, n$, that is the large bidder has a higher probability of having a low cost conditional on costs being above c than other bidders.

The hazard rate condition is satisfied if cartel and non-cartel firms are *ex ante* symmetric and the cartel is optimal. In this case the cost distribution of the cartel bidder is the first order statistic of k random variables with distribution function F, which is given by $1 - [1 - F(c)]^k$. The associated hazard rate of the cartel bidder equals $kf(c)/(1 - F(c)) = kH(c)$ and is larger than $H(c)$, the hazard rate of any non-cartel bidder. The hazard rate assumption is not satisfied if bidders are *ex ante* symmetric and the cartel randomly chooses a representative bidder. In this case there are no differences between cartel and non-cartel bidders. The cost distributions of the selected cartel bidder and a non-cartel bidder are identical.

To simplify notation it is assumed that there is only one contract for sale. This restriction is without loss of generality since costs are independently distributed across contracts.[7] Maskin and Riley (1992) establish the existence of equilibria in asymmetric auctions. In Pesendorfer (1995) it is established that under the above hazard rate condition, the bidding equilibrium is also unique. Remark 3 illustrates two properties of the bidding equilibrium that are useful for empirical purposes. These properties have originally been shown by Maskin and Riley (1993) and Waehrer (1999).

Remark 3. *The following properties hold in equilibrium for* $i = 2, \ldots, n$:

(i) $b_1(c) > b_i(c)$ *for all* $c \in (0, R)$;
(ii) $Pr(b_1(c) \leqq b) > Pr(b_i(c) \leqq b)$ *for all* $b \in (b_0, R)$.

The first property states that the small bidders bid identically, and more aggressively than the large bidder. The second property establishes that the *ex ante* bid distribution of any small bidder stochastically dominates the *ex ante* bid distribution of the large bidder.

The properties in Remark 3 can be used to examine the behaviour of a cartel based on bidding data. The predictions rely on the hazard rate assumption which requires that the cartel bidder has on average a lower cost than a non-cartel bidder. Alternatively, if the cartel is inefficient, then the predictions need not hold. For example, if bidders are *ex ante* symmetric and the cartel randomly chooses a representative bidder, then the cost distributions of the cartel and a non-cartel bidder are identical. In this case, cartel and non-cartel firms use the same bidding function. Thus, the predictions in Remark 3 enable a researcher to assess whether the cartel scheme is more efficient (in the sense of the hazard rate condition) than a random selection rule.

Selecting a cartel bidder randomly is not efficient, but may still enable substantial collusive gains for at least two reasons: First, it reduces the number of potential competitors in the procurement auction. Second, it reduces coordination efforts since it does not require communication about costs. McAfee and McMillan (1992) describe a number of cartels that used random selection rules.

7. An equilibrium of the single object auction is equivalent to an equilibrium of the multiple objects auction. This equivalence will not hold if there are bidding costs or firms are capacity constrained.

4. THE MARKET

This section describes the school milk market and the data. School milk contracts are awarded in a First-Price Sealed-Bid auction. In about April or May the Board of Education of the school district publicly announces a detailed description of upcoming contracts, containing a list of schools to be supplied, the contract period, the delivery times and estimated quantities for individual milk products. The potential vendors have approximately one month to submit their bids. All bidders have to sign a Non-Collusive Affidavit, which states that the undersigned bidder did not enter any agreement or understanding on either participation or price with any other person, and that the bidder did not and will not give or receive side payments. In May or June, on the day of the letting, sealed bids are opened and the identities of all bidders and the amounts of their bids are announced to those present. The lowest bid is accepted.

Using this auction mechanism, boards of education awarded about $200 million in school milk contracts from 1980 until 1989 in Florida and approximately $130 million in the Dallas–Fort Worth area.

According to a list of criminal cases filed by the Antitrust Division involving the milk industry, 25 bid-rigging cases for school milk were filed in Florida and 10 in Texas, where two are still pending. An indictment addresses an individual firm or a person. Florida indictments indicate that collusion started as early as the late 1960s and took place in 38 counties in Peninsular Florida. In all Florida cases the defendant pleaded guilty and the ten firms accused of colluding in Florida paid a total of about $15 million (not counting civil cases) in fines. Fourteen persons were sentenced for a total of 7·3 years in prison. Similarly, Texas indictments suggest that collusion began at least as early as 1975. In Texas seven firms accused of cheating paid a total of about $8 million in fines, and two persons were sentenced to 182 days in jail each.

In this paper collusion is thought of as an explicit scheme or implicit scheme designed by cartel members to limit competition and increase their profits. Several features in the school milk market in Florida and Texas may encourage collusion:

The terms of the contract and in particular the quality and quantity of individual milk items delivered to schools are fixed. Firms compete only on price. A potential ring has to coordinate only in the price dimension in order to succeed.

The school milk market is split into many small contracts, enabling a cartel to split the spoils fairly evenly. In Florida, there are 239 contracts every year on average. The Dallas–Fort Worth data contain 136 contracts every year.

The Boards of Education in every school district act independently and award contracts considering only the bids submitted to their Board. The lack of coordination may not be in the best interest of the Boards of Education because it makes it difficult to detect collusion.

In a given year the letting dates within a state are not the same but vary across school districts. The time span for the lettings is April until the end of July. On the day of a letting the bids and the identities of all bidders are publicly announced, so cartel members can immediately detect deviations from an agreement. The potential gains from deviation are rather limited because a deviation can be punished at subsequent auction dates during the same year.

The demand for school milk is very inelastic. The milk consumption of pupils below the poverty level is subsidized by the federal school milk/lunch programme. The aggregate quantity consumed per school district did not change significantly over the sample period. It may be difficult to use substitute products for milk. As a result, any inflation of winning bids due to collusion can be captured as increased profits.

The regional school milk market is highly concentrated. According to the data, in Florida three firms won 62% and eight firms won 90% of the dollar value of all contracts (in constant 1982 dollars). In Texas three firms won 56% of all contracts. In Florida a total of 45 firms and in Texas a total of 36 firms submitted bids on at least one contract.

To summarize, the characteristics of the school milk market tend to facilitate collusion. The particular auction mechanism employed consists of sequential and independent sales of many small contracts, the short-run demand elasticity is close to zero, and the market of potential suppliers is highly concentrated.

The relevant aspects of the milk market are the following: The major input in the production of milk is raw milk. Milk processors purchase raw milk either directly from farmers or more commonly from cooperatives. The minimum price of raw milk is regulated by federal milk marketing orders. It equals the upper Midwest price plus a fixed differential typically increasing in distance from the upper Midwest. Generally the raw milk price is above the minimum price. A comparison of the raw milk price (in constant 1982 dollars) of the largest cooperatives in Miami and Dallas during the sample period reveals that the two prices follow a very similar pattern even though the raw milk price in Florida is about 18% higher. This difference can be attributed to the longer distance from the upper Midwest. The milk processors remove all butterfat from the raw milk, pasteurize it, and put it together in varying proportions as whole milk, low fat milk, flavoured milk and other products. The various products are then packaged and delivered. No federal orders restrict milk movements, but long delivery distances increase costs. According to a report of the Department of Agriculture (1974) examining 56 metropolitan areas between 1969 and 1971, 66% of the milk supplied to retail stores came from plants within 50 miles of the market, 91% of the milk came from plants within 150 miles and less than 1% came from plants more than 250 miles away. Clearly, milk might be shipped farther in rural areas and technological changes may have increased delivery distances since the year 1971.

During the sample period, retail prices[8] in Dallas are about 10% higher than in Miami. This fact is surprising since, as noted above, the raw milk price is much lower. A possible explanation is that in the Dallas area, plant locations are farther apart. The average distance between the winning firm's plant and the school district is 16·3 miles in Florida and 66·7 miles in Texas.[9] Longer delivery distances increase costs and as a result retail prices will be higher. Of course this could also be due to other cost differences in milk processing or stronger competition in Florida.

Milk delivery trucks typically supply retail stores,[10] schools and other facilities at the same time. The cost of a milk processor for a school milk contract will to some extent depend on what other stores or facilities the processor supplies in that area. It will also be affected by his current capacity and processing cost. These cost-determining factors

8. The data sources are explained in Appendix B.

9. The comparison of the distance measures between Florida and Texas has to be interpreted with caution. Florida contracts are awarded on the county level and Texas contracts are awarded in specific towns within the county. The distance measure for Florida is based on one point within the county (usually the largest city) and is a coarser measure than in Texas. The difference in the construction of the MILES variable between the Florida and Texas data may result in smaller distance estimates in Florida than in Texas.

10. Large supermarket chains own milk processing plants which deliver milk products exclusively to their stores.

are known to the milk processor but are too complex to be common knowledge among all firms. Uncertain factors that affect the costs of all firms symmetrically such as the future raw milk price are qualitatively less important. As pointed out above, the minimum raw milk price is regulated and thus, to some extent, predictable. In this sense, the informational environment in school milk auctions is best viewed as private values.

The data

This study looks at school milk contracts awarded in Florida and Dallas–Fort Worth[11] between 1980 and 1991. The data include a total of 4077 contracts sold during this period, with 2392 contracts in Florida and 1685 in Texas (177 contracts that were not awarded or that had more than one winner are excluded). Texas contracts are awarded on the school district level and there is typically one contract per school district. Florida contracts are awarded on the county level and there are typically several contracts per county. The individual milk items delivered to schools are whole white milk, low-fat white milk, skim white milk, whole chocolate, low-fat chocolate milk and buttermilk.

The data set contains the following information for each contract: the date of the letting, the county and the location within the county (school district or county region); the identity of the bidding firms and the value of their bids overall and by individual milk items; the estimated quantity of each item, dummy variables that indicate whether a bid was firm or escalated[12] and whether a contract was awarded or not; the county population; the county area in square miles; and a variable that measures the distance between a firm's plant and the school district. The Texas data also include the number of meals and a dummy variable that indicates whether a firm must supply a cooler. The sources for the data are described in Appendix B.

Firms are classified into ''cartel'' and ''non-cartel'' firms according to legal evidence. All 10 firms in Florida and all 7 firms in Texas accused of cheating pleaded guilty and for this study we presume that these allegations were correct. Cartel firms are labelled with 1, 2, . . . , 10 in Florida and 1, 2, . . . , 7 in Texas.

Table 1 lists some of the variables. All monetary variables are denominated in 1982 dollars. QUANT is the aggregate quantity measured in half pints. BID is the price offered for a contract and WINBID is the lowest price per contract. NOBID equals the number of bidders per contract, CARTEL NOBID is the number of cartel bidders per contract and NON-CARTEL NOBID is the number of non-cartel bidders per contract. MILES measures the distance between the school district and the plant of the firm. WONLAST is a dummy variable that equals one if a firm won the same contract in the preceding period. The variable SCHOOLLUNCH measures the percentage of students whose lunches are paid by the school lunch programme. It is used to control for the poverty level in school districts.

The following sections analyze 1351 contracts in Florida and 1176 contracts in Texas. The remaining contracts were eliminated due to missing observations or undefined variables. Specifically, in Florida 122 contracts awarded in North-East Florida are left out from the sample because the cartel was detected only in Peninsular and Panhandle Florida, not in North-East Florida. WONLAST is not defined on 471 contracts and QUANT is missing on 301 contracts. For 147 contracts the plant location of a firm is missing. In

11. The Texas data include a set of contracts for the San Antonio area. We exclude these from the analysis.

12. An escalation clause permits an increase in the price of the product based on market conditions. This clause is tied to specific market conditions, such as a change in the Class I raw milk price based on monthly Federal Milk Order.

TABLE 1

Definition of variables

QUANT:	the estimated aggregate quantity measured in half pints
WIN BID:	the winning bid in 1982 dollars divided by QUANT
BID:	the bid in 1982 dollars divided by QUANT
POP:	the school district population in thousands
SQMILE:	the school district area measured in square miles
MEALS:	the number of school lunches per school district
COOLER:	a dummy variable that equals one if a firm has to supply a cooler
ESC:	the escalation factor, a dummy variable that equals one if prices are allowed to increase or decrease with the raw milk price
RAWMPRICE:	the monthly raw milk price in Florida respectively Texas in 1982 dollars
NOBID:	the number of bidders per contract
CARTEL NOBID:	the number of cartel bidders per contract
NON CARTEL NOBID:	the number of non-cartel bidders per contract
ONEBID:	a dummy variable that equals one if there is only one non-cartel bidder or if there are only cartel bidders
CAPACITY:	the maximum total dollar value won by the firm in any year
MILES:	the distance between the school district and the closest plant of the firm in miles
UTIL:	a utilization rate that measures the fraction of the firms CAPACITY committed by the date of the letting
NOBACKLOG:	a dummy variable that equals one if a firm has not won a contract prior to the letting date in that year
WONLAST:	a dummy variable that equals one, if a firm has won the same contract in the preceding period
SCHOOLLUNCH:	the percentage of pupils who are subsidized by the school lunch programme

Texas, 55 contracts awarded before 1980 or after 1991 are deleted. WONLAST is not defined on 264 occasions, and QUANT or BID are missing for 190 contracts.

Table 2 provides summary statistics on selected variables. The table reveals that there are significantly fewer bidders per contract in Texas than in Florida (t-statistic 19·2). According to the Florida data, there are on average 3·66 bidders per contract, and in Texas there are on average 2·60 bidders per contract. In both Florida and Texas, the maximum number of bidders is 7. In Florida a total of 45 firms submitted at least one bid and at most 31 firms were active in any given year. In Texas, 36 firms submitted bids on at least one contract.

TABLE 2

Summary statistics of selected variables

	Florida			Texas		
Variable	Obs	Mean	Std	Obs	Mean	Std
NOBID	1351	3·66	1·46	1176	2·60	1·30
CARTEL NOBID	1351	3·08	1·44	1176	2·05	1·18
NON CARTEL NOBID	1351	0·58	0·63	1176	0·55	0·62
WIN BID[1]	1351	−2·03	0·13	1176	−1·99	0·15
WIN MILES[2]	1351	2·79	1·28	1176	4·20	0·74
QUANT[3]	1351	9·80	2·51	1176	12·19	1·41
WONLAST[4]	1351	0·41	0·49	1176	0·79	0·41

[1] Variable in logarithm. Dollar figures in 1982 dollars.
[2] Variable in logarithm. It measures the distance of the closest plant of the winning firm to the school district in miles.
[3] Variable in logarithm. It is measured in half pints.
[4] WONLAST is a dummy variable that equals one if a firm wins the same contract it has won in the preceding period.

In both Florida and Texas there are on average many more cartel bidders than non-cartel bidders per contract. In Florida the mean number of cartel bidders is 3·08 and the average number of non-cartel bidders is 0·58. In Texas there are on average 2·05 cartel bidders but only 0·55 non-cartel bidders per contract. The sample mean of WIN BID in Florida is in logarithm $-2{\cdot}03$ or 13·1 cents per half pint of milk; in Texas it is in logarithm $-1{\cdot}99$ or 13·6 cents per half pint of milk.

A difference between the Florida and Texas data is revealed by the variable WON-LAST. In Texas 79% of contracts are won by the incumbent bidder, but only 41% in Florida. Moreover, splitting the sample of Florida contracts into contracts won by cartel firms and contracts won by non-cartel firms reveals that the incumbency effect differs significantly for the two groups. Cartel firms win only 35% of their incumbency districts, while non-cartel firms win 70% of their incumbency districts. In Texas, the incumbency factors are about the same for the two groups. This evidence suggests that the cartels in Florida and Texas use different schemes to rig their bids. The next section studies how the cartel operates.

5. DIVISION OF SPOILS

This section examines the allocation of contracts among cartel members. We document that Florida market shares exhibit stronger dispersion than Texas market shares and that the difference in dispersion is significant. We show that this difference is not explained by a lack of potential competition, or a lack of interaction between cartel firms, or due to correlation in costs in Florida. The difference in market share dispersion is explained by models of bidder collusion with and without side payments. In Florida there is also legal evidence of side payments, but not in Texas. We also describe the effects of mergers on the Texas cartel agreement.

Figures 1 and 2 report the annual market shares of individual cartel firms in Florida and Texas, respectively. Two variables are used to measure annual market shares: Figures 1(a) and 2(a) give the percentage of won contracts out of all contracts. Figures 1(b) and 2(b) present the dollar revenues measured in percent of the value of all contracts. The market share data are reported in Tables 3, 4, 5 and 6.

Evidently, in Florida market shares fluctuate substantially, while in Texas they appear fairly constant. To determine whether there is a significant difference in the variances of annual market shares between the two states, the following test is constructed: Let S_{it} be the market share of cartel firm i in year t, D_i a cartel firm specific dummy variable and ε_{it} idiosyncratic randomness. Suppose the market shares follow the relationship

$$\ln\left(\frac{S_{it}}{1-S_{it}}\right) = D_i\beta + \varepsilon_{it}. \tag{2}$$

The estimated variance of the error term in (2) captures variations in market shares around the firm-specific mean. The test is based on comparing these variances in Florida and Texas. Under the null hypothesis, the variances in Florida and Texas are equal. The alternative is that the variance in Florida is bigger than in Texas.

The data are the market shares of cartel firms with more than 1·5% in total revenues in Florida and with more than 1% in total revenues in Texas. To correct for a merger occurring in 1986 in Texas, the observations after the merger of the firms involved in the merger were deleted from the sample. In Florida three observations with zero market shares were deleted from the sample. The total number of observations equals 60 in Florida and 50 in Texas. In Florida, the resulting standard deviations of the residuals of

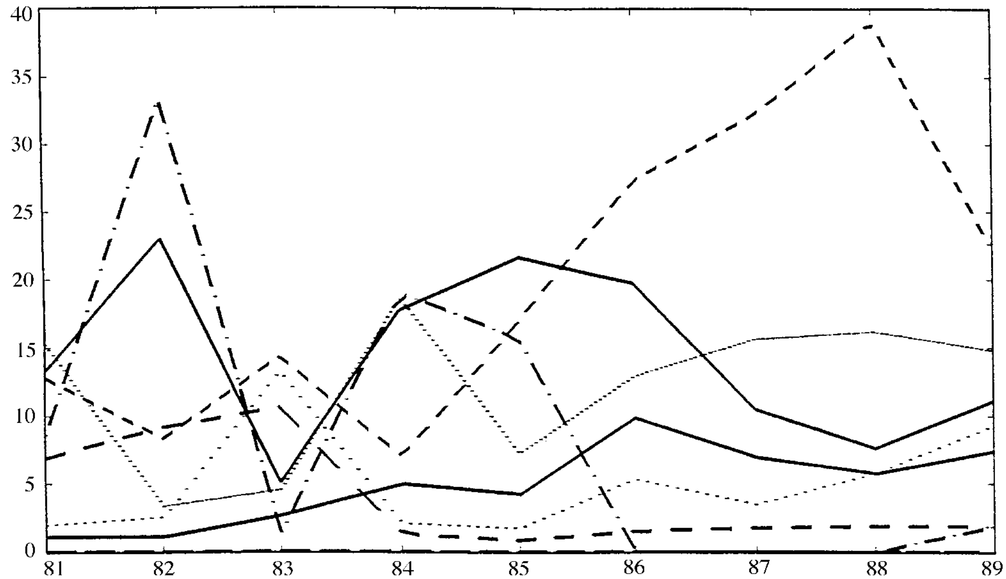

FIGURE 1(a)
Marketshares of cartel firms in total wins in Florida

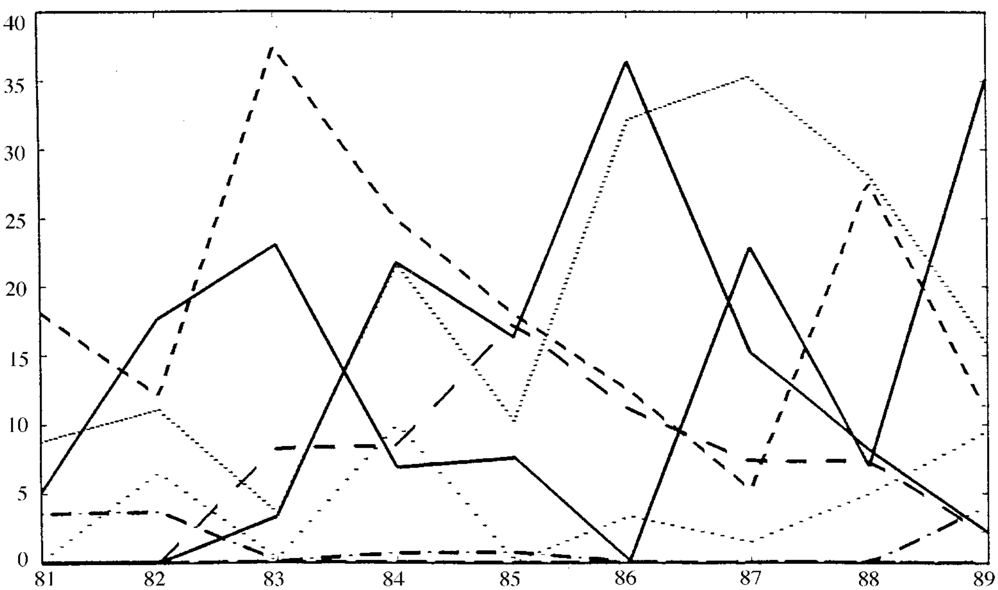

FIGURE 1(b)
Marketshares of cartel firms in total revenues in Florida

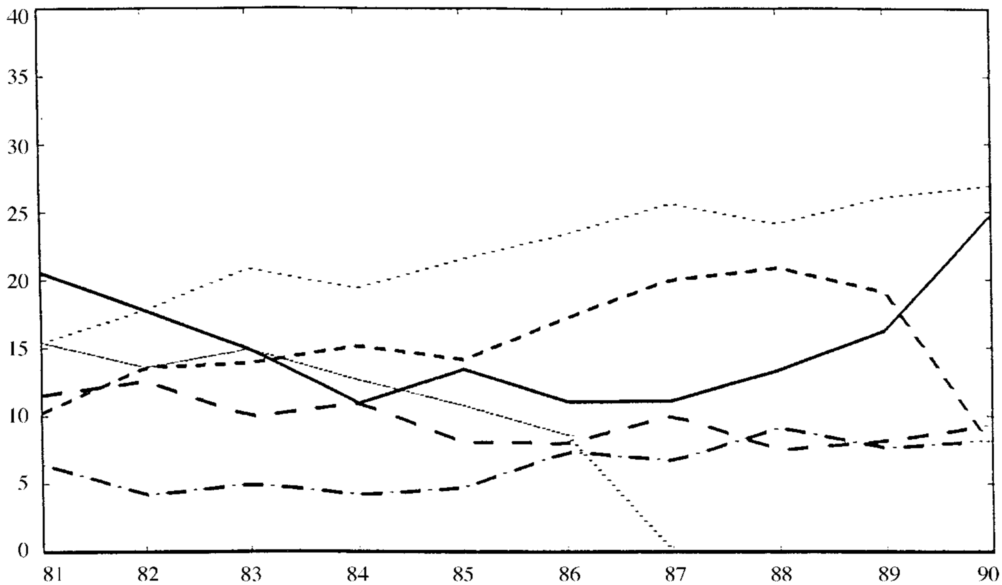

FIGURE 2(a)
Marketshares of cartel firms in total wins in Texas

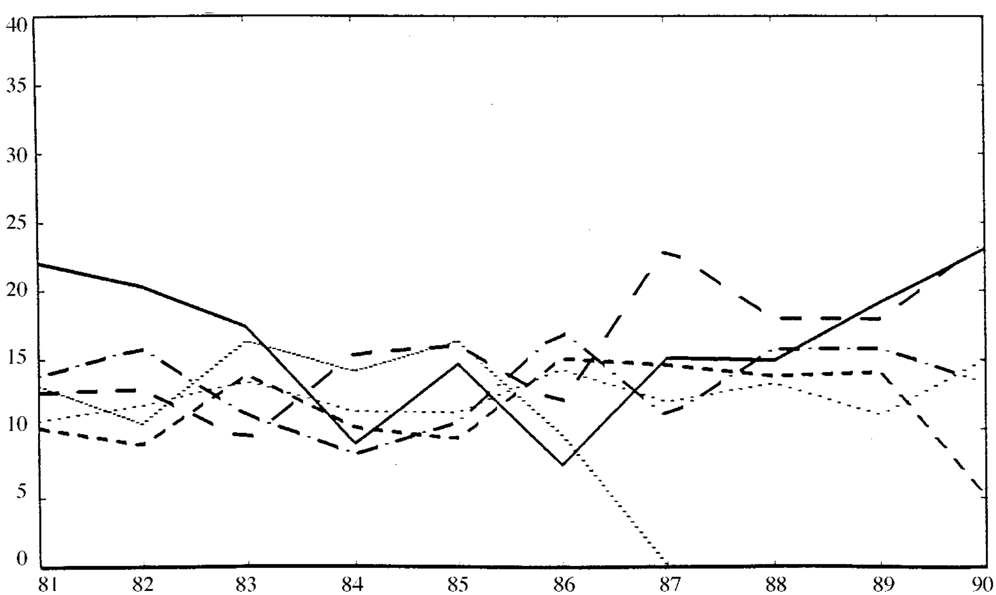

FIGURE 2(b)
Marketshares of cartel firms in total revenues in Texas

TABLE 3

Annual market shares in total wins: Florida[1]

	Year								
	81	82	83	84	85	86	87	88	89
All cartel firms	75·6	84·2	86·2	86·4	84·3	83·3	77·4	81·0	75·9
Firm 1	12·7	8·3	14·3	7·0	16·9	27·3	32·2	39·0	22·2
Firm 2	15·2	3·3	4·5	18·6	7·2	12·9	15·7	16·2	14·8
Firm 3	13·3	23·1	5·0	17·8	21·6	19·7	10·4	7·6	11·1
Firm 4	8·6	33·3	1·3	19·0	15·7	0·0	0·0	0·0	1·9
Firm 6	6·9	9·2	10·6	1·2	0·8	1·5	1·7	1·9	1·9
Firm 8	1·9	2·5	13·3	2·1	1·7	5·3	3·5	5·7	9·3
Firm 9	1·1	1·1	2·7	5·0	4·2	9·8	7·0	5·7	7·4
Total no. of contracts	361	360	377	242	236	132	115	105	54

[1] Cartel firms with more than 1·5% of total revenues.

TABLE 4

Annual market shares in total revenues: Florida[1]

	Year								
	81	82	83	84	85	86	87	88	89
All cartel firms	38·7	52·9	81·6	95·6	72·8	97·7	88·1	88·5	90·0
Firm 1	18·0	12·1	37·5	24·9	18·0	12·4	5·2	27·7	10·8
Firm 2	8·8	11·0	3·5	21·6	10·2	32·2	35·3	28·2	15·8
Firm 3	5·1	17·7	23·0	6·9	7·6	0·1	22·9	6·9	35·1
Firm 4	3·6	3·7	0·1	0·7	0·8	0·0	0·0	0·0	4·1
Firm 6	0·1	0·1	8·2	8·5	17·4	11·0	7·3	7·3	2·0
Firm 8	0·2	6·5	0·3	10·0	0·1	3·3	1·5	4·9	9·5
Firm 9	0·0	0·0	3·3	21·7	16·3	36·5	15·1	8·1	2·2
Million \$ revenues[2]	6·4	6·1	5·7	11·6	10·2	15·4	21·4	19·6	16·5

[1] Cartel firms with more than 1·5% of total revenues.
[2] Dollar figures in 1982 dollars.

TABLE 5

Annual market shares in total wins: Texas[1]

	Year									
	81	82	83	84	85	86	87	88	89	90
All cartel firms	79·5	79·2	79·2	73·9	73·2	76·7	73·3	74·9	77·2	76·9
Firm 1	15·4	17·7	20·8	19·3	21·5	23·3	25·6	24·1	26·1	26·9
Firm 2[2]	11·5	12·5	9·9	10·9	8·1	8·0	10·0	7·5	8·2	9·3
Firm 3[3]	20·5	17·7	14·9	10·9	13·4	11·0	11·1	13·4	16·3	24·7
Firm 4[3]	10·3	13·5	13·9	15·1	14·1	17·2	20·0	20·9	19·0	7·7
Firm 5	6·4	4·2	5·0	4·2	4·7	7·4	6·7	9·1	7·6	8·2
Firm 6[2]	15·4	13·5	14·9	12·6	10·7	8·6				
Total no. of contracts	78	96	101	119	149	163	180	187	184	182

[1] Cartel firms with more than 1% of total revenues.
[2] Firm 2 merged with firm 6 in 1986.
[3] Firm 3 merged with firm 4 in 1990.

TABLE 6

Annual market shares in total revenues: Texas[1]

	Year									
	81	82	83	84	85	86	87	88	89	90
All cartel firms	82·2	79·8	80·9	68·1	77·9	75·4	75·6	75·5	77·9	79·9
Firm 1	10·6	11·7	13·4	11·2	11·1	14·3	11·9	13·2	11·0	15·0
Firm 2[2]	12·6	12·8	9·1	15·4	16·0	11·8	23·2	17·9	17·9	23·5
Firm 3[3]	22·0	20·3	17·3	8·9	14·7	7·4	15·1	14·9	19·1	23·1
Firm 4[3]	10·0	8·8	13·9	10·1	9·2	15·1	14·6	13·8	14·1	5·1
Firm 5	13·8	15·8	10·9	8·2	10·5	17·1	10·8	15·7	15·8	13·3
Firm 6[2]	13·1	10·4	16·3	14·2	16·3	9·6				
Million \$ revenues[4]	13·3	13·9	11·0	12·4	12·9	13·2	13·4	13·2	12·7	12·5

[1] Cartel firms with more than 1% of total revenues.
[2] Firm 2 merged with firm 6 in 1986.
[3] Firm 3 merged with firm 4 in 1990.
[4] Dollar figures in 1982 dollars.

equation (2) equal 0·81 for market shares in total number of contracts and 1·75 for market shares in total dollar revenues. In Texas, the standard deviations equal 0·28 for market shares in total number of contracts and 0·30 for market shares in total dollar revenues.

To test whether the variances differ an F test can be constructed. Under the null the ratio of the variances is distributed as an F random variable with (59, 49) degrees of freedom. The test statistic equals 8·74 for the market share in total number of contracts and 35·14 for the market share in the total dollar revenues. For both measures, the null can be rejected. In Florida, market shares exhibit stronger dispersion than in Texas.

The conjecture that the Florida cartel uses side payments is also substantiated by legal evidence. The Florida indictment against Land-O-Sun Dairies (Criminal no. 89-116-CRT-13(a)) states: ‘‘The charged combination and conspiracy consisted of a continuing agreement, understanding, and concert of action among the defendant and co-conspirators, the substantial terms of which were: . . . to have the defendant and its co-conspirators provide compensation to another co-conspirator for submitting collusive, noncompetitive and rigged bids to the School Boards . . .’’. According to an article of The American School Board Journal (May 1993), the Florida cartel allegedly institutionalized a market to trade contracts that covered all of Florida.

In Texas, on the other hand, we could not find any legal evidence of the use of side payments. The evidence suggests that the Florida cartel uses side payments and the Texas cartel does not. However, there are additional explanations that may account for the difference in market share dispersion. We next consider a number of alternatives and assess their relevance.

Stable market shares in Texas may be a result of a lack of potential competition for contracts due to regional cost advantages of individual suppliers. To assess this hypothesis we count the number of potential competitors in counties which are small geographic areas. A total of 34 counties are contained in the Texas data. On average there are 5·8 bidders per county, and about 4·1 of these firms also win one or more contracts in the county during the sample period. The evidence does not indicate a lack of potential competition in small geographic areas.

The variance in Texas market shares may be downward biased if some cartel firms compete only on a small subset of contracts. We examined how many counties the average cartel firm bids on. The six cartel firms listed in Tables 5 and 6 bid on average in 68% of the counties. The largest four win at least one contract in 75% of the counties. This suggests that the magnitude of a bias would be small, if it exists at all.

A third explanation is that part of the variation in market shares in Florida may be due to changes in the number of contracts per school district. In Florida the number of contracts were reduced over time.[13] In addition, there are a number of missing observations in the data. In Florida, missing observations occur predominantly during the last year of the sample, while in Texas they occur during earlier years of the sample. An analysis of market shares performed for a balanced panel of school districts did not change the test results qualitatively. Again Florida market shares exhibit substantially stronger dispersion than Texas market shares.

A fourth explanation is that the market share dispersion in Florida may be amplified by correlation in costs within a year. Specifically, individual Florida firms may have high cost realizations for all contracts in one year and low cost realizations in another year. To assess the importance of potential correlation, we examine the magnitude of correlation in winning patterns. As described above, the number of contracts per county is an unbalanced panel, and we study the presence of correlation across counties, which is a balanced panel. We consider a variable that equals one, if the firm wins at least one contract in a specific county and year, and zero otherwise. The unpredicted winning probability for a county is regressed on a constant and the sum of unpredicted winning probabilities across all other counties in the same year for the same firm. Under the null of independence, the estimated coefficients are zero, while, under the alternative, coefficients different from zero are expected. There are 38 counties in the Florida data. The constructed test statistic is distributed as an F-distributed random variable with (2,4558) degrees of freedom and equals 2·54 in Florida. The null of independence cannot be rejected at the 5% level. Moreover, the coefficient for the sum, if anything, is negative.[14]

To summarize, the difference in market share dispersion is not a result of a lack of potential competition in Texas, or a lack of interaction between cartel firms, or a result of correlation in costs for Florida firms. On the other hand, the difference in market share dispersion is explained by the adoption of different cartel schemes, with and without side payments. Moreover, legal evidence in Florida indicates the use of side payments, while in Texas there is no legal evidence of side payments.

Collusion without side payments is not efficient, but can almost maximize collusive gains, provided two conditions are satisfied: The number of contracts is large and costs are independently distributed across contracts. As described before, there is no evidence of correlation in costs between Florida counties and, thus, the assumption of independent cost draws appears satisfied. Second, the Texas cartel operated on at least 34 counties, containing a total of 136 contracts awarded annually, which may enable the Texas cartel to perform quite well.

The fraction of contracts that are won by the incumbent firm is higher in Texas than in Florida. This may reflect stronger cost advantages of the incumbent firm, or, possibly, less efficiency gains in Texas. However, in Texas a number of contracts are reallocated every year, usually among cartel firms, which may indicate efficiency gains. Texas cartel firms lose about 21% of their incumbency districts. About 72% of the districts lost by an

13. In a subset of counties a number of small contracts were pooled and awarded in form of one large contract. Two changes occurred: The first in 1984 reducing the total number of contracts from 377 to 242. The second occurred in 1986 reducing the total number of contracts from 236 to 132.

14. There is some evidence of correlation in costs within a county. Specifically, tests of independence were constructed for individual counties. Some counties reject the null of independence. Possibly, the technology of milk delivery induces some correlation in costs locally.

incumbent cartel firm in the Dallas–Fort Worth area are won by another cartel member. A loss in one school district is typically accompanied by a gain in another cartel school district for the same firm in the same year keeping overall market shares constant. For example in 1985, firm A gains four contracts and loses three. Firm B gains one contract and loses one and firm C gains two and loses four, *etc*. The switching of contracts, on a contract for contract basis, appears in accordance with the outcome of a cartel that does not use side payments. In addition, it indicates attempts to improve efficiency.[15]

We next assess the effects of mergers on the Texas cartel agreement. During the sample period, three mergers took place in Texas: One in 1983, one in 1986 and one in 1990. The school milk market constitutes only a small share of the business of the merged firms and we consider the mergers as exogenous events. A number of tests are constructed to assess merger effects on cartel behaviour: First, an inspection of the sum of market shares of the two firms involved in the merger in 1986 reveals that the firms lose shares after the merger. This effect is stronger in the share of total wins than in the share of total revenues. A second test is constructed to determine whether mergers disrupt the division of spoils as measured by the market share dispersion.[16] We cannot report significant changes. Third, a number of tests are considered that compare the bidding behaviour before and after the merger.[17] None of these tests reveals significant changes in bidding behaviour before and after the merger. The evidence suggests that the mergers did not obstruct the activities of the cartel. This finding may be expected, since a merger affects bidding in much the same way as a cartel. Both, a cartel and a merged firm select the most efficient plant.

So far the analysis considers only cartel firms. Neither the Florida nor the Texas cartel are all-inclusive. An examination of market shares of individual firms reveals that non-cartel firms appear smaller than cartel firms in percent of won contracts and in their dollar value. The total market share of all cartel firms in Florida and Texas appear fairly stable at around 80%.[18] The largest five non-cartel firms in Florida won about 5% of the market in total number of contracts and about 12% of the total dollar revenues. The largest four non-cartel firms in Texas won about 20% of the market in total number of contracts and also of the total dollar amount.

The theoretical literature on collusion in auctions focuses on whether a cartel has the ability to select the lowest-cost firm. From the data, the efficiency properties of a cartel

15. As described before, in Florida a substantial fraction of contracts are reallocated every year. The incumbent cartel firm in Florida wins only about 35% of contracts. About 95% of the districts lost by the incumbent cartel firm are won by another cartel firm. However, in Florida the total number of contracts per cartel firm exhibits substantial year to year variation, as is evident in Figure 1(a).

16. To determine whether the merger resulted in changes in the dispersion of market shares a test is constructed based on a comparison of the variance of the residuals in relation (2) for two subsets of the data: Before and after the merger. Under the null hypothesis the variances for the subsets are equal. The alternative is that the variance increased after the merger. This test can only be performed for the merger occurring in 1986. For the other two incidence of mergers the data are too limited. To correct for the merger occurring in 1990 the last observation for the two firms involved in the merger is excluded. The test statistic which is distributed as an F random variable under the null with $(34, 18)$ degrees of freedom equals $0{\cdot}96$ for market shares in total wins and $1{\cdot}99$ for market shares in total revenues. The null hypothesis of no changes in the dispersion of market shares cannot be rejected at the 5% level.

17. First, correlation coefficients of the participation decision in individual auctions of cartel firms are examined before and after the merger. Second, changes in the distribution of bids are considered. Specifically, the standard deviation of bids around the contract specific mean before and after the merger are contrasted. Third, the bids of cartel firms are regressed on firm and contract specific variables and a set of time dummies. The coefficients of the time dummies before and after the merger are contrasted.

18. In Florida, there appears to be a trend in the early 1980s. Market shares of the cartel in revenues appear to increase from 1981 to 1983. In part, this trend is attributable to missing observations in the quantity variable.

agreement cannot be studied directly since there are no data on the cost of the selected cartel bidder and the cost distributions of individual cartel members. However, in the procurement auction the cartel bidder faces competition from non-cartel firms. The bidding data may be used to infer the effectiveness of the cartel scheme. The next section assesses cartel bidding behaviour.

6. BIDDING BEHAVIOUR

This section examines the bidding behaviour of cartels when there is competition from non-cartel firms. As noted above, a cartel scheme that is successful in selecting the lowest cost member induces asymmetries between cartel and non-cartel firms in the procurement auction. The cartel bidder has on average a lower cost than a non-cartel bidder. In addition, there may be *ex ante* asymmetries between cartel and non-cartel bidders. Alternatively, the presence of a cartel need not induce an asymmetry: If the cartel randomizes among members, and cartel and non-cartel firms are identical, then no differences in bidding behaviour are expected.[19] In this section we examine whether there is evidence of asymmetries between cartel and non-cartel bidders.

We report two results: First, we document that bidding rules of cartel and non-cartel firms differ. Second, we show that the distribution of cartel bids is first-order stochastically dominated by the distribution of non-cartel bids. These findings support the bidding model with cost asymmetries. We also assess whether there is evidence of differences in the effectiveness of the Texas and Florida cartel schemes. There are only minor differences in the bidding behaviour of the two cartels. The evidence on asymmetries in the bidding behaviour and the similarities between Florida and Texas are in accordance with optimal cartel agreements.

Of course, *ex ante* differences between cartel and non-cartel bidders may also account for cost asymmetries. The proposed test does not distinguish between cost asymmetries induced by the cartel scheme and asymmetries that existed *ex ante*. The test only reveals the presence of asymmetries.

Paarsch (1992) and Laffont, Ossard and Vuong (1995) conduct structural estimation of equilibrium bid functions. The structural approach permits estimation of privately known costs using the equilibrium bidding relationship. We do not follow this approach. Instead, we take the predictions of the model immediately to the data and test for the presence of asymmetries between bidders.

The first prediction of Section 3 is that cartel bidding behaviour differs from non-cartel bidding behaviour: it is less aggressive. According to Remark 3, for a given realization of costs, the cartel bid will be higher than a non-cartel bid. The second prediction is that the distribution of cartel bids is first-order stochastically dominated by the distribution of non-cartel bids.

We exclude complementary bids and analyze low cartel bids and all non-cartel bids. Complementary cartel bids may be intended to disguise the activities of the cartel or to create the impression of competition, but are not intended to win contracts. Thus, the lowest cartel bid for each contract is the only relevant information to characterize the bidding strategy of the cartel.

19. As described before, we assume that the convictions of bid-rigging were correct and do not examine the possibility of competitive behaviour by firms.

The selection of the lowest cartel bid is valid under the assumption that there is collusion. In auction markets with no presumption of collusion this selection may not be adequate.[20] The following tests cannot be applied to detect collusive behaviour.

We first examine whether there are differences in bidding functions between cartel and non-cartel firms. We consider the null hypothesis that bidding functions of cartel and non-cartel firms are identical. In order to test the null, we examine a regression of bids on observable contract and bidder characteristics. We assume that the residuals in the regression represent private cost shocks and are orthogonal to observed variables. Under the null of identical bidding functions, we expect no differences in the regression coefficients for cartel and non-cartel bidders.

Two assumptions are made to conduct the regression: First, we assume that the set of potential bidders for a contract is known to everybody.[21] Second, we consider the log linear regression,

$$\ln(b_{ij}^{c}) = X_{ij}\beta^{c} + \varepsilon_{ij}^{c}, \tag{3}$$

$$\ln(b_{ij}^{nc}) = X_{ij}\beta^{nc} + \varepsilon_{ij}^{nc}, \tag{4}$$

where (3) is the bid regression for the cartel bidder and (4) is the bid regression for non-cartel bidders. X_{ij} is a vector of firm and contract-specific variables affecting firm i's probability of winning or its cost for contract j and ε_{ij} represent private information, such as idiosyncratic cost effects for firm i on contract j.

Tables 7 and 8 report the determinants of bids in Florida and Texas, respectively. The dependent variable is the logarithm of bids divided by quantity, a measure of the per unit price. The set of explanatory variables includes contract-specific and firm-specific variables described in Table 1. All non-qualitative variables are in logarithms. In order to account for possible non-linearities, squared terms of the variables are included. In addition, each regression includes a set of year-specific dummies, a set of firm-specific variables, a set of geographic dummies and a set of variables measuring the share of individual milk items in total quantity. Ordinary Least Squares applied to the data will yield unbiased and efficient estimates under the assumption that all eligible bidders submit bids.

To test the null hypothesis of identical coefficients, $\beta^{c} = \beta^{nc}$, we consider three subsets of the sample data: low cartel bids plus all non-cartel bids; low cartel bids; and all non-cartel bids. A Chow test for equality of the coefficients can be constructed using two sets of estimates. First, the coefficients are estimated using low cartel bids in addition to all non-cartel bids. Second, the coefficients are estimated using non-cartel bids and cartel bids separately. Under the null hypothesis of no differences the estimates from the two sub-samples are identical to those from the full sample. In Florida the test statistic, which is distributed as an F random variable with (62, 1923) degrees of freedom under the null, equals 7·95. In Texas, the F-statistic has (68, 1656) degrees of freedom and equals 3·57. The null of no differences can be rejected in both Florida and Texas.

In general, the equations fit well. In both Florida and Texas about 80% of the variation in the dependent variable is explained. From Table 7 we conclude that in Florida the main differences between cartel and non-cartel bids are the following: RAWMPRICE

20. In a competitive market, the bidder with the lowest bid among a group of competitive bidders may also have on average a lower cost than other bidders.

21. The cartel submits a bid on almost all contracts. In Florida, only 27 contracts of a total of 1351 contracts did not receive a cartel bid. In Texas, 104 contracts of a total of 1176 contracts did not receive a cartel bid. In accordance with the model in Section 3, we assume that non-cartel bidders know of the existence of the cartel. This assumption appears reasonable since the cartel formed at least a decade prior to the sample period.

TABLE 7

Determinants of bids in Florida[1]

Data	Low cartel and all non-cartel bids	Low cartel bids	All non-cartel bids
Dependent variable:	BID	BID	BID
Observations:	2029	1324	705
Degrees of freedom:	1967	1269	654
R-squared:	0·87	0·90	0·88
Variable			
WONLAST	−0·0139	−0·0053	−0·0247
	(4·6)	(1·6)	(4·2)
RAWMPRICE	1·6701	2·2548	0·2186
	(8·8)	(9·8)	(0·8)
QUANT	0·0237	0·0267	0·0348
	(3·3)	(3·6)	(2·0)
QUANTSQ	−0·0009	−0·0011	−0·0015
	(2·5)	(3·4)	(1·8)
MILES	−0·0224	0·0047	−0·0751
	(3·2)	(0·6)	(3·4)
MILESSQ	0·0050	0·0019	0·0132
	(5·1)	(1·8)	(3·6)
NON CARTEL NOBID	0·0014	−0·0002	0·0709
	(0·3)	(0·1)	(5·4)
CARTEL NOBID	−0·0732	−0·0819	−0·0385
	(15·2)	(12·7)	(5·7)
ONEBID	0·0257	0·0246	0·0141
	(3·6)	(3·0)	(1·2)
POP	−0·0636	−0·0653	0·2721
	(4·9)	(4·9)	(5·0)
POPSQ	0·0075	0·0078	−0·0507
	(4·8)	(4·9)	(6·5)
SQMILE	0·0079	0·0053	0·0137
	(2·5)	(1·6)	(1·2)
ESC	−0·0053	−0·0012	−0·0706
	(0·8)	(0·2)	(3·9)
SCHOOLLUNCH	−0·2337	−0·3039	−0·4406
	(3·3)	(3·9)	(2·8)
UTIL	0·1772	0·2499	−0·5853
	(6·0)	(8·0)	(3·3)
UTILSQ	−0·3063	−0·4335	0·5307
	(6·3)	(8·1)	(2·4)
NOBACKLOG	0·0049	0·0190	−0·1916
	(1·1)	(4·0)	(7·0)

[1] All variables are in logarithms. Absolute values of *t*-statistics are displayed in parentheses. Each regression includes a set of 9 year-specific dummies, a set of 14 firm-specific dummies, a set of 16 county-specific dummies and a set of 6 variables measuring the share of individual milk items in total quantity.

has on average a significantly higher effect on cartel bids than on non-cartel bids. QUANT affects bids positively. As QUANT increases bids increase initially and then decrease. Evaluating the effect at the sample mean of QUANT reveals that the average effect of QUANT is stronger for non-cartel bids than for cartel bids. Evaluating the effect of POP at sample mean values reveals that POP affects cartel bids negatively, while it affects non-cartel bids positively. UTIL and NOBACKLOG have significant effects on bids. As UTIL

TABLE 8

Determinants of bids in Texas[1]

Data	Low cartel and all non-cartel bids	Low cartel bids	All non-cartel bids
Dependent variable:	BID	BID	BID
Observations:	1778	1072	706
Degrees of freedom:	1710	1007	649
R-squared:	0·78	0·78	0·84
Variable			
WONLAST	−0·0106	0·0081	−0·0244
	(2·6)	(1·4)	(4·0)
RAWMPRICE	1·1482	1·1968	1·0140
	(29·7)	(22·8)	(17·9)
QUANT	−0·0863	−0·0948	0·1568
	(1·7)	(1·5)	(1·8)
QUANTSQ	0·0028	0·0033	−0·0068
	(1·3)	(1·2)	(2·0)
MILES	0·0298	0·0354	0·0909
	(0·7)	(0·7)	(1·2)
MILESSQ	−0·0045	−0·0043	−0·0167
	(0·8)	(0·7)	(1·8)
NON CARTEL NOBID	−0·0046	−0·0143	0·0316
	(0·4)	(0·5)	(1·7)
CARTEL NOBID	−0·0264	−0·0296	−0·0331
	(3·6)	(2·8)	(3·3)
ONEBID	−0·0129	−0·0196	−0·0111
	(1·3)	(0·9)	(0·9)
POP	−0·0246	0·0134	−0·1508
	(0·9)	(0·4)	(3·1)
POPSQ	0·0031	0·0001	0·0117
	(1·7)	(0·0)	(3·9)
ESC	−0·0141	−0·0216	−0·0148
	(3·3)	(3·6)	(2·6)
MEALS	0·0055	0·0023	0·0213
	(0·8)	(0·3)	(2·4)
MEALSQ	−0·0005	−0·0003	−0·0016
	(1·1)	(0·5)	(2·7)
COOLER	0·0050	−0·0013	0·0152
	(1·1)	(0·2)	(2·0)

[1] All variables are in logarithms. Absolute values of t-statistics are displayed in parentheses. Each regression includes a set of 10 year-specific dummies, a set of firm-specific dummies, a set of 27 county-specific dummies and a set of 4 variables measuring the share of individual milk items in total quantity. The variables UTIL and NOBACKLOG are missing in Texas because the exact date of the letting is not known for a number of contracts.

increases, cartel bids initially increase and then decrease. Cartel bids are highest at a utilization rate of 29%. At this rate they are about 4% higher than at a utilization rate of 0% and about 22% higher than at a utilization rate of 100%. Non-cartel bids initially decrease in UTIL and then increase. Non-cartel bids are lowest at a utilization rate of 55% at which they are about 16% lower than at a utilization rate of 0%. The effect of NOBACKLOG is positive for cartel bids and negative for non-cartel bids.

Table 8 suggests that in Texas the differences between cartel and non-cartel bids are the following: The average effect of the raw milk price is positive and about 18% higher for cartel bids than for non-cartel bids. As MEALS increases bids increase and then decrease. The effect of MEALS evaluated at the sample mean is positive and stronger for non-cartel bids than for cartel bids. The average effect of QUANT is positive for non-cartel bids, while it is negative for cartel bids.

The coefficient of the variable RAWMPRICE indicates how an increase in the price of the main input affects the final bid price. In both states there is a stronger effect for cartel than for non-cartel firms. Thus, on average cartel firms charge a higher mark-up than non-cartel firms which is in accordance with less aggressive bidding.

The effect of the variable CARTEL NOBID may illustrate how cartel size affects the bidding outcome. Assuming that all cartel bidders submit a bid, the coefficient of the variable CARTEL NOBID measures the number of potential cartel bidders in the school district. The coefficient of the variable CARTEL NOBID is significant and negative in Tables 7 and 8. Increasing the cartel size lowers the bid which is in accordance with efficient cartel mechanisms.

The variable WONLAST illustrates some differences between the Florida and the Texas cartel scheme. WONLAST measures possible cost advantages of the incumbent supplier which are not attributable to transportation cost advantages. Remark 3 in Section 3 predicts that this effect should be negative. For non-cartel bids, the effect is negative in both Florida and Texas. For cartel bids, WONLAST has a negative effect in Florida but a positive effect in Texas. A possible explanation for the positive effect on Texas cartel bids may be that it reflects part of the allocation scheme of the Texas cartel.[22] The incumbency effect in the Texas data has originally been documented by Hewitt, McClave and Sibley (1993) who conclude that it is evidence of collusion.[23]

Next, the second prediction in Remark 3 is examined. The prediction is that the distribution of cartel bids is first-order stochastically dominated by the distribution of non-cartel bids. A direct comparison of the bid distributions is not meaningful, since the terms of contracts vary across school districts and, also, cost parameters may have changed across years. Instead, we attempt to filter out the variations across contracts and across time. Bids are projected on contract-specific variables and time-specific variables and then the residuals are examined. Specifically, we compute ordinary least squares residuals by regressing low cartel bids and all non-cartel bids on contract specific variables and time dummies (the same variables as listed in Tables 7 and 8 but excluding firm specific variables). The resulting residuals measure idiosyncratic variations in bids. The set of residuals is split into two subsets corresponding to low cartel bids and non-cartel bids. The prediction of the theory is that the distribution of cartel residuals is first order stochastically dominated by the distribution of non-cartel residuals. Figures 3 and 4 depict the distribution of the residuals in Florida and Texas.

A Rank test can be constructed to examine the equality of the distributions of cartel residuals and non-cartel residuals. Cartel and non-cartel residuals are ordered and each residual is assigned a rank corresponding to its position in the total ordering. Under the null hypothesis of identical distributions, the sum of non-cartel ranks equals the expected sum of randomly drawn ranks. In Florida, the test statistic, which is distributed as a

22. In Pesendorfer (1995) results of a regression explaining the determinants of winning bids are reported. The Texas cartel winning bids are on average 3·9% higher in incumbency school districts than non-incumbency districts. For Florida cartel bids there is no significant difference between incumbency and non-incumbency school districts.

23. The analysis by Hewitt, McClave and Sibley differs from the approach taken in this paper. They estimate bidding functions under the null that there is no cartel. Consequently, their estimation uses all bids including phony bids. Here the analysis differs. Complementary bids are excluded from the data and the focus is on the differences between cartel and non-cartel bids.

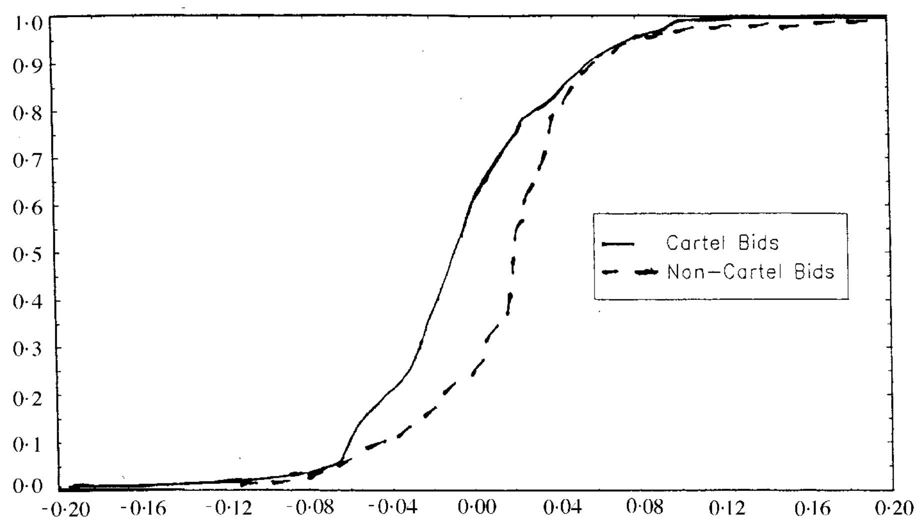

FIGURE 3
Distribution of cartel and non-cartel bids in Florida

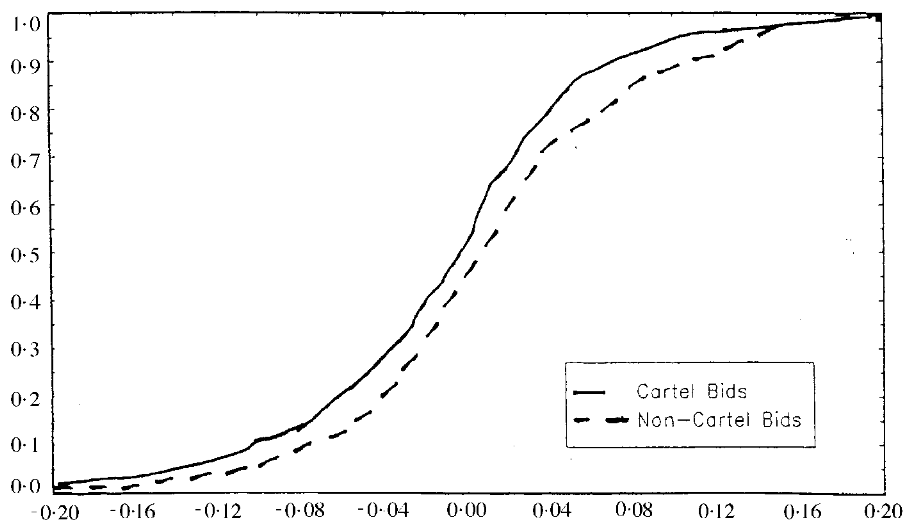

FIGURE 4
Distribution of cartel and non-cartel bids in Texas

standard normal random variable, equals 6·71. In Texas, it equals 5·26. The null hypothesis of identical distributions is rejected in both Florida and Texas.

To summarize, the results of tests are a confirmation of the theoretical predictions in Section 3.2: First, the bidding rules differ significantly between cartel and non-cartel firms. Cartel firms appear to bid less aggressively than non-cartel firms. Moreover, increasing cartel size tends to lower bids which is in accordance with efficient cartel mechanisms.

Second, the distribution of cartel bids is first-order stochastically dominated by the distribution of non-cartel bids. These findings are consistent with a model of bidding under asymmetries between bidders. The results are, in general, similar for Florida and Texas which is in accordance with the asymptotic efficiency result.

7. CONCLUSION

Antitrust investigations in the Florida and Texas school milk markets and subsequent convictions of cartel firms provide data for the study of bidder collusion. We find that there is a distinction in the operations of the Texas and Florida cartel. The two schemes are best described by theoretical models of bidder collusion with and without side-payments. We show that a cartel even without using side-payments is almost efficient, provided there are many contracts for sale. The school milk market lends itself to this form of collusion, since it is split into many contracts for sale every year enabling a weak cartel to be almost efficient.

In the buyer's auction the cartel bidder faces competition from non-cartel firms. The bidding data are studied to assess the optimality of cartels. A cartel that is successful in selecting the lowest cost member induces an asymmetry in the auction. The cartel bidder has on average a lower cost than a non-cartel bidder. In addition, there may exist *ex ante* asymmetries. We find that the evidence on collusive bidding supports predicted behaviour in asymmetric auctions. The evidence on asymmetries in Florida is in general similar to the evidence in Texas, which confirms the explanation that both forms of cartel agreements may be optimal. The determinants of bids differ between cartel and non-cartel firms and the distribution of cartel bids is first-order stochastically dominated by the distribution of non-cartel bids.

This study shows that markets with many contracts are susceptible to collusion. In addition, collusive agreements can take on different forms which makes cartel detection more difficult. If the buyer can choose whether to offer a project as a single contract or many small contracts, then, under the suspicion of collusion, the buyer should offer a single contract for two reasons: First, the expected cartel profits for weak cartels are smaller and thus may not exceed coordination costs. Second, the possible gains from defection are larger.

APPENDIX

A. *Proofs*.

Proof of Remark 1. Let $Q_i^j(c_i) = E_{-i} q_i^j(c_i, c_{-i})$ be the expected probability of receiving contract j for bidder i. The expected payoff for bidder i of announcing $\hat{c}_i$ is given by $U_i(c_i, \hat{c}_i) = \sum_{j=1}^m [R - c_i^j] Q_i^j(\hat{c}_i)$. First, we show that the incentive constraints imply that for costs on rays through the point $(R, R, \ldots, R)$ in $\Re^m$, *i.e.* for $d, d' \in [0, C]^m$ and a scalar $\alpha > 0$ such that $R - d^j = \alpha \cdot [R - d'^j]$ for all $j = 1, \ldots, m$, the following equality has to be satisfied,

$$\sum_{j=1}^m [R - d^j] Q_i^j(d) = \sum_{j=1}^m [R - d^j] Q_i^j(d'). \tag{A1}$$

To see this, let us consider the incentive constraint, which can be written as $\sum_{j=1}^m [R - d^j] Q_i^j(d) \geqq \sum_{j=1}^m [R - d^j] Q_i^j(d')$ and also $\sum_{j=1}^m [R - d'^j] Q_i^j(d') \geqq \sum_{j=1}^m [R - d'^j] Q_i^j(d)$. Using $R - d^j = \alpha \cdot [R - d'^j]$ in the second inequality, and cancelling α on both sides, yields $\sum_{j=1}^m [R - d^j] Q_i^j(d') \geqq \sum_{j=1}^m [R - d^j] Q_i^j(d)$. This inequality combined with the first inequality yields (A1).

Restrict next the pair of points d, d' to lie on the 45 degree line, *i.e.* $d^i = d^1$ for all $i = 1, \ldots, m$ and assume that $\alpha \in (0, 1)$. Factoring out and cancelling $[R - d^j]$ on both sides of (A1) yields $\sum_{j=1}^m Q_i^j(d) = \sum_{j=1}^m Q_i^j(d')$. This implies, that there exists a j, such that $Q_i^j(d) \geqq Q_i^j(d')$. Since $R - d'^j > R - d'^j$, this violates efficiency. ||

Proof of Theorem 1. First, the maximum *ex ante* expected payoff per contract is given by $R-\int_0^R cn[1-F(c)]^{n-1}f(c)dc$. Next consider the Ranking Mechanism: There are m contracts and *ex ante* each bidder ranks a contract in a particular place with probability $1/m$. Thus, the probability that a bidder ranks a contract at least in the ith place is i/m, the discrete uniform distribution on $\{1,\ldots,m\}$. The probability that at least one out of n bidders ranks a contract at least in the ith place is $1-[1-(i/m)]^n$, the distribution of the first-order statistic of the discrete uniform distribution (with n draws). So, under the Ranking Mechanism the probability that the winning bidder has ranked the contract in ith place is $[1-(i-1)/m]^n-[1-(i/m)]^n$. If a bidder ranks a contract in ith place, then the *ex ante* expected cost is the expected value of the ith order statistic (of m draws) from the distribution F and is given by $\int_0^R m!/((i-1)!(m-i)!)cF(c)^{i-1}[1-F(c)]^{m-i}f(c)dc$. Thus, the *ex ante* expected payoff per contract under the mechanism is given by $R-\sum_{i=1}^m[(1-(i-1)/m)^n-(1-(i/m))^n]\times\int_0^R m!/((i-1)!(m-i))cF(c)^{i-1}[1-F(c)]^{m-i}f(c)dc$. As $m\to\infty$, the term in square brackets converges to $n(1-x)^{n-1}dx$, the density of the first-order statistic of the uniform $[0,1]$ distribution (with n draws). The integral term converges to the asymptotic mean $F^{-1}(x)$ (for a proof see Arnold, Balakrishnan and Nagaraja (1992) p. 223). Thus, the *ex ante* expected payoff per contract under the Ranking Mechanism converges to $R-\int_0^1 n(1-x)^{n-1}F^{-1}(x)dx$. Using a change of variable, $c=F^{-1}(x)$, yields $R-\int_0^R n[1-F(c)]^{n-1}cf(c)dc$. ||

Before presenting the proof of Remark 2, we state three lemmas, which are used to establish Remark 2. The lemmas establish properties of the probability of winning of contracts under the Ranking Mechanism for a particular bidder. Lemma 1 characterizes the probabilities of winning. Lemma 2 shows that the probability of winning the contract ranked in the ith position does not diminish as the number of contracts increases. Lemma 3 establishes that the covariance between the event of winning contract i and the event of winning contract j is negative.

Lemma 1. *Consider a particular bidder and her probability of winning when there are a total of m contracts for sale. The probability of winning the contract ranked in the ith position equals $P(\text{win } i; m)=(m/n)[((m+1-i)/m)^n-((m-i)/m)^n]$. The probability of winning the contract ranked in the ith position conditional on winning the contract ranked in the jth position, with $i<j$, is given by $P(\text{win } i\,|\,\text{win } j)=P(\text{win } i; m-1)$.*

Proof. Consider the probability of winning the contract ranked in the ith position. This equals the probability that all other bidders ranked that contract lower plus the winning probability, when there is a tie for the ith contract. (With ranking a contract higher we mean assigning a higher preference for the contract.) Let k index the number of ties. The probability is given by

$$\sum_{k=0}^{n-1}\frac{(n-1)!}{k!(n-1-k)!}\frac{1}{k+1}\left(\frac{1}{m}\right)^k\left(\frac{m-i}{m}\right)^{n-k-1}.$$

Using a change of variable, this expression can be rewritten as

$$=\frac{m}{n}\sum_{k=1}^{n}\frac{n!}{k!(n-k)!}\left(\frac{1}{m}\right)^k\left(\frac{m-i}{m}\right)^{n-k}$$

$$=\frac{m}{n}\left[\left(\frac{m+1-i}{m}\right)^n-\left(\frac{m-i}{m}\right)^n\right].$$

The first equality changes the range of summation from k to $k+1$. The second equality uses the binomial sum $\sum_{k=0}^n n!/(k!(n-k)!)(1/m)^k((m-i)/m)^{n-k}=(1/m+(m-i)/m)^n$.

Consider next the conditional probability $P(\text{win } i\,|\,\text{win } j)$. It equals $P(\text{win } i;\ m-1)$. To see this observe, that winning contract j implies that all bidders ranked that contract in the jth position, or in a position larger than j. Since $i<j$, no bidder could have ranked contract j in position i, or in a position smaller than i. So after awarding contract j, one position in the ranking is deleted for all bidders, and $m-1$ positions are left. The conditional probability can be expressed as the winning probability of the contract ranked in the ith position with $m-1$ contracts. ||

Lemma 2. *The probability of winning the contract ranked in the ith position increases in the number of contracts, m: $P(\text{win } i; m-1)<P(\text{win i}; m)$.*

Proof. We show that $P(\text{win } i; m)$ is monotone increasing in m. From Lemma 1 the probability $P(\text{win } i; m)$ equals $(m/n)[((m+1-i)/m)^n-((m-i)/m)^n]$. Consider the partial derivative of $P(\text{win } i; m)$ with respect to m. It equals

$$\frac{\partial P(\text{win } i; m)}{\partial m} = \frac{[(m-i+1)^{n-1} - (m-i)^{n-1}]m^{n-1} - ((n-1)/n)[(m-i+1)^n - (m-i)^n]m^{n-2}}{m^{n-2}}.$$

Multiplying by $1/m^{n-2}$, and rearranging, yields that the derivative is proportional to

$$(m-i+1)^{n-1}\left[m - \frac{n-1}{n}(m-i+1)\right] - (m-i)^{n-1}\left[m - \frac{n-1}{n}(m-i)\right].$$

Rewriting this expression, yields that the derivative is positive, if

$$(m-i+1)^{n-1}\left[\frac{m}{n} + \frac{(n-1)\cdot(i-1)}{n}\right] > (m-i)^{n-1}\left[\frac{m}{n} + \frac{i\cdot(n-1)}{n}\right].$$

Multiplying by n, and rewriting, yields that the derivative is positive, if

$$\frac{m + i\cdot(n-1)}{m+(i-1)\cdot(n-1)} < \left(\frac{m-i+1}{m-i}\right)^{(n-1)}.$$

The binomial equation yields, $((m-i+1)/(m-i))^{(n-1)} = ((m-i)/(m-i))^{(n-1)} + (n-1)/(m-i) + \sum_{k=2}^{n-1}(n-1)!/(k!(n-1-k)!)(1/(m-i))^k$. This implies that the right-hand side in the above inequality is larger than $1^{(n-1)} + (n-1)\cdot 1/(m-1)$. Using this approximation, yields that the inequality is satisfied, if

$$1 + \frac{n-1}{m+(i-1)\cdot(n-1)} < 1 + \frac{n-1}{m-i},$$

or, if $1/(m+(i-1)\cdot(n-1)) < 1/(m-i)$. Rearranging yields $-i < (i-1)\cdot(n-1)$, which is clearly satisfied. ||

Lemma 3. *Let x_i be a random variable that equals one if the contract ranked in the ith position is won and zero otherwise. Then the covariance between x_i and x_j is negative.*

Proof. For brevity of notation let $P_{ww} = P(\text{win } i, \text{ win } j)$, $P_{lw} = P(\text{lose } i, \text{ win } j)$, and let $P_{\cdot w} = P(\text{win } j; m)$. In addition, let $\bar{x}_i$ denote the expected value of x_i. Using this notation, the covariance is given by

$$[1-\bar{x}_i][1-\bar{x}_j]P_{ww} - [1-\bar{x}_i]\bar{x}_j P_{wl} - \bar{x}_i[1-\bar{x}_j]P_{lw} + \bar{x}_i\bar{x}_j P_{ll}.$$

Observe that $\bar{x}_i = P_{w\cdot}$ and $\bar{x}_j = P_{\cdot w}$. Using these expressions, we can rewrite the covariance as

$$[1-\bar{x}_i][1-P_{\cdot w}]P_{ww} - [1-\bar{x}_i]P_{\cdot w}P_{wl} - \bar{x}_i[1-P_{\cdot w}]P_{lw} + \bar{x}_i P_{\cdot w}P_{ll}.$$

Using $P_{\cdot w} = 1 - P_{\cdot l}$, and rearranging yields,

$$\begin{aligned} &= [1-\bar{x}_i][P_{ww} - P_{\cdot w}(P_{wl} + P_{ww})] + \bar{x}_i[P_{ll} - P_{\cdot l}(P_{ll} + P_{lw})] \\ &= [1-\bar{x}_i][P_{ww} - P_{\cdot w}P_{w\cdot}] + \bar{x}_i[P_{ll} - P_{\cdot l}P_{l\cdot}]. \end{aligned}$$

From this expression, we see that the covariance is negative, if $P_{ww} - P_{\cdot w}P_{w\cdot} < 0$ and $P_{ll} - P_{\cdot l}P_{l\cdot} < 0$. We establish both inequalities:

The first inequality is obtained by applying Lemma 1 which gives $P_{ww} = P_{\cdot w}\cdot P(\text{win } j; m-1)$ and substituting this expression into $P_{ww} - P_{\cdot w}P_{w\cdot} < 0$. Cancelling yields, $P(\text{win } j; m-1) < P(\text{win } j; m)$ which is satisfied by Lemma 2.

We next show that $P_{ll} - P_{\cdot l}P_{l\cdot} < 0$. First, we find an expression for $P(\text{lose } i | \text{win } j)$. Consider the definition of P_{lw}. It can be written as

$$\begin{aligned} P_{lw} &= P(\text{lose } i | \text{win } j)[P(\text{win } j; m)] \\ &= [1 - P(\text{win } i; m-1)]P(\text{win } j; m). \end{aligned}$$

The first equality uses the definition of conditional probabilities. The second inequality uses Lemma 1. An alternative way of expressing P_{lw} is given by the definition of a conditional probability,

$$P_{lw} = P(\text{win } j | \text{lose } i)[1 - P(\text{win } i; m)].$$

Equating the first expression and the second expression for P_{lw}, and expressing $P(\text{win } j | \text{lose } i)$, yields

$$P(\text{win } j | \text{lose } i) = \frac{1 - P(\text{win } i; m-1)}{1 - P(\text{win } i; m)} P(\text{win } j; m).$$

Similarly, the definition of a conditional probability implies that P_{ll} is given by

$$\begin{aligned} P_{ll} &= P(\text{lose } j \mid \text{lose } i)P(\text{lose } i;\, m) \\ &= [1 - P(\text{win } j \mid \text{lose } i)][1 - P(\text{win } i;\, m)] \\ &= \left[1 - \frac{1 - P(\text{win } i;\, m-1)}{1 - P(\text{win } i;\, m)} P(\text{win } j;\, m)\right][1 - P(\text{win } i;\, m)]. \end{aligned}$$

Substituting into the inequality $P_{ll} - P_{\cdot l}P_{l\cdot} < 0$, yields

$$\left[1 - \frac{1 - P(\text{win } i;\, m-1)}{1 - P(\text{win } i;\, m)} P(\text{win } j;\, m)\right][1 - P(\text{win } i;\, m)] < [1 - P(\text{win } i;\, m)][1 - P(\text{win } j;\, m)].$$

Cancelling yields

$$-\frac{1 - P(\text{win } i;\, m-1)}{1 - P(\text{win } i;\, m)} < -1,$$

or,

$$P(\text{win } i;\, m-1) < P(\text{win } i;\, m),$$

which is satisfied by Lemma 2. ||

Proof of Remark 2. Let x_i be a random variable that equals one if the contract ranked in the ith position is won and zero otherwise. Consider the variance of $\sum_{i=1}^m x_i$,

$$\begin{aligned} \operatorname{Var}\left(\textstyle\sum x_i\right) &= E[x_1 - \bar{x}_1]^2 + 2E\{[x_1 - \bar{x}_1][\textstyle\sum_{j \geqq 2} x_j - \sum_{j \geqq 2} \bar{x}_j]\} + \cdots + E[x_m - \bar{x}_m]^2 \\ &\leqq \textstyle\sum_{j=1}^m E[x_i - \bar{x}_i]^2 \\ &= \textstyle\sum_{j=1}^m P(\text{win } i;\, m)[1 - P(\text{win } i;\, m)]. \end{aligned}$$

The first line is obtained from the definition of the variance. The second inequality uses Lemma 3, which establishes that the covariance between two arbitrary x_i and x_j is negative. The third inequality expresses the variance of x_i.

Lemma 1 establishes that

$$P(\text{win } i;\, m) = \frac{1}{n} \frac{((m+1-i)/m)^n - ((m-i)/m)^n}{1/m}.$$

Let $x = i/m$ and let $i, m \to \infty$ such that x is fixed. Then

$$P(\text{win } i;\, m) = \frac{1}{n} \frac{(1 - x + 1/m)^n - (1-x)^n}{1/m}.$$

Thus,

$$\lim_{i,m\to\infty} P(\text{win } i;\, m) = (1-x)^{n-1}.$$

Using this equation we find that

$$\begin{aligned} \lim_{i,m\to\infty} \frac{1}{m} \textstyle\sum_{i=1}^m P(\text{win } i;\, m)[1 - P(\text{win } i;\, m)] \\ = \int_0^1 (1-x)^{n-1}[1 - (1-x)^{n-1}]\,dx. \end{aligned}$$

Using a change of variable yields

$$\begin{aligned} &= \int_0^1 u^{n-1}[1 - u^{n-1}]\,du \\ &= \frac{1}{n} - \frac{1}{2n-1} \\ &= \frac{n-1}{n(2n-1)}. \end{aligned}$$

Winning a contract under the efficient mechanism with side-payments is distributed as a binomial random variable with mean $1/n$. The variance equals $(1/n)[1-1/n]$. Using this expression, we find that the ratio of the variance of the efficient mechanism with side-payments to the variance of the Ranking Mechanism is given by

$$\frac{n-1}{n^2}\bigg/\frac{n-1}{n(2n-1)}.$$

Cancelling yields,

$$\frac{2n-1}{n},$$

which is the desired expression. ||

Proof of Remark 3. A bidder of type c_i chooses b to maximize $[b-c_i]\prod_{j\neq i}[1-F_j(y_j(b))]$ where $y_j(\cdot)$ denotes the inverse of the equilibrium bidding function of a bidder j. The first-order condition for a maximum imply

$$\frac{1}{x-y_i}=\sum_{j\neq i}H_j(y_j)y_j' \quad \forall i, \tag{1}$$

where x denotes i's bid and $c_i=y_i(x)$. Pesendorfer (1995) establishes that finding an equilibrium is equivalent to finding a solution to (1) almost everywhere. Moreover, the solution satisfies the boundary conditions $(\exists x_0)$ $y_i(x_0)=0$ $(\forall i)$ and $y_i(R)=R$ $(\forall i)$.

Part (i): Subtracting the first equation from the ith in (1) yields

$$\frac{1}{x-y_i}+H_i(y_i)y_i'=\frac{1}{x-y_1}+H_1(y_1)y_1'. \tag{A3}$$

Thus, for all $x\in[x_0,R)$, if $y_i(x)=y_1(x)$, then $(y_i'/y_1')=(H_1(y_1)/H_i(y_i))>1$, since by assumption, $H_1(y_1)>H_i(y_1)$, and, thus, $y_1(x)'<y_i(x)'$. Since $y_1(x)$ and $y_i(x)$ intersect at x_0, it has to be that $y_i>y_1$ for all $x\in(x_0,R)$.

Part (ii): From (i) $y_1(x)<y_i(x)$ for all $x\in(x_0,R)$ and (1) implies $H_1(y_1(x))y_1'(x)>H_i(y_i(x))y_i'(x)$ for all $x\in(x_0,R)$. This can be written as $-(d/dx)\ln[1-F_1(y_1(x))]>-(d/dx)\ln[1-F_i(y_i(x))]$. Since $1-F_1(y_1(x_0))=1-F_i(y_i(x_0))$, this implies $\ln[1-F_1(y_1(x))]<\ln[1-F_i(y_i(x))$ for all $x\in(x_0,R)$, or, equivalently we have, $F_i(y_i(x))<F_1(y_1(x))$. ||

Sources of data by variables

1. Florida The variables QUANT, BID, WINBID, ESC, NOBID, CARTEL NOBID, NON CARTEL NOBID, CAPACITY, UTIL, NOBACKLOG, WONLAST were obtained from the data base on the supply of milk to schools made available by the Antitrust Division of the Department of Justice, Florida Division in Tampa, Florida. The variables BID, WINBID were deflated with the GDP deflator using 1982 as a base year. The source for the variables POP and SQMILE is the ''Florida Statistical Abstract 1993''. RAWMPRICE was obtained from the ''Florida Agricultural Statistics on Livestock, Dairy, Poultry'', 1992, by the Florida Agricultural Statistics Service, Orlando, Florida and deflated. The variable SCHOOLLUNCH was obtained from the ''Fiscal Report of the State of Florida'', 1988 (SRI number S-1725-2). The variable MILES was calculated based on a list of dairy plants provided by the Florida Department of Agriculture and Consumer Services, Dairy Division, Tallahassee, Florida, and the location of the school districts by using a map of Florida.

2. Texas. The variables QUANT, BID, WINBID, ESC, MEALS, NOBID, CARTEL NOBID, NON CARTEL NOBID, POP, WONLAST were obtained from the data base on the supply of milk to schools made available to David Sibley, University of Texas at Austin, Texas. The variables BID, WINBID were deflated with the GDP deflator using 1982 as a base year. RAWMPRICE for Dallas, Texas is listed in nominal terms in a United States General Accounting Office publication on ''Information on Farm and Retail Milk Prices'', 1991. It was deflated using the GDP deflator. The variable MILES was calculated based on a list of dairy plants provided by Dairy Division of the United States Department of Agriculture, Carrollton, Texas, and the location of the school districts by using a map of Texas.

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