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Mergers under entry

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I study merger incentives in a dynamic model under the presence of gradual entry. I consider a repeated game with merger decisions in every period and characterize the set of equilibria. I establish two properties: (i) a merger for monopoly may not be profitable; (ii) a merger in a nonconcentrated industry can be profitable. I illustrate the merger welfare implications in the Cournot model.

1. Introduction

■ Merger incentives in static oligopoly models have received considerable attention in the literature. Yet, little is known about merger incentives in dynamic oligopoly models. This article characterizes merger incentives in a dynamic model under the presence of gradual entry. I consider a two-stage game that is repeated over time: In the first stage, firms can make acquisition offers, and in the second stage, acquisition offers are accepted or rejected. I characterize the set of Markov equilibria.

If firms expect no additional mergers to arise in the future and firms are patient, then a single merger may not be profitable. I show that in the Cournot model with linear demand and constant average cost, even a merger for monopoly may not be profitable.

If future mergers are expected, then merger incentives may differ. I show that if a merger leads to additional mergers in the future, then the profitability of the merger is increased. I illustrate that frequent mergers can occur with Cournot payoffs. In particular, in the Cournot model with linear demand and constant (and identical) average cost, profitable mergers in nonconcentrated industries exist.

An example of an environment with frequent mergers is the software industry, which experiences a steady inflow of new firms. A possible strategy to cope with new competitors is to buy them up. According to *Merger Stat*, Microsoft acquired at least 24 firms between 1988 and 1998: 7 firms in 1998, 6 firms in 1997,[1] 3 firms in 1996, and 2 firms during each of the years 1995, 1992, 1990, and 1988.[2]

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[1] The acquired companies during the year 1998 include Coopers and Peter Inc., Interse Corp., LinkAge Software Corp., Web TV Networks Inc., Vxtreme Inc., Hotmail Corp., and DimensionX Inc. The acquisitions during the year 1997 include Electric Gravity Inc., Exos Inc., Aha Software Corp., Colusa Software Inc., Panorama Software Sys-On-Line, and Vermeer Technologies.

[2] The acquired firms during the year 1996 include Render Morphics Ltd., Microsoft India Pvt Ltd., and Network Managers PLC. The acquisitions between 1988 and 1995 include Softimage Inc., Next Base Ltd., Consumer Software, Fox Software Inc., Forthought Inc., and Intermail.

A second example is the toy manufacturing industry. This industry is essentially a duopoly with two large firms, Mattel and Hasbro. New firms enter frequently, but often they are bought immediately by existing firms. According to the *New York Times*, Hasbro added six companies during the two prior years and Mattel purchased three.[3]

Merger incentives and merger welfare effects in static models have been studied by a number of authors. Salant, Switzer, and Reynolds (1983), Perry and Porter (1985), Levin (1990), and Farrell and Shapiro (1990) consider the Cournot model. The latter three assume that the capital stocks of firms are exogenously fixed. A merger is defined as a combining of two capital stocks. The marginal production costs are a decreasing function of capital, and thus a merger achieves efficiency gains. Perry and Porter show that a merger is profitable if sufficiently large efficiency gains are achieved. Farrell and Shapiro provide conditions such that a profitable merger increases total welfare. Levin shows that any profitable merger among a group of firms with less than 50% of the industry output raises welfare. Salant, Switzer, and Reynolds assume that the capital stock can be adjusted at no cost. They show that in the absence of efficiency gains, mergers are not profitable unless they occur in an industry with a small number of firms.[4] Deneckere and Davidson (1985) consider mergers in a differentiated-products price-setting industry.[5] They show that a merger is profitable independent of the number of firms in the industry.

The described literature on merger incentives makes two assumptions: First, the occurrence of mergers is exogenous.[6] Second, firms take only short-term considerations into account. Stigler (1950) already emphasizes that short-run efficiency gains are negligible in the long run. In the long run, depreciation of capital stocks and adjustment by other firms may diminish any short-term efficiency advantages of mergers. Thus, provided firms are sufficiently patient, the long-run merger incentives may differ from short-run considerations.

The welfare implications of my analysis are in contrast to the literature that examines welfare effects of an exogenous merger. I find that there can be profitable mergers in nonconcentrated Cournot industries that *reduce* welfare. Levin (1990) and Farrell and Shapiro (1990) find that profitable mergers in nonconcentrated Cournot industries *raise* welfare. The reason for the opposite welfare implications is that Levin, as well as Farrell and Shapiro, assume that mergers are profitable in the short run. I show that profitable mergers exist that yield losses in the short run but gains in the long run. Thus, Levin's and Farrell and Shapiro's assumption of short-run profitability of mergers need not apply. Welfare-reducing mergers can arise in Cournot industries with many firms.

My formulation of the merger stage builds on the model by Kamien and Zang (1990), who endogenize merger decisions. Firms may acquire other firms by submitting bid-and-ask prices. Kamien and Zang show that monopolization is limited to industries with relatively few firms. In contrast to the one-shot game considered by Kamien and Zang, I consider a dynamic merger game in which a merger may lead to additional mergers in the future. Cheong and Judd (1992), Gowrisankaran (1999) and Gowrisankaran and Holmes (2004) examine dynamic models in which mergers arise endogenously. Cheong and Judd (1992) and Gowrisankaran (1999) use numeric methods to determine the implication of the model. Gowrisankaran and Holmes (2004) study acquisition incentives in the dominant-firm model.

[3] "The nation's toy giants are increasingly obtaining new products by buying the companies that come up with the original ideas. While there have been a rash of recent deals, one of the earliest examples was Mattel's purchase of Coleco's assets in 1989 to gain the Cabbage Patch Kids dolls" (*New York Times*, February 9, 1999, p. C1). The recent acquisitions are described as follows: Hasbro acquired Microsprose in August 1998 for $700 million, Galoob in September 1998 for $220 million, Atari in March 1998 for $5 million, Tiger Electronics in February 1998 for $335 million, and Oddzon and Cap Toys both in February 1997 for $166 million. Mattel acquired the Learning Company in 1999 for $3.8 billion, it acquired the Peasant Company in 1998 for $700 millionm and Tyco in November 1996 for $755 million.

[4] Baye, Crocker, and Ju (1996) study divestiture incentives in this model. They show that divestitures arise in equilibrium.

[5] They consider a merger in which one firm acquires the product line of another firm. The merged firm sets prices for the two product lines jointly.

[6] An exception is Kamien and Zang (1990, 1991) who endogenize merger activity.

The article is organized as follows. In Section 2, I describe the model and illustrate the relationship to the static single-merger condition. Section 3 analyzes merger equilibria under entry. I characterize necessary and sufficient conditions for (i) no merger and (ii) merger(s) to arise. In Section 4, I discuss the findings and illustrate welfare effects. Section 5 provides conclusions. All proofs are in the Appendix.

The model and the static merger condition

■ This section describes my model and illustrates how it relates to the literature on single mergers. The first subsection describes the model. The second subsection illustrates the static merger condition.

□ **Model.** This section describes the elements of the dynamic model. I describe the state space, the period game, the payoffs, the strategies, and the equilibrium concept.

I consider a dynamic game with discrete time, $t = 1, 2, \ldots, \infty$. The set of potential firms is denoted by $\mathbf{Z} = \{1, 2, \ldots\}$. Potential firms are identical, and a typical potential firm is denoted by $i \in \mathbf{Z}$. The set of potential firms is fixed and does not change over time. Every period the following stage game takes place.

Stage game. I consider a stage game in which the number of potential firms that are active at the end of the period is determined endogenously. Entry increases the number of active firms, and merger can reduce it. The timing of events is illustrated in Figure 1.

At the beginning of the period, a state vector $(\mathbf{e}^t, \mathbf{s}^t)$ is publicly observed. The state consists of a vector of potential firms' states, $\mathbf{e}^t = (e_i^t)_{i \in \mathbf{Z}}$ and a signal $\mathbf{s}^t = (s_i^t)_{i \in \mathbf{Z}}$. The state variable of a potential firm, e_i^t, has the following interpretation: $e_i^t = 0$ if the potential firm i is not active, $e_i^t = 1$ if the potential firm i is an incumbent, and $e_i^t = 2$ if the potential firm i is an entrant. A potential firm is called active if it is an incumbent or an entrant. Inactivity can arise because either the potential firm has not yet entered the industry, or the potential firm has already exited the industry. With n^t I denote the number of active firms at the beginning of the period, $n^t = |\{i \in \mathbf{Z} \mid e_i^t \geq 1\}|$. Initially, in period 1, all potential firms $i \in \mathbf{Z}$ are inactive, $e_i^1 = 0$, and the number of active firms equals zero, $n^1 = 0$. The signal $s^t \in S$ is a coordination device. It can be a realization of an identically and independently distributed random variable that is drawn every period anew.

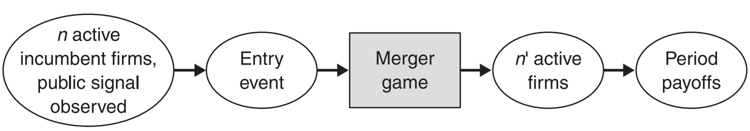
In the entry event, exactly one potential firm enters the industry. Without loss of generality, I assume that potential firm t enters in period t. The entry event alters the state of potential firm t from $e_t^t = 0$ to $e_t^t = 2$. At the end of the entry event the number of active firms equals $n_t + 1$.

The third event is the merger game. It is a game between active firms only. It consists of take-it-or-leave-it offers and takes place in two stages: In the first stage, all active firms simultaneously announce acquisition offers to other active firms that consist of a price at which they are willing to buy out any other active firm. Each offer is a commitment to buy one firm, and the offer is not tied to the identity of a specific firm. At the end of the first stage, offers are ordered in descending sequence, $B^{(1)} \geq B^{(2)} \geq \cdots \geq B^{(n+1)}$. The offer sequence, the identities of the offering firms, and their position in the ranking are publicly announced.

In the second stage of the merger game, after observing all the offers and their ranking, all active firms (including the entrant) announce an acceptance level, which means that any offer that exceeds the acceptance level is accepted. If one or more offers exceed one or more acceptance levels, then the following rationing rule is used: Buyers and sellers are matched sequentially in

FIGURE 1

SEQUENCE OF EVENTS



accordance with the first stage offer ranking. To describe the rationing rule, I relabel firms in accordance with the offer ranking such that firm i submits offer $B^{(i)}$. I use the index i to denote the offering firm and the index j to denote the receiving firm in the match. Initially, firm 1 is matched with firm $n+1$. Hence, $i = 1$ and $j = n+1$. If firm j accepts the offer, that is, $B^{(i)} \geq A_j$, then the acquisition takes place and a new match is found by increasing the index i by one and reducing the index j by one. If firm j does not accept the offer, then a new match is found by leaving the index i unchanged and decreasing index j by one. This process repeats until there are no offers left that exceed an acceptance level, hence $i \geq j$. If firm j accepts the offer by firm i, then the proposing firm pays the accepting firm the offer. The accepting firm receives a terminal payoff equal to the offer and exits the industry. The exit switches firm j's state to zero, $e_j^t = 0$.

Period payoffs. At the end of the period, all firms that remain active collect profits. The profit of a firm is described by a profit function, $\Pi(n)$, which maps the number of active firms into positive real numbers. I assume that an increase in the number of active firms reduces the profit function, $\Pi(n) > \Pi(n+1)$, and the sum of all possible profit realizations is finite, $\sum_{i=1}^{\infty} \Pi(i) < \infty$. The period payoffs may be determined by a quantity-setting Cournot game, a Bertrand game, or some other form of competition between active firms. The period payoff of an inactive potential firm is zero.

State transition. The transition of the state vector to the next period depends on the outcome of the merger game. Every potential firm i (including the entrant) that is active at the end of period t, $e_i^t > 0$, is an incumbent firm in the next period, $e_i^{t+1} = 1$. Every potential firm i that is inactive at the end of period t, $e_i^t = 0$, remains inactive in the next period, $e_i^{t+1} = 0$. An active firm that is acquired in period t exits the industry, $e_i^{t+1} = 0$, which means that the potential firm is inactive from then on.

Game payoff. I introduce a common discount factor parameter, $\beta \in (0, 1)$, as a measure of firms' patience with regard to future profit. The game payoff is defined as the discounted sum of future-period payoffs.

Strategies. In the first stage of the merger game, a strategy for active firm i is a real-valued offer, $B_i(\mathbf{s}^t, \mathbf{e}^t)$, that depends on the realization of the public signal and the state of all potential firms. The acceptance rule of active firm i in the second stage of the merger game is a real number $A_i(B_1, \ldots, B_{n+1}, \mathbf{s}^t, \mathbf{e}^t)$ that depends on the sequence of offers, the realization of the public signal, and the state of all potential firms. I consider pure strategies only. Further, I restrict attention to symmetric behavior. Under symmetry, the number of active firms, n^t, and the state of active firm i are a sufficient statistic for the state vector at the beginning of the merger game. Hence, I write the symmetric strategies as $A(B_1, \ldots, B_{n+1}, n^t, \mathbf{s}^t, e_i^t)$, $B(n^t, \mathbf{s}^t, e_i^t)$, where n^t denotes the number of active firms in the beginning of the merger stage and e_i^t indicates whether active firm i is an entrant, $e_i^t = 2$, or not, $e_i^t = 1$. Symmetry rules out that the identity of firms affects their behavior. I do permit asymmetries between incumbent firms and the entrant in the period of entry, measured by the state variable e_i to capture distinct behavior as in the examples in the Introduction.

The equilibrium. Following Maskin and Tirole (1997, 2001), I examine the set of Markov-perfect equilibria. The Markovian assumption allows me to abstract from calendar time, and subsequently I omit the time superscript.

Before proceeding with the analysis, I illustrate the profitability condition for static mergers.

□ **The static merger condition.** Salant, Switzer, and Reynolds (1983) derive the condition when an exogenous merger is profitable in the Cournot model with constant average cost and linear demand. This subsection describes the condition in my model.

I consider a more general demand function that includes the linear-demand function as a special case. My demand function is of the form $P(Q) = a - bQ^{\gamma}$ and $\gamma > 0$, which for $\gamma = 1$ is linear. Each active firm chooses a quantity level, q_i, to maximize profits, $[a - bQ^{\gamma} - c]q_i$, where c denotes the constant average cost. Industry production is the sum of individual quantity choices,

$Q = \sum q_i$. The first-order condition for optimal quantity choices yields $a - bQ^{\gamma} - \gamma bqQ^{\gamma}/Q - c = 0$. Summing over all active firms yields $Q^{\gamma} = n(a-c)/(nb + \gamma b)$. Substituting this expression into the demand equation and the equation for equilibrium production levels yields the following payoff, which I refer to in subsequent sections as the "Cournot payoff."

Cournot payoff. $\Pi^c(n) = \gamma[a-c]^{(1+\gamma)/\gamma} b^{-1/\gamma} n^{(1-\gamma)/\gamma}(n+\gamma)^{-(1+\gamma)/\gamma}$, with $\gamma > 0$. The function Π^c is an example of a payoff function that fits my Markovian framework. To see this, notice that the Markovian strategy space does not permit firms' quantity choices to affect future behavior. Hence, I could augment the period game of my dynamic game to include a Cournot game at the end of the period. The period output choices will not affect future play, and the Cournot output levels will be chosen leading to Cournot payoffs every period.

Salant, Switzer, and Reynolds (1983) show that an isolated (exogenous) merger, other than a merger in an industry with a small number of active firms, is not profitable. The following properties of the payoff function illustrate their result. (The proof of the properties and all subsequent proofs are in the Appendix.)

Properties of Cournot payoffs.

(i) Suppose $\gamma \geq 1$ and $n \geq 3$. Then $\Pi^c(n-1) < 2\Pi^c(n)$.

(ii) Suppose $0 < \gamma < 1$ and $n \geq 4$. Then $\Pi^c(n-1) < 2\Pi^c(n)$.

The properties show that an exogenous two-firm merger is profitable in concentrated industries only. The first property says that a merger by two firms in a concave-demand Cournot industry, $\gamma \geq 1$, with at least three firms is not profitable. The combined firm makes less profit than two stand-alone firms. The second property says that an exogenous merger in a convex-demand Cournot industry, $\gamma < 1$, with initially at least four firms is not profitable. Again, the combined firm makes less profit than two stand-alone firms.[7]

The measure of merger profitability based on a comparison of combined profits versus stand-alone profits is valid for an isolated exogenous merger only. The comparison may not be valid if mergers arise endogenously. The reason is that firms may expect additional mergers to take place following a merger. The next section characterizes endogenous merger equilibria in the dynamic game.

3. Equilibria

■ This section characterizes the set of equilibria. I provide the necessary and sufficient conditions for equilibria in which (i) no mergers take place and (ii) at least one merger takes place.

I begin the analysis with situations in which no mergers take place. Consider the continuation payoff of active firm i in period n when no mergers take place in this period and in future periods. Due to entry, the number of active firms will increase by one every period, and the sum of period profits, discounted to period n, equals $\Pi(n+1) + \beta \cdot \Pi(n+2) + \ldots$. This sum leads us to the following expression for the discounted sum of payoffs for active firm i:

$$W(n) = \sum_{t=1}^{\infty} \beta^{t-1} \Pi(n+t).$$

The sum $W(n)$ exists because, by assumption, the sum of profits $\sum_{t=1}^{\infty} \Pi(t)$ is finite and β is less than one.

The following proposition characterizes equilibria in which no mergers take place.

Proposition 1. There exists an equilibrium in which no mergers take place in this and future periods if and only if

$$W(n-1) \leq 2 \cdot W(n) \qquad \text{for all } n. \tag{1}$$

[7] The results differ for the exponential-demand function, $P(Q) = A \cdot \exp(-bQ)$. In this case, the profit function equals $\Pi^c(n) = (A/b) \cdot \exp(-n)$. A merger is profitable for any n.

The necessary and sufficient condition in Proposition 1 says that the continuation value of the combined firm cannot exceed the sum of continuation values of two stand-alone active firms in any period. The argument in the proof is the following: Suppose we are in an equilibrium with no mergers. Consider any period n. Each active firm receives a continuation value of $W(n)$, since there are no mergers. Hence, the reservation value in the second stage of the merger game equals $W(n)$ for all active firms. Next, consider the offer stage. A proposing firm can guarantee itself a continuation value of at least $W(n)$ by making an offer less than $W(n)$, which will be rejected. Suppose active firm i makes an offer equalling $W(n)$ or more. Then, the offer will be accepted and a merger takes place. The continuation value of the offering firm will then equal $W(n-1) - W(n)$. Now, by construction, the acquisition does not benefit the offering firm provided condition (1) is satisfied. Hence, in any equilibrium in which no mergers arise, condition (1) has to hold. Conversely, if condition (1) is satisfied, then the offers equalling zero and suitable acceptance rules can be shown to constitute a Markov equilibrium.

As an illustration of condition (1) for Cournot payoffs, consider the following example.

Example 1. Consider the linear-demand Cournot model with zero marginal costs. The properties of the Cournot payoffs, derived earlier, guarantee that the combined payoff $\Pi^c(n)$ exceeds the sum of stand-alone payoffs $2\Pi^c(n+1)$ in every period when there are three or more active firms. An isolated merger would make a loss in every period, and the inequality in condition (1) in Proposition 1 is satisfied for $n \geq 3$. With two active firms, the monopoly payoff exceeds twice duopoly payoffs. Whether the inequality in condition (1) applies in this case or not depends on two elements: (i) the relative magnitude of the first-period gain versus future losses, and (ii) the discount factor. As β approaches one, the continuation value becomes the sum of payoffs $\sum_{n=1}^{\infty} \Pi^c(n) = \sum_{n=1}^{\infty} [1/(n+1)^2]$, which is a negative power sum and equals $\pi^2/6 - 1$, or approximately .645. Now, $\Pi^c(1) = .25$, and I obtain that $.645 < 2[.645 - .25] = .79$. Hence, the inequality in condition (1) is satisfied for all n when β is close to one. It is an equilibrium that no merger takes place.

The example shows that a merger for monopoly in the linear-demand Cournot model is *not profitable* when firms are sufficiently patient. Hence, the Salant, Switzer, and Reynolds (1983) result fails to hold. The intuition is that in the long run, entry erodes short-term merger gains. A merger for monopoly achieves a one-time gain equal to monopoly profits minus twice duopoly profits. Thereafter, the merged firm makes losses. Stigler (1950) already describes the tradeoffs involved in a single merger for monopoly under entry. He considers a graphical analysis and emphasizes the role of the discount factor in the tradeoff between short-term gains and future losses due to entry for a merger for monopoly. My analysis confirms the intuition that with Cournot payoffs and sufficiently patient firms it is indeed an equilibrium that no merger takes place.

Condition (1) states that no coalition consisting of two active firms has an incentive to merge. A stronger requirement is that no coalition of active firms of any size $k \geq 1$ has an incentive to deviate,

$$W(n-1) \leq (1+k) \cdot W(n-1+k) \qquad \text{for all } k \geq 1 \text{ and all } n. \tag{2}$$

The stronger condition (2) implies that the no-merger outcome is a strong equilibrium, or a coalition-proof equilibrium.

For β close to one, the no-merger equilibrium characterized in Proposition 1 is in fact a strong equilibrium. To see this, notice that condition (1) can be written as $\Pi(n) \leq [1 + (1 - \beta)] \cdot \sum_{i=n+1}^{\infty} \beta^{i-(n+1)} \Pi(i)$ for all n. In the limit, as β approaches one, the condition reduces to $\Pi(n) \leq \sum_{i=n+1}^{\infty} \Pi(i)$ for all n. On the other hand, condition (2) becomes $\Pi(n) + \cdots + \Pi(n-1+k) \leq k \cdot \sum_{i=n-1+k}^{\infty} \Pi(i)$ for all k, n. Now, repeated substitution of condition (1) into condition (2) yields the desired result.

So far, I have characterized the necessary and sufficient condition for no merger to take place. Next, I characterize the necessary and sufficient condition a merger to take place. I begin the analysis with a general equilibrium existence result.

Proposition 2. An equilibrium exists.

The argument in the proof of Proposition 2 consists of two steps: First, I show that there exists an equilibrium in the period game for any continuation value. Second, I show that the equilibrium period payoff is consistent with the continuation payoff by using a fixed-point argument. I show that the set of expected period equilibrium profits is upper hemicontinuous and convex valued. Convexity follows from the publicly observable signal, as any randomization across equilibrium payoffs must be an equilibrium payoff. Then, by Kakutani's theorem, there exists a fixed point.

Proposition 2 combined with Proposition 1 characterizes a set of cases in which mergers arise. Specifically, mergers arise when condition (1) fails. Condition (1) fails if the continuation value of the merged firm exceeds the continuation value of the sum of two stand-alone active firms. The static analog of condition (1) played a prominent role in the literature studying welfare effects of "profitable" mergers in Levin (1990) and Farrell and Shapiro (1990). In this literature, an exogenous merger is considered profitable if the profit of the combined firm exceeds the sum of profits of stand-alone active firms. Notice, however, that the failure of condition (1) is a sufficient but not a necessary condition for a merger to take place in a dynamic setting. There may exist weaker conditions under which mergers can arise in equilibrium. Next I characterize the weakest conditions such that a merger can arise in equilibrium.

Before stating the result, I provide two lemmas.

Lemma 1. $W(n)$ is a lower bound for the equilibrium continuation value of active firms.

This lemma establishes that $W(n)$ is a lower bound on the continuation value of an active firm in any equilibrium. As has been shown by Abreu, Pearce, and Stacchetti (1986), the lower bound on the continuation value provides a useful tool to characterize the set of equilibrium paths and payoffs in repeated games. I use the minimum continuation value to explore in which states mergers can arise in the repeated game.

The second lemma considers the situation in which multiple mergers occur in a period.

Lemma 2. If there exists an equilibrium in which k mergers take place in a period that starts with n active firms, then the continuation value $V(n)$ satisfies:

$$V(n-k) \geq 2V(n+1-k). \tag{3}$$

Lemma 2 gives a necessary condition for mergers involving k active firms. It states that the continuation value with $n-k$ active firms has to exceed twice the continuation value with $n+1-k$ active firms.

I am now ready to state the main result.

Proposition 3. There exists an equilibrium in which a merger takes place if and only if

$$W(n-1) \geq \left[2 - \beta\frac{n-1}{n}\right] \cdot W(n) \qquad \text{for some } n. \tag{4}$$

Proposition 3 characterizes a necessary and sufficient condition for a merger to take place. Condition (4) says that the continuation value $W(n-1)$ has to exceed the continuation value of two stand-alone active firms, $2 \cdot W(n)$, minus the amount, $\beta[(n-1)/n] \cdot W(n)$, that depends on the discount factor and the number of active firms. Condition (4) is readily rewritten in terms of per-period payoffs:

$$\Pi(n) \geq 2W(n)(1-\beta) + \frac{\beta}{n}W(n) \qquad \text{for some } n. \tag{4a}$$

It has the interpretation that the per-period payoff, $\Pi(n)$, has to exceed the weighted average per-period payoff of the two stand-alone active firms, $2 \cdot W(n)(1-\beta)$, plus the expected per-period cost of future acquisitions, $\beta(1/n)W(n)$, where $1/n$ is the probability of being selected as a buyer.

Strategies that achieve the lower bound in Proposition 3 have the following features: In any period that begins with n firms, the entrant is acquired by an incumbent. A publicly observable flip of a coin decides which incumbent firm steps forward and acquires the entrant. Future realizations of the coin flip are random, and the burden of future mergers an individual incumbent firm has to carry is equally split. No acquisitions take place in periods that begin with $n' \neq n$ firms.

Along the equilibrium path, what is observed is that the number of active firms gradually increases from zero to n due to entry. From then on, in every period the entrant is acquired by an incumbent firm selected at random.

Condition (4) is weaker than the complement of condition (1). To see this, observe that the possibility of future mergers can enhance the benefits to an acquisition. Firms may expect high profits due to future mergers, while firms may expect low profits in the absence of future mergers due to entry. Thus, there is an additional reason for a merger. A merger can induce additional mergers in future periods.

Comparative statics for condition (4a) with respect to β reveal that an increase in β has two opposing effects on its right-hand side: First, the weighted average per-period payoff of a stand-alone active firm, $(1-\beta)\sum_{t=1}^{\infty}\beta^{t-1}\Pi(n+t)$, is reduced as the weight assigned to payoffs with a larger number of active firms is increased. Second, the expected per-period cost of future acquisitions, $(\beta/n)\sum_{t=1}^{\infty}\beta^{t-1}\Pi(n+t)$, is increased. Which of the two effects dominates is determined by the shape of the payoff function.

The following example illustrates condition (4) for Cournot payoffs.

Example 2. Consider the Cournot payoff function,

$$\Pi^c(n) = \gamma[a-c]^{\frac{1+\gamma}{\gamma}} b^{-\frac{1}{\gamma}} n^{\frac{1-\gamma}{\gamma}} (n+\gamma)^{-\frac{1+\gamma}{\gamma}},$$

described in Section 2.[8] Figures 2 and 3 report the region of the state variable n in which a profitable merger equilibrium exists. On the horizontal axis I vary the discount factor. The parameters are the following: $a = 10,000^{\gamma/(1+\gamma)}$, and $c = 0$. Figure 2 considers a linear-demand curve, $\gamma = 1$. Figure 3 considers a convex-demand curve with $\gamma = 1/2$. The lines characterize the smallest and largest n such that condition (4) holds. The region between the two lines denotes industry structures in which mergers can arise.

Somewhat surprisingly, Figure 2 shows that profitable mergers exist in the linear-demand Cournot model. Merger can be profitable provided the discount factor exceeds a threshold, $\beta >$ .966. Initially, mergers can arise in equilibrium when the number of active firms equals 6. As the discount factor increases, the range of states where mergers can arise equals $\{6, 7\}$, then $\{6, 7, 8\}$, and so on. The upper bound on the number of active firms such that a merger can arise in equilibrium increases very rapidly with the discount factor. The lower bound increases at a slower rate than the upper bound, at least initially, which creates a range of states such that a merger can arise. The lower bound equals 6 for a substantial range of the discount factor. For discount factors close to one, the lower bound increases at a fast rate.

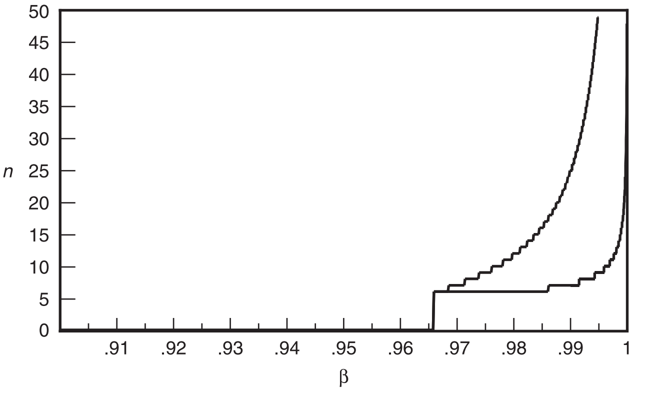
Figure 3 is qualitatively similar to Figure 2, although mergers arise already at a discount factor equal to .90. Both the upper and lower bounds on the number of active firms increase, as the discount factor increases. Initially, the upper bound increases at a faster rate than the lower bound, which creates a region of states in which mergers can arise. For discount factors exceeding .99, the region of mergers is very similar in Figures 2 and 3. Both figures illustrate also that the minimum number of active firms required to satisfy condition (4) increases with β.

There is a remaining issue as to whether multiple mergers or a single merger every period arise more easily. I define a merger cycle of length k as a path in which k mergers take place in some period n and are followed by $k-1$ periods without a merger. From the discussion following Proposition 3 we know already that the lower bound in condition (4) is achievable with a merger cycle of length 1, that is, a merger every period. Next, we must ask the question under what condition a merger cycle of length $k > 1$ can arise in equilibrium.

[8] Although the payoff function has an explicit expression, condition (4) is not easily verified. The reason is that, in general, the sum $W(n)$ has an untractable limit. For $\beta = 1$, the sum $W(n)$ corresponds to a negative power sum. For $\beta < 1$ these sums do not have explicit expressions. I thus examine condition (4) numerically.

FIGURE 2

$\alpha = 1$



Theorem 1. A merger cycle of length $k > 1$ occurs in equilibrium in some period n only if $W(n-k) \geq 2W(n-k+1)$.

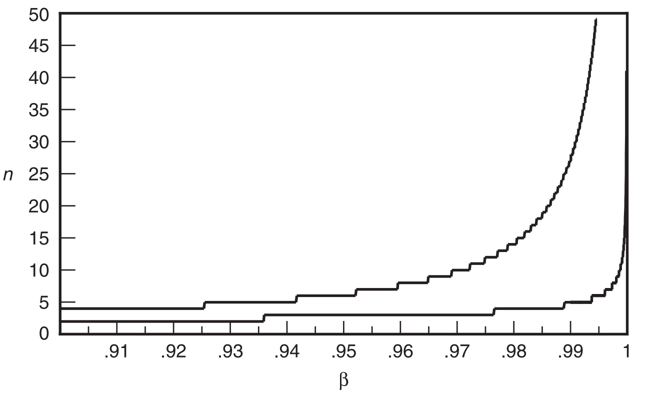
The theorem says that merger cycles of length $k > 1$ arise under stronger conditions than condition (4). They arise when the complement of condition (1) holds. The intuition is that there is a free-rider problem associated with merger cycles of length $k > 1$: Firms benefit from the acquisitions that rival firms are making. An active firm that is designated to acquire another active firm may also decide not to acquire now and to "free ride." If it does not acquire, then the number of active firms is reduced due to mergers by other active firms and cannot be distinguished from a situation in which the number of active firms after the mergers is increased due to entry. Thus, the free riding cannot be detected and the benefits of future mergers with multiple simultaneous mergers are limited. Mergers arise more easily when a single merger takes place every period.

The proof argument builds on the earlier results in Lemmas 1 and 2. Let $V(n)$ denote the continuation value of an active firm at the beginning of a period with n active firms. Since there is no merger in the period immediately following the mergers, it must be that $V(n-k) = \Pi(n-k+1) + \beta V(n-k+1)$. Using this equation for $V(n-k)$, condition (3) in Lemma 2 can be written as

$$\begin{aligned}\Pi(n-k+1) &\geq (2-\beta)V(n-k+1)\\ &\geq (2-\beta)W(n-k+1),\end{aligned}$$

FIGURE 3

$\alpha = 1/2$



where the second inequality uses Lemma 1. Since by definition $W(n-k) = \Pi(n-k+1) + \beta W(n-k+1)$, I can rewrite the outer inequality to obtain

$$W(n-k) \geq 2W(n-k+1),$$

which is the complement of condition (1) evaluated at $n-k$.

This section has characterized merger equilibria. The bounds on the number of active firms such that a merger can arise in equilibrium are derived. I have shown that a single merger every period arises more easily in equilibrium than multiple mergers followed by a period without merger. The next section examines the robustness of the results to changes in the assumptions and discusses merger welfare effects.

4. Discussion

■ This section first discusses the robustness of the results. Then I illustrate the welfare implications of endogenous merger decisions.

□ **Robustness.** In this subsection I analyze sunk entry costs, the strategy space, the definition of the merger game, and divestitures.

Entry cost. The model can be augmented by assuming that every entrant incurs a sunk cost F that is paid in the period of entry. Entry will then take place only if the expected continuation value of the entrant exceeds the entry cost. To see how this modification affects the results, consider the case in which no mergers take place. By monotonicity of the payoff function, it must be that there exists a $\overline{n}$ such that entry occurs only if $n < \overline{n}$. Hence, the continuation payoff for an active firm in the absence of mergers, with $n < \overline{n}$, becomes

$$\overline{W}(n) = \Pi(n+1) + \beta\Pi(n+2) + \ldots + \beta^{\overline{n}-n}\frac{\Pi(\overline{n})}{1-\beta}.$$

The condition characterizing no-merger equilibria in Proposition 1 remains valid when $W(n)$ is replaced with $\overline{W}(n)$. Similarly, there exists an equilibrium in which a merger takes place if condition (4) in Proposition 3 holds with $W(n)$ replaced by $\overline{W}(n)$.

Strategy space. The assumption of Markovian strategies rules out collusive behavior as a merger motive. I do not permit punishment as a merger enforcement device, say by charging a low price, if an active firm fails to merge. Permissible strategies are a function of state variables only. These variables include the number of active firms and whether an active firm entered this period or not.

Even a stricter state space definition, in which strategies depend on the number of active firms only and in which therefore the entrant and the incumbent are symmetric, yields qualitatively similar although weaker results. Under the modified strategy space, profitable mergers arise if the following condition holds:

$$W(n-1) \geq W(n) \cdot \left[2 - \beta\frac{n-1}{n+1}\right] \qquad \text{for some } n. \tag{5}$$

Condition (5) is qualitatively similar to but tighter than condition (4), implying that mergers are more difficult to sustain. The modification arises in the denominator of the last term in square brackets on the right-hand side, as the merger benefits are now split among $n+1$ active firms instead of n active firms as in condition (4). Hence, the distinctive behavior of incumbents and entrants mentioned in the introductory examples facilitates the occurrence of mergers.

Augmenting the state space by including for every firm an additional variable that measures the duration since the last acquisition yields a weaker but again qualitatively similar condition to condition (4). In this case, profitable mergers arise if the following condition holds:

$$W(n-1) \geq W(n) \cdot \left[2 - \beta + \beta^{n}\frac{1-\beta}{1-\beta^{n}}\right] \qquad \text{for some } n. \tag{6}$$

A comparison of conditions (4) and (6) reveals that the modified condition (6) is less tight than condition (4), permitting mergers to arise more easily. The reason is that active firms can alternate in acquiring the entrant. An incumbent incurs the merger cost every n periods only.

Merger stage. The two-stage offer game is not essential for the results. Indeed, in an earlier version, Pesendorfer (2000), I consider a bidding game in which every active firm simultaneously announces a bid and an ask price.[9] The existence condition of merger equilibria when the period game consists of a bidding game is identical to condition (4).

Relaxing the assumption that an active firm can acquire at most one active firm per period does not facilitate the occurrence of multiple mergers. The reason is that the free-rider problem remains. The active firm that is designated to make multiple acquisition has to be better off making the acquisitions than one less acquisition.

Divestiture. The model can be augmented to include divestiture decisions. Doing so can facilitate the occurrence of mergers. Divestitures can lead to profit erosion, which can be triggered if an active firm fails to make an acquisition. Thus, divestitures can have a stronger effect than entry, because divestitures can lead to competitive profit levels immediately, while entry leads to competitive profit levels gradually.

□ **Welfare.** This subsection discusses the relationship to the literature on merger welfare effects.

The welfare effects of mergers in my model are readily assessed. By assumption, a merger has an anticompetitive effect only, as it reduces the number of active firms. Hence, any merger reduces total welfare.

The seminal article by Farrell and Shapiro (1990) proposes a methodology for assessing merger welfare effects. The method is based on the external effects of a merger. The total merger welfare effect is decomposed into the insiders-profit effect and the external effect. The external effect equals the effect on the welfare of firms not participating in the merger and consumers. Farrell and Shapiro (1990, p. 116-117) motivate the decomposition in the following way:

> Privately unprofitable mergers will not be proposed, so proposed mergers should be permitted unless their external effects are “sufficiently” bad to outweigh their private profitability. In particular, if a proposed merger would have a beneficial external effect, then it should be allowed.

The external-effects argument by Farrell and Shapiro crucially relies on the premise that a merger is an isolated, exogenous, but profitable event. A profitable merger has the property that the profit of the merged firms (insiders) exceeds the sum of stand-alone profits before the merger. In my model, the isolated-merger condition corresponds most closely to the complement of condition (1).

My analysis shows that a merger can arise under much weaker conditions when it is not an isolated exogenous event. Indeed, the necessary and sufficient condition for a profitable endogenous merger is condition (4), which is weaker than the complement of condition (1). My analysis shows that the merger welfare assessment method proposed by Farrell and Shapiro (1990) may fail to give the correct assessment when merger decisions are endogenous.

The distinctions in welfare implications between my model and that of Farrell and Shapiro are most easily illustrated with an example. Consider the Cournot model in which active firms face a linear-demand curve and have identical constant marginal costs equal to zero. Suppose the intercept of the demand curve equals $a = 10,000$ and the slope coefficient equals $b = 1$. The first column in Table 1 reports the number of firms prior to the merger. Column 2 reports the net external effect using equation (16) in Farrell and Shapiro. Columns 3, 4, and 5 report the welfare

[9] The bidding game has the following features: If one or more bids exceeds the ask price of a firm, the following rationing rule is used to determine the winner. Firms are awarded to bidders in sequence according to their ask price beginning with the firm that submitted the lowest ask price. The highest bidder wins and a merger takes place at a price equal to the average between the bid and the ask price. This process repeats until there are no bids left that exceed an ask price. Ties of bids are resolved with a flip of a coin.

TABLE 1 Welfare Effect of a Merger

Number of Active Firms	Net External Effect	Average Per-Period Welfare Effect with β Equalling		
		.97	.98	.99
(1)	(2)	(3)	(4)	(5)
6	856.03	−856.21	−902.09	
7	742.67	−643.53	−681.35	−725.69
8	648.15		−531.29	−569.05
9	566.12		−424.84	−457.52
10	496.44		−346.74	−375.36
15	279.11			−171.36
20	176.39			−96.31

effect of the merger varying the discount factor. To illustrate the welfare effect of endogenous mergers, I select the equilibrium in which the entrant is acquired by an incumbent in every period. I report the difference in welfare between the equilibrium outcome in which a merger takes place every period and the equilibrium outcome in which no merger takes place in this and future periods. To make the welfare calculations comparable with the static net welfare measure, I report the average per-period welfare effect.

Table 1 shows that the net external welfare effect is positive for all mergers. It decreases as the number of active firms increases. A comparison between the net external effect and the welfare effect reveals that the welfare effect is of opposite sign but about the same magnitude as the external welfare effect. Missing entries in the table indicate that there is no equilibrium with profitable mergers. The welfare effect decreases in magnitude as the number of firms increases, and it increases in magnitude as the discount factor increases.

Table 1 illustrates that welfare implications of endogenous mergers can be of the opposite sign than is suggested by the existing merger literature. The reason is that the literature is built on the premise of an isolated and profitable merger. This premise need not be satisfied, as profitable mergers need not be isolated events.

5. Summary and conclusions

■ I examined a repeated game with endogenous merger decisions. The driving force of a merger is that it leads to additional mergers in the future. In other words, active firms are willing to carry the burden of an acquisition provided the future benefits are sufficiently large. This merger motive differs from the efficiency-improving argument emphasized in the merger literature.

The implications of my analysis are in contrast to implications obtained in a static Cournot model in a number of ways.

First, mergers in an industry with a number of oligopolists can be profitable even in the absence of merger efficiency gains. In particular, in the Cournot model with linear demand and constant average cost, mergers other than merger for monopoly can be profitable.

Second, mergers that are not profitable when there is a small number of active firms can become profitable as the number of active firms increases. This is in contrast to the results obtained by Salant, Switzer, and Reynolds (1983) for the same model in the absence of dynamics.

Third, the welfare effect of a profitable Cournot merger can be of the opposite sign than is suggested by the existing merger literature. Further, negative welfare effects can arise for Cournot mergers even in nonconcentrated industries.

Appendix

■ Proofs of the properties of Cournot payoffs, and proofs of Propositions 1–3, Lemmas 1–2, and Theorem 1 follow.

Proof of the properties of Cournot payoffs in Section 2. The properties can be seen by substituting the Cournot payoff function into the stated inequality, $\Pi^c(n) > 2\Pi^c(n+1)$ and cancelling. Doing so yields

$$\frac{n(n+\gamma+1)}{(n+1)(n+\gamma)} < \left[2 \cdot \frac{n(n+\gamma)}{(n+1)(n+\gamma+1)}\right]^{\gamma}.$$

Observe that the left-hand side of this inequality is less than one. Clearly, the right-hand side is bigger than one if the term in square brackets exceeds one, $2n(n+\gamma) > n(n+\gamma) + (n+1) + (n+\gamma+1)$. Rearranging this inequality yields $(n-1)(n+\gamma-1) > 1$. Since $\gamma > 0$, this inequality is satisfied for either $\gamma \geq 1$ and $n \geq 2$, or $0 < \gamma < 1$ and $n \geq 3$. *Q.E.D.*

Proof of Proposition 1. Consider any equilibrium in which no merger takes place in all periods. Consider any period that starts with n active firms. The expected sum of discounted payoffs equals $W(n)$ for all active firms. Thus, any offer in any period exceeding the expected discounted sum of payoffs, $B > W(n)$, will be accepted, and only an offer less than or equal to the expected discounted sum of payoffs,

$$B \leq W(n),$$

may be rejected. A potential buyer will not make an acquisition offer in excess of $W(n)$ provided

$$W(n-1) - B \leq W(n).$$

Combining the last two inequalities yields condition (1).

Next, suppose condition (1) is satisfied. I construct an equilibrium in which no merger takes place. The offer strategy is to bid zero in any state, $B = 0$. The acceptance rule has to specify which offers to accept and reject for all possible offer sequences, including offers that will not be chosen on the equilibrium path. To do so, I reorder the offers in descending order such that $B^{(1)} \geq B^{(2)} \geq \ldots \geq B^{(n+1)}$ and let $B^{(0)} = W(n+1)$. I rename active firms in accordance with their position in the offer ranking such that firm i submits offer $B_i = B^{(i)}$. I define the integer k^* as the largest number less than or equal to $(n+1)/2$ ($n/2$ if n is an even number) and with the property that the kth offer exceeds the continuation value when $k-1$ mergers take place:

$$k^* = \max\left\{k \in \left\{0, 1, \ldots \frac{n+1}{2}\right\} \;\middle|\; B^{(k)} \geq W(n+1-k)\right\}.$$

I specify the acceptance level as

$$A(B_1, B_2, \ldots B_n, B_{n+1}, n, s, e_i) = \begin{cases} W(n+1-k^*) & \text{if } i > n+1-k^* \\ W(n-k^*) & \text{otherwise.} \end{cases}$$

The acceptance strategy says that no offer will be accepted, $k^* = 0$, if $B^{(1)} < W(n)$. On the other hand, $k^* > 0$ offers are accepted if $B^{(k^*)} \geq W(n+1-k^*)$ and $B^{(k^*+1)} < W(n-k^*)$.

I need to establish that the strategies constitute an equilibrium. First, I verify the optimality of the acceptance rule. To do so, I begin by considering any firm i with $i \leq k^*$. By construction, this firm submits an offer $B_i \geq B^{(k^*)}$. By the rationing rule, firm i will not be chosen as an accepting firm. Hence, the acceptance level is optimal. Next, I consider any firm i with $n+1-k^* < i \leq n+1$. This firm receives $B^{(n+2-i)} \geq W(n+1-k^*)$. Any acceptance level less than or equal to $B^{(n+2-i)}$ yields the same payoff. If the firm increases the acceptance level above $B^{(n+2-i)}$, then the firm may not be acquired and will receive the continuation payoff of $W(n+1-k^*)$ instead. The acceptance rule is optimal. Finally, I consider any firm i with $k^* < i \leq n+1-k^*$. The firm is not acquired and receives a continuation value of $W(n-k^*)$. Lowering the acceptance level may result in the firm being acquired, but since $B^{(k^*+1)}$ is less than $W(n-k^*)$, the firm would be worse off.

Next, I verify the optimality of the offer strategy. If condition (1) holds with equality, then the proposing firm is indifferent between the offer $B = 0$ and an (acceptable) offer equal to $W(n)$. If condition (1) holds with a strict inequality, then the proposing firm prefers the offer $B = 0$ to any offer equal to or larger than $W(n)$. Hence, the strategies are optimal and constitute an equilibrium. *Q.E.D.*

Proof of Proposition 2. I show that an equilibrium exists in which the entrant and the incumbents use identical strategies. The proof consists of two parts: part (i) shows that for any continuation value there exists an equilibrium in the period game; part (ii) shows that there exists a consistent continuation value such that the period game equilibrium can be extended to a Markov equilibrium of the dynamic game.

Part (i). Suppose there are n active firms at the beginning of the period. Let the continuation value $V(n)$ measure the expected discounted sum of future payoffs evaluated at the beginning of period n. After the entry event there are $n+1$ active firms. To simplify notation, I denote with $\widehat{V}(n+1)$ the continuation value at the end of the merger game before payoffs are collected when there are $n+1$ active firms remaining:

$$\widehat{V}(n+1) = \Pi(n+1) + \beta V(n+1).$$

To show that an equilibrium exists in the stage game, I examine the stage game backward, beginning with the second stage of the merger game. I reorder the offers in descending order such that $B^{(1)} \geq B^{(2)} \geq \ldots \geq B^{(n+1)}$ and let $B^{(0)} = \widehat{V}(n+2)$. I rename active firms in accordance with their position in the offer ranking such that firm i submits offer $B^{(i)}$. I need to show that an optimal acceptance rule exists for all possible offer sequences including offers that will not be chosen on the equilibrium path. To do so, I define the integer k^* as the largest number less than or equal to $(n+1)/2$ ($n/2$ if n is an even number) such that the kth offer exceeds the continuation value when $k-1$ mergers take place:

$$k^* = \max\left\{k \in \left\{0, 1, \ldots \frac{n}{2}\right\} \,\middle|\, B^{(k)} \geq \widehat{V}(n+2-k)\right\}.$$

I specify the acceptance rule as

$$A(B_1, B_2, \ldots B_n, B_e, n, s, e_i) = \begin{cases} \widehat{V}(n+2-k^*) & i > n+1-k^* \\ \widehat{V}(n+1-k^*) & \text{otherwise.} \end{cases}$$

The acceptance strategy says that all offers are rejected, $k^* = 0$, if $B^{(1)} < \widehat{V}(n+1)$. On the other hand, $k^* > 0$ offers are accepted if $B^{(k^*)} \geq \widehat{V}(n+2-k^*)$ and $B^{(k^*+1)} < \widehat{V}(n+1-k^*)$.

The acceptance rule is optimal. To see this, consider firm i with $i \leq k^*$. By construction, firm i's offer satisfies $B_i \geq B^{(k^*)}$. By the rationing rule and given the acceptance rule of other firms, firm i will not be chosen as an accepting firm. Hence, the acceptance level is optimal. Consider any firm i with $n+1-k^* < i \leq n+1$. This firm receives $B^{(n+2-i)} \geq \widehat{V}(n+2-k^*)$. Any acceptance level less than $B^{(n+2-i)}$ yields the same payoff. If firm i increases the acceptance level to above $B^{(n+2-i)}$, then the firm receives the continuation payoff of $\widehat{V}(n+2-k^*)$ instead. Finally, consider firm i with $k^* < i \leq n+1-k^*$. The firm is not acquired, and it receives a continuation value of $\widehat{V}(n+1-k^*)$. Any acceptance level less than $\widehat{V}(n+1-k^*)$ can make the firm only worse off, as it then may accept an offer less than $\widehat{V}(n+1-k^*)$.

Next, I consider the offer stage. I need to show that an optimal offer strategy exists. Optimality of an offer requires that the continuation value minus the acquisition cost is at least as large as the continuation value in the absence of an acquisition for the acquiring firm. Hence, k offers are an equilibrium outcome if the kth acceptable offer is profitable and the $(k+1)$st acceptable offer is not profitable. The optimality condition for k mergers, with $1 \leq k < (n+1)/2$, leads to two inequalities:

$$\widehat{V}(n+1-k) - \widehat{V}(n+2-k) \geq \widehat{V}(n+2-k) \qquad \text{if } k \geq 1 \tag{A1}$$

$$\widehat{V}(n-k) - \widehat{V}(n+1-k) \leq \widehat{V}(n+1-k) \qquad \text{if } k < \frac{n+1}{2}. \tag{A2}$$

We may write the two preceding inequalities more compactly as

$$\widehat{V}(n+1-k) \geq 2\widehat{V}(n+2-k) \quad \text{and} \quad \widehat{V}(n-k) \leq 2\widehat{V}(n+1-k) \qquad \text{for } 1 \leq k < \frac{n+1}{2}. \tag{A3}$$

There are two boundary cases, $k = 0$ and $k = (n+1)/2$ (or $k = n/2$ if n is even). No merger takes place, $k = 0$, provided the continuation value evaluated at n firms minus an acceptable offer is less than the continuation value evaluated at $n+1$ firms:

$$\widehat{V}(n) \leq 2\widehat{V}(n+1). \tag{A4}$$

The maximal number of mergers, $k = (n+1)/2$ (or $k = n/2$ if n is even), is an equilibrium outcome, if the continuation value evaluated at $(n+1)/2$ firms minus an acceptable offer is at least as large as the continuation value evaluated at $(n+1)/2+1$ firms:

$$\widehat{V}\left(\frac{n+1}{2}\right) \geq 2\widehat{V}\left(\frac{n+1}{2}+1\right). \tag{A5}$$

An examination of the conditions (A3)–(A5) shows that at least one of the conditions must be satisfied for some k. To see this, I examine the conditions sequentially beginning with $k = 0$. If condition (A4) holds when $k = 0$, then the claim is established. Supposing otherwise, I observe that a violation of condition (A4) with $k = 0$ implies that the first inequality in condition (A3) holds when $k = 1$. If the second inequality in condition (A3) holds for $k = 1$, then the claim is established. Supposing otherwise, I observe that a violation of the second inequality in condition (A3) with $k = 1$ in turn implies that the first inequality in the condition holds when $k = 2$. Continuing the argument, I finally obtain the conclusion that a violation of the second inequality in condition (A3) for $k = (n-1)/2$ implies that condition (A5) must hold for $k = (n-1)/2$. Thus, at least one of the conditions (A3)–(A5) must be satisfied for some k. This leads to the conclusion that there exists an equilibrium in the period game for any continuation value function V.

Conditions (A3)–(A5) are weak inequalities. For any sequence of continuation values $V^j \rightarrow V$, if it is an equilibrium that k mergers take place for all V^j, then it is also an equilibrium that k mergers take place in the limiting continuation value V. The equilibrium correspondence is upper hemi continuous.

Part (ii). I extend the equilibrium of the stage game to an equilibrium of the dynamic game. I show that there exists a continuation value V consistent with the period-game equilibrium.

Suppose k acquisitions are made in a period that begins with n active firms. From the period game analysis I deduce that a buying firm receives $\widehat{V}(n+1-k) - \widehat{V}(n+2-k)$, a selling firm receives $\widehat{V}(n+2-k)$, and an active firm that is neither a buyer nor a seller receives $\widehat{V}(n+1-k)$. The *ex ante* expected period payoff in an equilibrium with k acquisitions, after cancelling, equals

$$t_k(n, V) = \frac{n+1-k}{n+1}\widehat{V}(n+1-k).$$

The set of equilibrium period payoffs is convex because the public signal implies that any mixture of equilibrium payoffs is again an equilibrium payoff. The set is given by

$$T(n, V) = \left\{ \sum_{k \in K(n)} \alpha_k t_k(n, V) \,\middle|\, \sum_{k \in K(n)} \alpha_k = 1 \text{ and } \alpha_k \geq 0 \right\},$$

where $K(n)$ denotes the set of the number of offers that are accepted in some equilibrium in the period game. I can define $T(V) = \{T(n, V)\}$. It is a mapping from the set of continuation values into the set of continuation values. The correspondence T is upper hemicontinuous and convex valued. By Kakutani's theorem, there exists a fixed point, $V = T(V)$. By construction, the fixed point and the equilibrium strategies in the period game constitute a Markov-perfect equilibrium. *Q.E.D.*

Proof of Lemma 1. I show that active firm i can guarantee itself at least $W(n)$. Consider the strategy where active firm i submits an acceptance level of $W(n)$ and an offer of zero for any state. If no mergers take place, then firm i receives exactly $W(n)$. Now suppose that in some period a merger occurs. I distinguish two cases: in case (i), firm i is not involved in any merger, and in case (ii), firm i is involved in some merger.

Case (i). Suppose firm i is not involved in any merger. Since $\Pi(n) > \Pi(n+1)$, any reduction in the number of firms due to merger can only increase per-period payoffs to firm i. The continuation value of firm i will be at least as large as $W(n)$.

Case (ii). If firm i is involved in some merger, then firm i cannot have made a loss. The reason is that if firm i was bought, then it received at least the acceptance level $W(n)$. Furthermore, if firm i purchased another firm, then the purchase price was zero and it did not cost anything to firm i. The result is a reduction in the number of firms which by the above argument leads to a continuation value of at least $W(n)$. *Q.E.D.*

Proof of Lemma 2. In any equilibrium with k mergers, the acceptance level of an acquired firm must equal at least $V(n-k+1)$, which is what the active firm will get if it rejects the offer. Hence, it must be that

$$B \geq V(n+1-k).$$

Similarly, the proposing firm cannot make a loss, and the equilibrium condition for the firm making an offer implies

$$V(n-k) - B \geq V(n+1-k).$$

Combining the two inequalities yields the statement in the lemma. *Q.E.D.*

Proof of Proposition 3. By Propositions 1 and 2, I can restrict attention to the case in which condition (1) is satisfied. The proof consists of two parts: part (i) constructs an equilibrium that satisfies condition (4), and part (ii) establishes that in any equilibrium in which a merger takes place, condition (4) is satisfied.

Part (i). I begin by describing the offer and acceptance strategies. Then I show that the strategies indeed constitute an equilibrium. The constructed equilibrium has the feature that every period, the public signal selects one incumbent firm that makes an acceptable offer to the entrant.

The offer strategies are the following: Let n' be the smallest n such that condition (4) is satisfied. The offer by active firm i is given by

$$B(n, s, e_i) = \begin{cases} W(n') & \text{if } n = n', e_i = 1 \text{ and } s_i = 1, \\ 0 & \text{otherwise}, \end{cases}$$

where the public signal s is a realization of an "n-dimensional coin flip," which means that $s_i \in \{0, 1\}$, $\sum_{i=1}^{n} s_i = 1$, and $\Pr(s_i = 1) = 1/n$. An incumbent firm receiving the signal $s_i = 1$ in period n' submits an offer equalling $W(n')$. All other active firms and the entrant submit offers equalling zero.

Before specifying the acceptance rule, I reorder the offers in descending order such that $B^{(1)} \geq B^{(2)} \geq \ldots \geq B^{(n+1)}$ and define $B^{(0)} = V(n+1)$ where the continuation value $V(n)$ equals

$$V(n) = \begin{cases} \pi(n+1) + \beta V(n+1) & \text{if } n < n', \\ \pi(n') - (1/n')W(n') + \beta V(n') & \text{if } n = n', \\ W(n) & \text{if } n > n'. \end{cases}$$

I need to specify an acceptance rule for all possible offer sequences, including offers that will not be chosen on the equilibrium path. To do so, I define the integer k^* as the largest number less than or equal to $(n+1)/2$ ($n/2$ if n is an even number) such that the offer exceeds the above continuation value for the marginal offer:

$$k^* = \max \left\{ k \in \left\{ 0, 1, \ldots \frac{n+1}{2} \right\} \middle| \; B^{(k)} \geq V(n+1-k) \right\}.$$

I define the acceptance level as

$$A(B_1, B_2, \ldots B_n, B_{n+1}, n, s, e_i) = \begin{cases} W(n) & \text{if } n = n', V(n) > B^{(1)} \geq W(n), e_i = 2, \text{and } B_i < B^{(1)}; \\ \overline{A}(i, n, k^*) & \text{otherwise,} \end{cases}$$

$$\text{with } \overline{A}(i, n, k^*) = \begin{cases} V(n+1-k^*) & \text{if } i > n+1-k^* \\ V(n-k^*) & \text{otherwise.} \end{cases}$$

By construction, $V(n)$ equals the *ex ante* expected continuation value of an incumbent firm when all firms adopt the above strategies. The acceptance rule specifies acceptance levels for all possible offer sequences. Along the equilibrium path, what is observed is that all firms make zero offers except in period $n = n'$. In period $n = n'$, one incumbent is selected at random to step forward and make the offer $W(n')$. The offer is accepted by the entrant but not accepted by other incumbent firms. Next, I establish that the described strategies are indeed an equilibrium.

I begin with an examination of the acceptance rule: I distinguish two cases depending on whether the first condition in the acceptance rule applies or not. First, consider the case in which the condition that says "if $n = n'$, $V(n) > B^{(1)} \geq W(n)$, $e_i = 2$, and $B_i < B^{(1)}$" does not apply for any firm. Any firm i with $i \leq k^*$ has submitted an offer $B_i \geq B^{(k^*)}$, and by the rationing rule, firm i will not be chosen as an accepting firm. Next, consider any firm i with $i > n+1-k^*$. This firm is acquired and receives $B^{(n+2-i)} \geq V(n+1-k^*)$. Any acceptance level less than or equal to $B^{(n+2-i)}$ yields the same payoff. If the firm increases the acceptance level above $B^{(n+2-i)}$, then the firm will not be acquired and receives the continuation payoff of $V(n+1-k^*)$ instead. Finally, consider firm i with $k^* < i \leq n+1-k^*$. The firm is not acquired and receives a continuation value of $V(n-k^*)$. Any acceptance level less than $V(n-k^*)$ can make the firm only worse off, as it may have to accept an offer less than $V(n-k^*)$.

Second, I consider the case in which the condition that says "if $n = n'$, $V(n) > B^{(1)} \geq W(n)$, $e_i = 2$, and $B_i < B^{(1)}$" applies for some firm. The acceptance rule of the entrant is optimal, as the continuation value in case of an offer rejection equals $W(n)$. The acceptance rule of incumbent firms is optimal, as $V(n) \geq W(n)$ by construction.

Now I consider the first stage of the merger game. I distinguish three cases depending on the number of firms in the beginning of the period: $n > n'$, $n < n'$, and $n = n'$. First, I consider the case $n > n'$. Only offers that exceed an acceptance level will be accepted. By condition (1), it would not be profitable for the offering firm to make an acceptable offer.

Second, I consider the case $n < n'$. Only offers that exceed the acceptance level $V(n)$ will be accepted. But such an offer is a profitable deviation for the offering firm only if $V(n-1) > 2V(n)$. The left-hand side equals $V(n-1) = \pi(n) + \beta V(n)$, and I obtain the condition

$$\begin{aligned} \pi(n) &> [2-\beta]V(n) \\ &\geq [2-\beta]W(n), \end{aligned}$$

where the second inequality uses Lemma 1. The condition $\pi(n) > [2-\beta]W(n)$ is exactly the complement of the inequality in condition (1), which yields the desired contradiction.

Third, I consider the case $n = n'$. An incumbent firm with signal $s_i = 1$ makes an offer equal to $W(n')$ and the offer will be accepted. The resulting payoff to the proposing firm equals $\pi(n') - W(n') + \beta V(n')$. Clearly, increasing the offer is not beneficial to the proposing firm. Lowering the offer results in a continuation value $W(n')$. By condition (4), the offer is optimal, as $\pi(n') - W(n') + \beta V(n') \geq W(n')$. Next I consider an incumbent firm with signal $s_i \neq 1$. Only an offer equal to at least $V(n'-1)$ would be accepted. But such an offer results in a loss to the offering firm by the argument under case $n < n'$. Finally, consider the entrant. Following the strategy the entrant receives $W(n')$. Consider a deviation: Only an offer of at least $\pi(n') - W(n') + \beta V(n')$ will be accepted. The deviation is not profitable, as the gains from making the offer $\pi(n') + \beta V(n') - [\pi(n') - W(n') + \beta V(n')]$ do not exceed $W(n')$. This completes the proof of part (i).

Part (ii). I show that condition (4) has to hold in any equilibrium. Suppose to the contrary, that is, there exists an equilibrium in which a merger takes place and

$$W(n-1) < 2 \cdot W(n) - \beta \frac{n-1}{n} W(n) \qquad \text{for all } n. \tag{A6}$$

In turn, Proposition 1 and Lemma 1 combined imply that the minimum continuation value equals $W(n)$ for any active firm in any equilibrium. Now, suppose a merger occurs and let n be the smallest integer in which at least one merger occurs. I distinguish two cases depending on the number of mergers that take place in period n: (a) single merger and (b) k simultaneous mergers. I obtain a contradiction to condition (A6) in both cases.

Case (a). Consider any equilibrium in which a single merger takes place when the state equals n. To permit the possibility that mergers are triggered randomly by the public signal, let α denote the *ex ante* probability that a merger occurs in period n. Let $V_i(n)$ denote the continuation value for active firm i when the number of active firms in the beginning of the period equals n. I can focus on the merger stage. Suppose the public signal designates active firm i as the acquiring firm and active firm j as the acquired firm. Firm i, which makes the acceptable offer for firm j with some probability, has to be better off making the offer, B_{ij}, than submitting a lower offer (that will be rejected). We may write this condition as

$$\Pi(n) + \beta V_i(n) - B_{ij} \geq \Pi(n+1) + \beta V_i(n+1). \tag{A7}$$

Similarly, firm j, which accepts the offer, has to prefer the acceptance level, A_{ji}, to the continuation value in the absence of a merger,

$$\begin{aligned} A_{ji} &\geq \Pi(n+1) + \beta V_j(n+1) \\ &\geq \Pi(n+1) + \beta W(n+1) \\ &= W(n). \end{aligned} \tag{A8}$$

The second inequality uses Lemma 1. The third line uses the definition $W(n) = \Pi(n+1) + \beta W(n+1)$.

For a merger to take place it has to be that the offer exceeds the acceptance level:

$$B_{ij} \geq A_{ji}. \tag{A9}$$

In equilibrium it must be that the offer equals the acceptance level, $B_{ij} = A_{ji}$. Otherwise, that is if $B_{ij} > A_{ji}$, firm i can profitably deviate by lowering the offer.

I next give an expression for the continuation value $V_i(n)$, which I then substitute into condition (A7). After the entry event there are $n+1$ active firms. Let P_i denote the probability that firm i is the buyer and let Q_i denote the probability that firm i is the seller. Observe that a firm can only be a buyer or a seller, $P_i + Q_i \leq 1$. Let $B_{i\cdot}$ and $A_{i\cdot}$ denote the average offer and acceptance level when firm i is designated as the buyer or seller. If there is a merger and firm i is the buyer, the average continuation value equals $\Pi(n) + \beta V_i(n) - B_{i\cdot}$. If there is a merger and firm i is the seller, then the average continuation value equals $A_{i\cdot}$. If there is a merger and firm i is neither a buyer nor a seller, then the continuation value equals $\Pi(n) + \beta V_i(n)$. Finally, the continuation value of firm i in the absence of a merger equals $\Pi(n+1) + \beta V_i(n+1)$. Since α denotes the probability of a merger, we can write the continuation value of firm i as

$$\begin{aligned} V_i(n) &= \alpha[P_i(\Pi(n) + \beta V_i(n) - B_{i\cdot}) + Q_i(A_{i\cdot}) + (1 - P_i - Q_i)(\Pi(n) + \beta V_i(n))] \\ &\quad + (1-\alpha)[\Pi(n+1) + \beta V_i(n+1)] \\ &= \frac{\alpha[Q_i A_{i\cdot} - P_i B_{i\cdot}] + \alpha(1 - Q_i)\Pi(n) + (1-\alpha)[\Pi(n+1) + \beta V_i(n+1)]}{1 - \alpha\beta(1 - Q_i)}. \end{aligned}$$

We may substitute the above expression into condition (A7). Moreover, since the acceptance level equals at least $W(n)$ by (A8) and the offer exceeds the acceptance level by (A9), we can substitute $W(n)$ for B_{ij}. Multiplying the resulting inequality by $1 - \alpha\beta(1 - Q_i)$ on both sides yields

$$\begin{aligned} \Pi(n) &\geq W(n)[1 - \alpha\beta(1 - Q_i)] + (\Pi(n+1) + \beta V_i(n+1))[1 - \alpha\beta(1 - Q_i) - \beta + \alpha\beta] \\ &\quad + \alpha\beta(P_i B_{i\cdot} - Q_i A_{i\cdot}) \\ &\geq W(n)[2 - \beta - \alpha\beta + 2\alpha\beta Q_i] + \alpha\beta(P_i B_{i\cdot} - Q_i A_{i\cdot}), \end{aligned}$$

where the second inequality uses $\Pi(n+1) + \beta V_i(n+1) \geq \Pi(n+1) + \beta W(n+1)$ from Lemma 1, and $\Pi(n+1) + \beta W(n+1)$ equals $W(n)$.

We may sum the above inequality over all active firms i that have some chance of being a buyer, $P_i > 0$. Doing so yields

$$\sum_{i,P_i>0} \Pi(n) \geq W(n)\left[\sum_{i,P_i>0}(2-\beta-\alpha\beta)+2\alpha\beta\sum_{i,P_i>0} Q_i\right]+\alpha\beta\left[\sum_{i,P_i>0} P_i B_{i\cdot}-\sum_{i,P_i>0} Q_i A_{i\cdot}\right].$$

Observe that the equilibrium condition (A9) implies that $B_{ij} = A_{ji}$, which in turn implies that the *ex ante* expected offer equals the *ex ante* expected acceptance level,

$$\begin{aligned}\sum_{i,P_i>0} P_i B_{i\cdot} &= \sum_{i,Q_i>0} Q_i A_{i\cdot} \\ &= \sum_{i,P_i>0} Q_i A_{i\cdot} + \sum_{i,P_i=0} Q_i A_{i\cdot}.\end{aligned}$$

We may use this relationship in the above inequality. Dividing by the sum $\sum_{i,P_i>0} 1$ on both sides of the inequality yields

$$\Pi(n) \geq W(n)\left[2-\beta-\alpha\beta+2\alpha\beta\frac{\sum_{i,P_i>0} Q_i}{\sum_{i,P_i>0} 1}\right]+\alpha\beta\frac{\sum_{i,P_i=0} Q_i A_{i\cdot}}{\sum_{i,P_i>0} 1}.$$

From (A8), $A_{i\cdot} \geq W(n)$, and I obtain

$$\Pi(n) \geq W(n)\left[2-\beta-\alpha\beta+2\alpha\beta\frac{\sum_{i,P_i>0} Q_i}{\sum_{i,P_i>0} 1}+\alpha\beta\frac{\sum_{i,P_i=0} Q_i}{\sum_{i,P_i>0} 1}\right].$$

The term in square brackets is minimized at $[2-\beta-\alpha\beta(n-1)/n]$, which yields condition (4) and, thus, violates condition (A6). To see this, notice that if $P_i > 0$ for all i, then the term involving $\alpha\beta$ in the square brackets equals $2/(n+1)-1$. If $P_i > 0$ for all but one i, then the term involving $\alpha\beta$ in the square brackets equals $1/n-1$. Finally, if $P_i > 0$ for m active firms, with $m < n$, then the term involving $\alpha\beta$ in the square brackets equals $1/m-1$. Thus, the expression is minimized at $1/n-1$. This yields the desired contradiction.

Case (b). Next, suppose $k > 1$ mergers occur in period n. Let $V_i(n)$ denote the continuation value of active firm i in state n. Since n is the smallest integer at which mergers occur, it must be that no mergers occur in periods $n-k$. Hence, $V_i(n-k) = \Pi(n-k+1)+\beta V_i(n-k+1)$. Using Lemma 2 yields

$$\begin{aligned}\Pi(n-k+1) &\geq (2-\beta)V_i(n-k+1) \\ &\geq (2-\beta)W(n-k+1),\end{aligned}$$

where Lemma 1 yields the second inequality. Since, by definition, $W(n-k) = \Pi(n-k+1)+\beta W(n-k+1)$, I obtain the condition

$$W(n-k) \geq 2W(n-k+1),$$

which is a contradiction to condition (A6). *Q.E.D.*

Proof of Theorem 1. Let $V(n)$ denote the continuation value. Since there is no merger in the period immediately following the period with k mergers, it must be that $V(n-k) = \Pi(n-k+1)+\beta V(n-k+1)$. Now condition (3) in Lemma 2 can be written as

$$\begin{aligned}\Pi(n-k+1) &\geq (2-\beta)V(n-k+1) \\ &\geq (2-\beta)W(n-k+1),\end{aligned}$$

where the second inequality follows from Lemma 1.

Since, by definition, $W(n-k) = \Pi(n-k+1)+\beta W(n-k+1)$, I can rewrite the outer inequality to obtain

$$W(n-k) \geq 2W(n-k+1).$$

Q.E.D.

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