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EC 533 Labour Economics
Problem Set 2
Answers

1. **(Signaling)**

   (a) Wages for high and low types will be given by

$$\begin{aligned} w_h &= \alpha_1 + \alpha_2 e_h \\ w_l &= \alpha_1 \end{aligned}$$

where e_h is the education level chosen by the high types in equilibrium. High types would like to maximize

$$\max_e w_h(e) - c_h(e)$$
$$\max_e \alpha_1 + \alpha_2 e - e^2.$$

The first order condition is

$$\begin{aligned} \alpha_2 - 2e'_h &= 0 \\ e'_h &= \frac{\alpha_2}{2}. \end{aligned}$$

For this to be a separating equilibrium, it is necessary that low types do not choose to obtain education, i.e.

$$\begin{aligned} w_h - w_l &\leq c_l(e_h) \\ \alpha_2 e_h &\leq \frac{3}{2}e_h^2 \\ \frac{2}{3}\alpha_2 &\leq e_h. \end{aligned}$$

The utility maximizing level of education e'_h for high types is too low, and they have to choose $e_h = (2/3)\alpha_2$ for a separating equilbrium to obtain. Choosing a higher level of education would be wasteful, so the efficient separating equilibrium is given by

$$\begin{aligned} e_h &= \frac{2}{3}\alpha_2 \\ e_l &= 0. \end{aligned}$$

   (b) Getting education has to be worthwhile for the high type, i.e.

$$\begin{aligned} w_h - w_l &> c_h(e_h) \\ \alpha_2 e_h &> e_h^2 \\ \alpha_2 &> e_h = \frac{2}{3}\alpha_2 \end{aligned}$$

which is clearly satisfied.

(c) Yes. In the perfect information case the high types would obtain the lower level of education e'_h. This level of education maximizes utility of the high types but it would be prefered by low types as well, if they could get the higher wage.

(d) Using the condition

$$\begin{aligned} w_h - w_l &\leq c_l(e_h) \\ \alpha_2 e_h &\leq 10e_h^2 \\ \frac{1}{10}\alpha_2 &\leq e_h \end{aligned}$$

again, we see that the perfect information level of education $e'_h = \alpha_2/2$ satisfies the condition for a separating equilibrium now. High types do not have to obtain an inefficiently high education level for a separating equilibrium to obtain. This is because education is so costly for the low types now.

(e) Since education is unproductive for the low types, in a separating equilibrium it is still true that

$$\begin{aligned} w_h &= \alpha_1 + \alpha_2 e_h \\ w_l &= \alpha_1. \end{aligned}$$

However, low types now have to obtain $\underline{e}$, and, hence, the condition for a separating equilibrium is now

$$\begin{aligned} w_h - c_l(e_h) &\leq w_l - c_l(\underline{e}) \\ w_h - w_l &\leq c_l(e_h) - c_l(\underline{e}) \\ \alpha_2 e_h &\leq \frac{3}{2}\left(e_h^2 - \underline{e}^2\right). \end{aligned}$$

In an efficient equlibrium, the education level of the high types will solve

$$\frac{3}{2}e_h^2 - \alpha_2 e_h - \frac{3}{2}\underline{e}^2 = 0$$

or

$$\begin{aligned} e_h &= \frac{\alpha_2 \pm \sqrt{\alpha_2^2 + 4\frac{3}{2}\frac{3}{2}\underline{e}^2}}{3} \\ &= \frac{\alpha_2 \pm \sqrt{\alpha_2^2 + 9\underline{e}^2}}{3}. \end{aligned}$$

The only positive education level is given by

$$= \frac{\alpha_2 + \sqrt{\alpha_2^2 + 9\underline{e}^2}}{3} > \frac{2\alpha_2}{3}$$

which is above the education level of the high types in part (a). Hence, compulsory education for the low types leads to an increase in education for the high types.

(f) In this case $y_l(e) = y_h(e)$, so the types do not differ in productivity. Hence, there is no reason for signalling. However, becasue of different costs, the two types of agents will get different levels of schooling. Low types maximize

$$\begin{aligned} &\max_e w_l(e) - c_l(e) \\ &\max_e \alpha_1 + \alpha_2 e - \frac{3}{2}e^2. \end{aligned}$$

The first order condition is

$$\begin{aligned} \alpha_2 - 3e_l &= 0 \\ e_l &= \frac{\alpha_2}{3}. \end{aligned}$$

High types maximize

$$\begin{aligned} &\max_e w_h(e) - c_h(e) \\ &\max_e \alpha_1 + \alpha_2 e - e^2. \end{aligned}$$

The first order condition is

$$\begin{aligned} \alpha_2 - 2e_h &= 0 \\ e_h &= \frac{\alpha_2}{2}. \end{aligned}$$

Both types get their first best levels of education in this case. This is higher than in (a) for the low types, because they have a positive return to education now, and lower than in (a) for high types, because they don't have to overinvest to signal.

(g) For separation, we need

$$\begin{aligned} w_h - c_l(e_h) &\leq w_l - c_l(e_l) \\ w_h - w_l &\leq c_l(e_h) - c_l(e_l) \end{aligned}$$

We have to check whether this level of education is worthwhile for the high types. The condition is

$$w_h - w_l > c_h(e_h) - c_h(e_l)$$

Putting these together yields

$$c_h(e_h) - c_h(e_l) < w_h - w_l \leq c_l(e_h) - c_l(e_l).$$

But $c_h(\bullet) = c_l(\bullet)$ here, so that this leads to a contradiction. No separating equilibrium exists.

(h) The return to schooling is given by

$$\frac{\ln w_h - \ln w_l}{e_h - e_l} \text{ or } \frac{(w_h - w_l)/w_l}{e_h - e_l}$$

Using the second expression yields

$$\begin{aligned} \frac{(w_h - w_l)/w_l}{e_h - e_l} &= \frac{(\alpha_1 + \alpha_2 e_h - \alpha_1)/\alpha_1}{e_h} \\ &= \frac{\alpha_2 e_h/\alpha_1}{e_h} \\ &= \frac{\alpha_2}{\alpha_1}. \end{aligned}$$

Using the ln expression and the $\ln(1+x) = x$ approximation yields the same result.