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EC 533 Labour Economics
Problem Set 3
Answers

1. **(Training With Labor-Market Frictions)**

   (a) The outside wage $v(t) = 1 + \tau - \theta$. The wage schedule $w(\tau)$ of the incumbent firm is a choice variable (well, function). So consider the firm's profits:

$$\pi = q(w(\tau))[1 + \tau - w(\tau)] - c(\tau).$$

The first order condition for the wage schedule is

$$\frac{\partial \pi}{\partial w} = \frac{\partial q}{\partial w}[1 + \tau - w(\tau)] - q(w(\tau)) = 0.$$

First find

$$\begin{aligned} q(w(\tau)) &= P(w(\tau) \geq 1 + \tau - \theta) \\ &= P(\theta \geq 1 + \tau - w(\tau)) \\ &= 1 - [1 + \tau - w(\tau)] \\ &= w(\tau) - \tau \end{aligned}$$

and hence

$$\frac{\partial q}{\partial w} = 1.$$

Now return to the first order condition for the wage schedule

$$\begin{aligned} \frac{\partial \pi}{\partial w} &= [1 + \tau - w(\tau)] - w(\tau) + \tau = 0 \\ 1 + 2\tau - 2w(\tau) &= 0 \\ w(\tau) &= \tau + \frac{1}{2}. \end{aligned}$$

The wage schedule is increasing in training and

$$w'(\tau) = 1.$$

Using this the quit function is

$$q(w(\tau)) = \tau + \frac{1}{2} - \tau = \frac{1}{2}.$$

   (b) Use the expression for profits again

$$\pi = q(w(\tau))[1 + \tau - w(\tau)] - c(\tau)$$

and look at the first order condition for training

$$\frac{\partial \pi}{\partial \tau} = \frac{\partial q}{\partial \tau}[1 + \tau - w(\tau)] + q(w(\tau))[1 - w'(\tau)] - c'(\tau) = 0$$

$$
\begin{aligned}
0\left[1+\tau-\tau-\frac{1}{2}\right]+\frac{1}{2}[1-1]-c'(\tau) &= 0 \\
-c'(\tau) &= 0.
\end{aligned}
$$

Training has the potential to contribute to profits in two ways here: it raises the productivity inside the firm (the second term in the FOC) and it could change the probability that a worker stays in the firm, and hence expected profit per worker at a given training level (the first term in the FOC). The second term in the FOC is zero because wages rise by the same amount as productivity. The first term is zero because $q(w(\tau))$ does not actually depend on τ. $w(t)$ and $v(t)$ increase in the same way with τ, so that the gap does not depend on training (and the probability of retaining a worker is constant). Since training does not contribute to profits, there is no sense in the firm bearing any of the costs.

(c) The firm's wage offer in $t=1$ remains $w(t)=\tau+1/2$. The worker pays for the training either directly or implictly through the wage in period $t=0$. In either case utility is

$$U=w(\tau)-c(\tau)$$

So the first order condition is simply

$$
\begin{aligned}
\frac{\partial U}{\partial \tau} &= w'(\tau)-c'(\tau)=0 \\
c'(\tau) &= 1.
\end{aligned}
$$

It is easy to see that this is the first best level of training. First best maximizes

$$1+\tau-c(\tau).$$

(d) The outside wage $v(t)=1+\tau(1-\theta)=1+\tau-\tau\theta$. The mobility cost is now $\tau\theta$, rather than just θ, so it increases with the level of training. This may happen, for example, because θ is a search cost and the market for more trained workers is thinner. The first order condition for wages is again

$$\frac{\partial \pi}{\partial w}=\frac{\partial q}{\partial w}[1+\tau-w(\tau)]-q(w(\tau))=0.$$

Now

$$
\begin{aligned}
q(w(\tau)) &= P(w(\tau)\geq 1+\tau(1-\theta)) \\
&= P(\tau\theta\geq 1+\tau-w(\tau)) \\
&= 1-\frac{1+\tau-w(\tau)}{\tau} \\
&= \frac{w(\tau)-1}{\tau}
\end{aligned}
$$

and

$$\frac{\partial q}{\partial w}=\frac{1}{\tau}.$$

Returning to

$$
\begin{aligned}
\frac{\partial \pi}{\partial w} &= \frac{1}{\tau}\left[1+\tau-w(\tau)\right]-\frac{w(\tau)-1}{\tau}=0 \\
1+\tau-w(\tau)-w(\tau)+1 &= 0 \\
w(\tau) &= 1+\frac{\tau}{2}.
\end{aligned}
$$

This implies

$$
q\left(w(\tau)\right)=\frac{1+\frac{\tau}{2}-1}{\tau}=\frac{1}{2}.
$$

So the first order condition for training is

$$
\frac{\partial \pi}{\partial \tau}=\frac{\partial q}{\partial \tau}\left[1+\tau-w(\tau)\right]+q\left(w(\tau)\right)\left[1-w'(\tau)\right]-c'(\tau)=0
$$

$$
\begin{aligned}
0\left[1+\tau-\tau-\frac{1}{2}\right]+\frac{1}{2}\left[1-\frac{1}{2}\right]-c'(\tau) &= 0 \\
c'(\tau) &= \frac{1}{4}.
\end{aligned}
$$

Increasing training now raises the profit earned from each worker who remains in the firm because wages rise by less than productivity with training (the inside wage schedule is compressed compared to productivity). Training still does not affect the probability of retaining a worker. It is also easy to see that the firm underinvests compared to first best because the firm is not the full residual claimant ($w'(\tau)>0$).

2. **(General and Specific Training Investments):**

   (a) A competitive outside market implies:

   $$
   v(\tau)=g(\tau,s)=1+\tau.
   $$

   Because the incumbent firm moves first, in order to retain the worker it has to offer exactly $v(\tau)$. Hence

   $$
   w(\tau,s)=v(\tau)=1+\tau
   $$

   and workers will stay at the firm in equilibrium. The firm has all the bargaining power in this set up.

   (b) If the firm does not invest $\tau=0$ and $w(0,s)=1$. The workers maximize

   $$
   \begin{aligned}
   U &= w(0,s)-s^2=1-s^2 \\
   &\Rightarrow\ s=0.
   \end{aligned}
   $$

   This is a typical hold up problem. The workers have to make their investment choice and the firm gets to set the wage afterwards. it extracts all the rents from the specific investment. Workers realize this and will therefore not make any investments.

(c) If the worker does not invest $s = 0$. Output $f(\tau, 0) = 1 + \tau$ and the wage is $w(\tau, 0) = 1 + \tau$. Hence profits are

$$\begin{aligned} \pi &= f(\tau, 0) - w(\tau, 0) - \tau^2 \\ &= 1 + \tau - (1 + \tau) - \tau^2 \\ &= -\tau^2. \end{aligned}$$

As a result $w(0, 0) = 1$. The firm does not reap any benefits from training because it faces a competitive outside market. Hence it does not invest in training; the standard Becker result.

(d) Start from the last period where wages are set. As before

$$w(\tau, s) = v(\tau) = 1 + \tau.$$

The firm's profits in period 2 are

$$\begin{aligned} \pi &= f(\tau, s) - w(\tau, s) - \tau^2 \\ &= (1 + \tau)(1 + s) - (1 + \tau) - \tau^2 \\ &= (1 + \tau) s - \tau^2 \end{aligned}$$

and the first order condition is

$$\begin{aligned} \frac{\partial \pi}{\partial \tau} &= s - 2\tau = 0 \\ &\Rightarrow \tau = \frac{s}{2}. \end{aligned}$$

Workers decide on their level of specific training in the first period. To them the training investment of the firm depends now on the specific investment they make, $\tau = \tau(s)$. This leads to the wage schedule

$$w(\tau, s) = w(\tau(s), s) = 1 + \tau(s) = 1 + \frac{s}{2}.$$

Hence

$$U = w(\tau, s) - s^2 = 1 + \frac{s}{2} - s^2.$$

The first order condition is

$$\begin{aligned} \frac{\partial U}{\partial s} &= \frac{1}{2} - 2s = 0 \\ &\Rightarrow s = \frac{1}{4} \\ &\Rightarrow \tau = \frac{1}{8}. \end{aligned}$$

Both the firm and the worker invest now. By making a specific investment, the worker creates a rent for the firm. Because $f(\tau, s) = (1 + \tau)(1 + s)$, the firm can increase this rent by investing in general training. This raises the outside wage but not by the full amount. General and specific skills are complementary and the firm captures this complementarity because it only pays the worker for the part of the skills valuable in the outside market. The workers realize this and hence invest in specific skills.

(e) If $f(\tau, s) = 1 + \tau + s$, there is no complementarity between specific and general skills, and this does not work. Investing in specific skills does not give rise to wage compression, and hence does not lead to general investment by the firm. The firm's profit is

$$\begin{aligned} \pi &= f(\tau, s) - w(\tau, s) - \tau^2 \\ &= 1 + \tau + s - (1 + \tau) - \tau^2 \\ &= s - \tau^2 \end{aligned}$$

so there is no payoff to the firm from investing in τ. But if $\tau = 0$, $w(\tau, s) = 1$, and the worker is in the same situation as in (b).