The LATE Theorem and Returns to Schooling

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LSE

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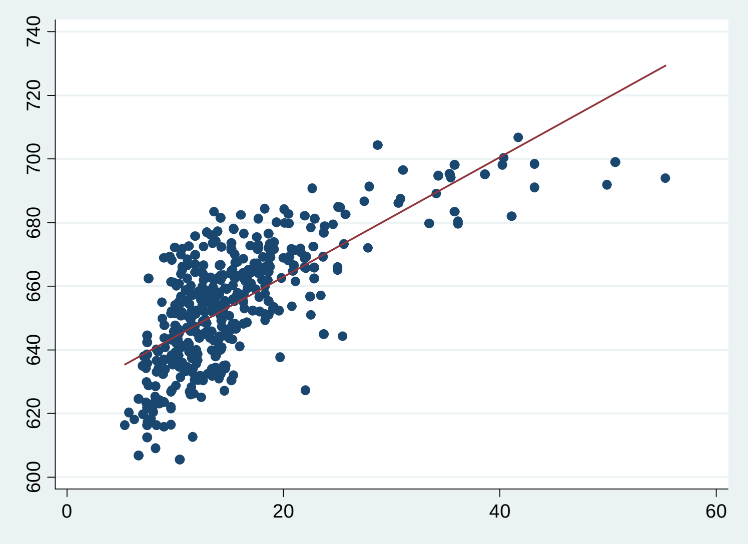
Linear regression produces an average slope



What does linear 2SLS produce?

Suppose returns to schooling are heterogeneous in the population. IV will produce an average of these returns, but what average?
To consider this, we will

- focus on the simplest case: a dummy endogenous variable, a single dummy instrument, and no covariates (2SLS is the Wald estimator)
- need some assumptions
- need some notation to state these assumptions

Recap counterfactual outcomes notation

- Scholing for individual i is described by $D_i = \{0, 1\}$, say $D_i = 1$ denotes completing high school, and $D_i = 0$ denotes dropping out.
- The outcome of interest, log earnings is denoted by y_i.
- Potential outcomes for earnings given schooling choices (D_i) are

$$\textit{potential outcomes} = \begin{cases} \mathrm{Y}_{1i} & \text{if } D_i = 1 \\ \mathrm{Y}_{0i} & \text{if } D_i = 0 \end{cases} .$$

More notation for the IV problem

- Introduce an instrument
$$Z_i = \{0, 1\}$$
which shifts schooling, e.g. $Z_i = 1$ denotes being born in the fourth quarter.
- D_i, schooling, now takes on the role of outcome of the process of being treated with the instrument.
- Hence we index D_i in terms of potential outcomes against Z_i

$$\textit{potential outcomes for schooling} = \begin{cases} \mathrm{D}_{1i} & \text{if } Z_i = 1 \\ \mathrm{D}_{0i} & \text{if } Z_i = 0 \end{cases} .$$

More notation for the IV problem

The ultimate outcome of interest, earnings, is now indexed against two treatments, schooling and the instrument value $\mathrm{Y}_i(d, z)$. There are four potential outcomes for earnings:

$$\textit{potential earnings} = \begin{cases} \mathrm{Y}_i(1,1) & \text{if } D_i = 1, Z_i = 1 \\ \mathrm{Y}_i(1,0) & \text{if } D_i = 1, Z_i = 0 \\ \mathrm{Y}_i(0,1) & \text{if } D_i = 0, Z_i = 1 \\ \mathrm{Y}_i(0,0) & \text{if } D_i = 0, Z_i = 0 \end{cases}.$$

Assumptions for the IV problem

We will need four assumptions for the IV problem:

- Assumption 1: Independence of the instrument

$$\left\{\mathrm{Y}_i\left(\mathrm{D}_{1i},1\right),\mathrm{Y}_i\left(\mathrm{D}_{0i},0\right),\mathrm{D}_{1i},\mathrm{D}_{0i}\right\} \amalg Z_i$$

- Assumption 2: Exclusion restriction

$$\mathrm{Y}_i(d,0) = \mathrm{Y}_i(d,1) \equiv \mathrm{Y}_{di} \text{ for } d = 0,1$$

- Assumption 3: First stage

$$E[\mathrm{D}_{1i} - \mathrm{D}_{0i}] \neq 0$$

- Assumption 4: Monotonicity

$$\mathrm{D}_{1i} - \mathrm{D}_{0i} \geq 0 \quad \forall i, \text{or vice versa}$$

The LATE Theorem

Theorem

THE LATE THEOREM.
Under assumptions 1 - 4:

$$\frac{E[y_i|Z_i=1]-E[y_i|Z_i=0]}{E[D_i|Z_i=1]-E[D_i|Z_i=0]} = E[\mathrm{Y}_{1i}-\mathrm{Y}_{0i}|\mathrm{D}_{1i}>\mathrm{D}_{0i}]$$

Proof.

Start with the first bit of the Wald estimator:

$$\begin{aligned} E[y_i|Z_i &= 1] \\ &= E[\mathrm{Y}_{0i}+(\mathrm{Y}_{1i}-\mathrm{Y}_{0i})D_i|Z_i=1] \quad \text{by the exclusion restriction} \\ &= E[\mathrm{Y}_{0i}+(\mathrm{Y}_{1i}-\mathrm{Y}_{0i})\mathrm{D}_{1i}] \quad \text{by independence} \end{aligned}$$

□

The LATE Theorem

Proof.

Similarly

$$E[y_i|Z_i = 0] = E[\text{Y}_{0i} + (\text{Y}_{1i} - \text{Y}_{0i})\text{D}_{0i}]$$

So the numerator of the Wald estimator is

$$E[y_i|Z_i = 1] - E[y_i|Z_i = 0] = E[(\text{Y}_{1i} - \text{Y}_{0i})(\text{D}_{1i} - \text{D}_{0i})]$$

Monotonicity means $\text{D}_{1i} - \text{D}_{0i}$ equals one or zero, so

$$E[(\text{Y}_{1i} - \text{Y}_{0i})(\text{D}_{1i} - \text{D}_{0i})] = E[\text{Y}_{1i} - \text{Y}_{0i}|\text{D}_{1i} > \text{D}_{0i}]P[\text{D}_{1i} > \text{D}_{0i}].$$

A similar argument shows

$$E[D_i|Z_i = 1] - E[D_i|Z_i = 0] = E[\text{D}_{1i} - \text{D}_{0i}] = P[\text{D}_{1i} > \text{D}_{0i}].$$

□

What does this mean?

Under the assumptions of the LATE theorem, the Wald estimator equals

$$E\left[\mathrm{Y}_{1i} - \mathrm{Y}_{0i} \middle| \mathrm{D}_{1i} > \mathrm{D}_{0i}\right]$$

i.e. the average treatment effect for a group defined by $\mathrm{D}_{1i} > \mathrm{D}_{0i}$. What is that group?
Since D_i is zero or one,

$$\mathrm{D}_{1i} > \mathrm{D}_{0i} \Leftrightarrow \left\{\mathrm{D}_{1i} = 1, \mathrm{D}_{0i} = 0\right\}.$$

This means $\mathrm{D}_{1i} > \mathrm{D}_{0i}$ is the group of individuals for whom the instrument changes the schooling decision.
IV with heterogenous returns estimates the return to schooling for that subgroup of the population.

The complier subpopulation

There are four possible responses to a change in the value of Z_i:

	$D_{0i} = 0$	$D_{0i} = 1$
$D_{1i} = 0$	$\underbrace{Y_i(0,1) - Y_i(0,0) = 0}_{\text{Never taker}}$	$\underbrace{Y_i(0,1) - Y_i(1,0)}_{\text{Defier}}$
$D_{1i} = 1$	$\underbrace{Y_i(1,1) - Y_i(0,0)}_{\text{Complier}}$	$\underbrace{Y_i(1,1) - Y_i(1,0) = 0}_{\text{Always taker}}$

- The schooling of never takers and always takers does not change when the instrument gets switched on and off (by the exclusion restriction): they don't contribute to the IV estimate.
- Defiers and compliers contribute, the IV estimate is the sum of those two effects
  - Compliers get more schooling with the instrument switched on
  - Defiers get less schooling with the instrument switched on: perverse group (ruled out by monotonicity)

Finally: What's LATE?

- The LATE theorem says that (under the assumptions) IV estimates the average causal effect of treatment on the subpopulation of compliers.
- LATE stands for *Local average treatment effect*: the average is not for all treated, but for an instrument specific subpopulation.
- You cannot identify individual compliers in the data (but you can characterize their attributes: see MHE section 4.4.4).
- Different instruments will produce different LATEs depending on the complier group.
    - Corrolary: with heterogeneous treatment effects, there is no over-id testing. Over-id tests might establish whether different LATEs are the same.
    - External validity of IV estimates more questionable (is the same LATE going to apply to another population?)

The LATE cup

Different applied econometrician have different views on LATE:

- Angrist, Imbens, and others: The LATE cup is half full. We don't get the average effect on a stable population (like the average treatment effect on the treated) but we get something that still makes some sense (particularly for policy).
- Heckman, Deaton, and others: The LATE cup is half empty (or all empty!): We don't get what we want, the average treatment effect on the treated, so IV is somewhat useless, and we need to augment it with something else to produce economically meaningful parameters (e.g. more structural econometric models).

Generalizing LATE

The LATE result is special. It could be extended in multiple directions (see MHE, section 4.5):

- to multivalued endogenous regressors
- to multiple instruments
- to IV with covariates

These extensions are often not completely straightforward but the basic LATE insights seem to survive in all cases.

- For example in the case of a multivalued endogenous regressor, e.g. years of schooling, IV produces first a series of LATEs for particular values of schooling (e.g. going from 10 to 11 years) and then averages all these LATEs for all the possible values of schooling for an individual.