Solving and analyzing DSGE models
TI field course 2010
Third homework

1 Administrative stuff:

- Due date: Friday December 31st
- You can work in groups
- The idea is *not* to just write the computer programs. Act as a researcher and write something that a responsible researcher would write if this was part of a real research project.

2 Objective

The objective is to learn that the tools you learned in this course can also be used to solve models with heterogeneous beliefs. In particular, we will look at models in which there are agents that are not rational and agents that are. "Our" rational agents are truly rational and take into account the presence of both the other rational and the irrational agents in the economy. In contrast, there are numerous models in the literature in which there are agents that are referred to as rational agents, but these agents just know a bit more than other agents. Typically these agents have some key information like the fundamental value of an asset, but also ignore aspects of the model that the model maker does have, that is, they are not rational. You'll see that solving for the behavior of the rational agents is not necessarily that difficult.

3 Model

Production function. The economy consists of a unit mass of risk neutral firms. In this version of the model, firms are either "rational" firms or "type A" firms *and* their type never changes. Moreover, half of them is rational and half of them is not. Technology is the same for all firms. The idea is that firms have to allocate labor to two different production lines and both face their own random productivity levels. The tricky part is that firms have to choose period $t+1$ employment levels in period t. Thus expectations play a key role. The production function is given by

$$y_i = z_i \left(z_1 n_{1,-1}^{\alpha} + z_2 n_{2,-1}^{\alpha} \right),$$

where z_i is an idiosyncratic productivity shock, z_1 and z_2 are aggregate productivity shocks, and $n_{1,-1}$ and $n_{2,-1}$ are the levels of employment used in production processes 1 and 2, respectively. These are set in period $t-1$.

Exogenous random variables. The laws of motion for the three productivity levels are given by

$$\begin{aligned} z_i &= 1-\rho_i+\rho_i z_{i,-1}+e_i \\ z_1 &= 1-\rho_1+\rho_1 z_{1,-1}+e_1 \\ z_2 &= 1-\rho_2+\rho_2 z_{2,-1}+e_2 \end{aligned}$$

Firm problems Firms face a quadratic adjustment cost in adjusting the firm employment level. That is, firms can costlessly switch labor from one process to another, but there is a cost in changing the total employment level. Rational firms solve the following problem

$$v(n_{1,-1},n_{2,-1},z)=\max_{n_1,n_2} \begin{array}{c} z_i\left(z_1 n_{1,-1}^{\alpha}+z_2 n_{2,-1}^{\alpha}\right)-w(n_{-1})-0.5\eta(n-n_{-1})^2 \\ +\beta \mathrm{E}_t\left[v(n_1,n_2,z_{+1})\right] \end{array}$$

where

$$n=n_1+n_2.$$

The set of first-order conditions is given by

$$\begin{aligned} \alpha\beta \mathrm{E}\left[z_{i,+1}\right]\mathrm{E}\left[z_{1,+1}\right]n_1^{\alpha-1}-\beta \mathrm{E}\left[w_{+1}\right]-\eta\left(n-n_{-1}\right)+\eta\beta \mathrm{E}\left[n_{+1}-n\right] &= 0 \\ \alpha\beta \mathrm{E}\left[z_{i,+1}\right]\mathrm{E}\left[z_{2,+1}\right]n_2^{\alpha-1}-\beta \mathrm{E}\left[w_{+1}\right]-\eta\left(n-n_{-1}\right)+\eta\beta \mathrm{E}\left[n_{+1}-n\right] &= 0 \end{aligned}$$

The first-order conditions for type A firms are assumed to be identical, except that the forecasts are not made using the true conditional expectation. Thus,

$$\begin{aligned} \alpha\beta \widehat{\mathrm{E}}\left[\hat{z}_{i,+1}\right]\widehat{\mathrm{E}}\left[z_{1,+1}\right]\hat{n}_1^{\alpha-1}-\beta \widehat{\mathrm{E}}\left[w_{+1}\right]-\eta\left(\hat{n}-\hat{n}_{-1}\right)+\eta\beta \widehat{\mathrm{E}}\left[\hat{n}_{+1}-\hat{n}\right] &= 0 \\ \alpha\beta \widehat{\mathrm{E}}\left[\hat{z}_{i,+1}\right]\widehat{\mathrm{E}}\left[z_{2,+1}\right]\hat{n}_2^{\alpha-1}-\beta \widehat{\mathrm{E}}\left[w_{+1}\right]-\eta\left(\hat{n}-\hat{n}_{-1}\right)+\eta\beta \widehat{\mathrm{E}}\left[\hat{n}_{+1}-\hat{n}\right] &= 0 \end{aligned}$$

A circumflex above a variable indicates it is a variable of a type A firm. Similarly for the expectations operator. Type A firms think that there is no autocorrelation in the values of z_1, z_2, and w. This means that type A firms do no longer respond to aggregate productivity shocks. They are assumed to be rational about their forecasts of idiosyncratic productivity and their own future employment levels.

Labor supply We assume that labor supply is determined by the following labor supply equation

$$w=\omega_0+\omega_1\left(N_{-1}-N_{ss}\right)$$

where N is the per capita employment level. Thus

$$N=\frac{\int n_i di+\int \hat{n}_i di}{2}$$

and N_{ss} equal to the steady value of N. We set

$$\omega_0=\alpha 0.5^{\alpha-1}$$

This ensures that in the steady state

$$\begin{aligned} n_1 &= n_2 = \hat{n}_1 = \hat{n}_2 = 0.5 \\ w &= \omega_0 \\ N &= 1 \end{aligned}$$

4 Equilibrium

The equilibrium consists of

1. a set of policy rules for both types of agents
2. a law of motion for aggregate employment
3. a wage rule

such that

1. the policy rules for each type of agent solve their optimization problem given the wage rate and the law of motion for aggregate employment
2. the aggregated individual policy rules add up to the assumed law of motion for aggregate employment.
3. the labor market is in equilibrium

5 Programs to write

The idea is solve this model using XPA and to solve the individual problem with Dynare. This means you have to write a *.mod file that solves for the individual policy rule of the rational agent. You also have to write a matlab program that will take the individual policy rules of the rational agent as the input to calculate the law of motion of the aggregate variables by explicit aggregation across the heterogeneous agents.

Hint: The *end* objective is to have a model with both rational and non-rational agents. When writing programs you typically want to move slowly. That is, you first want to solve simpler models. Think of the right sequence of models to solve.

6 What to do?

1. Describe how you solved the model. You do not have to write down stuff that is generally known (e.g. how perturbation analysis works). But you have to write it up so that somebody else who is trained in numerical techniques can exactly replicate your programs without having taken a look at your programs.

2. Describe the differences between the model in which all agents are rational and the model in which there are both rational and non-rational agents. Think as a researcher, be creative *and* be responsible.