Dynare UvA - Day 1 - Additional Assignments

October 9, 2008

1 Exercise #1

Consider the standard growth model

$$
\begin{aligned}
c_t^{-\nu} &= \mathrm{E}_t\left[\beta c_{t+1}^{-\nu}(\alpha \exp(z_{t+1})k_t^{\alpha-1} + 1 - \delta)\right] \\
c_t + k_t &= \exp(z_t)k_{t-1}^{\alpha} + (1-\delta)k_{t-1} \\
\ln(z_t) &= \rho \ln(z_{t-1}) + \varepsilon_t \\
& \varepsilon_t \sim N(0, \sigma^2)
\end{aligned}
$$

Parameter values:

- $\nu = 7$; $\beta = 0.99; \alpha = 0.33; \delta = 0.025; \rho = 0.95; \sigma = 0.1$. Note that the standard deviation as well as the the risk-aversion are chosen to be extremely (unrealistically) high

Exercises

1. Solve this model with Dynare *four* different ways, namely
   (a) a first and second-order solution in the *levels* of k_t and c_t
   (b) a first and second-order solution in the *levels* of $\ln(k_t)$ and $ln(c_t)$
2. Compare the four policy rules. For example, plot k_t as a function of k_{t-1} for a fixed value of z_t.

3. Compare the generated business cycle statistics. Do they differ?

4. Increase the value of σ. What happens with $\mathrm{E}k_t$ for the four different solutions?

2 Exercise #2

In this exercise, we replace the standard productivity shock by an investment shock.

$$\max_{\{c_t,k_t\}} \mathrm{E} \sum_{t=1}^{\infty} \beta^{t-1} \frac{c_t^{1-\nu}-1}{1-\nu}$$

s.t.

$$c_t + i_t = k_{t-1}^{\alpha}$$

$$k_t = \exp(z_t) i_t + (1-\delta) k_{t-1}$$

$$\ln(z_t) = \rho \ln(z_{t-1}) + \varepsilon_t$$

$$k_0 \text{ given}, \;\; \mathrm{E}_t[\varepsilon_{t+1}] = 0 \; \& \; \mathrm{E}_t[\varepsilon_{t+1}^2] = \sigma^2$$

Use the same parameter values as in the first exercise.

Exercises

1. How do the key business cycle properties of this model compare with those of the model from the first exercise.

3 Exercise #3

Consider, the following modification of the standard growth model

$$\max_{\{c_t,k_t\}} \mathrm{E} \sum_{t=1}^{\infty} \beta^{t-1} \frac{c_t^{1-\nu}-1}{1-\nu}$$

s.t.

$$c_t + k_t = \exp(z_{t-4}) k_{t-1}^{\alpha} + (1-\delta) k_{t-1}$$

$$\ln(z_t) = \rho \ln(z_{t-1}) + \varepsilon_t$$

$$k_0 \text{ given, } \mathrm{E}_t[\varepsilon_{t+1}] = 0 \ \& \ \mathrm{E}_t[\varepsilon_{t+1}^2] = \sigma^2$$

The idea is that the productivity level in period t is known in period $t-4$. The first-order conditions are given byExercise #1

The first-order condition is given by

$$\begin{aligned}
c_t^{-\nu} &= \mathrm{E}_t\left[\beta c_{t+1}^{-\nu}(\alpha \exp(z_{t-3}) k_t^{\alpha-1} + 1 - \delta)\right] \\
c_t + k_t &= \exp(z_{t-4}) k_{t-1}^{\alpha} + (1-\delta) k_{t-1} \\
\ln(z_t) &= \rho \ln(z_{t-1}) + \varepsilon_t
\end{aligned}$$

This can be written in Dynare form as follows:

$$\begin{aligned}
c_t^{-\nu} &= \mathrm{E}_t\left[\beta c_{t+1}^{-\nu}(\alpha \exp(\tilde{z}_t) k_t^{\alpha-1} + 1 - \delta)\right] \\
c_t + k_t &= \exp(\tilde{z}_{t-1}) k_{t-1}^{\alpha} + (1-\delta) k_{t-1} \\
\tilde{z}_t &= \tilde{z}_{1,t-1} \\
\tilde{z}_{1,t} &= \tilde{z}_{2,t-1} \\
\tilde{z}_{2,t} &= \tilde{z}_{3,t-1} \\
\tilde{z}_{3,t} &= \tilde{z}_{4,t-1} \\
\tilde{z}_{4,t} &= \tilde{z}_{5,t-1} \\
\ln(\tilde{z}_{5,t}) &= \rho \ln(\tilde{z}_{5,t-1}) + \varepsilon_t
\end{aligned}$$

Use the same parameter values as in the first exercise, but use $\sigma = 0.007$ and $\nu = 1$.

Exercises

1. Calculate the IRFs. What happens with the economy after the news shock hits the economy and before actual productivity increases?