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TESTING THE IMPLICATIONS OF LONG-RUN NEUTRALITY FOR MONETARY BUSINESS CYCLE MODELS

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SUMMARY

This paper compares sample fluctuations of the US business cycle with those predicted by a class of equilibrium monetary business cycle models. The predictions of the models are generated using the long-run neutrality restrictions implicit in the models. By imposing these restrictions on sample data, tests of the ability of the models to replicate the dynamics of the US business cycle are constructed. Although the predictions of the models for real side variables are rejected, there is evidence that the nominal side predictions of the models are not rejected.

1. INTRODUCTION

The sources of aggregate fluctuations in modern economies continue to vex students of the business cycle. One line of business cycle research uses theoretical restrictions on vector autoregressions (VARs) to recover structural disturbances to the economy. Once these disturbances are found, aggregate fluctuations can be decomposed into various structural components. The structural VAR (SVAR) method is not costless, however, since the results are conditional on the just-identified restrictions used to recover the structural shocks. King (1993) and Blanchard and Quah (1993) discuss reasons why there may be more than one way to identify the disturbances a SVAR yields.[1]

The real business cycle (RBC) methods of Kydland and Prescott (1982) and King *et al.* (1988) provide another approach to the study of aggregate fluctuations. A typical RBC model posits a set of real side shocks as the sources of business cycles, and these shocks are propagated over time as they interact with production technologies and household preferences. Critics argue that the absence of monetary factors is an important weakness of RBC models (for example, West, 1988). Christiano (1991) and Christiano and Eichenbaum (1992a,b) explore the quantitative importance of this critique using equilibrium monetary business cycle models developed by Lucas (1990) and Fuerst (1992).

[1] King (1993) discusses an alternative identification scheme of Gali's (1992) IS-LM SVAR. In their reply to Lippi and Reichlin (1993), Blanchard and Quah (1993) note that the Blanchard–Quah decomposition rules out nonfundamental moving-average representations.

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The purpose of this paper is to study the ability of four monetary business cycle models to replicate aggregate fluctuations found in the US economy. To accomplish this, we compare the theoretical dynamics generated by these models with appropriate sample analogues. Our approach combines RBC theory with SVAR methods. We exploit the fact that the monetary business cycle models under consideration satisfy the assumptions of Blanchard and Quah (1989). Thus, applying SVAR methods to data generated by those models recovers the correct theoretical dynamics, on average. Then, under the null hypothesis that US data are generated by one of the models, we can compute appropriate sample analogues by applying the same SVAR methods to actual data. To test this null hypothesis, we use formal statistical techniques, which are based on Monte Carlo methods.[2] In particular, we ask how often the monetary business cycle models generate impulse response functions that resemble those found in the US data.[3]

The monetary business cycle models under consideration satisfy two long-run neutrality restrictions. We use these restrictions to identify structural shocks. The two restrictions are (1) output is independent of nominal shocks in the long run and (2) money is independent of real shocks in the long run. The first long-run restriction identifies structural shocks in a dynamic output–inflation system. This system can be interpreted as a dynamic aggregate supply and demand model in which the long-run aggregate supply curve is vertical. The second long-run restriction identifies structural shocks in a dynamic money–hours worked system. This system can be interpreted as a dynamic Phillips curve model in which the long-run Phillips curve is vertical at the steady-state level of hours. That is, the long-run trend in money is determined by the nominal side shock.

Although the models have many implications about dynamics, the only implications used to identify shocks are orthogonality and long-run neutrality restrictions. Since we use these restrictions to just-identify the structural disturbances, we do not test the long-run neutrality restrictions. However, King and Watson (1992) report empirical evidence that supports the long-run neutrality restrictions studied in this paper.[4]

The evidence reported in this paper extends the results of Cogney and Nason (1995) to monetary business cycle models. As in our earlier paper, we find that real side endogenous propagation mechanisms are unable to match sample dynamics. However, there is some evidence that sample nominal side dynamics could have been generated by the monetary business cycle models. For example, in the output–inflation system, the theoretical responses of inflation to real and nominal shocks match the sample impulse responses fairly well. Likewise, in the money-hours worked system, the theoretical responses of money to the real and nominal shocks cannot be rejected in most cases. Thus, the models are partially successful.

The outline of this paper follows. Section 2 reviews the structure of the monetary business cycle models of Lucas, Fuerst, Christiano, and Eichenbaum. Our method for solving the models appears in Section 3. Section 4 presents the models' prediction for SVARs. Results from estimating sample SVARs, from computing theoretical SVARs, and from formally testing the models appear in Section 5. Section 6 contains a conclusion.

2. THE MODEL ECONOMIES

This section constructs the equilibrium monetary business cycle models which serve as theoretical data-generating processes (DGP). In these models, money is held because

[2] Sims (1989) argues that this approach is useful for evaluating theoretical models.
[3] Cogley and Nason (1993) study output dynamics in a slew of RBC models using this approach.
[4] King and Watson (1992) test long-run neutrality restrictions by imposing values on the impact and long-run multipliers. Their evidence favours long-run neutrality for M2 and a long-run vertical Phillips curve.

households face a cash in advance (CIA) constraint. Households can purchase the single consumption good with the cash that they carry over from the previous period and with their current period labour income. A typical household also has the option of depositing some of its income each period with a financial intermediary (FI).

Besides the deposits of households, the FI receives monetary injections from a monetary authority each period. The monetary injections take the form of an exogenous stochastic process. Deposits and injections are loaned by the FI to firms.

Firms combine capital and labour with an exogenous technology shock in a constant returns to scale production function to produce the economy's output. This good serves either as the investment flow into the capital stock of a firm or as the consumption good of households. Since firms own their capital, the only factor payment they make is to labour. Firms finance wage bills each period with the funds they borrow from the FI.

The Standard CIA Model

All the models that we study share the same generic structure. They differ according to preferences, technologies, and the information sets that households and firms possess when they make decisions. We adopt the standard CIA monetary business cycle model as our baseline model. The information structure of this model assumes that households make the deposit decision after observing current period innovations to technology and monetary injection growth shocks. This is the standard model in the sense that date t exogenous disturbances occur before date t decisions are made.

Exogenous disturbances

With real and nominal sectors, the models require at least two disturbances. The technology shock, $A(t)$, follows a random walk with drift

$$\ln[A(t)] = \gamma t + \ln[z(t)], \quad 0 < \gamma$$

where γ is the deterministic trend component of technology growth. The stochastic trend component is

$$\ln[z(t+1)] = \ln[z(t)] + \alpha(t+1), \quad \varepsilon(t+1) \sim N(0, \sigma(\varepsilon)^2) \tag{1}$$

Besides the technology shock, the models include an exogenous stochastic process for the growth rate of the monetary injection

$$\ln[m(t+1)] = (1-\rho)\ln[m^*] + \rho \ln[m(t)] + \eta(t+1), \quad |\rho| < 1, \quad \eta(t+1) \sim N(0, \sigma(\eta)^2) \tag{2}$$

where m^* is the unconditional mean of monetary injection growth. Monetary injection growth is defined as $\ln[m(t)] = \ln[M(t+1)/M(t)]$, where $M(t)$ is the stock of the money base at the end of date $t-1$. Innovations to the technology and monetary injection growth shocks are uncorrelated at all leads and lags.

Households

A typical infinitely lived household makes decisions regarding consumption, deposits, and labour supply. The typical household maximizes an expected utility function of the form

$$E_0\left\{\sum_{t=0}^{\infty} \beta^t((1-\psi)\ln[c(t)] + \psi \ln[1-h(t)])\right\}, \; 0 < \beta, \psi < 1 \tag{3}$$

over uncertain consumption, $c(t)$, and labour supply, $h(t)$, streams. The expectations operator conditional on date 0 information is $E_0\{\}$. It is assumed that the household owns a time endowment of one unit each period.

Households face two constraints. It is assumed that only cash carried over from the previous period net of current period nominal deposits, $d(t)$, and current labour income is available for current consumption purchases by the household. Hence, the CIA constraint is

$$P(t)c(t) \leqslant W(t)h(t) + M(t) - d(t), \quad 0 \leqslant d(t) \tag{4}$$

where $P(t)$ and $W(t)$ denote the price level of the consumption good and the nominal wage rate, respectively.

A typical household wants to use its resources to make deposits at the FI, consume, and purchase cash to carry into the future. The resources of the household are its dividend income, labour income, interest income on deposits, and current cash holdings. In this case, the resource constraint of the household becomes

$$M(t+1) \leqslant f(t) + b(t) + RH(t)d(t) + W(t)h(t) + M(t) - d(t) - P(t)c(t) \tag{5}$$

where $f(t)$ are the nominal dividends the household receives from firms, $b(t)$ are the nominal dividends the household receives from the FI, and $RH(t)$ is the gross nominal interest rate the household faces in the market for deposits.[5]

The financial intermediary

In the monetary business cycle model considered here, the FI has a trivial problem to solve. The FI maximizes the expected infinite horizon discounted stream of dividends it pays to households

$$E_0\left\{\sum_{t=0}^{\infty} \beta^{t+1} b(t)/[c(t+1)P(t+1)]\right\} \tag{6}$$

Since households value a unit of nominal dividends in terms of the consumption it brings during date $t+1$ and FIs are owned by households, date t nominal dividends are discounted by the date $t+1$ marginal utility of consumption.

The FI faces three constraints. First, there is a budget constraint

$$b(t) + RH(t)d(t) \leqslant RF(t)l(t) + d(t) + X(t) - l(t) \tag{7}$$

where $l(t)$ is the nominal amount of loans the FI makes to firms, $RF(t)$ is the gross interest rate charged on those loans, and $X(t)$ is the monetary injection during date t, $X(t) = M(t+1) - M(t)$. The second constraint defines the balance sheet of the FI:

$$X(t) + d(t) \leqslant l(t) \tag{8}$$

The liabilities of the FI must be less than or equal to its assets. Along the equilibrium path, the FI also obeys a zero profit condition

$$RH(t)d(t) = RF(t)[l(t) - X(t)] \tag{9}$$

Profits on loans to firms net of the monetary injection equals the principle and interest the FI owes to households period by period.

[5] As in Christiano (1991), it is assumed that households own shares in the FI and firms. The equity market in which shares are traded is suppressed for convenience.

Firms

The objective of a typical firm is similar to the objective of the FI:

$$E_0\left\{\sum_{t=0}^{\infty} \beta^{t+1} f(t)/[c(t+1)P(t+1)]\right\} \tag{10}$$

Since a firm owns its physical capital, it has a capital accumulation decision to make each period. The firm trades off paying the household larger dividends or accumulating more capital. For a typical firm, the budget constraint relates this decision, labour demand, and loan demand by

$$f(t) + RF(t)l(t) + W(t)n(t) - l(t) \leqslant P(t)[y(t) - i(t)] \tag{11}$$

where $n(t)$ is labour demand, $y(t)$ is physical output, and $i(t)$ is physical investment. Firms carry $k(t+1)$ unit of physical capital into date $t+1$ from date t. The law of motion of capital defines gross investment

$$i(t) = k(t+1) - (1-\delta)k(t), \quad 0 < \delta < 1$$

Output is generated with the constant returns to scale production function:

$$y(t) = k(t)^{\theta}[A(t)n(t)]^{(1-\theta)}, \quad 0 < \theta < 1$$

The other constraint the firm faces reflects the fact that the firm finances its current period wage bill by borrowing. Hence, the firm obeys

$$W(t)n(t) \leqslant l(t) \tag{12}$$

The Lucas–Fuerst Model

Lucas (1990) and Fuerst (1992) alter the information structure of the standard CIA model. In the Lucas–Fuerst model, households make current period deposit decisions conditional on previous period technology and monetary injection growth innovations. This restriction imposes an infinite within-period adjustment cost on household deposit decisions. The adjustment costs induce a propagation mechanism in the LF model controlled by an expected inflation effect.[6] Through the financial intermediation process, the propagation mechanism affects firm behaviour.

The Portfolio Cost of Adjustment Model

Christiano and Eichenbaum (1992a) start with the LF model and impose dynamic portfolio adjustment costs on households. The idea is to introduce a propagation mechanism into the LF model to generate persistence in the response of output and interest rates to a monetary shock. The dynamic costs appear in the utility function of households as a loss of leisure whenever the household adjusts its cash holdings dedicated to purchasing the consumption good relative to previous periods. That is, leisure equals the time endowment minus labour supply, $h(t)$, and the amount of time given to portfolio management, $p(t)$. In this way, money growth appears in the

[6] The cost of adjustment in deposits households face can generate either a liquidity effect or an expected inflation effect. It depends on model specification as Christiano (1991) and Christiano and Eichenbaum (1992a,b) make clear. However, Dotsey and Ireland (1993) suggest that economically implausible specifications are required for Lucas–Fuerst models to possess dominant liquidity effects.

period utility function of households

$$(1-\psi)\ln[c(t)]+\psi\ln[1-h(t)-p(t)]$$

Following Christiano and Eichenbaum, we make the cost of portfolio management a dynamic function of the money balances earmarked for consumption purchases

$$p(t)=\alpha(1)[\exp\{\alpha(2)[\Gamma(t)/\Gamma(t-1)-m^*]\}+\exp\{-\alpha(2)[\Gamma(t)/\Gamma(t-1)-m^*]\}-2]$$

where $\Gamma(t)=M(t)-d(t)$. We denote this model economy the Christiano and Eichenbaum portfolio cost of adjustment model (CEP).

The Imperfect Labour Substitutes Model

Christiano and Eichenbaum (1992b) construct an imperfect labour substitutes model out of the LF model. In this model, the firm faces an intra-period labour demand decision. That is, the firm makes two labour demand decisions during each period. The firm makes the first labour demand decision, $n(1, t)$, before the realization of the innovation of the monetary shock. After the realization of this shock, the firm makes the second labour demand decision, $n(2, t)$. To make the first labour demand decision matter for the firm, Christiano and Eichenbaum assume that the different labour inputs are imperfect substitutes in a constant elasticity of substitution technology generating total labour demand:

$$n(t)=[0{\cdot}5n(1,t)^{1/\phi}+0{\cdot}5n(2,t)^{1/\phi}]^{\phi}, \quad \phi>1$$

The final goods technology retains its usual form:

$$y(t)=k(t)^{\theta}[A(t)n(t)]^{(1-\theta)}$$

Given the two labour demand decisions, the firm faces two financing decisions when paying its wage bill:

$$W(1,t)n(1,t)\leqslant l(1,t), \quad \text{and} \quad W(2,t)n(2,t)\leqslant l(2,t)$$

where $W(1, t)(W(2, t))$ denotes the nominal wage of $n(1, t)$ $(n(2, t))$ and $l(1, t)$ $(l(2, t))$ is the nominal loan the firm takes from the FI to finance hiring $n(1, t)$ $(n(2, t))$. Since the firm cannot adjust $n(1, t)$ to a current period monetary shock, a propagation mechanism exists in the model that can generate a dominant liquidity effect. We denote this model economy the Christiano and Eichenbaum imperfect labour substitutes model (CEL).

For the CEL model, all three constraints of the FI require modification. To account for the FI making loans to the firm during each date, the FI's budget constraint becomes

$$b(t)+RH(t)d(t)\leqslant RF(1,t)l(1,t)+RF(2,t)l(2,t)+d(t)+X(t)-l(1,t)-l(2,t)$$

where $RF(1, t)$ $(RF(2, t))$ is a gross interest rate charged on $l(1, t)(l(2, t))$. This is reflected in the balance sheet of the FI as

$$X(t)+d(t)\leqslant l(1,t)+l(2,t), \quad l(1,t)\leqslant d(t)$$

Since the return on the liabilities of the FI must be less than or equal to the return on its assets, the FI obeys a zero profit condition

$$RH(t)d(t)=RF(1,t)l(1,t)+RF(2,t)[l(2,t)-X(t)]$$

along the equilibrium path.

3. SOLVING THE MODELS

This section outlines the methods used to solve the models. The standard CIA model serves as the basis of our outline. Our technical appendix contains more details.

Equilibrium

For the model economies, an equilibrium requires clearing in the goods, labour, credit, and money markets. All markets are assumed to be perfectly competitive. To begin with the credit market, it clears when equation (8) holds with strict equality. This implies $RH(t) = RF(t) \equiv R(t)$ from equation (9). If we combine these two results together with equation (7), we find the dividends, $b(t)$, the FI pays to households equals $R(t)X(t)$.

For the money market to clear, money demand and money supply must be equated. Nominal consumption demand can be equated with money demand. Money supply equals current nominal balances and monetary injections. The following equation represents equilibrium in the money market:

$$P(t)c(t) = M(t) + X(t) \tag{13}$$

Equation (13) requires labour market equilibrium, $h(t) = n(t)$, credit market clearing imposed on equations (4), and equation (12) to hold with strict equality. Finally, goods markets clearing means output equals consumption plus investment:

$$c(t) + k(t+1) - (1-\delta)k(t) = k(t)^{\theta}[A(t)n(t)]^{(1-\theta)} \tag{14}$$

This condition is found by combining equations (4), (5), and (11) along with market clearing in the credit, money, and labour markets.

To construct equilibrium decision rules, the typical household maximizes equation (3) with respect to equations (4) and (5). The FI maximizes equation (6) with respect to equations (7) and (8). A typical firm maximizes equation (10) with respect to equations (11) and (12). When computing decision rules, the typical household, firm, and FI treat the equilibrium process generating $A(t+1)$, $M(t+1)$, $P(t)$, $W(t)$, and $R(t)$ as given.

Optimality

Any candidate equilibrium must satisfy three optimality conditions. These restrict equilibrium paths in the goods, labour, money, and credit markets. Optimality in the goods market defines the trade-off the economy faces in moving the consumption good across time. The Euler equation

$$\begin{aligned} &E_t\{-P(t)/[c(t+1)P(t+1)] + \beta P(t+1) \\ &\qquad \times [\theta k(t+1)^{\theta-1}[A(t+1)n(t+1)]^{1-\theta} + (1-\delta)]/[c(t+2)P(t+2)]\} = 0 \end{aligned} \tag{15}$$

represents this trade-off. Since the CIA constraint binds in equilibrium, money has positive finite value and the intertemporal consumption trade-off is in terms of marginal utility one period ahead weighted by the purchasing power of money.

The intratemporal condition which restricts labour market optimality depends on the structure of the credit market. Since the firm must finance its current period wage bill with borrowed funds, credit market structure affects the labour demand of the firm. The firm's borrowing constraint (equation (12)) becomes

$$W(t) = l(t)/n(t)$$

in equilibrium. Using this condition, the intratemporal labour market optimality condition is

$$-[\psi/(1-\psi)][c(t)P(t)/(1-n(t))]+l(t)/n(t)=0 \tag{16}$$

Equation (16) equates labour supply, the marginal rate of substitution between leisure and consumption, and labour demand.

For the standard information (or CIA) model economy, the optimality condition in the credit market depends on date t information. Credit market optimality requires that the household's loss in current consumption from increasing its deposits at the FI match the discounted expected gain in future consumption from that deposit. That is, the intertemporal Euler equation that represents credit market optimality is

$$1/[c(t)P(t)]-\beta R(t)\mathrm{E}_t\{1/[c(t+1)P(t+1)]\}=0 \tag{17}$$

The equilibrium interest rate is determined by the borrowing decision of the firm. At the margin, the firm equates the increase in its nominal revenue generated by an extra unit of labour to the nominal cost of borrowing required to pay for that unit of labour. Hence, the equilibrium nominal gross interest rate which clears the credit market equals the ratio of the marginal revenue product of labour to the nominal wage rate:

$$R(t)=P(t)(1-\theta)k(t)^{\theta}A(t)^{1-\theta}n(t)^{-\theta}/W(t)$$

The numerical solution of the optimization problems of the household, FI, and firm are written in terms of stochastically detrended variables. Stochastic detrending of real side aggregates involves $\hat{q}(t)=q(t)/A(t)$, where $q(t)=[y(t)\ c(t)\ i(t)\ k(t+1)]$. For example, after detrending, the aggregate resource constraint (equation (14))becomes

$$\hat{c}(t)+\hat{k}(t+1)=\exp\{-\theta[\gamma+\varepsilon(t)]\}\hat{k}(t)^{\theta}n(t)^{(1-\theta)}+(1-\delta)\exp\{-[\gamma+\varepsilon(t)]\}\hat{k}(t) \tag{18}$$

Nominal side aggregates and prices are transformed by $X(t)/M(t)=M(t+1)/M(t)-1$, $\hat{P}(t)=P(t)A(t)/M(t)$, $\hat{Q}(t)=Q(t)/M(t)$, where $Q(t)=[d(t)\ l(t)\ W(t)]$. In this case, the money market condition becomes

$$\hat{P}(t)\hat{c}(t)=m(t) \tag{19}$$

after transforming equation (13) and the credit market equilibrium condition (equation (8)) yields

$$m(t)-1+\hat{d}(t)=\hat{l}(t) \tag{20}$$

after detrending.

The numerical solution to the standard CIA model ties together the equilibrium conditions (equations (18)–(20)) with the detrended versions of the optimality conditions (equations (15)–(17)). Along with the exogenous stochastic processes for technology and monetary injection growth shocks, this system of six nonlinear equations determines the equilibrium distributions for the six unknowns:

$$[\hat{k}(t+1)\ n(t)\ \hat{d}(t)\ \hat{c}(t)\ \hat{l}(t)\ \hat{P}(t)]$$

Given these equilibrium distributions, the equilibrium distributions for output, real wages, the inflation rate, and the nominal interest rate can be found. The technical appendix to this paper contains details.[7]

[7] The technical appendix describes the optimality conditions of the other model economies and the approximate log linear solution method. Model parameter values are calibrated to the sample data described below and appear in the technical appendix.

4. STRUCTURAL VARs

Each model economy places identifying restrictions on its VAR representation. Of particular interest here are the long-run neutrality restrictions of the models. However, the models possess only two sources of underlying uncertainty. This limits the dimension of nonsingular VAR representations.

The models predict long-run monetary neutrality for output. This restriction can be used to identify structural shocks in a VAR of output growth and inflation. In this case, the long-run restriction translates into a vertical long-run aggregate supply curve in $y - \Delta P$ space. The permanent component is determined by the technology shock and the transitory response of output and inflation capture the affects of monetary injection growth disturbances. The formal statement of the identifying restriction used with the output–inflation information set is:

The long-run level of output is independent of monetary injection growth shocks.

Since the innovations of structural shocks are orthogonal, this restriction is sufficient to just-identify the theoretical structural VAR.[8]

The money growth and hours worked information set predicts that money is independent of technology shocks in the long run. This information set can be thought of as a dynamic Phillips curve. The permanent component of money defines the system's long-run stochastic trend. Innovations to the technology shock drive the transitory dynamics of money and hours worked. The identifying restrictions which defines this bivariate relationship in the long run is:

The long-run level of money is independent of technology shocks.

Real shocks have no effect on the monetary aggregate in the long run.

Theoretical VARs

It is straightforward to construct the structural VAR predicted by the model economies. An example is presented for the structural VAR of output growth and inflation. Given the approximate log linear decision rules and the laws of motion for technology and monetary injection growth shocks, the theoretical VAR can be written as an infinite order structural vector moving-average, SVMA(∞), process:[9]

$$\begin{bmatrix} \Delta \ln[y(t)] \\ \Delta \ln[P(t)] \end{bmatrix} = \mathbf{C(L)}\begin{bmatrix} \varepsilon(t) \\ \eta(t) \end{bmatrix}, \ \mathbf{C(L)} = \sum_{j=0}^{\infty} C(j)\mathbf{L}^{j}, \ C(j) = \begin{bmatrix} c(11,j) & c(12,j) \\ c(21,j) & c(22,j) \end{bmatrix}$$

The models impose restrictions on the SVMA through the monetary neutrality of output. Each

[8] The approach to identifying neutrality restrictions adopted here differs from that of Shapiro and Watson (1988), King and Watson (1992), and Cogley (1993). Shapiro and Watson work with over-identified models in order to uncover the sources of business cycle fluctuations. This requires an instrumental variables estimator. King and Watson's interest is to test various neutrality propositions given integrated variables and prior information about contemporaneous impact and long-run multipliers. Problems arise in this environment which requires instrumental variable estimation as well. Cogley estimates an exactly identified model and then tests whether the impulse response functions satisfy the over-identifying restrictions that are necessary for neutrality.

[9] The log linear approximation of the models predict a unique finite order vector ARMA process for the output growth–inflation information set. It is straightforward to recast this model as the fundamental infinite order MA representation of this information set. Hence, the Blanchard–Quah decomposition will have no problems in finding this fundamental representation.

model predicts output growth follows

$$\Delta \ln[y(t)] = \Delta \ln[\hat{y}(t)] + \gamma + \varepsilon(t)$$

and inflation to be

$$\Delta \ln[P(t)] = \Delta \ln[\hat{P}(t)] + \ln[m(t-1)] - \gamma - \varepsilon(t)$$

Output is determined only by technology shocks in the long run. Inflation is driven by technology and monetary injection growth shocks in the long run. Hence, in this system, the long-run level of output is independent of nominal side disturbances while inflation is not. Long-run monetary neutrality of output implies

$$c(12) \equiv \sum_{j=0}^{\infty} c(12, j) = 0$$

which restricts the long-run multiplier matrix to be

$$\mathbf{C}(1) = \begin{bmatrix} c(11) & 0 \\ c(12) & c(22) \end{bmatrix}$$

The identifying restrictions of the money–hours worked information set is constructed in the same manner. In this case, the restriction is that the level of money is independent of technology disturbances in the long run. In the models, the monetary aggregate, M2, equals the sum of monetary injections at the end of date t plus date t deposits by the typical household at the FI, $\mathrm{M2}(t+1) \equiv \mathrm{M}(t+1) + d(t)$. The log approximation of this identity yields

$$\Delta \ln \mathrm{M2}(t+1) = \mu(1)\Delta \ln[m(t)] + \mu(2)\Delta \ln[\hat{d}(t)] + \ln[m(t-1)]$$

where the parameters $\mu(1)$ and $\mu(2)$ are nonlinear functions of the steady-state values of $m(t)$ and $\hat{d}(t)$.[10] In the long run, M2 is driven only by the monetary injection growth shock. After either a technology or monetary disturbance, hours worked returns to steady state in the long run.

Empirical VARs

In order to construct the stylized facts of the output–inflation information set, the sample analogue of each theoretical VAR needs to be constructed. For the output–inflation system, the unrestricted, finite order, sample VAR yields the VMA(∞):

$$\begin{bmatrix} \Delta \ln[y(t)] \\ \Delta \ln[P(t)] \end{bmatrix} = \Lambda(\mathbf{L})\mathbf{e}(t), \qquad \Lambda(0) = \mathbf{I}, \qquad \mathbf{E}\{\mathbf{e}(t)\mathbf{e}(t)'\} = \Omega$$

If the theoretical SVMA(∞) process is written

$$\begin{bmatrix} \Delta \ln[y(t)] \\ \Delta \ln[P(t)] \end{bmatrix} = \mathbf{\Phi}(\mathbf{L})\mathbf{u}(t), \qquad \mathbf{E}\{\mathbf{u}(t)\mathbf{u}(t)'\} = \mathbf{I}$$

[10] The definition of the monetary aggregate in the model appears to be closer to the sample analogue of M1, currency, and deposits. However, there are two reasons for using M2 as the sample analogue of the theoretical monetary aggregate. In terms of an inside–outside money split, inside money in the model, $I(t)$, represents a short-term financial asset. In this instance, the sample analogue of the theoretical monetary aggregate is M2, currency, deposits, and short-term financial assets. The other reason for working with M2 rather than M1 is that Stock and Watson (1988) find M1 contains a linear time trend in growth rates. As King and Watson (1992) argue, this makes interpretation of the long-run neutrality restrictions problematic.

then $\mathbf{u}(t) = \Phi(0)^{-1}\mathbf{e}(t)$ and $\Phi(j) = \Lambda(j)\Phi(0)$, $j = 1, 2, \ldots$.[11] Once $\Phi(0)$ is known, the structural disturbances and the structural impulse response functions (IRFs) can be generated. However, this requires that $\Phi(0)$ be identified.

Blanchard and Quah (1989) provide a natural method to identify $\Phi(0)$. Identification of the four elements of $\Phi(0)$ requires imposing four conditions on the unrestricted VMA(∞). The equation $\mathbf{u}(t) = \Phi(0)^{-1}\mathbf{e}(t)$ yields three restrictions because the symmetric matrix $\Omega = \Phi(0)\Phi(0)'$. The fourth condition relies on a restriction on the long-run multiplier matrix of the theoretical SVMA(∞):

$$\Phi(1) = \begin{bmatrix} \phi(11) & 0 \\ \phi(21) & \phi(22) \end{bmatrix}$$

Since $\Phi(1) = \Lambda(1)\Phi(0)$, the fourth restriction is

$$\lambda(11)\phi(12, 0) + \lambda(12)\phi(22, 0) = \phi(12) = 0$$

The nonlinear system of four equations determines uniquely the four unknowns of $\Phi(0)$.[12]

5. TESTS OF MODEL PREDICTIONS

Tests of the predictions of the CIA and LF models are conducted in a Monte Carlo environment. The models are treated as the DGP of the artificial time series for output growth, inflation, M2 growth, and hours worked.[13] Artificial time series are generated with 152 observations, the same as the sample length of 1954:1 through 1991:4.[14] Each model is simulated 5000 times. During each replication and for each information set, fourth-order VARs are estimated, the Blanchard–Quah decomposition is constructed, and the IRFs are computed and collected into empirical probability distributions. Since the null hypothesis is that the sample data are generated by one of the models, the empirical probability distributions are used to calculate the probability of observing the sample statistics. As Gregory and Smith (1991) discuss, this approach can be interpreted as a model specification test.

Stylized Facts: Sample IRFs

The sample structural IRFs derived from the output–inflation information set appear in Figure 1.[15] The output–inflation information set yields a dynamic aggregate supply and demand

---

[11] The normalization is $\mathbf{C}(\mathbf{L}) = \Xi\Phi(\mathbf{L})$, where Ξ is the Choleski factorization of

$$\mathbf{E}\{[\varepsilon(t)\ \ \eta(t)]'[\varepsilon(t)\ \ \eta(t)]\}$$

The Monte Carlo experiments are constructed to take account of the normalization.

[12] Blanchard and Quah (1989) provide a proof of this result. The idea is that $\Phi(0)$ is an orthonormal transformation of the unique Choleski factorization of Ω. The long-run neutrality restriction acts as an orthogonality condition which uniquely determines $\Phi(0)$. Since $\varepsilon(t)$ and $\eta(t)$ are orthogonal by construction, the Blanchard–Quah decomposition exists under the null.

[13] Sample $y(t)$, $P(t)$, $M2(t)$, and $n(t)$ are per capita real GNP (1987 dollars), the implicit GNP price deflator, per capita M2, and per capita hours worked (household survey). The source of $y(t)$, $P(t)$, $n(t)$, and the population series is the CITIBASE data bank. Per capita quantities are generated using total population which includes members of the armed services overseas. The M2 series is the same one King and Watson (1992) use.

[14] In fact, 356 observations are generated. The first 204 observations are discarded and the last 152 observations are used to compute theoretical dynamics. This is to remove any dependence on initial conditions.

[15] To approximate the sample SVMA processes, we estimate fourth-order VARs for the sample period 1954:1 through 1991:4.

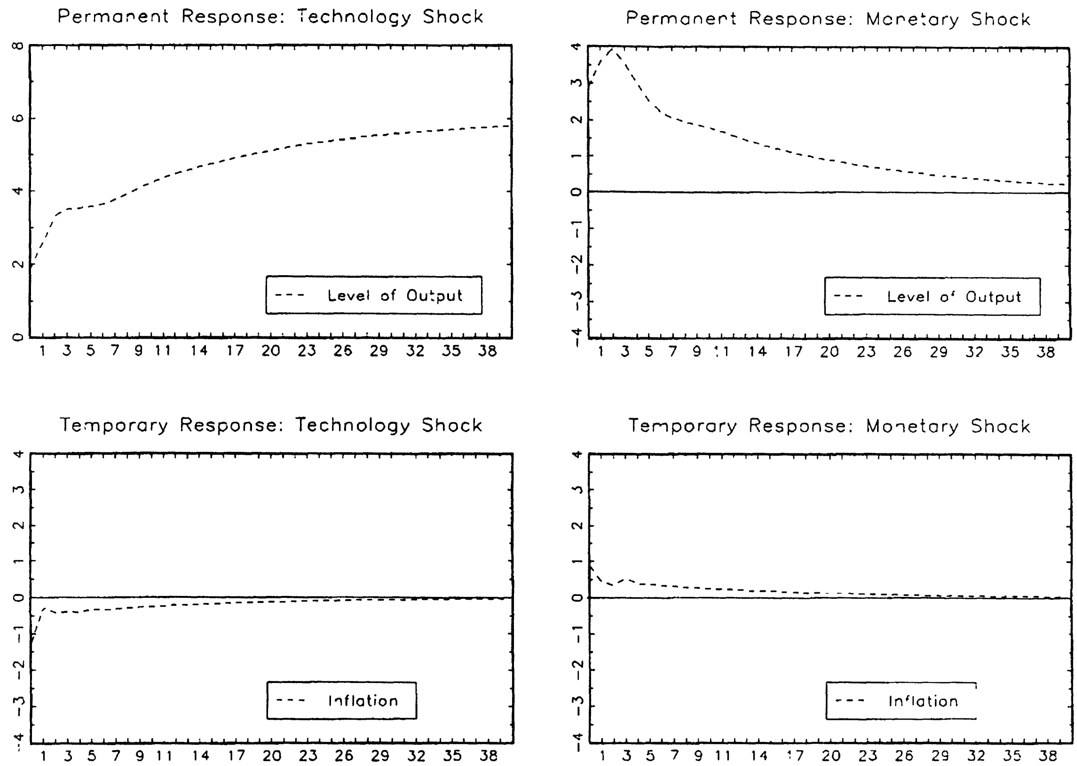

Figure 1. Sample dynamics from SVMA of $(Y, \Delta P)$

relationship. Output responds to the technology shock with a typical capital accumulation path. Technology shocks permanently shift the aggregate supply schedule.

The contemporaneous impact of the technology shock on inflation is negative. After this, inflation returns to its steady state with most of the recovery completed by the end of one year. This is the typical pattern for inflation's response to a technology disturbance in a dynamic aggregate supply and demand system. For example, Gali (1992) reports similar results.

The response of the output–inflation information set to the transitory monetary injection growth shock is typical for an aggregate supply and demand system. Initially, a positive monetary injection growth shock raises output and inflation. Subsequently, inflation experiences a gradual adjustment to its steady state except for a small wiggle between lags two and four.

Output exhibits a hump-shaped response to the monetary shock with the peak at lag two. Beyond lag two, output's response to the monetary shock slowly decays. Over half of the decay is accomplished within two years. The temporary increase in anticipated inflation lowers production costs initially. Afterwards, factor costs rise and this reduces output growth until the long-run equilibrium is reestablished.

Figure 2 contains the sample structural IRFs of the M2–hours worked information set. The M2–hours worked system is a type of dynamic Phillips curve. However, the twist is that the monetary injection growth disturbance is the permanent component. That is, only monetary shocks have a permanent impact on the hours worked–M2 Phillips curve. M2's accumulated response looks rather like the capital accumulation path for output found in Figure 1. In

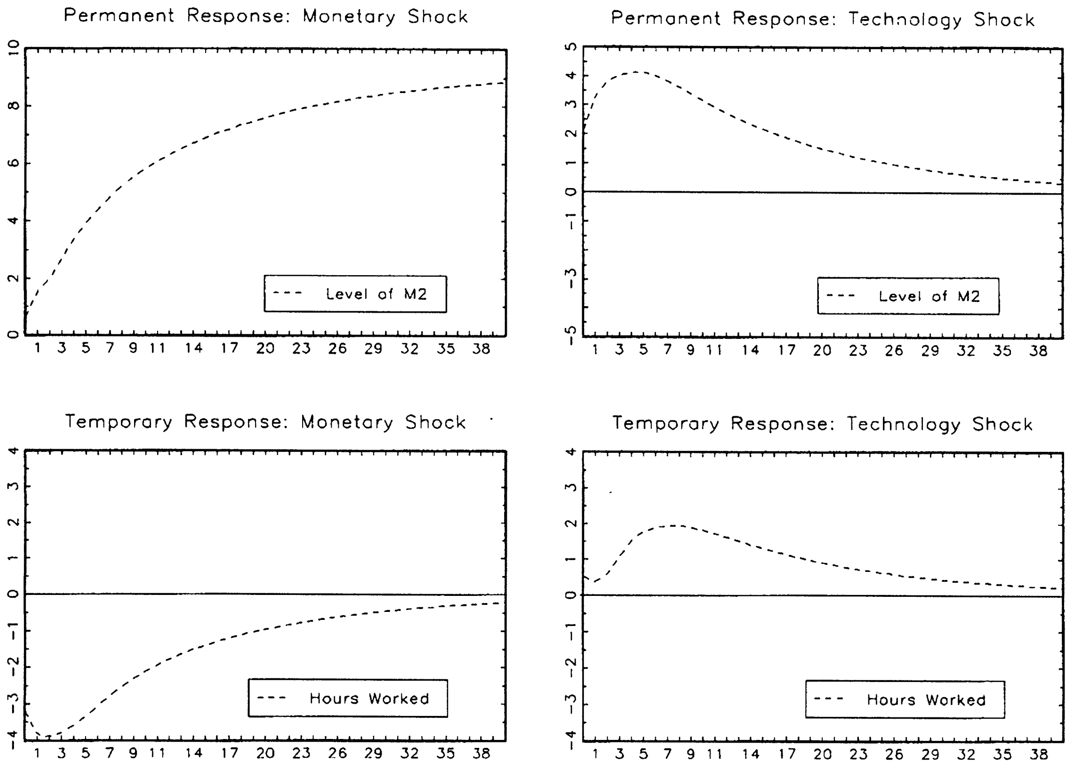

Figure 2. Sample dynamics from SVMA of (M2, N)

response to the permanent disturbance, hours worked are lower on impact and remain lower for ten years or more. Since innovations to the monetary shock drive the permanent component, permanent increases in M2 are equivalent to shifting the short-run hours worked–money Phillips curve to the right. That is, at any level of hours worked, M2 balances are higher. For a standard unemployment–inflation Phillips curve, monetary shocks lead to a higher inflation rate at any unemployment rate.

The technology disturbance causes M2 and hours worked to be higher at impact and generates a hump-shaped response in M2 and hours worked. The difference in the pattern of M2 and hours worked is that hours worked exhibits a cyclical response that peaks at two years. M2's hump-shaped pattern resembles the response of output to the monetary shock shown in Figure 1. However, the IRFs of M2 and hours worked both exhibit a long slow decay as the economy approaches its long-run equilibrium. Technology shocks shift the hours worked–M2 Phillips curve to the left temporarily.

Studying Propagation Mechanisms: Theoretical IRFs

The theoretical IRFs are generated numerically. To compute the theoretical IRF, the ensemble of artificial replications is averaged:

$$\mathbf{F} = N^{-1} \sum_{j=1}^{N} \mathbf{F}(j), \qquad N = 5000$$

where F is the mean IRF and F(j) is the IRF of replication j. The theoretical IRFs of the output–inflation (M2–hours worked) system of the standard CIA, LF, CEP, and CEL model economies appear in Figures 3–6 (7–10), respectively. In each figure, the left- (right-) hand column of each figure contains responses to the permanent (transitory) disturbance of the system. The IRFs of the integrated (stationary) variables appear in the top (bottom) row. Within each panel of the figures, solid (dashed) lines denote theoretical (sample) IRFs.

The output–inflation information set reveals the dynamics of the four model economies fall into two groups. As is apparent from inspection of Figures 3–6, the CIA and CEP models possess very weak real side-propagation mechanisms, as evidenced by the fact that output dynamics are nearly the same as the exogenous shock dynamics. The real side-propagation mechanisms in the LF and CEL models are somewhat stronger, and they generate wiggles at low-order lags. As for the nominal side, weak inflation dynamics are found for the CIA, LF, and CEP models. For the CEL model, inflation dynamics contain evidence of a propagation mechanism.

As discussed in Section 2, the model economies possess a common structure. Within this structure, two competing mechanisms operate on output, inflation, M2, and hours worked given a monetary shock. The competing mechanisms are the anticipated inflation and liquidity effects. As Christiano (1991) discusses, the anticipated inflation effect controls the response of output to monetary disturbances in the standard CIA and LF models. Since adjustments to household deposits dominate the impact of monetary shocks on loan demand by firms, the anticipated inflation effect dominates dynamics in the standard CIA and LF models. This holds true for the CEP model but not for the CEL model. In the latter the liquidity effect dominates the anticipated inflation effect at impact. However, across the CIA, LF, and CEP models output and hours worked respond quite differently to a monetary disturbance.

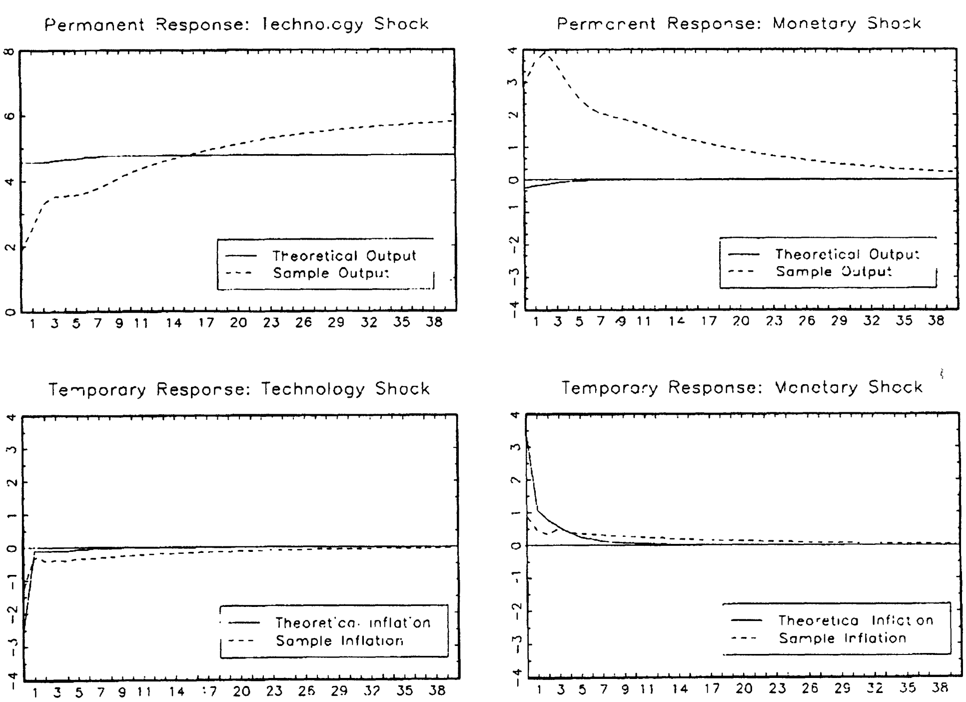

Figure 3. Theoretical dynamics of the CIA model

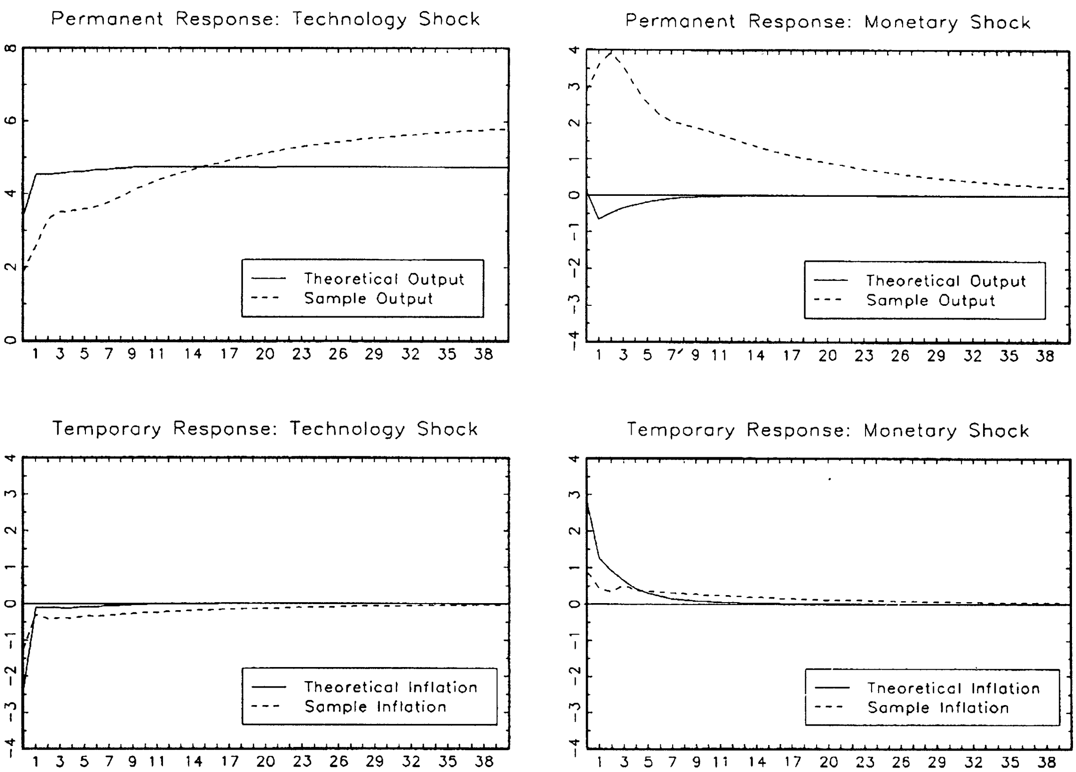

Figure 4. Theoretical dynamics of the LF model

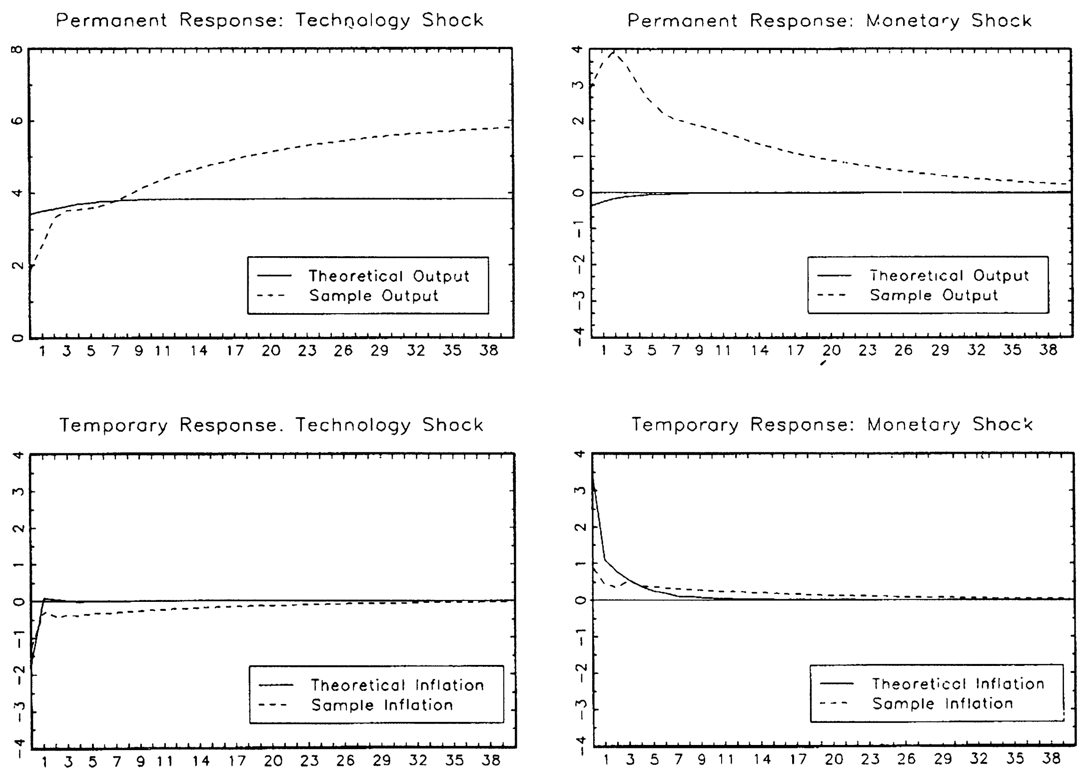

Figure 5. Theoretical dynamics of the CEP model

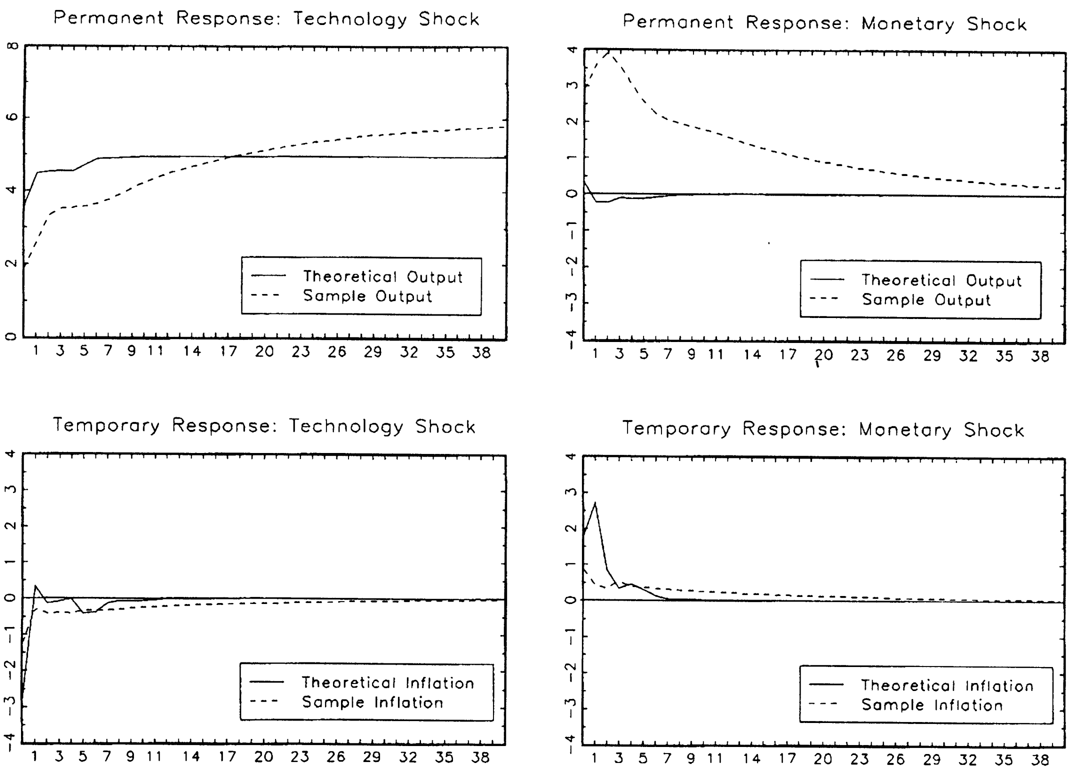

Figure 6. Theoretical dynamics of the CEL model

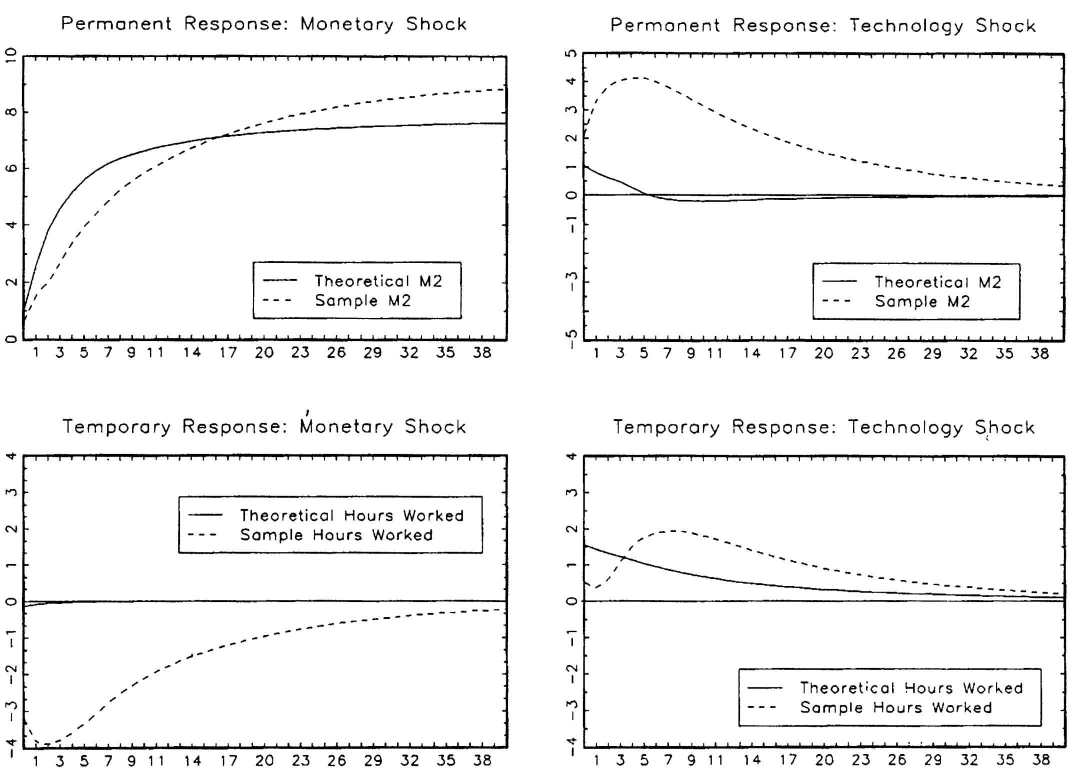

Figure 7. Theoretical dynamics of the CIA model

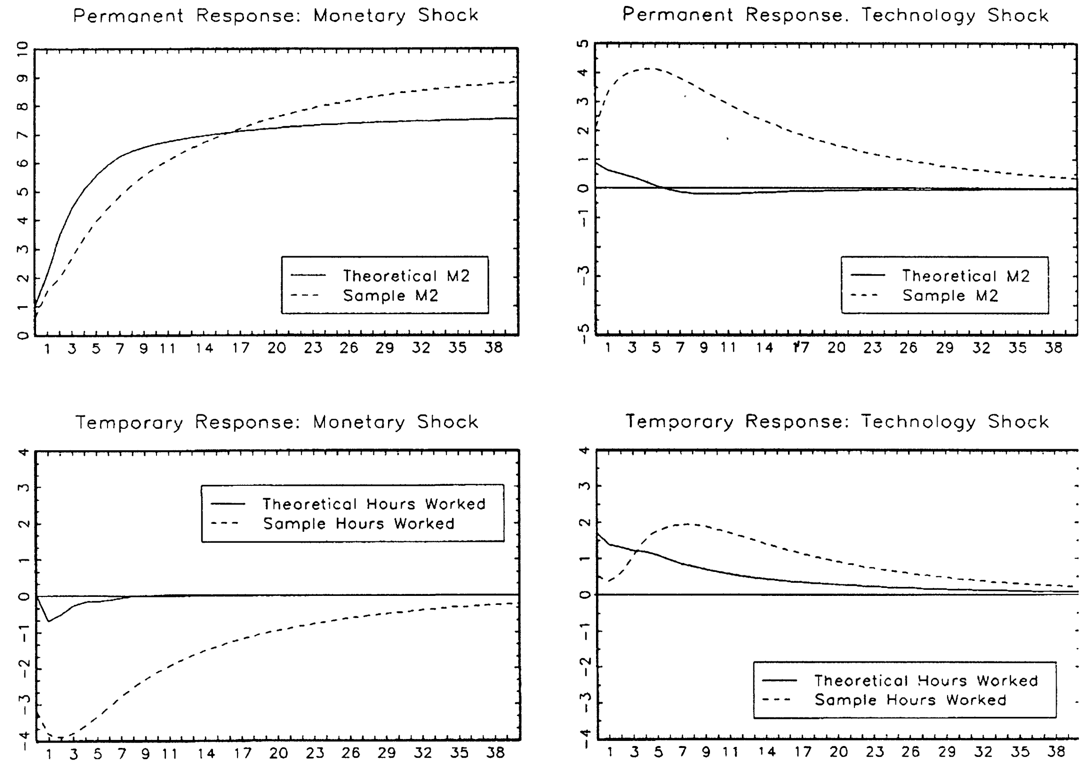

Figure 8. Theoretical dynamics of the LF model

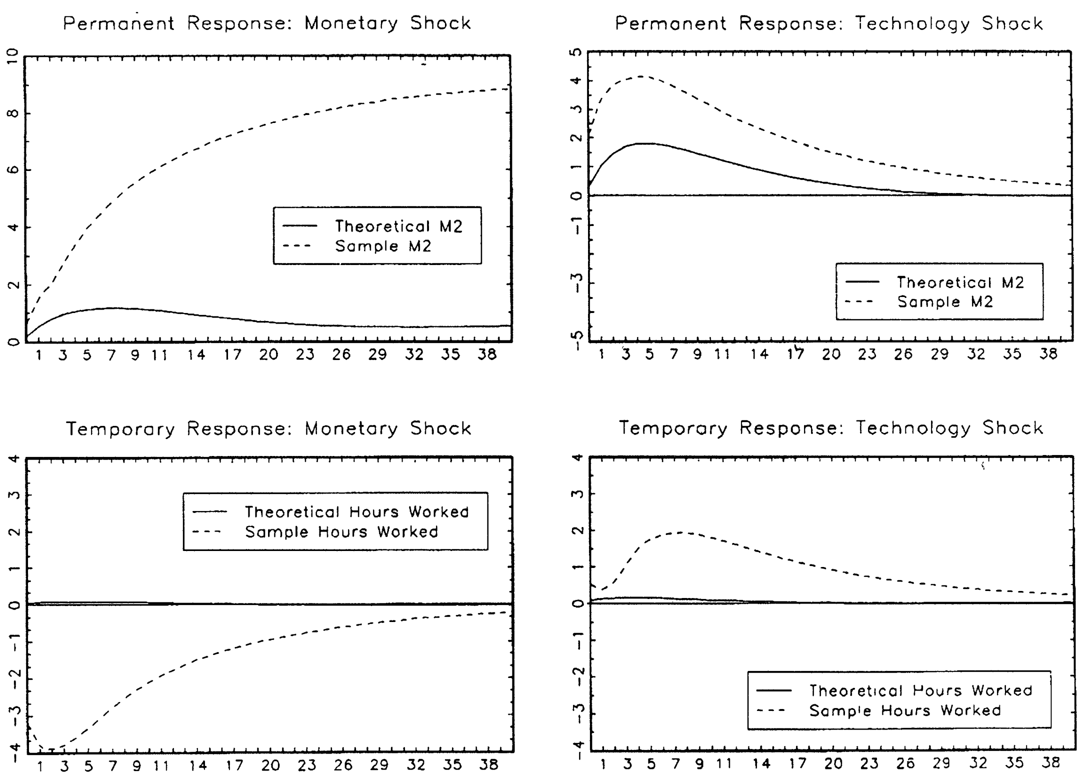

Figure 9. Theoretical dynamics of the CEP model

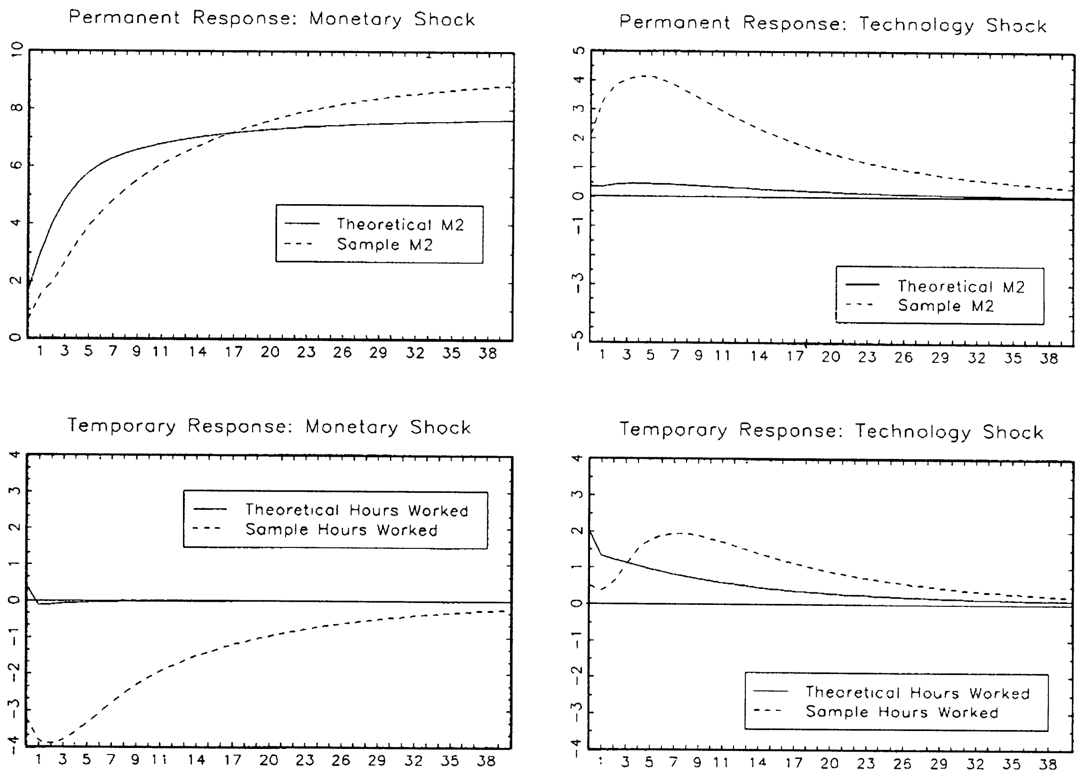

Figure 10. Theoretical dynamics of the CEL model

The source of the propagation mechanism of the LF model is its information structure. In the LF model, the household chooses the amount of funds to deposit with the FI before date t shocks occur in the LF model. Since the household faces an in-period infinite cost of altering its deposit decision, all household deposit adjustments to period t shocks occur during date $t+1$. This propagates shocks through the financial intermediation channel between the household and the firm. As the household adjusts its deposits the firm's borrowing decision and its labour demand decision are affected. Hence, monetary disturbances as well as technology shocks can have real side effects in the LF model. Without the infinite cost of adjustment in deposits, the anticipated inflation effect is unable to propagate monetary shocks in the standard CIA model.[16]

In the LF model, output is unchanged at impact but a negative spike is generated at lag one as shown in Figure 4. After lag one, the response of output decays completely in about 2 years. The spike in output is generated by a combination of the financial intermediation propagation mechanism, the anticipated inflation effect, and the information structure of the LF model. Since a positive unanticipated nominal shock causes the household to revise its inflation expectations upward, it deposits fewer funds with the FI. This results in less borrowing and labour demand by the firm. However, all this occurs in the period after the innovation to the monetary shock. Christiano (1991) reports similar results.

[16] This fact about the standard CIA and LF models originates with Christiano (1991).

As Figure 4 shows, the impact of the technology shock on output is short-lived in the LF model. Output's response to this shock is much steeper between impact and lag one than anywhere else. The infinite cost of adjustment in household deposits causes the firm to wait one period to alter labour input. The result is higher output during this period. After period one, the effect of the technology shock on output disappears.

The preference and technology structures of the CEP model and CEL models differ from the CIA and LF models. However, within the CEP model, output dynamics resembles those of the CIA model as appears in Figure 5. Given a technology shock, output follows a typical capital-accumulation path. For the monetary shock, the anticipated inflation dominates the response of output to the monetary shock at impact. A classic aggregate supply–demand system with a vertical long-run supply curve generates output dynamics in the CEP model.

Figure 6 contains the response of output to the technology and monetary shocks for the CEL model. One dimension along which the CEL model differs from the other models is that output is higher at impact given a monetary shock. That is, the liquidity effect dominates the anticipated inflation effect at impact. However, the dominance of the liquidity effect does not last beyond lag zero. At lag one and beyond, the dynamic response of output to either shock in the CEL model exhibits, in general, the same response as we find for the LF model.[17] Although the costs the firm faces in the adjustment in total labour demand create a dominant liquidity effect at impact, the propagation mechanism of the CEL model is not able to sustain a dominant liquidity effect beyond impact.

Across the CIA, LF, CEP, and CEL models, inflation dynamics appear very similar as shown in Figures 3–6. A technology shock reduces inflation at impact in these models. Subsequently, inflation exhibits little response to a technology disturbance in these models. The response of inflation to the monetary shock is also similar in these models with one exception. Inflation is higher at impact and then gradually returns to its steady state. The lone exception is that at lag one inflation exhibits a spike before beginning its return to steady state in the CEL model. Since the liquidity effect dominates the anticipated inflation effect at impact in the CEL model, inflation is lower at impact in response to a monetary shock in this model than in the other models. Nonetheless, across the four models, the theoretical response of inflation to either shock matches the predictions of a classical aggregate supply and demand system.

The theoretical IRFs predicted by the M2–hours worked information set appear in Figures 7–10 for the CIA, LF, CEP, and CEL models, respectively. Figures 7, 8, and 10 show M2 responding to a monetary shock with a capital-accumulation-like path in each model. Since an increase in monetary injections is persistent, the level of M2 increases over time.

The response of M2 to a monetary disturbance in the CEP model does exhibit a capital-accumulation-like path, but the accumulation is much smaller and exhibits a hump shape. The dynamic cost of adjustment in the household's portfolio decision produces this pattern. When the household alters its deposit decision, it gives up some leisure. However, the dynamic cost function causes this decision to be spread over several periods. This serves to smooth household deposits over time and reduce the response of monetary shocks on M2 at impact and beyond in the CEP model.

M2 responds to a technology shock in the same way in the CIA and LF models. As appears in Figures 7 and 8, M2 rises at impact given a technology shock. Subsequently, M2 reverts to trend

[17] When the IRFs of Figure 6 are computed using eighth-order VARs, the wiggles in these IRFs disappear. In this case, the IRFs resemble those shown in Figure 4 for the LF with one exception. At impact, the response of output to the monetary shock is positive. That is, the liquidity effect dominates the anticipated inflation effect at impact. Subsequently, output returns to trend smoothly from below.

with some overshooting. Once again, the dominance of the anticipated inflation effect creates this response in M2 to a technology shock.

For the CEP model, its unique propagation mechanism dominates the anticipated inflation effect for the response of M2 to a technology shock. Figure 9 shows that the dynamic portfolio cost of adjustment mechanism of the CEP model generates a hump-shaped response in M2 given a technology shock. Since the household adjusts its portfolio over several periods in the CEP model, deposits respond in a smooth fashion to a technology shock. In Figure 8, this produces the hump-shaped IRF of M2 to a technology shock.

As shown in Figure 10, the response of M2 to either shock generated by the CEL model are somewhat different. At impact, M2's response to the technology shock shows that the liquidity effect dominates the anticipated inflation effect in the CEL model. Subsequently, a small hump appears in M2's response to a technology shock. Since the technology shock causes firms to hire more labour at impact, the household acquires deposits. Given the information structure of the CEL model, it takes some time for the firm to adjust its labour demand and for the household to work off its accumulated assets. Hence, the liquidity effect is the source of the small hump-shaped response of M2 to the technology shock.

The impact of the two shocks on hours worked differs across the CIA, LF, CEP, and the CEL models. In the CIA model, hours worked exhibits almost no response to the monetary injection growth shock. This is not true for hours worked in the LF model. At impact, hours worked is left unchanged in response to a technology shock. Subsequently, the financial intermediation-propagation mechanism of the LF model takes over and the nominal side shock produces a negative response with a trough at lag one in hours worked. The monetary shock creates a negative wealth effect which is amplified by the costs of adjustment in household deposits. For the CEL model, the dominant liquidity effect drives the response of hours worked to the monetary shock at impact. At lag one and beyond, the response of hours worked in the CEL model to this shock resembles the CIA model.

In the CIA, LF, and CEL models, hours worked respond to the technology shock by increasing at impact and then showing a long, slow decline to steady state. The distinguishing feature of the response of hours worked to the technology shock are the kinks at lag one in the LF and CEL models. Since the household is not able to smoothly adjust its deposits in response to the technology shock, labour demand by the firm does not adjust smoothly over time in the LF model. For the CEL model, the same dynamics operate with the addition of the information structure imposed on firm labour demand decisions.

Hours worked exhibits little or no response to either shock in the CEP model. The portfolio cost of adjustment function the household faces provides incentives to smooth deposit flows in response to either shock. Hence, the firm smooths its labour demand over time. The smoothing incentive negates any other mechanism that might affect hours worked.

Statistical Tests: Comparing Theoretical and Sample IRFs

A visual comparison of the theoretical and sample IRFs gives an overall impression that the data are at odds with themodes. However, a closer inspection suggests that the models have a few successes. For example, in the lower-left panels of Figures 3–5 the theoretical and sample IRFs of inflation to a technology shock appear to be quite similar. The upper-left panels of Figures 7, 8, and 10 contain the theoretical response of M2 to the monetary shock and these objects appear to have the same shape as the sample response of M2 to the monetary shock. This suggests that, at least, the models do a reasonable job of matching nominal sample side dynamics. However, all the models generate real side dynamics that show little resemblance to real side sample dynamics.

We conduct formal tests of these conjectures by computing Quasi-Lagrange multiplier (Q-LM) tests. Q-LM test statistics take the quadratic form

$$\text{Q-LM}(p) = [\hat{\mathbf{F}} - \mathbf{F}]\mathbf{V}^{-1}[\hat{\mathbf{F}} - \mathbf{F}]'$$

where p denotes the number of lags of the IRF used to compute the statistic, $\hat{\mathbf{F}}$ is the sample IRF, and $\mathbf{V}$ is the empirical covariance matrix of the theoretical IRF. The empirical covariance matrix of the theoretical IRF is computed as the ensemble average of the outer product of the theoretical IRF:

$$\mathbf{V} = N^{-1} \sum_{j=1}^{N} [\mathbf{F}(j) - \mathbf{F}][\mathbf{F}(j) - \mathbf{F}]'$$

This test statistic measures the distance between the theoretical and sample IRFs after imposing the long-run neutrality restriction. Since the sample data are quarterly, Q-LM statistics are computed with p equal to four and eight lags. With four (eight) lags, there are five (nine) elements in $\mathbf{F}$ because the elements of $C(0)$ are included.

The results of Mittnik and Zadrozny (1993) imply that Q-LM statistics are asymptotically chi-square. However, this approximation does not appear to be accurate in our sample. For example, Figures 11 and 13 (12 and 14) contain empirical densities for the Q-LM(4) (Q-LM(8)) statistics

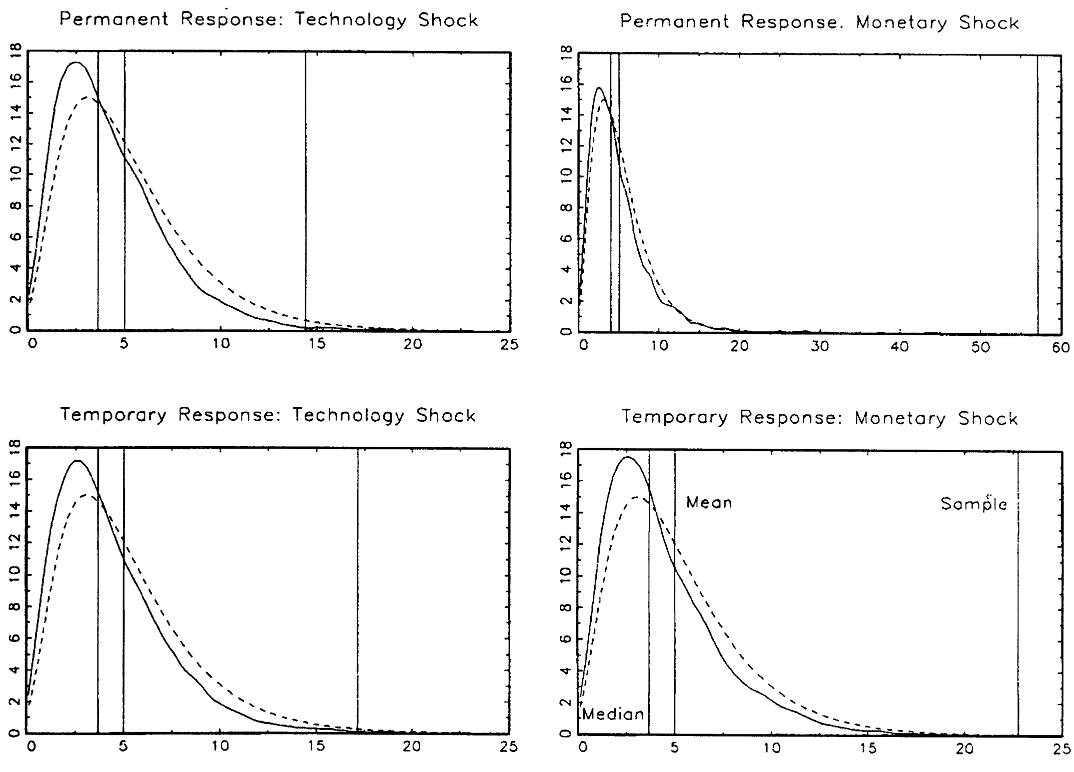

Figure 11. Densities of Q-LM(4) statistics

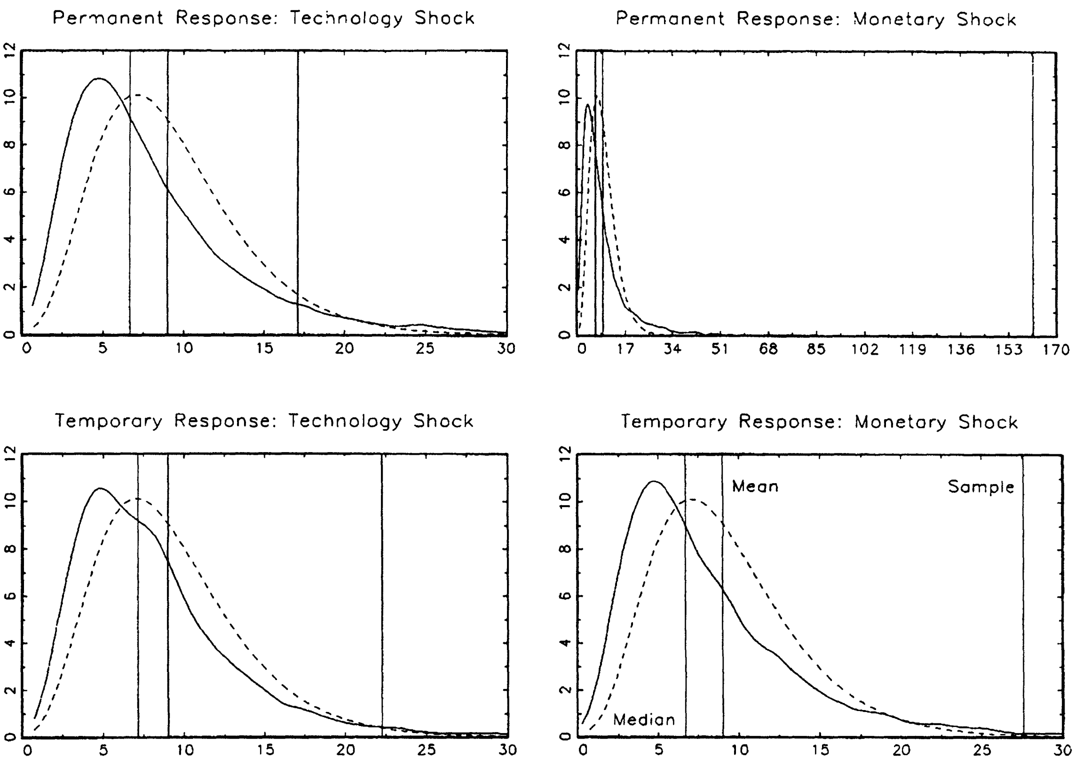

Figure 12. Densities of Q-LM(8) statistics

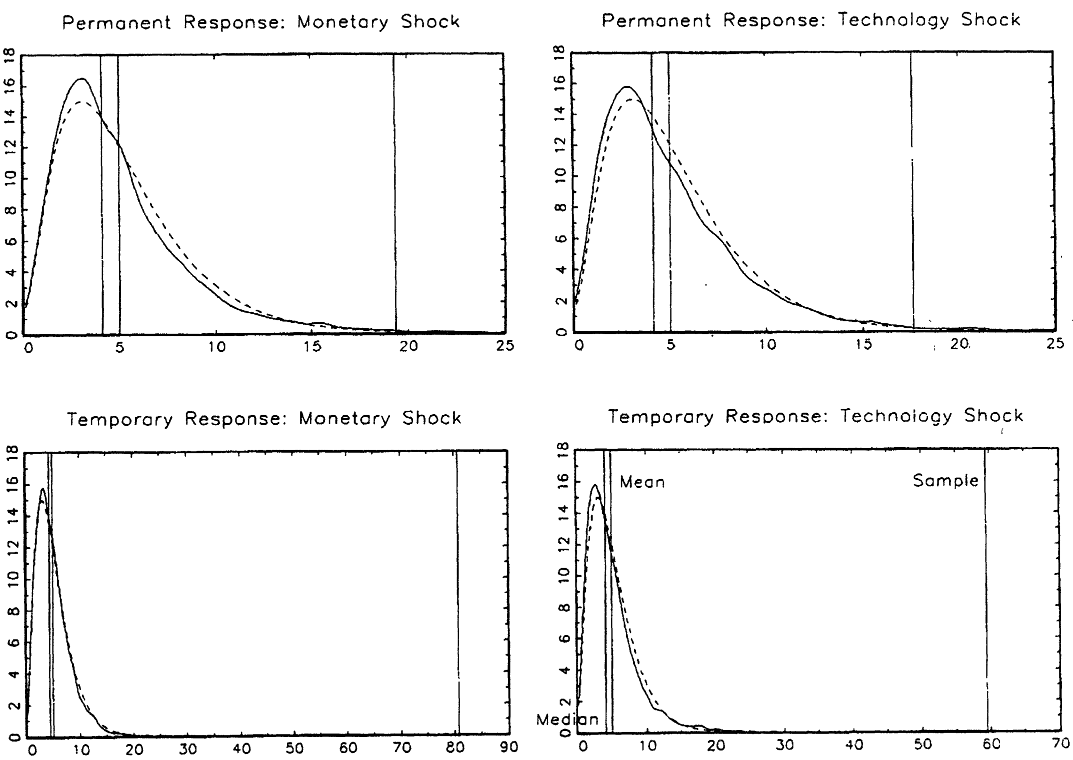

Figure 13. Densities of Q-LM(4) statistics

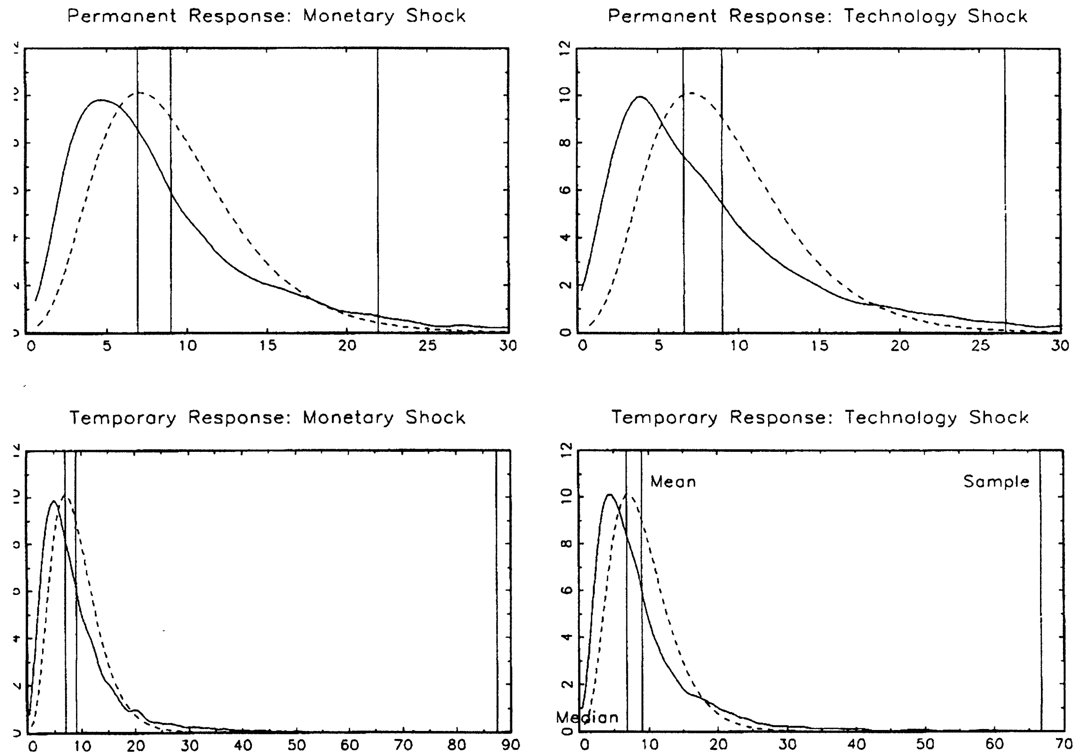

Figure 14. Densities of Q-LM(8) statistics

generated by the CEL model.[18] In each panel of Figures 11 and 13 (12 and 14), the dashed curve is the density of the chi-square distribution with five (nine) degrees of freedom, the right-most vertical line represents the sample Q-LM statistic, the middle vertical line denotes the mean of the theoretical Q-LM statistic, and the left-most vertical line is the median of the theoretical Q-LM statistic. Inspection of the densities in these figures make it clear that in most cases these densities are not close to chi-square. Hence, we compute the fraction of replications in which the theoretical Q-LM statistics equals or exceeds the sample Q-LM statistic.[19] That is, we compute the marginal probability value of the Q-LM statistic. At replication j, we calculate the theoretical Q-LM statistic as

$$\text{Q-TLM}(j) = [\mathbf{F}(j) - \mathbf{F}]\mathbf{V}^{-1}[\mathbf{F}(j) - \mathbf{F}]'$$

The ensemble of Q-TLM statistics produce the theoretical mean and median of the Q-LM statistic.

Test results appear in Tables I and II. The null hypothesis is that the sample data are generated by a particular model. Table I (II) reports Q-LM statistics and marginal probability values for

[18] We compute the densities of the Q-LM statistics using a normal kernel with window width computed automatically as outlined in Silverman (1986). The technical appendix to this paper contains the details of these computations.

[19] It would also be inappropriate to use rules of thumb for normal random variables, such as whether the sample statistic Q-LM lies within one or two standard errors of the model generated mean of the Q-TLM statistics. Although the means of the Q-TLM statistics generated by the CEL model equal their degrees of freedom, in all but five cases the variances of these statistics exceed twice their degrees of freedom by at least a factor of three.

Table I. Quasi-Lagrange multiplier tests of output–inflation IRFs

Model	No. of lags	Y/T	$\Delta P/T$	Y/M	$\Delta P/M$
CIA					
	4	68·914	6·613	69·541	23·332
		(0·000)	(0·251)	(0·000)	(0·001)
	8	72·255	21·810	296·026	29·616
		(0·001)	(0·051)	(0·000)	(0·019)
LF					
	4	38·172	9·215	34·938	17·977
		(0·000)	(0·096)	(0·000)	(0·004)
	8	40·702	18·210	135·904	20·212
		(0·006)	(0·079)	(0·000)	(0·063)
CEP					
	4	43·069	5·028	96·792	38·432
		(0·000)	(0·414)	(0·000)	(0·000)
	8	51·170	21·985	463·756	45·747
		(0·004)	(0·051)	(0·000)	(0·005)
CEL					
	4	14·406	17·155	57·148	22·744
		(0·011)	(0·007)	(0·000)	(0·006)
	8	17·114	22·262	161·858	27·592
		(0·078)	(0·028)	(0·000)	(0·020)

Note:
Here and in Table II the Q-LM test statistics appear as the first number in each stack. The null hypothesis is that the sample data are generated by the model economy. The probability values from the empirical distribution of the Q-LM statistics appear in parentheses. The probability value equals the fraction of artificial samples in which the sample Q-LM statistic is equal to or larger than the theoretical Q-LM statistic.

the output–inflation(M2–hours worked) information set. For example, the CIA model generates a Q-LM statistic equal to 15·270 for the response of the first four lags of M2 to the monetary shock, labelled M2/M, in Table II. The marginal probability value of this test statistic equals 0·026.

The probability values of the Q-LM tests indicate that the sample IRFs reject the real side dynamics predicted by the models at the 5% level in almost all cases.[20] For the response of output or hours worked to either shock, the models generate a marginal probability value larger than 5% only once.[21] The probability value equals 0·078 for the response of output to the technology shock at eight lags for the CEL model.

This supports the results of Cogley and Nason (1995). Cogley and Nason find that many RBC models lack the internal propagation mechanisms necessary to replicate the observed short-run dynamics of post-war US output. In order to produce the observed short-run dependence on US output in RBC models Cogley and Nason report that it is necessary to resort to exogenous mechanisms.

[20] Besides the reported test statistics, we computed scaled tests of the Q-LM statistics. A test of an unscaled Q-LM statistic measures the shape and height of a theoretical IRF relative to a sample IRF. Tests of scaled Q-LM statistics measure only the shape of the IRFs. Scaled Q-LM tests are constructed so that the theoretical variance of the growth rate of the integrated variable of each information set equals its sample counterpart. Scaling the IRFs has little (if any) impact on the test statistics. These results are available from the authors on request.
[21] This is true for the scaled Q-LM test statistics as well.

Table II. Quasi-Lagrange multiplier tests of M2–hours worked IRFs

Model	No. of lags	M2/M	H/M	M2/T	H/T
CIA					
	4	15·270	68·597	9·337	50·402
		(0·026)	(0·000)	(0·103)	(0·000)
	8	15·694	74·090	10·794	57·100
		(0·127)	(0·001)	(0·268)	(0·002)
LF					
	4	29·665	41·967	55·968	46·346
		(0·001)	(0·000)	(0·000)	(0·000)
	8	33·481	42·728	67·807	53·455
		(0·011)	(0·005)	(0·000)	(0·002)
CEP					
	4	50·589	5439·259	5380·991	4794·229
		(0·000)	(0·000)	(0·000)	(0·000)
	8	107·187	5478·598	6186·284	5920·539
		(0·000)	(0·000)	(0·000)	(0·000)
CEL					
	4	19·348	80·809	17·610	59·505
		(0·006)	(0·000)	(0·009)	(0·000)
	8	21·951	87·550	26·615	66·758
		(0·053)	(0·001)	(0·037)	(0·002)

This is not true of nominal side dynamics. About one-third of the Q-LM tests do not reject the null at the 5% level for these dynamics. However, the information sets produce different results across the four models. The CIA model generates almost half of the Q-LM tests that are not rejected at the 5% level on the nominal side. In particular, statistically significant responses of M2 to the technology shock appear only in the CIA model.[22] For the LF model, only once does it generate a Q-LM test with a probability value greater than 10% for M2's response to the monetary shock. Between the CEP and CEL models, only one Q-LM statistic possesses a probability level greater than 1% for nominal side dynamics.

The Q-LM tests contain more information about the ability of the models to replicate sample dynamics than do visual inspection of the IRFs. For example, the theoretical and sample responses of M2 to a technology shock that appear in the upper-left panel of Figures 7, 8, and 10 possess somewhat similar shapes. However, Q-LM statistics constructed with four lags provide evidence to reject all the models at the 5% level. Only the CIA model generates an eight-lag IRF not rejected at the 10% level. Hence, visual inspection of the IRFs can lead to incorrect inferences concerning the ability of the models to replicate sample dynamics.

The results of this section reveal that some of the model economies have some success in replicating the sample responses of M2 and inflation to technology and monetary shocks. Sample responses of output and hours worked to these shocks cannot be replicated by any of the models. A comparison across the models provides evidence that the CIA model achieves better

[22] Although the theoretical IRFs of M2 to a technology shock produced by the CIA and LF models appear to be quite similar, only the CIA model generates Q-LM statistics for this IRF that are significant at the 5% level. In this case, all the power of the test falls on the impact multiplier (at lag zero). Since the information structure of the LF model creates greater sampling uncertainty, the Q-LM statistics of the LF model are rejected more frequently than the Q-LM statistics of the CIA model.

results at replicating the sample response of M2 and inflation to technology and monetary shocks. Although the CIA model lacks the infinite in-period cost of adjustment in deposits of the Lucas–Fuerst model, the portfolio cost of adjustment function of the CEP model, and the imperfect labour substitutes technology of the CEL model, the CIA model does a better job overall of matching sample nominal side dynamics.

6. CONCLUSION

This paper studies the ability of four equilibrium monetary business cycle models to replicate sample dynamics of the aggregate US economy. Two long-run neutrality restrictions predicted by the models are used to generate the dynamic predictions which are tested. The long-run neutrality restrictions are that (1) output is independent of monetary injection growth shocks in the long run and (2) money is independent of technology shocks in the long run.

This paper follows the approach of Cogley and Nason (1995) by tying together the literatures on structural VARs and business cycle calibration techniques. However, it studies monetary business cycle models rather than RBC models. Stylized facts are constructed as the IRFs of the empirical structural VARs under the null of the long-run neutrality restrictions. The theoretical IRFs are computed using the models as DGPs. The long-run neutrality restrictions form the bond between model and sample. Model evaluation is conducted as a series of Monte Carlo experiments which compare the theoretical and sample IRFs with formal statistical procedures. In this case, the Monte Carlo experiments can be viewed as a model-respecification exercise.

The evidence reported in this paper extends our earlier results. Just as our other paper found that many RBC models lack interesting endogenous propagation mechanisms, this one shows that monetary business cycle models also have weak real side-propagation mechanisms. However, the monetary business cycle models have some success in replicating nominal side dynamics. In particular, the CIA model, which lacks the Lucas (1990) and Fuerst (1992) information restriction, the Christiano and Eichenbaum (1992a) portfolio cost of adjustment function, and the Christiano and Eichenbaum (1992b) imperfect labour substitutes technology, matches nominal side dynamics more than half the time. The Lucas–Fuerst and Christiano–Eichenbaum models perform less well along this dimension.

These results suggest that research on business cycle fluctuations needs to devote more attention to modelling propagation mechanisms. Whether these mechanisms are to be found in the real or financial sectors remains open to future study. As an aid in this research, the empirical methods used here and in Cogley and Nason (1995) should prove helpful.

TECHNICAL APPENDIX

This appendix outlines the methods we use to solve and simulate the models, compute structural vector autoregressions, and estimate nonparametric density functions. GAUSSi (1993), version 3.1.4, serves as the software platform for these computations. Computer programs containing these procedures are available from the authors on request.

Model Optimality Conditions

This section describes the optimality conditions of the monetary business cycle (MBC) models studied in the paper. The optimality conditions are presented in terms of stochastically detrended variables. Since numerical solutions of the models are constructed using stochastically detrended versions of the models, we present optimality conditions in that form. We begin with the standard CIA model and then describe how the special features of the other models alter the optimality conditions.

1. The standard CIA model

For the standard CIA model, any candidate equilibrium must satisfy three optimality conditions, which restrict equilibrium paths in the goods, labour, money, and credit markets. Optimality in the goods market defines the trade-off that the economy faces in moving consumption across time. From the text, the Euler equation (15) represents this trade-off. In stochastically detrended terms, equation (15) becomes

$$\mathrm{E}_t\{\hat{P}(t)/[\hat{c}(t+1)\hat{P}(t+1)m(t)] + \beta\hat{P}(t+1)[\theta\exp\{-\theta[\gamma+\varepsilon(t+1)]\}\hat{k}(t+1)^{\theta-1}n(t+1)^{1-\theta} + (1-\delta)\exp\{-[\gamma+\varepsilon(t+1)]\}]/[\hat{c}(t+2)\hat{P}(t+2)m(t+1)]\} = 0 \quad \text{(A1)}$$

The intratemporal condition that restricts labour market optimality depends on the structure of the credit market. Since the firm must finance its current-period wage bill with borrowed funds, credit market structure affects labour demand. After stochastic detrending, the firm's borrowing constraint (equation (12)) becomes

$$\hat{W}(t) = \hat{l}(t)/n(t)$$

in equilibrium. Using this condition, the intratemporal labour market optimality condition (equation (16)) becomes

$$-[\psi/(1-\psi)][\hat{c}(t)\hat{P}(t)/(1-n(t))] + \hat{l}(t)/n(t) = 0 \quad \text{(A2)}$$

after stochastic detrending.

Equation (17) represents optimality in the credit market and this condition depends on date t information in the standard CIA model. Subsequent to imposing the equilibrium stochastically detrended interest rate,

$$R(t) = \hat{P}(t)(1-\theta)\exp\{-\theta[\gamma+\varepsilon(t)]\}\hat{k}(t)^{\theta}n(t)^{-\theta}/\hat{W}(t)$$

and the firm's borrowing constraint the stochastically detrended intertemporal Euler equation which represents credit market optimality is written

$$1/[\hat{c}(t)\hat{P}(t)] - \beta[\hat{P}(t)(1-\theta)\exp\{-\theta[\gamma+\varepsilon(t)]\}\hat{k}(t)^{\theta}n(t)^{1-\theta}/m(t)\hat{l}(t)] \times \mathrm{E}_t\{1/[\hat{c}(t+1)\hat{P}(t+1)]\} = 0 \quad \text{(A3)}$$

The solution to the CIA model ties together the optimality conditions (equations (A1)–(A3)) and the equilibrium conditions (equations (18)–(20)). Along with the exogenous stochastic processes for technology and monetary injection growth shocks (equations (1) and (2)) this system of six nonlinear equations determines the equilibrium distributions for the six unknowns:

$$[\hat{k}(t+1)\ n(t)\ \hat{d}(t)\ \hat{c}(t)\ \hat{l}(t)\ \hat{P}(t)]$$

Given these equilibrium distributions, the equilibrium distributions for output, real wages, the inflation rate, and the nominal interest rate can be found.

2. The Lucas–Fuerst model

The only difference between the CIA model and that of Lucas (1990) and Fuerst (1992) is the information structure that the household faces when making its deposit decision. The

Lucas–Fuerst model assumes that households make their date t deposit decision before the realization of the date t monetary shock. Hence, the household bases its date t deposit decision on date $t-1$ information. Consequently, the optimality condition (A3) is written

$$\mathrm{E}_{t-1}\{1/[\hat{c}(t)\hat{P}(t)] - \beta\hat{P}(t)(1-\theta)\exp\{-\theta[\gamma+\varepsilon(t)]\}\hat{k}(t)^{\theta}n(t)^{1-\theta}/[\hat{c}(t+1)\hat{P}(t+1)m(t)\hat{l}(t)]\} = 0 \tag{A4}$$

Otherwise, the CIA and Lucas–Fuerst models share an identical structure.

3. The portfolio cost of adjustment model

Christiano and Eichenbaum (1992a) take the Lucas–Fuerst model and impose dynamic portfolio adjustment costs on households. These costs appear in the period utility function as $(1-\psi)\ln[c(t)] + \psi\ln[1-h(t)-p(t)]$, where the dynamic portfolio cost of adjustment is

$$p(t) = \alpha(1)[\exp\{\alpha(2)[\Gamma(t)/\Gamma(t-1) - m^*]\} + \exp\{-\alpha(2)[\Gamma(t)/\Gamma(t-1) - m^*]\} - 2]$$

and $\Gamma(t) = M(t) - d(t)$. In stochastically detrended terms, this dynamic adjustment cost function appears as

$$\begin{aligned} p(t) = \alpha(1)[&\exp\{\alpha(2)[m(t-1)\hat{\Gamma}(t)/\hat{\Gamma}(t-1) - m^*]\} \\ &+ \exp\{-\alpha(2)[m(t-1)\hat{\Gamma}(t)/\hat{\Gamma}(t-1) - m^*]\} - 2] \end{aligned}$$

where $\hat{\Gamma}(t) = 1 - \hat{d}(t)$. For this model, dynamic portfolio adjustment costs alters the credit market optimality condition (equation (A4)) of the Lucas–Fuerst model. In the Christiano and Eichenbaum portfolio cost of adjustment model, the credit market optimality becomes

$$\begin{aligned} \mathrm{E}_{t-1}\{&(1-\psi)/[\hat{c}(t)\hat{P}(t)] - \psi\alpha(1)\alpha(2)m(t-1)q(t)/(\hat{\Gamma}(t-1)[1-h(t)-p(t)]) \\ &- \beta(1-\psi)R(t)/[\hat{c}(t+1)\hat{P}(t+1)m(t)] + \beta\psi\alpha(1)\alpha(2)q(t+1) \\ &\times [m(t)\hat{\Gamma}(t+1)/\hat{\Gamma}(t) + R(t)]/(\hat{\Gamma}(t)[1-h(t+1)-p(t+1)]) - \beta^2\psi\alpha(1)\alpha(2)R(t) \\ &\times m(t+1)q(t+2)\hat{\Gamma}(t+2)/(m(t)\hat{\Gamma}(t+1)^2[1-h(t+2)-p(t+2)])\} = 0 \end{aligned} \tag{A5}$$

where

$$q(t) = \exp\{\alpha(2)[m(t-1)\hat{\Gamma}(t)/\hat{\Gamma}(t-1) - m^*]\} - \exp\{-\alpha(2)[m(t-1)\hat{\Gamma}(t)/\hat{\Gamma}(t-1) - m^*]\}$$

We set $\alpha(1) = 5\cdot0\times10^{-5}$ and $\alpha(2) = 1000$ following Christiano and Eichenbaum (1992a). Otherwise, the portfolio cost of adjustment and Lucas–Fuerst models share a common structure.

4. The imperfect labour substitutes model

Christiano and Eichenbaum's (1992b) imperfect labour substitutes model also starts with the Lucas–Fuerst model. In this model, a firm faces two labour demand decisions during each period. The first labour demand decision, $n(1, t)$, is made prior to the realization of the innovation of the monetary shock. The second, $n(2, t)$, is made after the realization of this shock. The different labour inputs are assumed to be imperfect substitutes in a constant elasticity of substitution technology generating total labour demand:

$$n(t) = [0\cdot5n(1, t)^{1/\phi} + 0\cdot5n(2, t)^{1/\phi}]^{\phi}, \quad \phi > 1$$

Hence, the firm must hire both types of labour each period. We follow Dotsey and Ireland

(1993) and set $\phi = 10/9$. In stochastically detrended terms, the final goods technology is

$$\hat{y}(t) = \exp\{-\theta[\gamma + \varepsilon(t)]\}\hat{k}(t)^{\theta} n(t)^{(1-\theta)}$$

Given the two labour demand decisions, the firm faces two financing decisions when paying its wage bill. In equilibrium, the finance constraints of the firm become

$$\hat{W}(1, t) = \hat{l}(1, t)/n(1, t), \quad \text{and} \quad \hat{W}(2, t) = \hat{l}(2, t)/n(2, t)$$

The constraints of the FI require stochastic detrending. After stochastic detrending, the budget constraint, the balance sheet constraint, and the equilibrium zero profit condition of the FI become

$$\hat{b}(t) + \mathrm{RH}(t)\hat{d}(t) = \mathrm{RF}(1, t)\hat{l}(1, t) + \mathrm{RF}(2, t)\hat{l}(2, t) + \hat{d}(t) + [m(t) - 1] - \hat{l}(1, t) - \hat{l}(2, t)$$

$$m(t) - 1 + \hat{d}(t) = \hat{l}(1, t) + \hat{l}(2, t)$$

and

$$\mathrm{RH}(t)\hat{d}(t) = \mathrm{RF}(1, t)\hat{l}(1, t) + \mathrm{RF}(2, t)[\hat{l}(2, t) + 1 - m(t)]$$

respectively.

The firm's intra-period labour demand decision creates additional optimality conditions for the imperfect labour substitutes model. The extra decisions the firm makes are for $n(1, t)$, $n(2, t)$, $\hat{l}(1, t)$, and $\hat{l}(2, t)$. This creates the need to define the information sets $\Theta(1, t)$, and $\Theta(t)$. $\Theta(1, t)$ contains all information dated $t-1$ and earlier as well as $\varepsilon(t)$. The monetary shock innovation, $\eta(t)$, and $\Theta(1, t)$ appear in $\Theta(t)$. That is, define $\mathrm{E}_{1t}\{\cdot\} = \mathrm{E}\{\cdot \mid \Theta(1, t)\}$ and $\mathrm{E}_t\{\cdot\} = \mathrm{E}\{\cdot \mid \Theta(t)\}$.

The optimality conditions of this model are as follows. Optimality in the goods market is represented by

$$\mathrm{E}_{1t}\{-\hat{P}(t)/[\hat{c}(t+1)\hat{P}(t+1)m(t)] + \beta\hat{P}(t+1)[\theta\exp\{-\theta[\gamma + \varepsilon(t+1)]\}\hat{k}(t+1)^{\theta-1}n(t+1)^{(1-\theta)}$$
$$+ (1-\delta)\exp\{-[\gamma + \varepsilon(t+1)]\}]/[\hat{c}(t+2)\hat{P}(t+2)m(t+1)]\} = 0 \quad \text{(A6)}$$

the optimality condition in the market for beginning of period labour is

$$\mathrm{E}_{1t}\{(1-\psi)[1/(1 - n(1, t) - n(2, t))] - \psi\hat{l}(1, t)/[m(t)n(1, t)]\} = 0 \quad \text{(A7)}$$

and the optimality condition in the market for end-of-period labour is

$$(1-\psi)[1/(1 - n(1, t) - n(2, t))] - \psi\hat{l}(2, t)/[m(t)n(2, t)] = 0 \quad \text{(A8)}$$

Once we account for $\mathrm{RH}(t) \neq \mathrm{RF}(1, t)$ and $\mathrm{RH}(t) \neq \mathrm{RF}(2, t)$, the credit market optimality condition becomes

$$\mathrm{E}_{t-1}\{1/[\hat{c}(t)\hat{P}(t)] - \beta\mathrm{RH}(t)/[m(t)\hat{c}(t+1)\hat{P}(t+1)\} = 0 \quad \text{(A9)}$$

The unique credit market structure of the imperfect labour substitutes model also introduces arbitrage possibilities for the FI. Since the FI operates in two different loan markets, an optimality condition is needed to rule out the possibilities of expected returns on loans being different across markets. Hence, the FI's no-arbitrage condition is represented by

$$\mathrm{E}_{1t}\{\beta\psi(\mathrm{RF}(1, t) - \mathrm{RF}(2, t)]/[m(t)m(t+1)]\} = 0 \quad \text{(A10)}$$

Constructing the Linear Approximate Solutions

The MBC economies are solved using the method of undetermined coefficients (MUC) developed by Christiano (1991). It is illustrated here for the Lucas (1990) and Fuerst (1992) information structure.

Before the MUC solution can be constructed, the steady state of the model must be found and the model parameters and steady-state ratios chosen by calibration. The calibration exercise requires some model parameters and steady-state ratios be set using information from the sample data. Given these parameters and steady-state ratios, the remaining parameters and steady-state ratios are found by imposing the nonstochastic steady state on equations (A1), (A2), (A4), and (18)–(20). For all the models, the steady-state investment–capital ratio and capital–output ratios are

$$i^*/k^* = [1-(1-\delta)/\gamma^*], \quad \gamma^* = \exp\{\gamma\}$$

and

$$k^*/y^* = \beta\theta\gamma^*/[\gamma^* - \beta(1-\delta)]$$

respectively. Steady state per capita deposits are

$$d^* = \beta(1-\theta)(y^*/c^*) + 1 - m^*$$

Given d^*, steady state per capita hours are

$$n^* = (1-\psi)(d^* + m^* - 1)/[\psi m^* + (1-\psi)(d^* + m^* - 1)]$$

In the steady state of the imperfect labour substitutes model, $n^*(1) = n^*(2) = n^*/2$ and $l^*(1) = l^*(2) = l^*/2$.

The last four equations serve as the basis for calibrating model parameters. From the sample data on output and hours worked, we compute $\gamma = 0{\cdot}00315$ and $n^* = 0{\cdot}206$ as sample averages. To compute m^* and ρ, the first-order autoregressive coefficient for the growth rate of monetary injections, the monetary base series of the Federal Reserve Bank of St Louis is used.[23] For the sample period 1954:1 to 1991:4, a first-order autoregression of this series yields

$$\ln[m(t+1)] = \underset{(0{\cdot}28792)}{1{\cdot}17060} + \underset{(0{\cdot}05624)}{0{\cdot}72809} \ln[m(t)],\ R^2 = 0{\cdot}528$$

where standard errors appear in parentheses. This regression implies $m^* = 1{\cdot}01088$ and $\sigma(\eta) = 0{\cdot}05453$. To complete the calibration, the steady-state ratio $c^*/y^* = 0{\cdot}74867$ for our sample period. Further, we assume $k^*/y^* = 10$. Given γ, n^*, m^*, and c^*/y^*, we calculate θ, δ, ϕ, and i^*/k^*. To tie down $\sigma(\varepsilon)$, we set it equal to $0{\cdot}0135$ to force the theoretical standard deviation of output growth in the Lucas–Fuerst model to match its sample analog. All the parameters across all the models appear in Table AI.

After solving for the steady state and calibrating the parameters, we take a first-order log linear Taylor expansion of the optimality and market-clearing conditions around the steady state. That is, the MUC solution algorithm yields a system of linear expectational difference equations. We solve this system by positing approximate log linear decision rules for $\hat{k}(t+1)$, $n(t)$, and $\hat{d}(t)$

$$\tilde{k}(t+1) = K(k)\tilde{k}(t) + K(\varepsilon)\varepsilon(t) + K(m)\tilde{m}(t) + k(\varepsilon 1)\varepsilon(t-1) + K(m1)\tilde{m}(t-1)$$
$$\tilde{n}(t) = \mathrm{N}(k)\tilde{k}(t) + N(\varepsilon)\varepsilon(t) + N(m)\tilde{m}(t) + N(\varepsilon 1)\varepsilon(t-1) + N(m1)\tilde{m}(t-1)$$

and

$$\tilde{d}(t) = D(k)\tilde{k}(t) + D(\varepsilon 1)\varepsilon(t-1) + D(m1)\tilde{m}(t-1)$$

[23] This is the only monetary base series that covers the 1954:1 to 1991:4 sample period.

Table AI. Parameter values of models

Cash in advance model:

$$[\beta\ \psi\ \theta\ \gamma\ \delta\ \sigma(\varepsilon)\ m^*\ \rho\ \sigma(\eta)]' = [0{\cdot}993\ 0{\cdot}773\ 0{\cdot}345\ 0{\cdot}003\ 0{\cdot}022\ 0{\cdot}014\ 1{\cdot}011\ 0{\cdot}728\ 0{\cdot}005]'$$

Lucas–Fuerst model:

$$[\beta\ \psi\ \theta\ \gamma\ \delta\ \sigma(\varepsilon)\ m^*\ \rho\ \sigma(\eta)]' = [0{\cdot}993\ 0{\cdot}773\ 0{\cdot}345\ 0{\cdot}003\ 0{\cdot}022\ 0{\cdot}014\ 1{\cdot}011\ 0{\cdot}728\ 0{\cdot}005]'$$

Christiano and Eichenbaum (1992a) portfolio cost of adjustment model:

$$[\beta\ \psi\ \alpha(1)\ \alpha(2)\ \theta\ \gamma\ \delta\ \sigma(\varepsilon)\ m^*\ \rho\ \sigma(\eta)]' = [0{\cdot}993\ 0{\cdot}773\ 5{\cdot}0\times 10^{-5}\ 1000{\cdot}000\ 0{\cdot}345\ 0{\cdot}003\ 0{\cdot}022\ 0{\cdot}014\ 1{\cdot}011\ 0{\cdot}728\ 0{\cdot}005]'$$

Christiano and Eichenbaum (1992b) imperfect labour substitutes model:

$$[\beta\ \psi\ \theta\ \phi\ \gamma\ \delta\ \sigma(\varepsilon)\ m^*\ \rho\ \sigma(\eta)]' = [0{\cdot}993\ 0{\cdot}773\ 0{\cdot}345\ 1{\cdot}111\ 0{\cdot}003\ 0{\cdot}022\ 0{\cdot}014\ 1{\cdot}011\ 0{\cdot}728\ 0{\cdot}005]'$$

where, for example, $k(t+1) = \ln[\hat{k}(t+1)/k^*]$. Hence, we want to solve for the unknown parameter vector

$$[K(k)\ K(\varepsilon)\ K(m)\ K(\varepsilon 1)\ K(m1)\ N(k)\ N(\varepsilon)\ N(m)\ N(\varepsilon 1)\ N(m1)\ D(k)\ D(\varepsilon 1)\ D(m1)]'$$

After taking a first-order Taylor expansion and applying the three market-clearing conditions, the three optimality conditions (A1), (A2), and (A4) yield the linear stochastic difference equations

$$\Delta_1\tilde{k}(t+1) + \Delta_2\tilde{k}(t) + \Delta_3\tilde{n}(t) + \Delta_4\alpha(t) + \Delta_5\tilde{m}(t) + \mathrm{E}_t\{\Delta_6\tilde{k}(t+2) + \Delta_7\tilde{n}(t+1) + \Delta_8\varepsilon(t+1) + \Delta_9\tilde{m}(t+1)\} = 0$$

$$\Phi_1\tilde{n}(t) + \Phi_2\tilde{d}(t) + \Phi_3\tilde{m}(t) = 0$$

and

$$\mathrm{E}_{t-1}\{\Psi_1\tilde{k}(t+1) + \Psi_2\tilde{k}(t) + \Psi_3\tilde{n}(t) + \Psi_4\tilde{d}(t) + \Psi_5\varepsilon(t) + \Psi_6\tilde{m}(t+1) + \Psi_7\tilde{m}(t)\} = 0$$

where the Δ's and Φ's and Ψ's are nonlinear functions of the parameters of the model economy and the unknown parameter vector

$$[K(k)\ K(\varepsilon)\ K(m)\ K(\varepsilon 1)\ K(m1)\ N(k)\ N(\varepsilon)\ N(m)\ N(\varepsilon 1)\ N(m1)\ D(k)\ D(\varepsilon 1)\ D(m1)]'$$

First, apply the decision rules for $\tilde{k}(t+2)$, $\tilde{k}(t+1)$, $\tilde{n}(t+1)$, $\tilde{n}(t)$, and $\tilde{d}(t)$, the law of motion for $\tilde{m}(t+1)$, and $\mathrm{E}_t\varepsilon(t+1) = 0$, to the last three equations, and then combine terms to produce

$$A_1\tilde{k}(t) + A_2\varepsilon(t) + A_3\tilde{m}(t) + A_4\varepsilon(t-1) + A_5\tilde{m}(t-1) = 0$$

$$B_1\tilde{k}(t) + B_2\varepsilon(t) + B_3\tilde{m}(t) + B_4\varepsilon(t-1) + B_5\tilde{m}(t-1) = 0$$

and

$$D_1\tilde{k}(t) + D_2\varepsilon(t-1) + D_3\tilde{m}(t-1) = 0$$

The last three conditions imply that

$$A_1 = A_2 = A_3 = A_4 = A_5 = B_1 = B_2 = B_3 = B_4 = B_5 = D_1 = D_2 = D_3 = 0$$

Hence, we have 13 zero conditions in 13 unknowns. By applying a nonlinear equation solver to the last set of equalities, we can solve for the unknown parameter vector

$$[K(k)\ K(\varepsilon)\ K(m)\ K(\varepsilon 1)\ K(m1)\ N(k)\ N(\varepsilon)\ N(m)\ N(\varepsilon 1)\ N(m1)\ D(k)\ D(\varepsilon 1)\ D(m1)]'$$

We do this using a quasi-Newton nonlinear equation solution method.[24] To satisfy the transversality condition, it is required that $K(k) < \beta^{1/2}$. Once the parameters of the decision rules are known, any endogenous variable of the model economies can be generated.

Structural VAR Decomposition

The structural VAR (SVAR) computations follow from Blanchard and Quah (1989). We begin by estimating fourth-order VARs for each system. The next step is to invert the VAR to obtain the reduced-form vector moving-average representations (VMA). The reduced-form innovations are orthogonalized by imposing the restrictions that the structural shocks are uncorrelated and that one of them is neutral in the long run. Finally, structural impulse response functions (IRFs). are computed by feeding the structural shocks through the reduced-form dynamics.

In particular, let $\mathbf{x}(t)$ denote a bivariate stationary time series. Since $\mathbf{x}(t)$ is stationary, it has a Wold representation

$$\mathbf{x}(t) = \Lambda(\mathbf{L})\mathbf{e}(t)$$

where $\mathbf{e}(t)$ has mean zero and covariance matrix Ω.[25] We estimate the reduced-form VMA representation by estimating and inverting a fourth-order VAR. We assume that the structural shock, $\mathbf{u}(t)$, is linearly related to the reduced-form innovations:

$$\mathbf{e}(t) = \mathbf{C}(0)\mathbf{u}(t)$$

This implies that the structural (SVMA) representation can be written as

$$\mathbf{x}(t) = \Lambda(\mathbf{L})\mathbf{C}(0)\mathbf{u}(t) \equiv \mathbf{C}(L)\mathbf{u}(t)$$

Two kinds of identifying restrictions are used to recover $\mathbf{u}(t)$ from $\mathbf{e}(t)$. First, we assume that the elements of $\mathbf{u}(t)$ are uncorrelated and have unit variance

$$\mathbf{E}\{\mathbf{u}(t)\mathbf{u}(t)'\} = \mathbf{I}$$

This assumption serves as a normalization. Second, we assume that $u(22, t)$ has no long-run effect on $x(1, t)/(1-\mathbf{L})$. For example, the output–inflation system produces the long-run neutrality restriction that $\eta(t)$ has no long-run effect on $\ln[y(t)]$. These conditions give rise to the following system of nonlinear equations:

$$c(11, 0)^2 + c(12, 0)^2 - \Omega(11) = 0$$
$$c(21, 0)^2 + c(22, 0)^2 - \Omega(22) = 0$$
$$c(11, 0)c(21, 0) + c(12, 0)c(22, 0) - \Omega(12) = 0$$
$$\lambda(11)c(12, 0) + \lambda(12)c(22, 0) = c(12) = 0$$

[24] The GAUSS™ nonlinear equation solver, version 3.1.2, contains the quasi-Newton method; see GAUSS Applications (1993) for details.
[25] The mean of $\mathbf{x}(t)$ has been suppressed.

This system is solved for the elements of $\mathbf{C}(0)$ using a quasi-Newton method. Given estimates of $\mathbf{C}(0)$, the structural IRFs are computed from the SVMA polynomial

$$\mathbf{C}(\mathbf{L}) = \Lambda(\mathbf{L})\mathbf{C}(0)$$

This procedure is used to generate the sample and theoretical IRFs. The statistical analysis of the theoretical IRFs is based on Monte Carlo simulation. When the models serve as the data-generating processes, the normalization $\mathbf{C}(\mathbf{L}) = \Xi\Phi(\mathbf{L})$ relates the SVAR to the model's restriction, where Ξ is the Choleski factorization of

$$\mathbf{E}\{[\varepsilon(t) \quad \eta(t)]'[\varepsilon(t) \quad \eta(t)]\}$$

At each replication, the decision rules are used to generate 356 observations of the elements of $\mathbf{x}(t)$. The first 204 observations are discarded and the remaining 152 observations are used to examine structural IRFs. The theoretical structural IRFs and their covariances are constructed by averaging over the ensemble of model-generated structural IRFs. These results are also used to compute the means and medians of the theoretical Q-LM statistics. The results reported in this paper are based on 5000 replications of the models.

Nonparametric Density Estimation

Estimation of the densities of the Q-LM statistics follow the methods described in Silverman (1986). Since the Q-LM statistics are asymptotically distributed as chi-square random variables, the probability density functions of the Q-LM statistics possess fat tails. Non-parametric estimates were made with the normal kernel given by

$$K(x) = \exp\{-0{\cdot}5x^2\}/\sqrt{2\pi}$$

where x is the distance between two elements of the density of the Q-LM statistic; see Silverman (1986, pp. 42–3). We compute the densities of the Q-TLM statistics by

$$f(x) = N^{-1}\sum_{j=1}^{N} k\{h^{-1}[x \quad - \quad \text{Q-TLM}(j)]\}$$

The bandwidth, h, used to compute $f(x)$ is determined by the automatic bandwidth methods Silverman (1986, pp. 43–8) suggests. To make this procedure operational, we use 500 points to compute the density estimates. A measure of the spread also follows the ideas presented in Silverman (1986, pp. 47–8).

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