Portfolio problem

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1 Idea

The idea of this assignment is use projection methods to solve a portfolio problem. The important thing to keep in mind is that portflio problems are often difficult. The reason is that choices are determined by second-order properties (variances and covariances) and often they are not that large. So it is essential to (i) have an excellent minimization routine, (ii) check for having found a global optimum, and (iii) check for accuracy. I sometimes even wonder whether multiplicity is more likely to occur in these types of problems, at least in terms of "pretty good" numerical solutions.

2 Model

This is a simple endowment model in which the agent can save against idiosyncratic shocks by investing in a safe asset with return r_r and a risky asset with return r. The model is characterized by the following equations:

$$(c_t - \bar{c})^{-\nu} = \eta_1 \exp(-\eta_0 s_t) - \eta_1 + \mathrm{E}_t \left[\beta \left(c_{t+1} - \bar{c}\right)^{-\nu} \left(1 + a_t(r - r_{t+1})\right)\right] \quad (1)$$

$$0 = \theta_0(a - 0.5) + \mathrm{E}_t \left[\left(c_{t+1} - \bar{c}\right)^{-\nu} \left(r - r_{t+1}\right)\right] \quad (2)$$

$$c_t = y_t + w_t - s_t \quad (3)$$

$$w_t = s_{t-1}(1 + a_{t-1}(r - r_t)) \quad (4)$$

$$y_{t+1} = \mu_y \exp\left(\rho_y \log y_t + \sigma_y \varepsilon_{y,t+1}\right) \qquad \varepsilon_{y,t+1} \sim N(0,1) \quad (5)$$

$$r_{t+1} = \mu_r + \sigma_r \varepsilon_{r,t+1} \qquad \varepsilon_{r,t+1} \sim N(0,1) \quad (6)$$

Equation 2 makes clear that the comovement between the marginal utility of consumption and the excess return plays a key role. By putting in a constant habit, $\bar{c} > 0$, it is possible to make this correlation quantitatively a bit more important. The parameter μ_y is chosen to ensure that the unconditional mean of y_t is equal to 1. Thus,

$$\mu_y = \frac{1}{1 + 0.5 * \sigma_y^2/(1 - \rho_y^2)}$$

To make the numerical problem less challenging, the model has two penalty terms. The larger the value of η_0 the costlier it is to choose a non-zero savings level and the larger the value of θ_0 the costlier it is to deviate from a fifty-fifty portfolio. The idea is to start with large values and then lower them. The problem is often not that the program doesn't run for low values of these two parameters, but it becomes trickier to determine whether you have found the true solution.