Idea

The idea of this assignment is to show you that solving a model with projection methods is actually quite simple. There is one somewhat tricky step, namely the numerical integration of the conditional expectation.

Model

The standard growth model characterized by the following equations

$$
\begin{aligned}
c_t^{-\nu} &= \mathrm{E}_t[\beta c_{t+1}^{-\nu}(\alpha \exp(z_{t+1})k_t^{\alpha-1} + 1 - \delta)] \\
c_t + k_t &= \exp(z_t)k_{t-1}^{\alpha} + (1-\delta)k_{t-1} \\
z_{t+1} &= \rho z_t + \sigma\varepsilon_{t+1} \qquad \varepsilon_{t+1} \sim N(0,1)
\end{aligned}
$$

Of course you can choose your own parameters but to make our programs comparable let's consider

case	B
α	0.36
ρ	0.95
β	0.99
δ	0.10
ν	5
σ	0.005

Preliminaries

Grid

- First calculate the steady state for capital, $\bar{k}$.
- For k take values in the range from $0.5\bar{k}$ to $1.5\bar{k}$.
- For z construct a grid going from 3 times the standard deviation of z below the mean of z to 3 times above. Note that the standard deviation of z is not equal to the standard deviation of ε.

Approximating function

You always have to take a stand on which function you are going to approximate with what kind of approximating function. Here we approximate the consumption function and use the following second-order polynomial

$$
\ln c = \begin{array}{c} \gamma_1 + \gamma_2 \ln k_{-1} + \gamma_3 z \\ +\gamma_4(\ln k_{-1})^2 + \gamma_5 z \ln k_{-1} + \gamma_6 z^2 \end{array}
$$

The matlab function consfun.m defines this function

To do

1. Write a Matlab function that calculates the sum of squared Euler equation errors on the grid taking as variable input the values of the six γ coefficients
2. Write a Matlab program that finds the values of γ that minimizes the sum of the squared Euler equation errors. As initial values for γ you can use

$$\begin{bmatrix} -0.6889 & 0.4829 & 0.7228 & 0.0297 & 0.0834 & -0.0933 \end{bmatrix}$$

These are the values that solve the model when $\nu = 1$.

Numerical integration

Evaluation of the error on the grid requires evaluation of the conditional expectation, i.e., numerical integration. Don't be scared of this. Just realize the following:

- The stochastic variable inside the conditional expectation is ε_{+1}. Note that ε_{+1} shows up *three* times, because z_{+1} shows up three times. Once directly, once as an argument of next period's consumption choice, c_{+1}, and once as an argument of next period's capital choice, k_{+1}.
- Numerical integration basically comes down to thinking of ε_{+1} as a random variable with discrete support with each realization having a probability. That is, in the slides we showed that

  $$\mathrm{E}[h(\varepsilon_{+1})],$$

  with $\varepsilon_{+1} \sim N(\mu, \sigma^2)$, can be approximated as follows:

  $$\begin{aligned} \mathrm{E}[h(\varepsilon_{+1})] &= \int_{-\infty}^{\infty} h(\varepsilon_{+1}) \frac{\exp\{-(\varepsilon_{+1} - \mu)^2/(2\sigma^2)\}}{\sigma\sqrt{2\pi}} d\varepsilon_{+1} \\ &\approx \sum_{j=1}^{J} h(\mu + \sigma\sqrt{2}\,\zeta_j) \frac{\omega_j}{\sqrt{\pi}} \end{aligned}$$

  where ω_j and ζ_j are the Hermite weights (sort of like probabilities) and nodes, respectively. Note that in our application $\mu = 0$.