Projection methods; TI field course 2010

Wouter J. Den Haan

November 9, 2010

1 Idea

The idea of this assignment is to show you that solving a model with projection methods is actually quite simple. There is one somewhat tricky step, namely the numerical integration of the conditional expectation.

2 Model

The standard growth model characterized by the following equations

$$
\begin{aligned}
c_t^{-\nu} &= \mathrm{E}_t\left[\beta c_{t+1}^{-\nu}\left(\alpha \exp(z_{t+1})k_t^{\alpha-1}+1-\delta\right)\right] \\
c_t + k_t &= \exp(z_t)k_{t-1}^{\alpha}+(1-\delta)k_{t-1} \\
z_{t+1} &= \rho z_t + \sigma\varepsilon_{t+1} \qquad \varepsilon_{t+1}\sim N(0,1)
\end{aligned}
$$

Of course you can choose your own parameters, but to make programs comparable let's consider

case	B
α	0.36
ρ	0.95
β	0.99
δ	0.10
ν	5
σ	0.005

3 Preliminaries

Grid

- First calculate the steady state for capital, $\bar{k}$.
- For k take values in the range from $0.5\bar{k}$ to $1.5\bar{k}$.
- For z construct a grid going from 3 times the standard deviation of z below the mean of z to 3 times above. Note that the standard deviation of z is not equal to the standard deviation of ε.

Approximating function You always have to take a stand on which function you are going to approximate with what kind of approximating function. Here we approximate the consumption function and use the following second-order polynomial

$$\ln c = \begin{array}{c} \gamma_1 + \gamma_2 \ln k_{-1} + \gamma_3 z \\ +\gamma_4 \left(\ln k_{-1}\right)^2 + \gamma_5 z \ln k_{-1} + \gamma_6 z^2 \end{array}$$

The matlab function consfun.m defines this function

4 To do

1. Write a Matlab function that calculates the sum of squared Euler equation errors on the grid taking as variable input the values of the six γ coefficients
2. Run the Matlab program that finds the values of γ that minimizes the sum of the squared Euler equation errors. As initial values for γ you can use

$$[-0.6889\ 0.4829\ 0.7228\ 0.0297\ 0.0834\ \ -0.0933]$$

   These are the values that solve the model when $\nu = 1$.

5 Numerical integration

Evaluation of the error on the grid requires evaluation of the conditional expectation, i.e., numerical integration. Don't be scared of this. Just realize the following:

- The stochastic variable inside the conditional expectation is ε_{+1}. Note that ε_{+1} shows up *three* times, because z_{+1} shows up three times. Once directly, once as an argument of next period's consumption choice, c_{+1}, and once as an argument of next period's capital choice, k_{+1}.

- Numerical integration basically comes down to thinking of ε_{+1} as a random variable with discrete support with each realization having a probability. That is, in the slides we showed that

$$\mathrm{E}\left[h\left(\varepsilon_{+1}\right)\right],$$

with $\varepsilon_{+1} \sim N(\mu, \sigma^2)$, can be approximated as follows:

$$\begin{aligned}
\mathrm{E}\left[h\left(\varepsilon_{+1}\right)\right] &= \int_{-\infty}^{\infty} h(\varepsilon_{+1}) \frac{\exp\left\{-(\varepsilon_{+1}-\mu)^2/(2\sigma^2)\right\}}{\sigma\sqrt{2\pi}} d\varepsilon_{+1} \\
&\approx \sum_{j=1}^{J} h(\mu + \sigma\sqrt{2}\zeta_j) \frac{\omega_j}{\sqrt{\pi}}
\end{aligned}$$

where ω_j and ζ_j are the Hermite weights (sort of like probabilities) and nodes, respectively. Note that in our application $\mu = 0$.